Towards Better Stability and Control Properties of the Conventional Helicopter Using Inherent Main Rotor Couplings

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The influence of three main rotor couplings, collective- torque, pitch-flap and blade C.G. offset, on inherent stability and control properties of the conventional helicopter is presented in terms of the effects on rotor damping, cross-coupling and angle of attack derivatives. It is shown that all three couplings change the nature of derivatives positively.

Notation

\( A.C \) Blade aerodynamic centre
\( A_t \) Lateral cyclic pitch
AFCS Automatic Flight Control Systems
\( B_t \) Longitudinal cyclic pitch
\( C.G \) Centre of gravity
\( C_T \) Rotor thrust coefficient
\( K_{CG} \) Ratio of blade flap to blade torsional inertia, \( \frac{I_{bf}}{I_{bt}} \)
NOE Nap of the Earth
\( p,q,r \) Aircraft Roll, pitch and yaw velocities
\( u,v,w \) Aircraft velocity components in x, y and z directions
\( \mu_x, \mu_y, \mu_z \) Non-dimensional velocity components in the x, y, z directions
\( \beta \) Tip speed ratio, \( \frac{\mu}{\mu_T} \)
\( \overline{\beta}, \overline{\gamma}, \overline{\Omega} \) Normalised aircraft angular velocities, \( \frac{\beta}{\beta_T}, \frac{\gamma}{\gamma_T}, \frac{\Omega}{\Omega_T} \)
\( n_b \) A blade flap inertia number, \( \frac{I_{bf}}{I_{bt}} \)
\( S_\beta \) Stiffness number, \( \frac{\mu^2 - 1}{n_b} \)
\( S_{\beta,T+1} \) Effective Stiffness number
\( U_p \) Flow velocity normal to the rotor disc
\( U_t \) Flow velocity parallel to the rotor disc
\( V_t \) Rotor blade tip speed
\( x \) \( \frac{\beta}{\beta_T} \)
\( y_c \) Ratio of local blade C.G. offset, \( y_c \), to blade local radius, \( r \)
\( \beta(\psi) \) Blade flapping angle, \( \beta(\psi) = a_0 - a_1 \cos \psi - b_1 \sin \psi \)
\( \gamma \) Blade Lock number, \( \frac{\alpha \cos \Omega \tau}{\beta} \)
\( \lambda_b \) Non-dimensional blade flapping frequency, \( \omega_b / \Omega \)
\( \lambda_t \) Non-dimensional blade torsional frequency, \( \omega_t / \Omega \)
\( \Omega \) Rotational speed of the rotor
\( \theta(\psi) \) Blade pitch angle applied by the control system, \( \theta = \theta_0 - A_1 \cos \psi - B_1 \sin \psi \)

Introduction

It is an accepted fact that the conventional helicopter has poor inherent stability and control properties. The result is a vehicle that is difficult to fly and which subjects the pilot to a high control workload. Therefore a major design consideration for helicopters is the ease with which the helicopter is flown.

The main rotor is used to generate lift and propulsive moments for trim and directional control. The magnitude of these moments depends on rotor properties such as stiffness of blade attachment and hub design. Any perturbations of the rotor will cause blade flapping and generate moments that will displace the helicopter from its trim condition. In addition, instabilities intrinsic to the helicopter, can cause the helicopter to destabilize unless the pilot takes corrective action. This can be a demanding task.

Conventional helicopter control is highly cross-coupled, dynamic motions are under damped and at high speed, unstable in pitch because of the main rotor behaviour. An articulated rotor (\( \lambda_3 = 1 \)) with blades hinged at the centre is less cross-coupled then hingeless rotor helicopters. The flapping response to cyclic pitch of the articulated rotor helicopter occurs 90° later (phase lag) which allows pure longitudinal or lateral control. On the other hand, the hingeless rotor with a flapping frequency greater than one, has a phase lag of less then 90° between cyclic pitch application and flapping response. This makes the hingeless rotor more cross-coupled. Damping of dynamic motions increases the natural period of the motion, which helps the pilot by allowing more time for corrective action. Rotor damping in pitch and roll is caused by the gyroscopic nature of the rotor and is proportional to blade inertia.

Pitch instability in helicopters exists for rotors with all

Subscripts

eff effective

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‡Presented at the European Rotorcraft Forum, Bristol England, September 17-20, 2002. Copyright ©2002 by A. N. Modha and S. J. Newman. All rights reserved
flapping frequencies and gets worse as the stiffness of blade attachment increases. When a helicopter is disturbed such that the rotor tilts backward, there are no restoring forces apart from those due to the tailplane. This instability is worse particularly at high speeds, for hingeless rotor helicopters because of the capability to generate hub moments proportional to the stiffness.

Significant changes in the blade flapping behaviour are possible by the introduction of various couplings in the rotor system such as pitch-flap, blade C.G. offset, pitch-lag etc. Rotor couplings can be aeroelastic, kinematic and electronic and can involve positive or negative feedback of variables. Rotor system couplings have a significant effect on rotor properties such as flapping frequency and flapping magnitude. The couplings also influence the inherent stability and control properties of the aircraft.

Hohenemser\textsuperscript{1} investigated a particular form of pitch-flap $\delta_2$ coupling in which collective pitch is coupled to blade coning and leaving cyclical flapping independent. With large values of coupling (70°), the helicopter is shown to become neutrally stable in attitude. Other forms of couplings include mechanical devices such as the Bell bar and the Lockheed gyro-rotor which use the concept of gyroscopic rigidity. Chen et al\textsuperscript{2} studied the effects of pitch-flap coupling amongst other parameters, on the handling qualities of the several helicopter configurations in NOE flight. Several different rotor configurations were investigated with all showing improvements in handling qualities although none was found to be superior. Miller\textsuperscript{3} studied the effects of viscous, elastic restraints and blade C.G. offset on basic dynamic properties of the helicopter. A rotor with visco-elastic restraint in the control system, was shown to have stability properties resembling a helicopter with pitch and roll rate augmentation.

The Bolkow\textsuperscript{4} and Lynx\textsuperscript{5} experience with torsion-flap-lag coupling is interesting. The Westland design philosophy for the Lynx was to remove aeroelastic couplings to reduce the risk of blade aeroelastic instability. The development work of the hingeless rotor was done on the Westland Research Scout.\textsuperscript{6} In addition to minimising blade instability, the design of the Lynx accommodated for AFCS in the control system. Bolkow’s design of the Bo105 helicopter, resulted in a hingeless rotor with torsion-flap-lag coupling and blade C.G. offset. This coupling in the Bo105 has resulted in improved stability and control properties.

Away from stability and control, several studies have investigated the role of main rotor couplings on blade aeroelastic instabilities. The use of pitch-flap coupling was shown to minimise flap-lag instability by Gaffney.\textsuperscript{7} A review of blade instability involving some main rotor couplings is given by Ormiston.\textsuperscript{8}

With the advent of the Automatic Flight Computers systems (AFCS), it is possible to introduce sophisticated control laws that tailor stability, controllability and response properties. Such systems are expensive and complex because of airworthiness requirements of redundancy in case of failure of the primary flight computer. Therefore the benefits of incorporating main rotor couplings that aid the stabilisation and control workload are quite obvious, as they would result in a reduction in complexity and expense of AFCS.

Considerable advances have been made in advanced rotor design in the last 20 years in terms of improved aerodynamics such as delaying blade stall, reducing aeroelastic instabilities and improving vibration characteristics. However, not much work has been done on the application of main rotor couplings to improving inherent stability and control properties in the last decade or two and is worth a revisit. In addition, a reduction on the dependence of AFCS for stabilisation and control will be a significant improvement on the current situation where the use of AFCS for medium to large helicopters is taken for granted. There is also a lack of information on the physical effects of these couplings either individually or when combined, on helicopter stability and control properties.

The primary aim of the research project at Southampton University is to assess semi-analytically, the influence of several main rotor couplings individually and to investigate combinations of these couplings on stability and control properties.

To assess the impact of main rotor couplings on the dynamic characteristics, in this paper, the use of basic stability and control derivatives is made. In terms of basic stability and control properties, it can safely be said that reduction of cross-coupling, increase in direct damping and reduction in instability is highly desirable. In terms of rotor derivatives, this corresponds to the-

\begin{itemize}
  \item[(i)] Minimisation of Cross-coupling derivatives, $\frac{\delta Y}{\delta q^r}$, $\frac{\delta Y}{\delta q^l}$,
  \item[(ii)] Maximisation of Direct damping derivatives, $\frac{\delta Y}{\delta q^r}$, $\frac{\delta Y}{\delta q^l}$,
  \item[(iii)] Minimisation of destabilising derivatives, $\frac{\delta Y}{\delta q^r}$,
  \item[(iv)] Control power, $\frac{\delta C_{l\alpha}}{\delta \delta q^r}$, $\frac{\delta C_{l\alpha}}{\delta \delta q^l}$ and $\frac{\delta b_{\alpha}}{\delta \delta q^l}$ should not be unduly affected
\end{itemize}

**Theoretical Background**

A six degree-of-freedom analytical model of the single main and tail rotor helicopter was devised with the aim of calculating stability, control and flapping derivatives. The model allows for explicit definition of main

80.2
rotor couplings and enables analytical evaluation of expressions for stability and control derivatives. The model also allows the evaluation of eigenvalues and eigenvectors for stability calculations and helicopter response to control inputs.

A number of assumptions and simplification in the model have been made in order to keep the formulation tractable. The rotor is assumed to act in a quasi-steady manner i.e. when disturbed the rotor is assumed to reach its new equilibrium instantaneously. The main and tail rotor aerodynamic model is based on standard blade element theory.\textsuperscript{9} The simplifications mentioned below restrict the application of the helicopter model to values of tip speed ratios of up to 0.35.\textsuperscript{10} The assumptions made are:

(i) A rigid constant chord blade with a constant lift curve slope and linear twist is assumed.

(ii) Effects of compressibility, stall, tip-loss, reverse flow and unsteady aerodynamics have been ignored.

(iii) Blade flapping and rotor inflow angles are assumed to be small.

(iv) The rotor blade is assumed to be attached at the hub centre with a spring restraint to embody hinge offset, enabling the simulation of various rotor configurations.

(v) Only the primary blade flapping and torsional mode affects rotor forces and moments.

(vi) The Glauert model for induced velocity distribution has been assumed.

(vii) Interaction between the rotor wake and fuselage and empennage has been ignored.

A further consideration in the application of main rotor couplings is the torsional degree of freedom. Conventionally the tie control system is assumed to be stiff and blade pitch is not a degree of freedom. However, the torsional degree of freedom does play a role in rotor blade behaviour as shown by Kamau\textsuperscript{11} and Bolkow\textsuperscript{12} and needs to be incorporated in order to study couplings such as blade C.G. and aerodynamic centre (A.C.) offset. The torsional degree of freedom can be incorporated in the model by assuming that the blade is rigid in pitch with flexibility allowed in the control system.\textsuperscript{10} This assumption allows changes in pitch of the blade about the feathering hinge to be included in blade flapping analysis. The equations for blade flapping and torsion are given below.

\[ \beta'' + \lambda^2 \beta = \frac{M_{X_{\text{cm}}}}{J_{\beta} \Omega^2} + 2(\tau \cos \psi - \tau \sin \psi) \]  

(1)

Through careful design, aerodynamic torsional moments \((M_{X_{cm}})\) as shown in equation 2, acting on the blade are made to be small. This term for the present analysis has thus been ignored.

Although the gyroscopic pitch and roll rate terms in the blade flapping equation 1 are generally incorporated into analysis, the term \(2(\tau \sin \psi + \tau \cos \psi)\) is not. This term is similar to that in the blade flapping equation. Gyroscopic inertia forces act along the chord of the blade and impose blade torsional (feathering) moments. This term is generally not included in helicopter rotor dynamics modelling because the assumption of a torsionally stiff rotor blade with mass concentrated uniformly along the blade span is generally made. On the other hand, a torsionally soft blade, will be subject to Gyroscopic Feathering Moments\textsuperscript{13} that cause blade twisting. This particular phenomenon and the relationship to the Bell-bar rotor design has been investigated as part of this project and is reported separately.

A trim model of the conventional helicopter has also been devised based on moment balance about aircraft C.G., to obtain trim information needed for the calculation aircraft stability and control derivatives. The evaluation of stability and control derivatives is based on the method of Hansen.\textsuperscript{14}

A limited attempt was made to validate the resulting trim and the derivative model. Trim attitudes, stability and control derivatives and eigenvalues for three aircraft, Puma, Boeing and Lynx, were compared with those computed using Helsim.\textsuperscript{15} Figure 1 shows a comparison of calculated trim collective and cyclic pitch variation with advance ratio for the Bell-Boeing 105 helicopter. Basic properties as represented by trim and stability/control derivatives gave reasonable agreements numerically and physical trends with those published.\textsuperscript{15–17} The model was therefore deemed adequate for the purposes of this project.

### Main Rotor Couplings

The effects of three main rotor system couplings are outlined here. These are Collective-cone \((\delta_0)\) coupling, Cyclic pitch-flap \((\delta_3)\) coupling and Blade section C.G. offset.

#### Collective-Cone coupling

Collective-cone \((\delta_0)\) coupling involves the reduction in collective pitch in response to an increase in coning. Hohenemser\textsuperscript{1} shows that the coupling can be used to decrease pitch instability in forward flight. The change
in blade pitch due to Collective-cone coupling is defined in the same way as pitch-flap \( \delta_3 \) coupling, where \( \frac{\partial \theta}{\partial \beta} = \tan \delta_3 \).

\[
\Delta \theta = a \frac{\partial \theta}{\partial \beta}
\]

When a collective-cone coupled rotor is disturbed, the rotor blades will experience by convention a reduction in the collective pitch in response to an increase in blade coning, the magnitude of reduction depending on the amount of gain \( (\delta_3) \) in the coupling.

The Lynx Computer Acceleration Control (C.A.C.) system first tested on the Research Scout is also a negative feedback system, which uses an accelerometer to sense vertical acceleration \( (g) \) which is used to reduce the collective pitch of the blades. The research Scout rotor had a C.A.C. rating of 1.5\(^{\circ}\) of collective reduction for every unit of normal \( (g) \). Since thrust is directly related to vertical acceleration, both collective-cone coupling and C.A.C. are identical in function.

**Pitch-flap coupling**

Kinematic pitch-flap, \( \delta_3 \) coupling is commonly used on tail rotors to reduce cyclic flapping amplitude mainly to reduce blade lagging stresses particularly at high speeds and in maneuvers that result from the lack of lagging hinges. When applied to the main rotor, the magnitude of cyclic flapping caused by a change in flight state, is reduced. The convention for pitch-flap coupling used here is that positive coupling reduces blade cyclic pitch angle when the blade flaps up. For mathematical flexibility, pitch-flap coupling is only applied to cyclic flapping as shown by the equation 4 and leaving blade coning uncoupled. The term \( \frac{\partial \theta}{\partial \beta} \) is generally expressed as \( \tan \delta_3 \) where \( \delta_3 \) is an angle made by for example a kinked\(^{9}\) flapping hinge.

\[
\Delta \theta = (-a_1 \cos \psi - b_1 \sin \psi) \frac{\partial \theta}{\partial \beta}
\]

If the aerodynamic term in blade flapping equation given in equation 1 is solved for hovering flight,

\[
\frac{M_{y_{app}}}{I_\beta} = \frac{1}{2} \frac{\rho U_r^2 R^2 a}{I_\beta} \left[ (\theta - \frac{\partial \theta}{\partial \beta}) - \lambda_\theta - \beta \right] dx
\]

and then substituted back into the flapping equation, the following equation for blade flapping results (aircraft pitch and roll rate have been ignored).

\[
\beta'' + \frac{\gamma}{8} \beta'' + (\chi_3 + \frac{\gamma}{8} \frac{\partial \theta}{\partial \beta}) \beta = \frac{\gamma}{8} (\theta - \frac{4}{3} \lambda_\theta)
\]

The above equation shows that \( \delta_3 \) is an effective spring term in the flapping equation therefore it changes the flapping frequency.

**Blade C.G. Offset**

To incorporate blade C.G. offset from the feathering axis, the inclusion of torsional degree of freedom is necessary because the inertial coupling of pitch and flap degrees of freedom. Assuming initially arbitrary flap and torsional frequencies, the equations for flap and torsional degrees of freedom are

\[
\beta'' + \chi_3 \beta - \bar{g}_c (\theta'' + \theta) = \frac{M_{y_{app}}}{I_\beta} + 2(\bar{\tau}\cos \psi - \bar{g}_c \sin \psi) + 2\bar{g}_c (\bar{\tau}\sin \psi + \bar{\tau}
\]

\[
\theta'' + \chi_3 \beta - K_{CG} \bar{g}_c (\beta'' + \beta) = \frac{M_{y_{app}}}{I_\beta} - 2(\bar{\tau}\sin \psi - \bar{\tau}_c \cos \psi - 2K_{CG} \bar{g}_c (\bar{\tau}\cos \psi - \bar{\tau}_c \sin \psi)
\]

The full derivation of the above equations is given in standard textbooks\(^{10,20}\) although the gyroscopic feathering moment term is not included in the derivation.

In order to simplify the analysis, the C.G. offset away from the feathering axis figure 2, has been assumed to be proportional to the local radius. Blade C.G. offset parameter, \( \bar{g}_c \) is the ratio of C.G. offset at the blade tip and the blade radius. Note the similarity between the gyroscopic inertial forcing terms in both equations. By offsetting the C.G. from the feathering axis, the blade can be thought of as being swept i.e. the effective azimuth position of the blade is \( (\psi + \Delta \psi) \), therefore additional gyroscopic forcing proportional to the offset is experienced.

Coupling of blade flapping with blade pitch is represented by the term \( K_{CG} \), which is a ratio of flap to torsional inertia. This term is known and can be assumed to be constant for conventional helicopter rotor blades. The final term of significance is the torsional stiffness as represented by the torsional frequency, \( \lambda_\theta \), typically \( 3 \Omega - 5 \Omega \). Noticeable coupling between blade flapping and pitch can only occur at low torsional stiffness.\(^{11}\) When a blade flaps, two forces namely centrifugal force acting at the C.G. and gyroscopic inertial force, act on the blade causing a change in pitch. The centrifugal force (C.F.) has a component of acting in the direction normal to the blade span and causes a change in blade pitch through twisting while the gyroscopic force acts in the chordwise direction and has the same effect. The centrifugal force is the dominant of the two forces when the blade C.G. is offset. When a disturbance results in blade flapping, an aft (positive)
C.G. offset results in a blade nose-up moment and vice versa. Both C.F. and gyroscopic forces become very small when the blade C.G. is on the feathering axis.

**Results**

The results of the above couplings are given below. To simplify analysis, calculations of the derivatives were done using a the GKN-Westland Lynx helicopter, with the flapping frequency and the blade flap inertia number \( n_b \) set to unity. Note that in some of the results in the appendix, derivatives with respect to pitch and roll are normalised with respect to the rotor velocity, \( \Omega \), while the vertical velocity is normalised with respect to the rotor tip speed, \( V_T \).

**Effects of Collective-cone coupling**

The nature of collective-cone coupling means that it has a considerable effect on rotor flapping derivatives that are altered by changes in coning. Rotor coning is largely affected by disturbances in vertical velocity that alters the incidence of all the blades simultaneously. The coupling does not affect the Stiffness number \( S_3 \). Figures 3 - 6 show the effect on rotor flapping and thrust derivatives. The results shown are for values of positive \( \delta_0 \) coupling of 0°, 10°, 45° and 70° degrees.

Collective-cone coupling neither affects the direct damping derivatives e.g. \( \frac{\delta \alpha_1}{\delta \theta} \) figure 3 nor significantly alters the control cross-coupling derivatives \( \frac{\delta \alpha_1}{\delta \alpha_3} \) and \( \frac{\delta \beta}{\delta \alpha_3} \), figure 3 and 4. Since rotor damping in pitch or roll depends on gyroscopic forcing requires cyclic flapping. The reason for off-axis flapping response to on-axis control input is unchanged is that collective-cone coupling only changes the collective angle when the coning changes.

There is a significant change to the thrust available per degree of collective input \( \frac{\delta C_T}{\delta \theta} \) figure 6, cross coupling of fore-aft flapping to collective pitch fig. 3.

Since positive collective cone coupling tends to resist changes in collective pitch, the thrust per unit collective pitch is reduced requiring more applied collective pitch for control. In an uncoupled rotor, a change in collective pitch induces cyclic fore-aft flapping because of lift dissymmetry. Collective-cone reduces this particular cross-coupling.

The effect on the destabilising derivative \( \frac{\delta \beta}{\delta \theta} \) is quite pronounced, figure 5. Pitch instability results from the rotor disc flapping backward which requires for-aft flapping. However, when a rotor experiences a vertical disturbance, the blades initially cone first because all blades experience an increase in incidence which is followed by fore-aft flapping because of lift dissymmetry. Because the coupling resists changes in coning, there is a reduction in fore-aft flapping. It is seen that a considerable reduction in pitch instability is obtained with between 30° - 45° of coupling. Hohenemser suggested a typical value of \( \delta_0 = 73^\circ \), as verified by figure 5. However, this level of \( \delta_0 \) coupling, there is a significant reduction in the thrust per unit collective i.e. low control power, which renders large coupling angles unacceptable.

Figures 3 and 4 also show that the flapping response to off-axis angular motion i.e. \( \frac{\delta \beta}{\delta \alpha_3} \) and \( \frac{\delta \alpha_1}{\delta \alpha_3} \) is not affected much.

**Effects of Pitch-flap coupling**

Application of pitch-flap \( \delta_3 \) coupling alters 'virtually' the flapping frequency of the rotor which is expected to affect the damping, cross-coupling and the destabilising derivatives. Figures 7 - 12 show the effects on these rotor derivatives of some positive and negative values of \( \delta_3 \) coupling.

Positive \( \delta_3 \) coupling reduces damping in pitch and roll, figures 7 and 8. Positive coupling initially increases damping, however a maximum increase is reached at about 15° followed by a decrease. Rotor damping in pitch and roll results from the rotor lagging behind the shaft caused by the gyroscopic nature of the rotor. When the rotor is pitching up at a given rate, there is a moment tending to tilt it advancing side down. To balance this, an aerodynamic moment is required which is generated through cyclic flapping, the response of which is 90° later causing the disc to lag. By positively coupling cyclic pitch to flapping, the blades experience less flapping thus reducing the amount of direct damping. The equation for direct damping in the pitch sense is given below.

\[
\frac{\delta \alpha_1}{\delta \theta} = -\frac{1}{1 + S_{\beta,ff} \left( \frac{2}{n_b} + S_{\beta,ff} \right)}
\]

Damping in roll shows a similar trend, figure 8.

The off-axis flapping response to on-axis angular motion on the other hand is decreased when positive \( \delta_3 \) is applied because of a reduction in flapping amplitude. On the other hand, negative coupling results in an increase in flapping response initially followed by a decrease in values of \( \frac{\delta \beta}{\delta \alpha_3} \) (see equation below) and \( \frac{\delta \alpha_1}{\delta \theta} \) to 0 as shown in figures 7 and 9.

\[
\frac{\delta \alpha_1}{\delta \theta} = -\frac{1}{1 + S_{\beta,ff} \left( \frac{2}{n_b} \right) - 1}
\]

Figures 10 and 12 shows that the direct flapping response to cyclic pitch e.g. \( \frac{\delta \alpha_1}{\delta \alpha_3} \) is decreased by both positive and negative pitch-flap coupling. The cross-coupled flapping response increases with with either positive or negative pitch-flap coupling. Since the application of both positive and negative \( \delta_3 \) coupling
increases the stiffness number, the above behaviour is expected.

Although fore-aft flapping response to collective pitch reduces with increasing negative $\delta_3$ figure 10, increasing positive $\delta_3$ again reaches a maximum terms of increasing fore-aft flapping which is followed by a reduction.

Figure 11 shows the change in the destabilizing derivative. It is possible with moderate positive coupling, to change the sign of the derivative over the low speed regime with moderate reduction in the derivative value at the high speed end. The application of negative coupling, the derivative is made more positive over the low speed regime while the instability at higher speed is considerably decreased. In both cases large coupling angles are required in order to obtain significant benefits in terms of reducing pitch instability albeit at the expense of control power and increasing cross-coupling.

**Effects of Blade C.G. Offset**

Blade C.G. offset does not alter the flapping frequency of the rotor. By blasing the blade chordwise mass distribution to obtain the required C.G. offset, the blade is subject to two blade pitch change mechanisms in response to flapping namely feathering moments due to centrifugal (propeller moment) and gyroscopic inertial forces. Figures 13 and 18 show the effect of C.G. offset on rotor derivatives. The results shown are for arbitrarily chosen values for forward offset of C.G. from the feathering axis of 6.5% and an aft offset of 3.3%.

Forward offset increases the magnitude of direct rotor damping in pitch and roll and the damping is decreased for an aft C.G. offset as shown by figures 13 and 14.

Figures 13 and 14 show that C.G. offset does not unduly affect flapping response due to off-axis angular motion. This means that aircraft pitching (rolling) motion will not change the magnitude of lateral (fore-aft) flapping. The cross-coupled flapping response to cyclic pitch input shows an interesting behaviour. The lateral flapping induced by longitudinal ($B_1$) cyclic pitch application is slightly altered by C.G. offset. Figure 16 while longitudinal flapping due to lateral ($A_1$) cyclic pitch application remains unchanged. In addition, C.G. offset also alters the cross-coupling of collective pitch to cyclic flapping, where forward offset reduces the cross-coupling while aft offset increases this coupling. Both effects are possible due to the higher lift generated on the advancing side due to blade pitch changes.

Figure 17 shows the effect on control power per unit blade pitch input. There is a small change in the thrust per unit longitudinal cyclic pitch and no change in the thrust per unit lateral cyclic pitch. However thrust per unit collective shows a noticeable decrease with forward blade C.G. offset, which means that an increase in applied collective would be required for trim for example.

In figure 18, the change in thrust when the aircraft is pitch is quite pronounced while the effect of rolling motion is small. Ideally there should be no change in thrust when the aircraft is subject to angular motion.

**Discussion**

The effects of three main rotor couplings namely Collective-coupe, Pitch-flap and blade C.G. offset on main rotor cross-coupling, damping and angle of attack derivatives for a helicopter with an articulated rotor ($\lambda_0=1$) has been shown. The objective was to confirm changes possible in these derivatives and to identify changes to the derivatives that would provide the helicopter with desirable stability and control properties.

All three couplings have mixed effects on the stability and control properties. No weighting has been given to any of the three types of derivatives shown. With large amounts of these couplings, a significant reduction in the angle of attack derivative is possible meaning that the helicopter may be made neutrally stable. The application of forward C.G. offset also reduces the angle of attack instability.

Positive $\delta_3$ and aft C.G. offset will reduce the damping of the rotor when pitching or rolling, while collective-coupe does not affect damping significantly. The effect of $\delta_3$ coupling, positive and negative, does not decrease or increase the magnitude of derivatives in either direction. Instead there is a ‘knee’ in the effect.

Positive $\delta_3$ significantly increases control cross-coupling, the largest effect being on the flapping response to cyclic pitch input. This is primarily because of the increase in the effective stiffness number of the rotor. Whereas positive collective-cone coupling does not affect cyclic to flapping response much, it does decrease the thrust per unit collective commanded. All three couplings in large amounts, have a significant effect on control power.

**Conclusions**

Based on a six degree of freedom single-main and tail rotor helicopter model, the effects of three main rotor couplings on stability and control properties has been shown. Although the individual coupling cannot improve on all areas of stability and control, the results show that significant changes to damping, cross-coupling and destabilising derivatives is possible. The selection of the type of feedback needs to be carefully selected particularly in the case of pitch-flap ($\delta_3$) cou-
pling, since its effects do not linearly increase for both positive and negative feedback.

The next step in the research is to complete the analysis on several other couplings such as A.C. offset and torsion-flap-lag coupling followed by a comprehensive study aimed at identifying the dynamic properties of the rotor and investigating combinations of couplings that can result in a helicopter with desirable stability and control properties.

References

Fig. 4  Effect of $\delta_0$ on lateral flapping derivatives

Fig. 5  Effect of $\delta_0$ on angle of attack flapping derivative

Fig. 6  Effect of $\delta_0$ on and rotor thrust derivatives

Fig. 7  Effect of $\delta_3$ on fore-aft flapping derivatives

Fig. 8  Effect of $\delta_3$ on lateral flapping and rotor thrust derivatives with respect to roll rate

Fig. 9  Effect of $\delta_3$ on lateral flapping and rotor thrust derivatives with respect to pitch rate
Fig. 10  Effect of $\delta_3$ on fore-aft flapping derivatives with respect to collective and longitudinal cyclic pitch

Fig. 11  Effect of $\delta_3$ on angle of attack derivative

Fig. 12  Effect of $\delta_3$ on flapping response due to cyclic control input

Fig. 13  Effect of C.G. offset on fore-aft flapping response due to pitch and roll rates

Fig. 14  Effect of C.G. offset on lateral flapping response due to pitch and roll rates

Fig. 15  Effect of C.G. offset on fore-aft flapping response to collective and cyclic control input
Fig. 16 Effect of C.G. offset on lateral flapping response to collective and cyclic control input

Fig. 17 Effect of C.G. offset on thrust variation due to flapping collective and cyclic control input

Fig. 18 Effect of C.G. offset on thrust variation due to pitch and roll rates