VIBRATION ANALYSIS OF A TILTROTOR AIRCRAFT IN CRUISE FLIGHT

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Abstract

The objective of this paper is the vibration analysis of a tiltrotor aircraft in cruise flight, and in particular the investigation of the impact of the aerodynamic model accuracy on the aeroelastic response predicted. A simplified structural model, composed of a bending-torsion semirigid wing and flap-lag semirigid proprotor-blades, has been coupled with a 2-D quasi-steady strip-theory for both wing and blades, a 2-D unsteady strip-theory, and a 3-D unsteady BEM for incompressible flows. Preliminary numerical results have been obtained comparing the aeroelastic responses of the three aforementioned models to an atmospheric gust disturbance.

1 Introduction

Tiltrotors are very complex machines, and in the analysis of their vibrations it is needed to study both the typical fixed wing and the typical rotary-wing aeroelastic phenomena. In this type of aircraft, an additional important role is played by the strong mechanical coupling between wing and proprotor, that affects the aeroelastic behaviour of the overall configuration. Hence, the importance of using sophisticated structural and aerodynamic formulations, to achieve a satisfying accuracy in aeroelastic response analysis. For instance, it has been evidenced that an accurate aeroelastic modelization is also of interest in the analysis of fuselage structural vibrations and internal sound disturbance, induced by wing and proprotor motion (see, e.g., ref. [1]).

During the last twenty years several models for tiltrotor aeroelastic analysis have been developed. After the early work of W. Johnson [2], who developed a semirigid wing-proprotor model, a finite element formulation for tiltrotor with advanced blade configurations has been developed by M.W. Nixon [3], both the formulations making use of a 2-D quasi-steady aerodynamic model. A multi-body formulation, including rigid as well as elastic bodies, has been developed by G.L. Ghiringhelli, P. Masarati and P. Mantegazza [4], using a 2-D strip theory aerodynamic model, with aerodynamic coefficients based on interpolation of experimental data.

The aim of this paper is the aeroelastic analysis of a wing-proprotor configuration perturbed from the cruise-flight trim condition, using several aerodynamic models. Specifically, in this work it will be investigated the influence of the choice of different aerodynamic models on the wing-proprotor aeroelastic response to a gust disturbance.

To this aim a simple nonlinear structural model has been developed, including a flap-pitch semirigid wing connected with a semirigid three-bladed proprotor, with each blade having flap-lag degrees of freedom.

Unsteady aerodynamic loads are obtained from several aerodynamic models having different accuracy and complexity, in order to assess the impact of the choice of the aerodynamic solution approximation on the aeroelastic behaviour predicted. Specifically, from the use of a simple 2-D quasi-steady aerodynamic model, we will go through more sophisticated aerodynamic load predictions, using a 2-D unsteady strip-theory, approximated in finite state form, up to the application of a 3-D unsteady boundary element solution for potential flows around the overall complex configuration [5].

The final aeroelastic model will be expressed in terms of a set of periodic-coefficient ordinary differential equations in the structural lagrangean variables, forced by aerodynamic loadings. These will be given either in explicit form in terms of the system variables (simple 2-D analytical models) or in terms of aerodynamic flow solutions implicitly dependent on them (BEM model). The numerical aeroelastic solution will be obtained
integrating the resulting set of ordinary differential equations by the Cranck-Nicholson time-marching algorithm.

2 Structural Model

The wing-propeller is modeled as a system of rigid bodies connected by hinges and springs, to simulate elastic effects (semi-rigid body approximation). Wing motion is represented by flap and pitch angles, in a frame of reference connected with the fuselage, whereas flap and lag angles are the degrees of freedom of proprotor blades motion, relative to a frame of reference connected with the rotating shaft. The governing equations for structural dynamics are the following Euler equations applied at each rigid body

\[
\begin{align*}
\dot{Q} &= f \\
\dot{H}_o + \nabla_o \times Q &= m_o
\end{align*}
\]

with

\[
\begin{align*}
Q &= M_v_o \\
H_o &= J_o \omega + M_{\omega_o} \times \nabla_o
\end{align*}
\]

(1)

(2)

where \(Q\) and \(H_o\) are the linear and angular momenta about point \(O\). In addition, \(\nabla_o\) and \(\nabla_o\) are respectively point \(O\) and center-of-mass, \(G\) velocities, whereas \(M\) and \(J_o\) are the rigid body mass and inertia moment about \(O\). External forces \(f\) and moments \(m_o\) are given by the superposition of weight loadings, aerodynamic generalized forces and constraint reactions at hinge joints. Constraints forces and moments, between wing and proprotor blades, cause the equations to have time-variant coefficients, due to blades rotational motion. Hence eq. (1) represents a set of nonlinear periodic equations with unknowns wing and blades rotation angles. Being wing and blades lagrangean variables defined with respect to the undeformed configuration, we assume that perturbation angles and their time derivatives are small (e.g., order \(\varepsilon^2\)) and consequently apply an ordering scheme retaining terms up to order \(\varepsilon^2\). Following this assumption, the nonlinear periodic equations of motion may be recast as

\[
\begin{align*}
\dot{M}^* (t, \bar{q}) \ddot{\bar{q}} + \ddot{C}^* (t, \bar{q}) \dot{\bar{q}} + \ddot{K}^* (t, \bar{q}) \bar{q} &= \\
= w_0 + \bar{m}_{uw} (\bar{q}) + \sum_{n=1}^{3} \bar{N}^{(n)} (t, \bar{q}) \bar{b}^{(n)} (\bar{q})
\end{align*}
\]

(3)

where \([\bar{q}^T, \bar{q}_w^T, \bar{q}_b^T]\) is the row matrix of wing (\(\bar{q}_w\)) and blades (\(\bar{q}_b\)) degrees of freedom whereas \(\bar{M}^*, \bar{C}^*, \bar{K}^*\) are the periodic, state-dependent, mass, damping/gyroscopic and stiffness structural matrices. Forcing terms are given by weight loadings in the undeformed configuration, \(w_0\), wing aerodynamic moments \(\bar{m}_{uw}\), and periodic matrix, \(\bar{N}^{(n)}\), which takes into account the effects (via joint reactions) of blade aerodynamic loadings, \(\bar{b}^{(n)}\), on wing dynamics.

3 Aerodynamic Models

Unsteady aerodynamic loads have been obtained from three aerodynamic models having different accuracy and complexity, in order to investigate the impact of the choice of the aerodynamic solution approximation on the aeroelastic response predicted. A simple \(2-D\) quasi-steady aerodynamic model, for wing and blades, is first considered, then wing aerodynamics is enriched with the use of a \(2-D\) unsteady strip-theory based on a finite state approximation of Theodorsen aerodynamic model. Finally we will consider a \(3-D\) unsteady boundary element solution for potential flows around complex configurations.

3.1 Two-dimensional Analytical Models

3.1.1 Wing

Wing aerodynamics has been modeled by a \(2-D\) strip-theory, based on Theodorsen theory for a thin symmetrical airfoil [6]. In particular, considering Theodorsen relations for section aerodynamic lift and pitch moment, and integrating along wing span to obtain wing aerodynamic flap and pitch moments (about the elastic center), one obtains in Laplace domain\(^1\)

\[
\bar{m}_{wu} = (s^2 A_2 + s A_1) \ddot{q}_w + C(s) (s A_1^* + A_0^*) \ddot{q}_w
\]

(4)

The first wing aerodynamic model considered is a quasi-steady approximation, obtained from eq. (4) assuming the lift deficiency function \(C(s)\) equal to unity. A more sophisticated model can be obtained considering a finite-state approximation of the transcendental Theodorsen expressions for generalized forces. Let us consider lift deficiency function \(C(s)\) approximation presented in [7]

\[
C(s) = 0.5 \frac{(s - z_1) (s - z_2)}{(s - p_1) (s - p_2)}
\]

(5)

where transfer function zeroes and poles are given by \(z_1 = -0.135 U_\infty/b, \quad z_2 = -0.651 U_\infty/b, \quad p_1 = -0.0965 U_\infty/b, \quad p_2 = -0.4555 U_\infty/b, \quad U_\infty\) is the tiltrotor advancing speed and \(b\) is the wing

\(^1\)In the following, the symbol \(\hat{a}\) will denote the Laplace transform of function \(a\).
semichord. Using such approximation, wing aerodynamics can be analytically obtained in finite-state form,

\[ \ddot{m}_{aw} = (s^2 A_2 + s A_1) \ddot{q}_{aw} + 0.5(s - z_1)(s - z_2) \dot{r} \quad (6) \]

introducing the additional aerodynamic states \( r \) defined by relation

\[ (s - p_1)(s - p_2) \dot{r} = (s A_1^c + A_0^c) \ddot{q}_{aw} \quad (7) \]

Hence returning to time domain one obtains the finite-state approximation

\[
\begin{align*}
\dot{m}_{aw} &= A_2 \ddot{q}_{aw} + A_1 \dot{q}_{aw} \\
&+ F_2 \dot{r} + F_1 \dot{r} + F_0 r \\
E_2 \dot{r} + E_1 \dot{r} + E_0 r &= A_1^c \ddot{q}_{aw} + A_0^c \dot{q}_{aw} \\
\end{align*}
\]

of the 2-D unsteady model for wing aerodynamic loadings, with obvious expressions for matrices \( F_i, E_i \). Forcing term \( \ddot{m}_{aw} \) in equation (3) is finally obtained as \( \ddot{m}_{aw}^T = \{ \ddot{m}_{aw}, 0^T \} \).

### 3.1.2 Proprotor

Unsteady aerodynamic loadings on proprotor blades have been obtained by a 2-D strip theory based on the simple quasi-steady approximation of Greenberg theory [8], which is an extension of Theodorsen theory to pulsating free-stream velocity. Greenberg theory is used to introduce in the aerodynamic model the in-plane blade motion (lag). Using the quasi-steady (low frequency) assumption, \( C(s) = 1 \), airfoil aerodynamic loads may be expressed, for a flap-lag blade model, as functions of blade section velocity components \( U_l, U_p \), tangent and perpendicular to blade chord \( c \) (see, e.g., [9])

\[
\begin{align*}
T &= -a \left( U_p^2 - \frac{c \phi}{c_{\alpha}} U_l^2 \right) \\
P &= a \left( U_p U_l - \frac{c}{4} \dot{U}_p \right) \\
M_a &= a \left( \frac{c}{4} \right)^2 \dot{U}_p \\
\end{align*}
\]

where \( T \) and \( P \) are blade section aerodynamic forces tangent and normal to blade chord, \( M_a \) is the aerodynamic pitching moment about the airfoil aerodynamic center, \( c_{\alpha} \) and \( c_{\alpha 0} \) are the airfoil aerodynamic lift and drag coefficients, \( \rho \) is air density, and \( a = \rho c c_{\alpha}/2 \).

From eq. (9) and blade sections twist distribution, it is possible to derive the in-plane and out-of-plane aerodynamic forces components, together with aerodynamic pitching moment about the elastic axis, as functions of blade section kinematics, represented by \( U_l \) and \( U_p \). Hence, considering blade section velocity as a function of blade and wing degrees of freedom \( \vec{q} \) and integrating along blade span, one obtains the n-th blade aerodynamic generalized forces \( \dot{\vec{b}}^{(n)} = \{ f_b^{(n)} T, m_b^{(n)} T \} \), which approximated to second order in \( \varepsilon \) become

\[
\dot{\vec{b}}^{(n)} = \ddot{\vec{b}}^{(n)} + \ddot{\vec{M}}_b^{(n)}(t, \vec{q}) \ddot{\vec{q}} + \ddot{\vec{C}}_b^{(n)}(t, \vec{q}) \dot{\vec{q}} \\
+ \ddot{\vec{K}}_b^{(n)}(t, \vec{q}) \vec{q} \\
\]

where blade mass, damping and stiffness aerodynamic matrices are state-dependent, periodic matrices. It must be noted that blade generalized aerodynamic forces depends not only on blade degrees of freedom, but also from wing degrees of freedom.

### 3.2 Three-dimensional BEM Model

In tiltrotor aeroelasticity the complex unsteady aerodynamic field generated by proprotor and wing motion, plays a fundamental role. In particular, the velocity field induced by blades and wing wake vorticity, their mutual aerodynamic interaction, as well as 3-D effects due to wing and blades finite aspect ratio, have a significant impact on aerodynamic loading and hence on the overall wing-proprotor aeroelastic response.

In order to include an accurate description of these phenomena in the aerodynamic model we use a Boundary Element Method for potential flow incompressible aerodynamics. Indeed, under irrotational flow assumption, the perturbation velocity field due to wing and blades elastic motion is given in terms of potential \( \phi \), obtained solving the boundary integral equation (see, e.g., ref. [5])

\[
\phi = \sum_{i=1}^{4} \iint_{S_{B_i}} \left( G \frac{\partial \phi}{\partial n} - \phi \frac{\partial G}{\partial n} \right) dS \\
- \sum_{i=1}^{4} \iint_{S_{W_i}} \Delta \phi \frac{\partial G}{\partial n} dS \\
\]

where \( G \) is the unit source, \( S_{B_i} \) and \( S_{W_i} \) (\( i = 1, 4 \) represent respectively body and wake surfaces of wing and blades.
Figure 1: Bodies and wakes surfaces BEM discretization

Boundary conditions are obtained imposing body surface impermeability

\[
\frac{\partial \phi}{\partial n}(x) = v_{B_i}(x) \cdot n(x) \quad \forall x \in S_{B_i}
\]  
(12)

where \(v_{B_i}\) is the \(i\)-th body surface velocity field (see, e.g., ref. [5] for details on wake boundary conditions). Pressure field on each body surface is obtained from velocity potential \(\phi\), through Bernoulli's theorem. Then, integrating pressure field on rigid-body modes, aerodynamic generalized forces \(\bar{m}_{aw}\) for wing and \(\bar{B}^{(n)}\) for each blade have been determined.

4 Aeroelastic Analysis

The nonlinear aeroelastic system, approximated to second order in \(\varepsilon\), is obtained substituting in equation (3) the generalized aerodynamic forces from 2-D analytical models or 3-D BEM model.

For each of these approaches a different aeroelastic system representation is obtained. Using quasi-steady aerodynamic models, equations (3)-(4)-(10) may be recast as

\[
\bar{M}(t, \bar{q})\ddot{\bar{q}} + \bar{C}(t, \bar{q})\dot{\bar{q}} + \bar{K}(t, \bar{q})\bar{q} = w_0 + b_0
\]  
(13)

where \(\bar{M}\), \(\bar{C}\), \(\bar{K}\) are the aeroelastic, state-dependent, periodic matrices, given by the superposition of structural matrices and wing/proprotor aerodynamic matrices, and

\[
b_0 = \sum_{n=1}^{3} \bar{N}^{(n)}(t, 0) \bar{B}^{(n)}(0)
\]  
(14)

represents blades aerodynamic forcing term relative to the undeformed configuration. It must be noted that, after summation over the three blades, \(b_0\) become time-invariant.

The time-invariant equilibrium solution \(\bar{q}_e\) has been obtained neglecting blade weight terms on structural stiffness matrix, and solving the non-linear equation

\[
\bar{K}_e(\bar{q}_e)\bar{q}_e = w_0 + b_0
\]  
(15)

by an iterative Newton-Raphson algorithm, where \(\bar{K}_e\) is obtained by the superposition of structural and aerodynamic stiffness matrices in equilibrium condition. Then, for the analysis of perturbation dynamics we have assumed \(\bar{q} = \bar{q}_e + \bar{q}\) and linearized equation (13) about the equilibrium solution \(\bar{q}_e\)

\[
M_e(t)\ddot{\bar{q}} + C_e(t)\dot{\bar{q}} + K_e(t)\bar{q} = G_e(t)v_g
\]  
(16)

where mass, damping/gyroscopic, and stiffness matrices are given by \(M_e = \bar{M}(\bar{q}_e)\), \(C_e = \bar{C}(\bar{q}_e)\), \(K_e = 2\bar{K}(\bar{q}_e) - \bar{K}(0)\), and where an aerodynamic forcing term due to a gust atmospheric disturbance has been added.

If the 2-D unsteady wing aerodynamics is considered, eq. (3) is changed into the following equation for the aeroelastic dynamics

\[
\bar{M}(t, \bar{q})\ddot{\bar{q}} + \bar{C}(t, \bar{q})\dot{\bar{q}} + \bar{K}(t, \bar{q})\bar{q} = w_0 + b_0 + \bar{F}_2\ddot{r} + \bar{F}_1\dot{r} + \bar{F}_0 r
\]  
(17)

where the dynamics of the additional states, \(r\), is governed by

\[
E_2\ddot{r} + E_1\dot{r} + E_0 r = \bar{A}_1^T\ddot{\bar{q}} + \bar{A}_0^T\dot{\bar{q}}
\]  
(18)

where \(\bar{F}_e^T = [F_e^T, 0^T]\) and \(\bar{A}_e^T = [\bar{A}_1^T, 0^T]\). For the definition of the perturbation dynamics, a procedure similar to that described for the quasi-steady aerodynamic case has been applied.

Finally we consider the 3-D unsteady BEM aeroelastic model. In this case the generalized forces \(\bar{m}_{aw}\) and \(\bar{B}^{(n)}\) are not given as analytical functions of structural degrees of freedom, and the aeroelastic solution must be obtained with a direct integration of nonlinear equation (3) or linearizing about the equilibrium solution and integrating the following aeroelastic perturbation equation

\[
M_e^\varepsilon(t)\ddot{\bar{q}} + C_e^\varepsilon(t)\dot{\bar{q}} + K_e^\varepsilon(t)\bar{q} = m_{aw}
\]  
\[
+ \sum_{n=1}^{3} N_e^{(n)}(t) b^{(n)} + \sum_{n=1}^{3} N_e^{(n)}(t) b_c^{(n)}
\]  
\[+ G_e(t)v_g
\]  
(19)

All of the linearized aeroelastic models presented above, may be recast in the form of as a set of first order differential equations that in this work have been integrated by using the Crank-Nicolson unconditionally-stable algorithm.

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5 Numerical Results

Preliminary numerical results obtained with the formulations described above, are presented in this section. In the present work the emphasis is on the impact of the aerodynamic model, on the aeroelastic response to atmospheric gust disturbance.

The characteristics of the wing-propeller refer to the Boeing experimental model presented in [2]. The three bladed propeller has a radius of 3.97 m, and a rotational speed of 386 RPM. The propeller is mounted on the tip of a rectangular wing, having a semispan of 5.08 m, a chord of 1.58 m, and an advancing speed of 50 m/s. The geometrical twist distribution of the blades is given in order to obtain a thrust coefficient $C_T = 0.0045$.

The atmospheric perturbation is assumed to be a $(1 - \cos)$-type vertical gust, whose effects are present within a time interval 5 times larger than the propeller revolution period $T_n = 0.15$ s.

In the aeroelastic models presented, the equilibrium solution has been always obtained using quasi-steady aerodynamic model, whereas perturbation solution has been obtained with the three different aerodynamic models (2-D quasi-steady, 2-D unsteady, 3-D BEM). Note also that in the case of the BEM model the wake shapes are defined as the surface swept by the lifting body trailing-edges.

Figures 2, 3, 4, 5, depict the comparison between gust response obtained using a 2-D strip-theory and a 2-D unsteady model respectively for wing flap and pitch, blade flap and lag lagrangian variables. As expected, the wing shows significant differences on the aeroelastic response, being directly influenced by the different aerodynamic model (quasi-steady/unsteady). The maximum value of lagrangian variables (which for this semirigid model are proportional to root elastic moments) is almost unchanged, whereas the unsteady model predict a more damped aeroelastic solution, especially for flap degree of freedom. Blades response, being indirectly influenced through wing different dynamic response, shows slight variations in flap response, and significant variations in lag response.

Figures 6,7,8,9, represent gust response obtained using quasi-steady model and 3-D BEM model for the aforementioned wing and blades degrees of freedom. In this case it is possible to observe that the two responses are very different, both in terms of amplitude and damping. This is particularly evident in the blade response depicted in figs. 8 and 9. This is only a preliminary analysis obtained through BEM code, but it seems that, for an accurate aeroelastic description of a tiltrotor system, in addition to a sophisticated structural model it is important to take account of all unsteady and 3-D aerodynamic effects.

![Figure 2: Gust response. 2-D Quasi-steady (QS) vs 2-D Unsteady (FS). Wing flap](image1)

![Figure 3: Gust response. 2-D Quasi-steady (QS) vs 2-D Unsteady (FS). Wing pitch](image2)

![Figure 4: Gust response. 2-D Quasi-steady (QS) vs 2-D Unsteady (FS). Blade flap](image3)
Figure 5: Gust response. 2-D Quasi-steady (QS) vs 2-D Unsteady (FS). Blade lag

Figure 6: Gust response. 2-D Quasi-steady (QS) vs 3-D BEM. Wing flap

Figure 7: Gust response. 2-D Quasi-steady (QS) vs 3-D BEM. Wing pitch

Figure 8: Gust response. 2-D Quasi-steady (QS) vs 3-D BEM. Blade flap

Figure 9: Gust response. 2-D Quasi-steady (QS) vs 3-D BEM. Blade lag

6 Conclusions

The aeroelastic analysis of a tiltrotor aircraft, perturbed from the cruise steady flight condition by an atmospheric disturbance, has been performed. The aeroelastic response has been obtained using three different aerodynamic models (2D quasi-steady and unsteady strip-theories, 3-D unsteady BEM).

Preliminary results have been obtained, indicating that, for the cases considered, the aeroelastic response is strongly dependent on the aerodynamic model used. This suggests that, in combination with the sophistication of the structural model, it is worth developing also an aerodynamic model having an equivalent accuracy.

Hence future work and developments will be dedicated to the introduction, in the structural model, of wing and blade elasticity, as well as to further developments of the aerodynamic model and finally to the direct integration of nonlinear equations of motion.
References


