AEROElastic COMPUTATIONS OF FLAPPED ROTORS

Florent Dehaeze, René Steijl and George N. Barakos
CFD Laboratory, Department of Engineering
University of Liverpool, L69 3GH, U.K.
http://www.liv.ac.uk/flightscience/PROJECTS/CFD/ROTORCRAFT/RBD/index.htm
Email: Florent.Dehaeze@liverpool.ac.uk, R.Steijl@Liverpool.ac.uk, G.Barakos@liverpool.ac.uk

Abstract
A method able to model the aerodynamics and aeroelastics of flapped rotors is presented in this paper. The structural model of the blade and the associated mesh deformation algorithms are tested for several cases with good results. In addition, a 1-DOF model is introduced for the flaps and several techniques are put forward for the coupled CFD/CSD solution. The demonstration results are promising and highlight the need for a dedicated set of experiments to provide an ultimate validation case. The selected test cases include flapped sections, wings, and even rotors with multiple trailing edge flaps.

NOTATION

\(a_\infty\) = freestream speed of sound, m/s
\(AR\) = \(R/c\), rotor aspect ratio
\(c\) = (mean) rotor chord, m
\(C_N\) = section normal force coefficient (blade
\(C_p\) = \((p - p_\infty)/q_\infty\), pressure coefficient
\(C_T\) = rotor thrust coefficient, \(T/(\rho_\infty(\Omega R)^2 \pi R^2)\)
\(M_{tip}\) = \(\Omega R/\rho_\infty\), rotational Mach number
\(M_\infty\) = freestream Mach number
\(M_r\) = \(M_{tip}[r/R \cos(\lambda) + \mu \sin(\psi - \lambda)]\), relative Mach number
\(M^2 C_P\) = \((p - p_\infty)/(\gamma p_\infty)\)
\(p_\infty\) = freestream pressure, Pa
\(q_\infty\) = \(\rho_\infty U_\infty^2\), free-stream dynamic pressure, Pa
\(q_r\) = \(\rho_\infty(\Omega r + U_\infty \sin(\psi - \lambda))^2\), dynamic pressure, Pa
\(r\) = radial coordinate along rotor blade, m
\(R\) = rotor radius, m
\(U_\infty\) = freestream velocity, m/s
CFD = Computational Fluid Dynamics
CSD = Computational Structural Dynamics
\(\alpha\) = wing incidence angle, deg.
\(\delta_f\) = flap deflection angle, deg.
\(\lambda\) = (local) blade sweep angle, deg
\(\mu\) = \(M_\infty/M_{tip}\), rotor advance ratio
\(\Omega\) = rotor rotation rate, rad/sec
\(\rho_\infty\) = freestream density, kg/m\(^3\)
\(\sigma\) = \(N_{blades}/\pi AR\), rotor solidity
\(\psi\) = blade azimuth (0° at rear of rotor disk), deg.

\(section\ axes\)

1 INTRODUCTION

This paper presents developments in the Helicopter Multi-Block (HMB) necessary for predicting the effect of flaps present on rotor blades. Flaps on rotors [23] can be used for different purposes: improved performance, vibration and/or noise reduction, or even as primary controls. However, due to the complexity of the flow field, comprehensive methods based on lookup tables and 2D-aerodynamics show limitations in predicting the rotor performance, particularly at challenging conditions. The use of CFD may allow for better simulations, especially if coupled with a structural model to account for the in-flight structural deformations of blades and flaps. This calls for a coupling between CFD and CSD. The reaction of the flap to the airloads is also of interest, because the flap deployment does not always correspond to the actuation law.

Most aeroelastic simulations have separate structural and flow solvers, that are coupled and exchange information during computations. Various coupling methods have been put forward in the literature, though two approaches are mainly used: weak and strong coupling. The former consists of an exchange of information between both solvers every rotor revolution or every fraction of a revolution, while the latter consists of exchanging information between the two solvers at the end of each time step or even more often. Representative examples of previous attempts are shown in Table 1. A study by Altmikus et al. [1] compared the strong and weak coupling strategies. While the predictions for the blade deformation were similar, the strong coupling proved more time consuming and less robust. The weak coupling strategy proved more popular, and recently, the interest moved from isolated rotors to full helicopter configurations [8, 15]. However, some attempts were made in using a strong coupling strategy. The main interest of the strong coupling strategy comes from the lack of forced periodicity, which allows the simulations of manoeuvring rotors, as demonstrated by Sitaraman and Roget [32]. Furthermore, coupling method including more advanced 3D-FEM structural models were studied by Ortun et al. [25] and Datta and Jonhson [11], in order to improve the prediction for newer rotor blades of advanced planforms.

The flaps may or may not be actuated during parts of the flight, and will have a finite stiffness and structural damping. It is therefore well possible that a flap may exhibit flutter, limit-cycle oscillation or even couple with the unsteady loads on the blade as these vary around the azimuth. This problem is common in fixed-wing aeroelasticity and for this reason clearance of all flapped sections with respect to their dynamic stability is necessary. Flap analyses has so far been attempted by several authors and with a variety of modelling techniques. Amongst these works, Milgram et al. [23] as well
as Friedman [17] and Shen [30] employed a range of comprehensive methods that allow for some aspects of the flap to be modeled. In a recent paper Cesnik [9] also reported on the use of comprehensive tools for flapped rotor analysis. Works with full Navier-Stokes based CFD also appear in the literature and these adopt several techniques to allow for modeling of the flap. The work by Jose and Baeder [20] shows several techniques including local mesh refinement and adaptation near the flap. Sitaraman et al. [31] show the use of over-set mesh techniques for a range of applications, including an airfoil-flap-slat test case where separate grids for each of the three components are combined to model the flow including the gaps between airfoil and flap and slat. All these works attempt to develop methods for the analysis of flaps or combine predictions and theory with experiments aiming to design rotors that out-perform conventional designs. In addition to the numerical studies, the potential of reducing rotor vibrations by active flaps on blades was investigated recently in wind-tunnel tests [10]. In the tests, a Mach-scaled rotor was equipped with two active flaps per blade, i.e. an inboard flap at approximately 75% span-wise positions and an outboard flap centred around 85%. The chord-wise and span-wise extents were 15% and 10% respectively. Predefined flap schedules as well as closed-loop control systems for the flap were investigated. For the closed-loop control system, a notable reduction in vibratory air-loads was reported. In addition, all the above works, identify the potential issues related to the aeroelasticity of the flap and the effect of the flap structural properties in the overall rotor performance. It is exactly this concern that motivates the current work. The overall objective is to develop an appropriate method for modelling flap aeroelasticity within the framework of Navier-Stokes CFD methods and establish the best coupling strategy for the aerodynamics and structural analysis. In earlier efforts to couple CFD and structural analysis, the integration of the governing equations in time has been studied [16] and several works exist presenting results for wing sections with one, two and some times more degrees of structural freedom. An example of these works can be found in [5]. For blade sections of a rotor in forward flight, the tangential velocity and blade pitch change periodically as the blade rotates. In the present work, these effects are modeled for two-dimensional sections using a combination of oscillatory translations as well as a periodic pitch schedule. For rotors, this simultaneous variation of the Mach number and incidence may need a different approach than for fixed-wing applications and for this reason, the coupling methods are revisited here for the problem of a rotor section equipped with a flap with one degree of freedom in pitch.

The work presented in this paper first concentrates on the validation of the Helicopter Multi-Block solver of the University of Liverpool for flapped sections and then compares and evaluates coupling methods for the aero-elastic analysis of a flap with 1-DOF. The coupling problem is far from trivial since the non-linearity of the aerodynamic loads must be resolved and at the same time, the flap deflection and velocity must be computed in a time-accurate fashion. The paper begins with the HMB validation, then presents the mathematical models and numerical schemes of the 1DOF problem, its non-dimensionalisation and the implementation of the algorithm. The results for several coupling methods are then presented and evaluated and the work shows that limit cycle oscillation of the coupled system is possible. The paper then proceeds with the analysis of flapped rotors.

In the next section, the numerical methods are described, including the HMB solver, flap modelling and CFD/CSD coupling strategy. Then, the various methods are tested for suitable test cases: 1-degree-of-freedom flap, static and oscillating aerofoils, fixed flap deployment on a wing model, and demonstration of the CFD/CSD coupling strategy for rotors with flaps in forward flight.

## 2 Numerical Methods

### 2.1 HMB Flow Solver

The Helicopter Multi-Block 2 (HMB2) CFD code [3,4,34,35] was employed for this work. HMB2 solves the unsteady Reynolds-averaged Navier-Stokes equations on multi-block structured mesh topologies using a cell-centred finite-volume method for spatial discretisation. Implicit time integration is employed, and the resulting linear systems of equations is solved using a pre-conditioned Generalised Conjugate Gradient method. For unsteady simulations, an implicit dual-time stepping method is used, based on Jameson’s pseudo-time integration approach [19]. The HMB2 method has been validated for a range of rotorcraft applications and has demonstrated good accuracy and efficiency for very demanding flows. Examples of work with HMB2 can be found in references [33–35]. Several rotor trimming methods are available in HMB2 along with a blade-actuation algorithm that allows for the near-blade grid quality to be maintained on deforming meshes [35].

The HMB2 solver has a library of turbulence closures including several one- and two-equation turbulence models and even non-Boussinesq versions of the $k−\omega$ model. Turbulence simulation is also possible using either the Large-Eddy or the Detached-Eddy approach. The solver was designed with parallel execution in mind and the MPI library along with a load-balancing algorithm are used to this end. For multi-block grid generation, the ICEM-CFD Hexa commercial meshing tool is used and CFD grids with 10-20 million points and thousands of blocks are commonly used with the HMB2 solver.

For forward flying rotors, the HMB2 method [3,4,34,35] solves the compressible-flow Reynolds-Averaged Navier-Stokes equations in an inertial frame of reference. The employed finite-volume discretisation accounts for moving and deforming meshes in time-accurate simulations. Consequently, a rotor in forward flight is modelled in a ‘helicopter-fixed frame of reference’, where the forward flight velocity is introduced through the definition of the ‘free-stream’ conditions. For isolated rotors, as well as, rotor/fuselage or rotor/wind-tunnel cases, the rotor and rotor blade motions are then accounted for using mesh velocities. For rotor/fuselage or rotor/wind-tunnel cases, the relative motion of the rotor and the fixed fuselage or tunnel is accounted for using the sliding-plane approach [34].
2.2 Modelling of wings with part-span flaps

Figure 1 illustrates the definition of flap surface and hinge positions used in the present model. For each flap a separate boundary is used. In the example shown in Figure 1(a) the 4 shaded surfaces all have the same boundary condition and thus define a single flap. The flapping motion is around the hinge line shown in the figure, which is defined by input parameters. Figure 1(b) shows the example of two neighbouring flaps. Here, the two shaded surfaces on the right (B1, B2) define flap 1, while flap 2 is defined by the left surfaces (B3, B4). The two flaps have separate hinge lines as shown in the figure. Flap 1 is hinged along a line close to the lower wing surface, while the second flap is hinged along a line close to the camber line of the section. The flap implementation includes blending in both chord-wise and span-wise directions in the vicinity of any flap edge neighbouring a fixed-surface or a surface belonging to another flap. The blending methods involve a gradual reduction of the flap deflection towards the flap edge. In the spanwise direction a simple algebraic method is used that reduces the deflection in an approximately parabolic way. For the single flap, the spanwise blending regions are indicated in Figure 1(a). The two flaps in Figure 1(b) are free to move independently. Since the flap surfaces are immediate neighbours along one edge, spanwise blending is employed normal to this edge with a zero deflection along the edge dividing the flaps. In the chordwise direction, the blending is more critical since it may introduce severe pressure disturbances near the flap edge due to the change of the blade surface curvature. In the present method, a parabolic spline is created to approximate the flap surface in the vicinity of the flap edge so that \(C^{(1)}\) continuity is achieved at the edge and \(C^{(2)}\) continuity in the point where the spline joins the un-blended part of the flap surface. Figure 2 demonstrates the blended-flap method for a 2D mesh around a NACA0012 section. In the examples shown, the flap has a chord-wise extent of 20% \(c\) and the flap deflection is 5\(^\circ\). The figure shows the effect of varying hinge locations. The left column shows the results for the mesh deformation without chord-wise blending. For Figure 2(a)-(f), the hinge is located at the symmetry line, with a variation in the chord-wise position. As can be seen, the smoothness of the airfoil section is quite well preserved for the method based on spline-fitting. The difference with the non-blended case is most pronounced for the flap hinge located at the 25\% flap chord station, shown in Figures 2(e)-(f). The examples in Figures 2(g)-(j) show examples where the hinge is located close to the aerofoil lower surface. The main challenge then becomes preventing grid folding near the flap edge on the upper surface for fine grids.

For flap surfaces that are completely separate from the main wing surface, the blending is not used. Examples of such geometries are shown in Reference [18] for a flapped rotor blade based on the HIMARCS blade [24]. The gap between the main and flap elements of the rotor is small but finite and should be refined to allow for the details of the flow to be resolved. This technique was successful in modelling flapped wings and rotors as detailed in [18].

In the model, a Trans-Finite Interpolation method is used to impose the flap surface deflection onto the volume mesh. In time-accurate simulations, the mesh velocities are computed using a second-order accurate finite-difference expression. Alternatively, for prescribed flap deflections an analytical velocity could be calculated.

2.3 1-Degree-of-Freedom Flap Model

The HMB2 flow solver includes models for incorporating active trailing edge flaps on aerofoils and wings, as well as, rotors in forward flight. Most of the time, the use of these models imposes a pre-defined flapping schedule in time-accurate flow simulations, typically by means of a Fourier series development of the flapping angles as function of time. However, the finite stiffness of the flap hinges introduces the potential for aero-elastic coupling of the flap with the surrounding unsteady flow field. For this reason, a 1-degree-of-freedom model was implemented in HMB2. The main objective of this type of simulations will be investigations of the flap stability. In the 1-degree-of-freedom aero-elastic model, the rotational motion of the flap around its hinge is governed by an ODE of the following form:

\[
I_h \frac{d^2 \delta}{dt^2} + \frac{d \delta}{dt} + k_h \delta = M_h \tag{1}
\]

where \(t\) is time, \(\delta\) is the flap deflection (positive down), \(I_h\) is the moment of inertia about the hinge, \(d\) is the damping coefficient, \(k_h\) is the spring stiffness and \(M_h\) the aerodynamic forcing, i.e. the aerodynamic moment about the hinge. In Equation (1), all quantities are dimensional and per unit flap span. In the following, a non-dimensionalisation is introduced and subsequently, omission of the tilde denotes a change to dimensionless quantities.

Equation (1) can be non-dimensionalised using the following definitions:

\[
t = \frac{t c}{U_\infty} \quad ; \quad b = \frac{c}{2} \quad ; \quad r^2 = \frac{I_h}{mb^2} \quad ; \quad \mu = \frac{m}{\rho_\infty \pi b^2} \tag{2}
\]

\[
\tilde{\omega}_0 = \sqrt{k_h/I_h} \quad ; \quad \omega_0 = \frac{c}{U_\infty} \quad ; \quad M_h = c_m \frac{1}{2} \rho_\infty U_\infty^2 c^2 = 2c_m \rho_\infty U_\infty^2 b^2 \\
\zeta = \frac{1}{2} \frac{d\delta}{\sqrt{k_h \omega_0}}
\]

where \(t\) is the non-dimensional time, \(b\) the half-chord of the aerofoil section, \(r\) the radius of gyration, \(m\) the mass of the section per unit span, \(\mu\) the mass ratio. The natural frequency of the flapping motion is \(\omega_0\), with \(\omega_0\) its non-dimensional equivalent. The scaled non-dimensional structural damping is defined through \(\zeta\). Introducing these definitions in Equation (1), one can write,

\[
\frac{d^2 \delta}{dt^2} + \frac{d \delta}{dt} \frac{U_\infty}{I_h} \frac{c}{dt} + \omega_0^2 \delta = \frac{2c_m \rho_\infty U_\infty^2 c^2}{r^2 m b^2} \tag{3}
\]

\[
\frac{d^2 \delta}{dt^2} + 2 \zeta \omega_0 \frac{d \delta}{dt} + \omega_0^2 \delta = 8 \frac{c_m}{\pi r^4 \mu}
\]

For a single harmonic forcing on the right-hand side,
Equation (3) has the following exact solution,

\[
\frac{d^2 \delta}{dt^2} + 2 \zeta \omega_0 \frac{d \delta}{dt} + \omega_0^2 \delta = F_0 \sin(\omega t)
\] (4)

\[
\delta(t) = \frac{F_0}{Z \omega} \sin(\omega t + \phi),
\]

\[
Z = \sqrt{(2 \cdot \omega_0 \zeta)^2 + \frac{1}{\omega_2^2} (\omega_0^2 - \omega^2)},
\]

\[
\phi = \arctan \left( \frac{2 \omega_0 \omega \zeta}{\omega^2 - \omega_0^2} \right)
\]

2.3.1 Leap-frog time integration method

In this model, the motion of the flap around its hinge is integrated in time using a second-order accurate leap-frog method for Equation (4). This integration involves a single aerodynamic load evaluation per time step. In a time-accurate simulation using this model, the integration from time step \( n \) to \( n + 1 \) involves the following steps. First, the flap is moved to the current position and the mesh is updated to account for this change. Then, the grid velocity at the present time step is evaluated. In the dual-time stepping approach employed in HMB2, a quasi-steady flow for this step is then solved, followed by an integration of the aerodynamic forces and moments acting on the main section and flap. Then, the flap position and velocity are updated according to,

\[
a = \frac{8 c_m}{\pi r^2 \mu} - 2 \zeta \omega_0 \left( \frac{d \delta}{dt} \right)^{(n)} - \omega_0^2 \delta^{(n)}
\]

\[
\delta^{(n+1)} = \delta^{(n)} + \delta t \left( \frac{d \delta}{dt} \right)^{(n)}
\]

\[
\left( \frac{d \delta}{dt} \right)^{(n+1)} = \left( \frac{d \delta}{dt} \right)^{(n)} + \frac{1}{2} \delta t \left( \frac{d^2 \delta}{dt^2} \right)^{(n)} + a
\] (5)

\[
\left( \frac{d^2 \delta}{dt^2} \right)^{(n+1)} = a
\]

In the above, dimensionless quantities are used, and \( \delta t \) is the non-dimensional time step in the simulation, while \( a \) represents the angular acceleration of the flap around the hinge.

2.3.2 Implicit fluid-structure coupling method

The leap-frog method discussed previously coupled the structural response and the aerodynamics in the ‘outer’ loop of the dual-time stepping method. Thus, in effect, during the flow field computation in the ‘inner’ relaxation loop, the structural response was kept constant. In the leap-frog method, the aerodynamics and structural response are effectively ‘staggered’ in time by one time-step. In a 4-stage Runge-Kutta method, the sub-iterations would create a more direct coupling between the aerodynamics and the structural response. This of course would be at the expense of the flow-field evaluations at each of the Runge-Kutta steps. For this reason Runge-Kutta method were not considered in this work.

The second 1-degree-of-freedom model implemented in HMB employs a fully implicit coupling of the aerodynamics and the structural response. Here, the coupling takes place at the levels of the ‘inner’ loop of the dual-time stepping method.

The second-order ODE for the flap deflection is first recast in a system of two first-order ODEs:

\[
\frac{dy}{dt} + 2 \zeta \omega_0 y + \omega_0^2 \delta = \frac{8 c_m}{\pi r^2 \mu}
\]

\[
\frac{d \delta}{dt} = y
\]

In the implementation in the dual-time marching method, the system is discretised with time-implicit finite-differences as:

\[
\left( \frac{dy}{dt} \right)^{(n+1)} = \left[ 3y^{(n+1)} - 4y^{(n)} + y^{(n-1)} \right] / (2 \delta t)
\]

\[
\left( \frac{d \delta}{dt} \right)^{(n+1)} = \left[ 3\delta^{(n+1)} - 4\delta^{(n)} + \delta^{(n-1)} \right] / (2 \delta t)
\] (6)

Taking the aerodynamic forcing at the new time level, the following system is obtained:

\[
\left[ \begin{array}{c}
\frac{dy}{dt} \\
\frac{d \delta}{dt}
\end{array} \right]^{(n+1)} = \left[ \begin{array}{c}
\frac{1}{2 \delta t} + 2 \zeta \omega_0 - \omega_0^2 \\
4 \delta^{(n+1)} - 4 \delta^{(n)} - \delta^{(n-1)} / (2 \delta t)
\end{array} \right]
\]

\[
\left[ \begin{array}{c}
\frac{dy}{dt} \\
\frac{d \delta}{dt}
\end{array} \right]^{(n+1)} = \left[ \begin{array}{c}
\frac{\omega_0^2}{2 \delta t} \\
\omega_0^2
\end{array} \right]
\]

\[
\left[ \begin{array}{c}
\frac{dy}{dt} \\
\frac{d \delta}{dt}
\end{array} \right]^{(n+1)} = \left[ \begin{array}{c}
\frac{dy}{dt} \\
\frac{d \delta}{dt}
\end{array} \right]^{(n)}
\]

\[
\frac{d^2 \delta}{dt^2} = \frac{1}{2 \delta t}
\]

\[
\frac{d^2 \delta}{dt^2} = \frac{1}{2 \delta t}
\]

where the system in Equation (7) is solved for every iteration in the dual time-stepping loop, i.e. the air-loads are re-evaluated every time the flap deflection angle is updated. Upon convergence of the inner-loop in this dual-time stepping method, the converged flow solution at the new time step will be consistent with the structural response obtained at the same time level. Compared to the leap-frog method discussed previously, the implicit fluid-structure coupling method involves more evaluations of the flap equation and corresponding mesh deformations per time-step. However, the stability added by the implicit treatment should enable larger time-steps and hence fewer time-steps in the aero-elastic simulation.

2.3.3 Solution of structural response in Fourier space

In the three previously discussed coupling methods, the flow state and structural response are integrated in time, using either a coupling per time-step or pseudo-time step in the dual-time stepping method. For cases with a ‘periodic’ flow forcing, an alternative method for the temporal integration was developed. In this method, the motion of the flap around its hinge is obtained from solving Equation (3) in Fourier space. The flap deflection and its derivatives, as well as, the forcing,
are expanded in the following Fourier series,

\[
\delta(t) = a_0 + a_1 \sin(\omega t) + b_1 \cos(\omega t) + a_2 \sin(2\omega t) + b_2 \cos(2\omega t) + \ldots
\]

\[
\frac{d\delta(t)}{dt} = \omega a_1 \cos(\omega t) - \omega b_1 \sin(\omega t) + 2\omega a_2 \cos(2\omega t) - 2\omega b_2 \sin(2\omega t) + \ldots
\]

\[
\frac{d^2\delta(t)}{dt^2} = -\omega^2 a_1 \sin(\omega t) - \omega^2 b_1 \cos(\omega t) - 4\omega^2 a_2 \sin(2\omega t) - 4\omega^2 b_2 \cos(2\omega t) + \ldots
\]

\[
s_{\text{cm}} = c_0 + c_1 \sin(\omega t) + d_1 \cos(\omega t) + c_2 \sin(2\omega t) + d_2 \cos(2\omega t) + \ldots
\]

Matching the Fourier coefficients for different frequencies gives,

\[
\begin{bmatrix}
(\omega_0^2 - k^2\omega_0^2)

-2k\zeta\omega_0

2k\zeta\omega_0

(\omega_0^2 - k^2\omega_0^2)
\end{bmatrix}
\begin{bmatrix}
a_k

b_k
\end{bmatrix}
= \begin{bmatrix}
c_k

d_k
\end{bmatrix},
\]

where \( k = 1, \ldots, n_{\text{harm}} \) is the number of Fourier harmonics used in the series expansions and \( \omega \) is the frequency of the 'imposed' flow unsteadiness. Solving for the Fourier coefficients gives,

\[
a_0 = \frac{c_0}{\omega_0^2} \] 

\[
\begin{bmatrix}
(a_k)

(b_k)
\end{bmatrix}
= \frac{1}{\Delta} \begin{bmatrix}
(\omega_0^2 - k^2\omega_0^2)

-2k\zeta\omega_0

2k\zeta\omega_0

(\omega_0^2 - k^2\omega_0^2)
\end{bmatrix}
\begin{bmatrix}
c_k

d_k
\end{bmatrix},
\]

\[
\Delta = (\omega_0^2 - k^2\omega_0^2)^2 + 4k^2\zeta^2\omega_0^2, \] 

\( k = 1, \ldots, n_{\text{harm}} \) \hspace{1cm} (9)

This scheme is a hybrid time-domain/Fourier-domain method, since the Navier-Stokes equations are solved in the time-domain. In the current implementation, the method proceeds along the following steps:

1. Set \( a_i = 0, b_i = 0 \), for \( i \in [0, n_{\text{harm}}] \); and compute the flapping schedule for the first two cycles based on these coefficients,

2. Solve the unsteady flow for the first two cycles and record the flap hinge moment for each time step,

3. Transform the hinge moment for the last cycle into Fourier space, which defines \( c_i, d_i \), for \( i \in [0, n_{\text{harm}}] \). Using Equation \((10)\), compute \( a_i, b_i \), for \( i \in [0, n_{\text{harm}}] \).

4. Compute the flapping schedule for next two cycles based on these coefficients. In case an under-relaxation is used, the new flapping schedule combines the ‘old’ Fourier coefficient and the ‘new’ coefficients to prevent excessively large changes in the flapping schedule from one structural solution to the next,

5. Solve the unsteady flow for the next two cycles and record the flap hinge moment for each time step,

6. Repeat from step 3.

The structural response is updated every 2nd cycle to enable the flow to adjust to each new flapping schedule. In the first cycle of a new flapping schedule, the flow will develop artificial transients which will need to decay before an accurate hinge moment can be recorded in the 2nd cycle after a new flapping schedule was imposed. The above method is a hybrid time/frequency-domain integration and resembles the harmonic balance technique used in aero-elasticity. An alternative would be ‘harmonic balance method’, as will be used in the following sections.

2.4 Aeroelastic Coupling method for Forward Flying Rotors

For forward flying rotors, a modal approach is used. The modal approach allows a reduction of the problem size by modelling the blade shape as the sum of a limited number of dominant eigenmodes, which are obtained using NASTRAN. The blade shape is described as follows:

\[ \phi = \phi_0 + \sum_{i=1}^{n_m} \alpha_i \phi_i, \] \hspace{1cm} (11)

where \( \phi \) is the blade shape, \( \phi_0 \) the blade static deformation and \( \phi_i \) is the \( i \)-th mass-scaled eigenmode of the blade. The amplitude coefficients \( \alpha_i \) are obtained by solving the equation:

\[ \frac{\partial^2 \alpha_i}{\partial t^2} + 2\zeta_i \omega_i \frac{\partial \alpha_i}{\partial t} + \omega_i^2 \alpha_i = \vec{f} \cdot \phi_i, \] \hspace{1cm} (12)

where \( \omega_i \) and \( \zeta_i \) are respectively the eigenfrequency and the eigenmode damping ratio. \( \vec{f} \) is the vector of external forces.

To solve Equation \((12)\) in time, along with the flow solution around the rotor, a strong coupling method was used.

The strong coupling approach does not force periodicity in the blade deformation and may need more time to solve a problem and be less stable. However, it allows more flexibility for complex motions of the rotor which are not linked to a steady flight (like manoeuvres). Two approaches were tested and compared: a leap-frog method computed the modes amplitudes between each time step, and an implicit method computed the mode shape amplitudes between each pseudo-time step. A flow chart showing the different steps for each method is shown in Figure 3a.

2.4.1 The Leap-Frog Method

Equation \((12)\) is solved at the end of each time step, as shown in Figure 5a. The \( i \)-th mode forcing is extracted from the computed time step \( t \) as:

\[ f_i^s = \vec{f} \cdot \phi_i \] \hspace{1cm} (13)

The \( i \)-th amplitude \( \alpha_i \) is then assessed for time step \( t + 1 \) as:

\[ [\alpha_i]_{t+1} = [\alpha_i]_t + \left[ \frac{\partial \alpha_i}{\partial t} \right]_t \Delta t + \frac{1}{2} \left[ \frac{\partial^2 \alpha_i}{\partial t^2} \right]_t \Delta t^2 \] \hspace{1cm} (14)

The time derivative of the amplitudes are then computed as:

\[ \left[ \frac{\partial^2 \alpha_i}{\partial t^2} \right]_t = [f_i^s]_t - \omega_i^2 [\alpha_i]_t - 2\zeta_i \omega_i \left[ \frac{\partial \alpha_i}{\partial t} \right]_t \] \hspace{1cm} (15)
\[
\frac{\partial \alpha_i}{\partial t} = \frac{1}{2} \left( \frac{\partial^2 \alpha_i}{\partial t^2} + \frac{\partial^2 \alpha_i}{\partial t^2} \right) \Delta t \tag{16}
\]

using the non-updated amplitudes derivatives estimates from the previous time step \( t \).

2.4.2 The Implicit Method

The discretisation of the derivatives of the modal amplitudes is expressed as follows:

\[
\frac{\partial \alpha_i}{\partial t} = \frac{[\partial \alpha_i]}{\partial t}_{t-1} + \Delta t \frac{[\partial \alpha_i]}{\partial t^2}_{t-1} \tag{17}
\]

\[
= 4[\alpha_i]_t - 4[\alpha_i]_{t-2} + [\alpha_i]_{t-1} + [\alpha_i]_t \tag{18}
\]

\[
= \frac{3[\alpha_i]_t - 4[\alpha_i]_{t-2} + [\alpha_i]_{t-1} + [\alpha_i]_t}{2\Delta t} \tag{19}
\]

which, if applied to the modal amplitudes and their time derivatives, gives:

\[
\frac{\partial \alpha_i}{\partial t} = \frac{\partial^2 \alpha_i}{\partial t^2} \tag{20}
\]

\[
\frac{\partial^2 \alpha_i}{\partial t^2} = \frac{3[\alpha_i]_t - 4[\alpha_i]_{t-2} + [\alpha_i]_{t-1} + [\alpha_i]_t}{2\Delta t} \tag{21}
\]

Equation 17 is discretised as:

\[
2\zeta \omega_1 + \frac{\omega_1^2}{\omega} L_1 = \left( \frac{\partial \alpha_i}{\partial t} \right)_{t-1} + \Delta t \frac{\partial \alpha_i}{\partial t^2} \tag{22}
\]

and solved at the end of each time step. The matrix is inverted using Cramer’s rule and the modal amplitudes coefficient updated for the following pseudo-time step. This method implies the computation of the new grid at each pseudo-time step, as shown in Figure 5(a) compared to each time step for the leap-frog method, but was deemed more robust.

2.4.3 Aeroelastic Coupling method for Hovering Rotors

For hovering rotors, CFD/CSD coupling is realised using an iterative method. The loads are first extracted from the fluid solution and are then used in NASTRAN to obtain the required blade shape using a static deformation approach. The blade is then deformed based on the structural shape using the method described in the next section. This process is repeated until convergence. Results from this approach are reported in an earlier work of the authors [13].

2.5 Grid Deformation Method

The method developed for HMB first deforms the blade surface according to the structural deformation using the Constant Volume Tetrahedron (CVT) method, then obtains the updated block vertex positions via spring analogy (SAM) and finally generates the full mesh via Transfinite Interpolation (TFI). It is extensively described in [14]. The TFI first interpolates the block edges and faces from the new vertex position and then interpolates the full mesh from the surfaces. This method uses the properties of multi-block meshes and maintains efficiency as the number of blocks increases, particularly in the spanwise blade direction.

The method to deform the blade surface to account for the flap motion, for rotor blades with active flaps, is very similar to the method for wings with active flaps, previously described in Section 2.2. The flap and hinge line locations are now to be defined for the blade in the ‘datum’ position, i.e. for the blade at \( \psi = 0^\circ \) and with the built-in coning and collective angle removed. The assumption is made that all blades have the same flap geometry. The flap angles are then defined as a Fourier series in the blade azimuthal angle. Again, flap blending is employed to maintain a smooth surface geometry.

2.6 Example mesh deformations and mesh quality preservation

Figure 4 shows the relative cell volume changes due to mesh deformation accounting for rotor trimming and trailing-edge flap actuation for the ‘rigid’-block mesh deformation method. A spanwise cut through mid-span of inboard flap is shown for three different conditions. Figure 5(a) shows results for the relative cell volume change resulting from applying a trailing-edge flap deflection of 5\(^\circ\). In Figure 5(b), an 8\(^\circ\) nose-down pitch change is added, while in Figure 5(c) an additional 2\(^\circ\) blade flapping angle is imposed. Figures 5(a)-(c) show the equivalent results for a chord-wise cut through the blade mesh. The mesh deformation method based on the spring analogy approach is demonstrated in Figure 6(a) for an 8\(^\circ\) nose-down pitch change as well as a 5\(^\circ\) trailing-edge flap deflection. Comparing this figure with Figure 4(a), it can be seen that using the ‘rigid’-block mesh deformation method, the mesh is not deformed at all in the blocks surrounding the leading edge of the blade, while for the spring analogy approach, the relative cell volume changes were limited to around 10\% in the leading-edge region. For the ‘rigid’-block mesh deformation method, the mesh defomation resulting from the blade pitch change is restricted to the blocks surrounding the ‘rigid’ blocks. For the spring analogy method, these blocks similarly take the majority of the mesh deformation. For comparison, Figure 4(b) shows results which were obtained by imposing the blade deflections onto the first layer of mesh blocks surrounding the blade surface by TFI. In the present method, this was achieved by re-setting the ‘internal’ vertex displacements to zero. As demonstrated previously [35] for a rotor without flaps, such an approach leads to unacceptable mesh deformations for typically rotor control angles.

3 Results and Discussion

In this section, demonstrations of the previously introduced methods are presented. The first part focuses on aerofoil simulation using a 1-degree-of-freedom flap model, followed by a demonstration of the flap actuation on a wing. The aeroelastic coupling strategy is then demonstrated on the UH-60A rotor, followed by a demonstration of the flapped rotor approach using a model rotor.
3.1 Results for 1-degree-of-freedom flap model - static aerofoil

As a first test of the 1-degree-of-freedom aero-elastic flap model, the accuracy of the temporal integration method was assessed by performing a series of simulations in which the coupling with the aerodynamic loads was switched off. In this case, the flap was given an initial deflection of 2.0° down. The non-dimensional flap stiffness $k_h$ was chosen as $0.1$ and the non-dimensional inertia $I_h$ about the flap hinge as $10.0$. This gives a scaled natural frequency $\omega_0 = \sqrt{k_h/I_h} = 0.10$, corresponding to a reduced frequency of $0.05$. Time-dependent simulations were set such that the considered time-interval spans 3 cycles of the flap at the natural frequency. Cases were computed with 360, 720 and 1440 time steps per cycle. No structural damping was imposed.

The evolution of the flap angle for the cases without aero-elastic coupling is shown in Figure 7(a), using the leap-frog 1-degree-of-freedom method. It can be seen that for all three temporal resolutions, the natural frequency of the flap oscillation is very well captured, i.e. the flap completes three cycles in the considered time interval. The effect of the temporal integration accuracy mainly shows through a slowly increasing flap amplitude for larger time-step cases. For this undamped case without aero-elastic coupling this amplitude should remain constant. For the 3 cycles considered, the simulation with 1440 time steps per cycle preserves the amplitude to a good level, while the simulation with 360 steps per cycle gives a clear over-prediction of the flap amplitude at later stages in the simulation.

The simulations were repeated with aero-elastic coupling, with the results shown in Figure 7(b). From comparison with the cases without aero-elastic coupling, it can be seen that the aero-elastic coupling increases the oscillation frequency somewhat. This is a manifestation of the added ‘stiffness’ due to the aerodynamic moments around the flap hinge for the considered conditions. As expected, the aero-elastic coupling appears to introduce a small amount of damping into the system, since the flap amplitude stays closer to the initial amplitude for the considered time interval.

3.2 Results for 1-degree-of-freedom flap model - oscillating aerofoil

Table 5 presents the test cases for an oscillating OA209 airfoil studied in the present work. In all cases, the airfoil oscillation schedule represents a blade section at 90% span-wise position on the ONERA 7AD 4-bladed model rotor assuming a rotor tip Mach number $M_{tip} = 0.60$. Test cases 1 and 2 assume a constant pitch angle of the section, 2.0° and 4.0°, respectively. The unsteady nature of the aerodynamics are therefore purely the result of the translation motion of the section. Figure 8 shows the surface pressure distribution for test case 2, when the airfoil is assumed fully rigid, i.e. without flap deflection. The formation of the strong normal shock on the advancing side of the cycle is apparent. The reduced dynamic head encountered by the section on the retreating side of the cycle leads to much reduced air-loads through that part of the cycle.

Test cases, 3, 4a and 4b assume a representative pitch schedule defined by $\theta(2kt) = \theta_0 - \theta_{1s} \sin(2kt) - \theta_{1c} \cos(2kt)$, with $\theta_{1s} = 8.0°$ and $\theta_{1c} = -2.0°$. For case 3, $\theta_0 = 2.0°$, while for 4a and 4b, $\theta_0 = 6.0°$. Test case 3 has a negative pitch angle through parts of the advancing side, leading to the formation of a strong shock wave on the lower surface of the section. For a blade station at 90% R, this situation can arise in high-speed forward flight [33]. The larger mean angle in cases 4a and 4b result in much weaker shocks on the advancing side of the rotor disk, while for these cases, the large blade pitch will cause dynamic stall through parts of the retreating side. For these different test cases, simulations were conducted for the different aero-elastic coupling methods and a wide range of different numerical parameters, as shown in Tables 3 and 4.

3.2.1 Consistency and stability of different schemes

For the fixed-pitch test cases 1 and 2, the predictions for the flap deflection angle from the leap-frog and direct implicit coupling methods are compared in Figures 9 and 10 for inviscid and viscous flow models. Results are shown for a range of time-steps as well as pseudo-steps. It can be seen that the results from the direct implicit coupling method converge well even for a small number of time-steps, i.e. for 720 or more time steps per translation cycle, the results are very close. The leap-frog method can also deliver good results that converge to the implicit solution at the expense of pseudo time steps. For the cases shown here, around 100 pseudo-steps are needed for the inviscid flow model and 200 for the viscous option. This result alone shows that the implicit method may be more suitable for computations and could offer savings. It is also evident that the combination of real time steps with the small number of pseudo-steps that the implicit method offers, is the key to efficient computations. This conclusion about the implicit method, is further supported by its robustness in comparison to the leap-frog method.

Figures 11 and 12 show how the results for the direct implicit coupling method depend on the number of pseudo-time steps and time-steps for cases 1a and 2. From the analysis it can be seen that with 50 pseudo-steps and CFL of 20, the inner iteration in the dual-time stepping method converges sufficiently to obtain a good temporal evolution in the outer loop with increasing number of time steps. Figure 11 also compares the result for the harmonic method for test case 1a with a single mode with the results of the direct implicit coupling method reconstructed with the first Fourier harmonics. The comparison shows that the harmonic method captures the first harmonic component of these solutions to a good level. As can be seen, the higher-frequency flap deflection modes present in the direct implicit coupling solutions feed back into the first-harmonic excitation. This higher-frequency feedback cannot be captured by the harmonic method.

For the leap-frog method, temporal convergence was seen to require more time steps per translation cycle. The results presented in Figure 10 indicate that at least 2880 time steps per translation cycle are required for the two fixed-pitch cases. For these cases, the results of the leap-frog methods for very small time-steps appear to approach those of the direct implicit coupling method. This means that for these two cases, both methods can be used effectively, however, with four times more steps per translation cycle required for the leap-frog method than for the direct implicit coupling method.
Test case 3 was simulated using the direct implicit coupling method as well as the harmonic method. Figure shows the predicted flap deflection angles. It was observed that the harmonic method produced a stable solution only when a single Fourier harmonic was used. Due to the selection of the natural flap frequency, the second Fourier harmonic exceeded the natural frequency of the flap. For two or more Fourier modes in the harmonic balance method, a resonance instability occurs. For this test case, the direct implicit coupling method produced results which showed a temporal convergence for the selected range of time steps. Also, the results for the direct implicit coupling method appear to be consistent with those for the 1-mode harmonic balance method, i.e. the component at 1/cycle for the results from the direct implicit coupling method are close to the prediction from the harmonic balance method.

Test cases 4a and 4b were also computed using the direct implicit coupling method as well as the harmonic balance methods. Figure shows the predicted flap deflection angles. It was observed that the harmonic method produced a stable solution only when a single Fourier harmonic was used, as was the case for the test case 3 discussed previously. For two or more Fourier modes in the harmonic balance method, a resonance instability occurs. For cases 4a and 4b, the direct implicit coupling method produced results which showed a temporal convergence for the selected range of time steps. Also, the results for the direct implicit coupling method appear to be consistent with those for the 1-mode harmonic balance method, i.e. the component at 1/cycle for the results from the direct implicit coupling method are close to the predictions from the harmonic method. Figure presents results from a more detailed investigation of the convergence behaviour of the direct implicit coupling method for cases 4a and 4b. In the analysis the number of time steps per translation cycle as well as the number of pseudo-steps in the dual time-stepping method was varied. For all cases, a CFL of 20 was used. The results show that at least 50 pseudo-steps per time-step were required to achieve the required temporal convergence with respect to the time-step in the dual-time stepping method. Also, the results indicate that for this challenging test case, well-resolved results can be obtained with the direct implicit coupling method with 1440 steps per translation cycle. This resolution is similar to what is used with HMB for rotors in forward flight.

3.2.2 Flap excitation during dynamic stall

The pitch/translation schedule used in test cases 4a and 4b was chosen such that stall would occur for the rigid airfoil at identical conditions. The 1-degree-of-freedom flap aeroelastic model can be seen to give a significant flap deflection for the selected conditions. One of the effects of the flap deflection is a change of the airfoil camber. During the part of the cycle associated with the 'retreating' side of a helicopter rotor disk, the highest angle of attack usually occurs around $2kt = 270^\circ$. For the direct implicit coupling method, the flap schedule for these cases is shown in Figure A large component at the harmonic nearest to the natural frequency of the flap can be observed for both natural frequencies of the flap. The main effect of the natural frequencies of the flap can be seen in the frequency content at higher frequencies as well as the phasing. The phasing of the flapping schedule will now largely determine whether during the part of the cycle with the highest angle of attack, the camber of the airfoil can be reduced or increased. For the test cases 4a and 4b, the only difference is the flap eigenfrequency $\omega_0$, which for case 4a is 0.05, while it is 0.10 for case 4b. As can be seen from Figure (a) and (b), the flap deflection curves are consistent with those for the 1-mode harmonic balance method, i.e. the component at 1/cycle for the results from the direct implicit coupling method are close to the prediction from the harmonic balance method.

3.3 Results for Wing with Active Part-Span Flaps

In this section, a wind tunnel model for a wing with two active trailing edge tabs is considered. The wing was constructed from a segment of a Sea King main rotor blade, which gave the model a small linear twist. Figure shows the CFD geometry created for this test case. An ellipsoidal wind tunnel wall is assumed with flat end plates to mimic the geometry of the AgustaWestland wind tunnel at Yeovil used for the tests. The active tabs are denoted by the red and green surfaces on the wing trailing edge. The CFD mesh for the flap is shown in Figures (b) and (c). A C-type mesh is used with separate block surfaces for the flaps. The mesh size was 4.7 million cells and a wall distance of $10^{-5}$ was used based on past experience with cases at similar Re. The conditions of the test cases computed are summarised in Table 8.

Figure presents results for the effect of the flap on the surface pressure coefficient distribution over the flapped sec-
tion. For zero incidence of the main wing, the flap effect is well-captured for deflections at $+/-3$ degrees and the CFD results fit well with the different measurements obtained during the test at four stations near the leading edge of the wing. The same holds for a case where the incidence of the wing is 8 degrees.

Figure 22 shows the span-wise effect of the flap. The vertical dashed lines indicate the flap ends and one can see that the effect of the flap dies gradually along the wing span. This is expected since for this case the flap has been modelled as a blended surface and the decay of the $C_p$ away of the flap is slow. The lift measured in the tunnel is compared against CFD on Figure 23. Three flap deflection angles are shown with minor differences between the CFD and experiments. The results simply validate that the steady flow model in HMB is capable of capturing the effect of the mean flap deflection.

3.4 Coupled CFD/CSD Simulation of the UH-60A Rotor in High-Speed Forward Flight

The UH-60A rotor [2] was chosen to assess the aeroelastic coupling strategy. This rotor was tested in flight by NASA and the US Army [21]. In the high-speed flight (Flight Counter 8534), it has been shown [12, 36] that the torsional deformation played an important role in the loading predictions. This torsional deformation is triggered by the movement of a shock on the advancing-side and the formation of a shock on the blade lower surface. This case was used by Steijl et al. [36], who showed that the inclusion of torsional deformation extracted from flight test data allowed for an improvement of the loads on the advancing side, that were mainly driven by a high amplitude pitch-down torsion around $\Psi = 140$ degrees. It was therefore deemed as an interesting test-case to assess the coupling method.

The flight conditions and control angles are summarised in Table 9. The grid contained 8.0 million nodes and the $k-\omega$ BSL turbulence model [22] was used. A first simulation was carried out using a structural damping of $\zeta = 0.3$ for every structural mode and an azimuthal step of $\Delta \Psi = 0.25$ degree. The implicit coupled method was used. The first half of the revolution was run as a rigid case, before the blade was allowed to elastically deform. Three revolutions allowed for convergence of the deformations.

The blade geometry was estimated from information in the literature [2], however, uncertainties still exist on the exact blade geometry, twist distribution and structural model.

The blade deformations were extracted from the coupled simulations and are shown in Figure 24. The most noticeable property of the blade deformation was the strong dip in torsion deformation at the advancing side. This deformation was caused by a shock formed on the lower surface, shown in Figure 25. With the torsional deformation, the shock moves on the lower surface and increases the amplitude of the blade deformation. The blade recovered from the torsional deformation when the local free stream velocity decreased enough for the strength of the shock to lower. Small oscillations also appeared on the retreating side and a slight increase of torsion was also noticed around $\Psi = 25$ degrees. The amplitude of the second torsional mode seemed negligible compared to the amplitude of the first torsional mode. The flapping deformation also seemed to be dominated by the second flapping mode, with a strong 1/Rev component, leading to a dip of the tip flapping at $\Psi = 135$ degrees.

The Mach-scaled sectional normal force and pitching moments were extracted and are shown in Figure 26. The influence of the torsional deformation around $\Psi = 160$ degrees is clearly visible, with negative normal force. High frequency oscillations can also be noticed on the advancing side. These were caused by BVIs. Looking at the pitching moments, the transition between aerofoil sections and the start of the sweep can be noticed through the moment discontinuities in the radial direction. The BVI area is also visible with the high-frequency changes on the advancing side. The higher moments due to the SC1094-R8 seemed to trigger the dip in the torsional moment, due to the higher amplitude of the pitching moment between $\Psi = 45$ degrees and $\Psi = 120$ degrees.

The sectional normal force was compared with flight-test measurements [21] at $r/R = 0.675$ and $r/R = 0.865$ in Figure 27. The dip in the sectional forces on the advancing side appeared stronger in the simulations than in the flight test measurements, and was delayed by 15 degrees. However, the loads on the retreating side agreed better with the flight test measurements. At $r/R = 0.675$, the BVIs predicted by the simulations did not seem to occur in the flight tests on the advancing-side, but at $r/R = 0.865$, their locations and amplitudes seemed to agree with the flight test data. The mean normal force in the first quadrant is, however, over-predicted.

The predicted loads were also compared to the ones obtained with a rigid blade and the ones obtained by Steijl et al. [36], using a prescribed torsion closer to the flight tests data. The effect of the blade in-flight deformation was mainly located on the advancing side. On the retreating side, the coupled case, predicted a higher increase of the Mach-scaled loading in the forth quadrant, in better agreement with flight test measurements. On the advancing-side, the differences in the dip amplitude and phase between the coupled simulation and the flight test measurements appeared to be due to different torsional levels between simulation and flight test data, as the simulation with a prescribed torsion agreed better with the flight tests. The BVIs around $\Psi = 85$ degrees also appeared to be stronger in the coupled simulation compared to the others, which may come from the inclusion of the flapping deformation. Clearly, the approximate blade shape and the lack of detailed data for the structural properties have an influence on the results. The mesh deformation method, however, managed to produce good quality grids, and resulted in no loss of code stability.

It was then decided to study the influence of the structural damping coefficient $\zeta$ (see Equation 12) on the blade in-flight deformation. Therefore, damping coefficients of $\zeta = 0.1$ and $\zeta = 0.02$ were compared to the original value of 0.3. The evolution of the blade tip deformation with the damping coefficient can be seen in Figure 28. The main features of the blade deformation did not change. The tip flapping showed a difference in the recovery from the dip on the advancing side. With the lower damping, the recovery happened at a higher speed, and the overshoot was also more pronounced. Higher differences can also be noticed on the tip torsion. The first remark deals with the converged state. While all blades con-
verged to the same equilibrium state at $\zeta = 0.3$, it was noticed that for lower damping coefficients, the converged deformation of blades 1 and 3 was marginally different from the one of blades 2 and 4. Also, the aerodynamic damping on the retreating side proved low, and a decrease in the structural damping allowed the blade to vibrate at the frequency of the first torsional mode.

The influence of the azimuthal time step was also studied, using $\Delta \Psi = 1$ degree and $\Delta \Psi = 0.25$ degree. The obtained tip deformation is shown in Figure 29. The difference in the blade deformation predictions with $\Delta \Psi = 1$ degree and $\Delta \Psi = 0.25$ degrees was limited to the advancing side in tip torsion, with an earlier recovery from the dip when using $\Delta \Psi = 0.25$ degree.

Therefore, a time step $\Delta \Psi = 0.25$ degrees was used to compare the two proposed coupling methods: the leap-frog and the implicit method. Figure 30 shows the tip deformation for the two methods. The difference between the two methods proved limited, with almost identical results.

Finally, the influence of the turbulence model was assessed, with the use of the $k - \omega$ BSL and SST turbulence models [22]. A time step of $\Delta \Psi = 25$ degrees, and damping ratio of $\zeta = 0.3$ were used. The tip deformation predictions are shown in Figure 31. The main difference was located on the dip: a 0.7 degree difference was visible in the dip of the tip torsion, the torsional deformation being more important with the $k - \omega$ BSL model. This difference is due to the small differences in strength of the shock appearing on the blade tip area.

3.5 Results for rotor with active trailing-edge devices

In this section, a 4-bladed model rotor is considered with straight blades of NACA0012 section and aspect ratio of approximately 12.2. Each blade is equipped with two active trailing-edge tabs: an inboard one of 15% spanwise extent centred around 60% R, and an outboard one with 10% spanwise extent centred around 80% R. In the chordwise direction, the flap had a dimension of 15%c. Based on these basic rotor parameters, a CFD geometry was defined with a generic rotor hub and wind tunnel support, with tunnel dimensions modelled on a typical subsonic tunnel return section ($15 \times 10$ R) at the Politecnico di Milano. The rotor geometry as well as the flap locations are shown in Figure 32. For the CFD analysis presented here, the sliding-plane method was used to couple a stationary background mesh (which includes the far-field boundaries and the cylindrical wind tunnel support) with a drum around the rotor blades and the hub. In this section, the application of the mesh motion/deformation method described in Section 2.3 for rotor simulations including active trailing-edge tabs is discussed. Table 10 presents a summary of the selected test cases.

Figure 33 shows the pressure in a slice of the flow field perpendicular to the blade spanwise direction. The location of the sliding planes is shown with the thick black lines. The effect of the sliding planes on the pressure is limited, as shown by the continuity of the isobar curves.

Figure 34 presents the rotor normal loading evolution with the flaps, using an inviscid simulations. The effect of the flaps proved to be strong at the flap locations (between the dot-dashed lines), but also showed an influence on the normal loading of the non-flapped sections. When using a viscous simulation, using the $k - \omega$ turbulence model, similar effect on the loads can be observed, as shown in Figure 35.

4 Conclusions

A CFD method for predicting the flow field around aerelastic flapped rotor has been introduced. This method is using a mesh deformation algorithm based on the use of trans-finite interpolation and spring analogy, and was shown to preserve the mesh quality in the area near the aerofoil, where a high mesh quality is necessary to accurately predict flow features, such as the boundary layer, and stall.

A first study dealt with a freely moving flap attached to a 2D-aerofoil. The aerodynamic environment of blade sections on a rotor in forward flight were approximated using a combination of an oscillatory translation and pitching motion, mimicking the effect of the tangential velocity changes and blade pitch schedule of sections on a rotor in forward flight, respectively. The unsteady aerodynamics of the blades creates a time-dependent flap deflection. For the cases considered, the flap deflection was shown to be bounded. Various time integration methods in the 1-degree-of-freedom were investigated in detail. The fully implicit coupling method was found to be the faster in convergence, allowing higher CFL, more robust than the leap-frog method and less dependent on the number of pseudo-steps. Time marching methods with a less direct coupling, i.e. the leap-frog method were found to give similar results to the fully implicit method when a much smaller time-step was used. As an alternative to the time-marching methods, a harmonic balance method was coupled to the time-marching CFD method. This method was found to be quite effective in establishing a time-periodic solution assuming an appropriate choice of an under-relaxation factor was used. With this choice of under-relaxation factor, the method was shown to be capable of resolving most of the flap dynamics of time-marching solutions, when a reduced number of time-steps per cycle were used compared to the time-marching simulations. However, the coupled harmonic balance time-marching CFD method was proved to be prone to instabilities. To obtain a stable simulation, the number of Fourier modes needed to be cut-off below the harmonic closest to the natural frequency of the flap. For stability analysis and accurate load evaluation, the method with direct implicit coupling appears to be the most reliable and accurate of the methods investigated in this work. The effect of the flaps was also tested for a wing, using a prescribed flap position. Trailing edges flaps were added on a part of a Seaking blade, and the loads predictions were compared to wind tunnel measurements. The effect of the flaps on the wing lift proved to be accurately captured by the CFD simulations. A strong coupling method was then applied to the UH-60A rotor in high-speed forward flight. The method proved to be able to predict the strong torsion peak on the advancing side, and allowed to get converged deformations in three rotor revolutions for a damping coefficient of $\zeta = 0.3$. Finally, the effect of flaps on rotors was demonstrated, using a model rotor. Two trailing edge flaps were located on each blade, and all flaps underwent the

10
same actuation as a function of the azimuth. The flaps clearly influenced the spanwise loading on the full blade, instead of having a local effect only.

The HMB method was found capable of dealing with flapped rotors and their aeroelastics. Further studies should deal with the effect of the flap actuation on the acoustic and vibrations coming from the rotor, as well as improvements of the rotor performance. Freely moving flaps for rotor cases should also be tested, in order to assess the flap stability, and the influence of the flaps on the aeroelastic blade deformations should also be assessed and quantified.

Acknowledgements

The financial support via the REACT project of AgustaWestland and the Business Innovation and Skills Department of UK is gratefully acknowledged.

REFERENCES


Table 1: CFD/CSD coupling methods in the literature

<table>
<thead>
<tr>
<th>Authors</th>
<th>Coupling Strategy</th>
<th>CFD Code</th>
<th>CSD Code</th>
<th>Fluid model</th>
<th>Test case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servera et al. [29], 2001</td>
<td>Weak</td>
<td>WAVES</td>
<td>HOST</td>
<td>Euler</td>
<td>ONERA 7A and 7AD rotors ($\mu = 0.4$)</td>
</tr>
<tr>
<td>Beaumier et al. [6], 2001</td>
<td>Weak</td>
<td>CANARI, FLOWer</td>
<td>R85</td>
<td>RANS</td>
<td>Hovering ONERA 7A and Bo-105 rotors</td>
</tr>
<tr>
<td>Altmikus et al. [1], 2002</td>
<td>Weak, Strong</td>
<td>WAVES, FLOWer</td>
<td>HOST</td>
<td>Euler</td>
<td>ONERA 7A rotor ($\mu = 0.4$)</td>
</tr>
<tr>
<td>Pomin and Wagner [27], 2004</td>
<td>Strong</td>
<td>INROT</td>
<td>DYNROT</td>
<td>Hybrid RANS-Euler</td>
<td>ONERA 7A rotor (hover and $\mu = 0.4$)</td>
</tr>
<tr>
<td>Potsdam et al. [28], 2004</td>
<td>Weak</td>
<td>OVERFLOW-D</td>
<td>CAMRAD-II</td>
<td>RANS</td>
<td>UH-60A rotor (flight counters 8534, 8513 and 9017)</td>
</tr>
<tr>
<td>Pahlke and Van der Wall [26], 2005</td>
<td>Weak</td>
<td>FLOWer</td>
<td>S4</td>
<td>RANS</td>
<td>ONERA 7A and 7AD rotors ($\mu = 0.4$)</td>
</tr>
<tr>
<td>Biedron and Lee-Rausch [7, 8], 2008, 2011</td>
<td>Weak</td>
<td>FUN3D</td>
<td>CAMRAD-II</td>
<td>RANS</td>
<td>HART-II ($\mu = 0.1509$) and UH-60A (flight counters 8534 and 9020) rotors</td>
</tr>
<tr>
<td>Ortun et al. [25], 2008</td>
<td>Strong</td>
<td>elsA</td>
<td>MSC.Marc</td>
<td>RANS</td>
<td>ONERA 7A ($\mu = 0.4$) and ERATO ($\mu = 0.423$) rotors</td>
</tr>
<tr>
<td>Dietz et al. [15], 2008</td>
<td>Weak</td>
<td>FLOWer</td>
<td>HOST</td>
<td>RANS</td>
<td>GOAHEAD helicopter configuration ($\mu = 0.0956$)</td>
</tr>
<tr>
<td>Sitaraman and Roget [32], 2009</td>
<td>Strong</td>
<td>UMTURNS</td>
<td>DYMORE</td>
<td>RANS, Vorticity transport</td>
<td>UH-60A rotor (pull-up manoeuvre flight counter 11029)</td>
</tr>
</tbody>
</table>

Table 2: Test cases considered for 1 degree-of-freedom model

<table>
<thead>
<tr>
<th>case</th>
<th>$M_{\text{mean}}$</th>
<th>$\mu_{\text{rotor}}$</th>
<th>$\mu$</th>
<th>$\omega_0$</th>
<th>$\zeta$</th>
<th>$\theta_0$</th>
<th>$\theta_{1s}$</th>
<th>$\theta_{1c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>0.5553</td>
<td>0.333</td>
<td>100</td>
<td>0.100</td>
<td>0.0</td>
<td>2.0°</td>
<td>0.0°</td>
<td>0.0°</td>
</tr>
<tr>
<td>1b</td>
<td>0.5553</td>
<td>0.333</td>
<td>10</td>
<td>0.100</td>
<td>0.0</td>
<td>2.0°</td>
<td>0.0°</td>
<td>0.0°</td>
</tr>
<tr>
<td>2</td>
<td>0.5553</td>
<td>0.333</td>
<td>100</td>
<td>0.100</td>
<td>0.0</td>
<td>4.0°</td>
<td>0.0°</td>
<td>0.0°</td>
</tr>
<tr>
<td>3</td>
<td>0.5553</td>
<td>0.333</td>
<td>100</td>
<td>0.100</td>
<td>0.0</td>
<td>2.0°</td>
<td>8.0°</td>
<td>−2.0°</td>
</tr>
<tr>
<td>4a</td>
<td>0.5553</td>
<td>0.333</td>
<td>100</td>
<td>0.050</td>
<td>0.0</td>
<td>6.0°</td>
<td>8.0°</td>
<td>−2.0°</td>
</tr>
<tr>
<td>4b</td>
<td>0.5553</td>
<td>0.333</td>
<td>100</td>
<td>0.100</td>
<td>0.0</td>
<td>6.0°</td>
<td>8.0°</td>
<td>−2.0°</td>
</tr>
</tbody>
</table>

Table 3: Simulations performed for test case 1a

<table>
<thead>
<tr>
<th>method</th>
<th>steps/cycle</th>
<th>pseudo-steps</th>
<th>harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leap-frog</td>
<td>360, 720, 1440, 2880</td>
<td>25, 50</td>
<td>-</td>
</tr>
<tr>
<td>Direct-implicit</td>
<td>360, 720, 1440, 2880</td>
<td>50, 100</td>
<td>-</td>
</tr>
<tr>
<td>Periodic</td>
<td>360</td>
<td>50</td>
<td>1, 2, 3, 4, 10, 32</td>
</tr>
</tbody>
</table>

Table 4: Simulations performed for test case 2

<table>
<thead>
<tr>
<th>method</th>
<th>steps/cycle</th>
<th>pseudo-steps</th>
<th>harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leap-frog</td>
<td>1440, 2880, 5760</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>Direct-implicit</td>
<td>360, 720, 1440, 2880</td>
<td>50, 100</td>
<td>-</td>
</tr>
<tr>
<td>Periodic</td>
<td>360</td>
<td>50</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 5: Simulations performed for test case 3

<table>
<thead>
<tr>
<th>method</th>
<th>steps/cycle</th>
<th>pseudo-steps</th>
<th>harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leap-frog</td>
<td>360, 720, 1440, 2880</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>Direct-implicit</td>
<td>360, 720, 1440</td>
<td>25, 50, 100</td>
<td>-</td>
</tr>
<tr>
<td>Periodic</td>
<td>360, 720, 1440</td>
<td>50</td>
<td>1, 10</td>
</tr>
</tbody>
</table>

Table 6: Simulations performed for test case 4a

<table>
<thead>
<tr>
<th>method</th>
<th>steps/cycle</th>
<th>pseudo-steps</th>
<th>harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leap-frog</td>
<td>360, 720, 1440</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>Direct-implicit</td>
<td>360, 720, 1440</td>
<td>25, 50, 100</td>
<td>-</td>
</tr>
<tr>
<td>Periodic</td>
<td>360</td>
<td>50</td>
<td>1, 10</td>
</tr>
</tbody>
</table>

Table 7: Simulations performed for test case 4b

<table>
<thead>
<tr>
<th>method</th>
<th>steps/cycle</th>
<th>pseudo-steps</th>
<th>harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leap-frog</td>
<td>360, 720, 1440, 2880, 5760</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>Direct-implicit</td>
<td>360, 720, 1440</td>
<td>25, 50, 100</td>
<td>-</td>
</tr>
<tr>
<td>Periodic</td>
<td>360</td>
<td>50</td>
<td>1, 10</td>
</tr>
</tbody>
</table>

Table 8: Wind-Tunnel cases - Wing Configuration

<table>
<thead>
<tr>
<th>model</th>
<th>α</th>
<th>u_∞ (ft/s)</th>
<th>M_∞</th>
<th>Reynolds no.</th>
<th>δ_f</th>
<th>freq. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RANS k - ω</td>
<td>0.0°</td>
<td>110</td>
<td>0.10</td>
<td>1.0 × 10^6</td>
<td>±3.0°</td>
<td>0</td>
</tr>
<tr>
<td>RANS k - ω</td>
<td>4.0°</td>
<td>110</td>
<td>0.10</td>
<td>1.0 × 10^6</td>
<td>±3.0°</td>
<td>0</td>
</tr>
<tr>
<td>RANS k - ω</td>
<td>8.0°</td>
<td>110</td>
<td>0.10</td>
<td>1.0 × 10^6</td>
<td>±3.0°</td>
<td>0</td>
</tr>
<tr>
<td>RANS k - ω</td>
<td>12.5°</td>
<td>110</td>
<td>0.10</td>
<td>1.0 × 10^6</td>
<td>±3.0°</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 9: UH-60A flight conditions and trimming for flight counter 8534. The angles are given in degrees.

<table>
<thead>
<tr>
<th>μ</th>
<th>M_∞</th>
<th>Re_∞</th>
<th>α_S</th>
<th>θ_0</th>
<th>θ_1c</th>
<th>θ_1s</th>
<th>β_0</th>
<th>β_1c</th>
<th>β_1s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.368</td>
<td>0.256</td>
<td>2.733 × 10^6</td>
<td>-7.31</td>
<td>11.6</td>
<td>-2.39</td>
<td>8.63</td>
<td>3.43</td>
<td>-0.70</td>
<td>-1.00</td>
</tr>
</tbody>
</table>

Table 10: Model rotor test case parameters

<table>
<thead>
<tr>
<th>model</th>
<th>M_{tip}</th>
<th>μ</th>
<th>θ(ψ)</th>
<th>δ_f(ψ)</th>
<th>gridsize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler (rigid)</td>
<td>0.50</td>
<td>0.15</td>
<td>8° − 2°sin(ψ) + 2°cos(ψ)</td>
<td>0°</td>
<td>9.6 × 10^6</td>
</tr>
<tr>
<td>Euler (active)</td>
<td>0.50</td>
<td>0.15</td>
<td>8° − 2°sin(ψ) + 2°cos(ψ)</td>
<td>5°sin(5°·ψ)</td>
<td>9.6 × 10^6</td>
</tr>
<tr>
<td>k - ω (elastic)</td>
<td>0.50</td>
<td>0.15</td>
<td>8° − 2°sin(ψ) + 2°cos(ψ)</td>
<td>0°</td>
<td>13.6 × 10^6</td>
</tr>
<tr>
<td>k - ω (active)</td>
<td>0.50</td>
<td>0.15</td>
<td>8° − 2°sin(ψ) + 2°cos(ψ)</td>
<td>5°sin(5°·ψ)</td>
<td>13.6 × 10^6</td>
</tr>
</tbody>
</table>
Figure 1: Definition of flap surfaces on wing. Flapping motion around a user-defined hinge line. In the vicinity of the edge, surface blending is used when a neighbouring surface is fixed or the surface belongs to a different flap.
Figure 2: Mesh deformation for blended trailing-edge flap. Examples show 2D mesh for NACA0012 section, with flap hinge at different locations. Solid circle denotes hinge location. The amplitude of the flap deflection is 5.0° in all cases.
Figure 3: Aeroelastic coupling strategies tested for a forward-flying rotor.
Figure 4: Relative cell volume changes due to mesh deformation accounting for rotor trimming and trailing-edge flap actuation. Spanwise cut through mid-span of inboard flap.

Figure 5: Relative cell volume changes due to mesh deformation accounting for rotor trimming and trailing-edge flap actuation. Chordwise cut through mid-chord of flaps.
(a) Spring analogy: $\beta = 0^\circ, \theta = -8^\circ, \delta_f = 5^\circ$

(b) Spring analogy: $\beta = 2^\circ, \theta = -8^\circ, \delta_f = 5^\circ$

(c) TFI in blade blocks: $\beta = 0^\circ, \theta = -8^\circ, \delta_f = 5^\circ$

Figure 6: Relative cell volume changes due to mesh deformation accounting for rotor trimming and trailing-edge flap actuation. Spanwise cut through mid-span of inboard flap.

Figure 7: Flap 1DOF aero-elastic model. Cambered section (OA209) with finite trailing-edge thickness. $M = 0.550375$. Flap has an initial $2.0^\circ$ downward deflection. Effect of temporal accuracy of structural response without aero-elastic coupling is shown in (a), while the results with aero-elastic coupling are shown in (b).
Figure 8: Surface pressure distribution. ONERA OA209 section in oscillatory translation. $M_\infty = 0.5553$. Advance ratio $\mu = 0.333$. Pitch angle is kept fixed at 4.0°. A rigid section is assumed with $\delta = 0°$. Test case 2.
Figure 9: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Fixed pitch at $\theta = 2.0^\circ$, mass ratio $\mu = 10.0$. Inviscid simulations - Case 1b.

Figure 10: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Fixed pitch at $\theta = 2.0^\circ$, mass ratio $\mu = 10.0$. Turbulent flow - SST model - Case 1b.
Figure 11: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Fixed pitch at $\theta = 2.0^\circ$, mass ratio $\mu = 100.0$ - Case 1a.

Figure 12: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Fixed pitch at $\theta = 4.0^\circ$. Comparison of effect of time-step and employed scheme - Case 2.
Figure 13: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory pitch/translation. Pitch schedule $\theta = 2.0^\circ - 8.0^\circ \sin(2kt) + 2.0^\circ \cos(2kt)$ - Case 3.

Figure 14: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Pitch schedule $\theta = 6.0^\circ - 8.0^\circ \sin(2kt) + 2.0^\circ \cos(2kt)$ - Cases 4a, b.
Figure 15: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Pitch schedule $\theta = 6.0^\circ - 8.0^\circ \sin(2kt) + 2.0^\circ \cos(2kt)$ - Cases 4a, b.
Figure 16: Mach contours. Rigid ONERA OA209 section without flapping deflection in oscillatory translation. Pitch schedule $\theta = 6.0^\circ - 8.0^\circ \sin(2kt) + 2.0^\circ \cos(2kt)$. 
Figure 17: Mach contours. Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Pitch schedule 
\[ \theta = 6.0^\circ - 8.0^\circ \sin(2kt) + 2.0^\circ \cos(2kt). \] \[ \omega_0 = 0.050. \]

Figure 18: Mach contours. Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Pitch schedule 
\[ \theta = 6.0^\circ - 8.0^\circ \sin(2kt) + 2.0^\circ \cos(2kt). \] \[ \omega_0 = 0.100. \]
Figure 19: Flap 1DOF aero-elastic model. ONERA OA209 section in oscillatory translation. Pitch schedule $\theta = 6.0^\circ - 8.0^\circ \sin(2kt) + 2.0^\circ \cos(2kt)$.

(a) sectional normal force

(b) sectional pitching moment
Figure 20: Definition geometry of wing wind tunnel model. Dark surface denotes active flap surface. (a) The wing is positioned in an elliptical cross-section tunnel, (b) mesh has C-topology around the wing, (c) surface mesh on wing surface.
Figure 21: Wing wind-tunnel model with active trailing edge flap. Steady flow test cases. CFD results for chordwise pressure coefficient at flap mid-span location are compared with experimental data.
Figure 22: Wing wind-tunnel model with active trailing edge flap. Steady flow test cases. Spanwise loading for different wing incidences. Undeflected flap case compared with $2.61^\circ$ flap down and $2.85^\circ$ flap up. $M = 0.133$, $Re = 1.13 \cdot 10^6$.

Figure 23: Wing wind-tunnel model with active trailing edge flap. $0.0^\circ$ and $8.0^\circ$ incidence - 'high q'. Steady flow test cases. Undeflected flap case compared with $2.61^\circ$ flap down and $2.85^\circ$ flap up. $M = 0.133$, $Re = 1.13 \cdot 10^6$. 

(a) $0.0^\circ$ incidence - 'high q'
(b) $8.0^\circ$ incidence - 'high q'
Figure 24: Predicted UH-60A blade deformation during a revolution for Flight 8534.

Figure 25: Comparison of the pressure coefficient on the blade lower surface between a rigid blade assumption and an elastic blade for Flight 8534.
Figure 26: Loading of the UH-60A for Flight 8534.

Figure 27: Comparison of the sectional normal force of the UH-60A with flight test measurements for Flight 8534. The prescribed twist predictions were obtained by Steijl et al. [36].
Figure 28: Predicted blade tip deformations for several structural damping coefficients (\(\zeta\)) for Flight 8534.

Figure 29: Effect of the time step on the predicted blade tip deformations for Flight 8534. Method 2 is the implicit coupled method.
Figure 30: Comparison of the predicted blade tip deformations with for two coupling methods for Flight 8534. Method 1 represents the leap-frog method and Method 2 the implicit coupling method.

Figure 31: Influence of the turbulence model on the predicted UH-60A blade tip deformation for Flight 8534. The implicit coupling method was used.
Figure 32: Geometry of 4-bladed WT model rotor, with idealised rotor head and WT support. Each blade has two active trailing-edge tabs, indicated as blue and green in plots.
Figure 33: Rotor with active trailing edge flaps. Pressure coefficient in slices through mid-span stations of flaps. $\mu = 0.15$, $M_{tip} = 0.5$. 

(a) $\psi = 180^\circ$ - mid-span inboard flap

(b) $\psi = 180^\circ$ - mid-span outboard flap
Figure 34: Rotor with active trailing edge flaps. Spanwise loading. Inviscid flow. Results for blade with two active flaps are compared with base-line rotor. $\mu = 0.15$, $M_{tip} = 0.5$. Flap deflection $\delta_f(\psi) = 5.0^\circ \sin(5\psi)$.
Figure 35: Rotor with active trailing edge flaps. Spanwise loading. Turbulent flow. Results for blade with two active flaps are compared with base-line rotor. In both cases, elastic torsional deflection is included. $\mu = 0.15, M_{tip} = 0.5$. Flap deflection $\delta_f(\psi) = 5.0^\circ \sin(5\psi)$. 

\[ \psi = 20^\circ: \text{no flap} \] 
\[ \psi = 20^\circ: \delta_f = 4.9^\circ \] 
\[ \psi = 55^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 20^\circ: \text{no flap} \] 
\[ \psi = 90^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 125^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 55^\circ: \text{no flap} \] 
\[ \psi = 125^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 165^\circ: \delta_f = 4.8^\circ \] 
\[ \psi = 200^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 200^\circ: \text{no flap} \] 
\[ \psi = 235^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 235^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 270^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 270^\circ: \text{no flap} \] 
\[ \psi = 305^\circ: \delta_f = 5.0^\circ \] 
\[ \psi = 340^\circ: \delta_f = 4.9^\circ \] 
\[ \psi = 340^\circ: \text{no flap} \] 
\[ \psi = 340^\circ: \delta_f = 5.0^\circ \]