FEASIBILITY ASSESSMENT: A CYCLOIDAL ROTOR TO REPLACE CONVENTIONAL HELICOPTER TECHNOLOGY

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Abstract

A mathematical model is presented for the solution of the aerodynamic performances of a cycloidal rotor. It is solved algebraically for the cases where the forces and incoming wind velocity share the same direction. The model is validated against experimental data coming from three different sources. It is then used to evaluate the viability of using a cycloidal rotor as a replacement for the traditional tail rotor of a helicopter. By doing so, power is saved by the helicopter having non-null advance ratios. The savings reach 50% at an advance ratio of 0.33. It was found that imposing a constant cycloidal rotor angular velocity does not reduce the efficiency and that high pitch angles are the most efficient. The experimental data indicates that three dimensional effects have an influence on the cycloidal rotor performance. A three dimensional Euler fluid dynamic analysis confirms the experimental findings.

1. INTRODUCTION

Cycloidal rotors are a viable means to produce aerodynamic forces whose direction can be rapidly varied in a plane normal to the axis of rotation. The working principle is illustrated on Fig. 1 and a graphical representation of is shown on Fig. 2.

Figure 1: Cycloidal rotor working principle.

Figure 2: A 6-blade cycloidal rotor viewed from the side.

In a cycloidal rotor, a drum carries a set of wings, or blades, and their axes are aligned with the rotation axis of the drum. Each blade can pitch about a feathering axis which is also aligned with the rotation axis of the drum. By pitching the blades with a period equal to that of the drum, a net aerodynamic force normal to the axis of rotation is generated. The direction of the force is controlled by changing the phase of the periodic pitch. Thus, thrust that can be vectored in a plane without moving large mechanical parts. Cycloidal rotors have been studied for their application in vertical axis wind and water turbines.[1–3] Recent studies also investigated their application in unmanned micro aerial vehicles.[4–6] This paper, however, focuses on a manned aircraft application of such rotors.

In this work, a hover-capable vehicle is proposed. It makes use of a conventional helicopter rotor for lift both in hover and forward flight, and uses one or more cycloidal rotors for anti-torque, additional propulsion, and some con-
In the explored design configuration, power demanding tasks like lift in hover and in forward flight are delegated to an efficient device, the rotor. The cycloidal rotor replaces existing anti-torque, propulsion and control devices. For such a vehicle, three design solutions were considered and are shown in Fig. 3.

In the first case, Figure 3(a), a cycloidal rotor is mounted on the tail of a conventional helicopter design with the rotor axis along the yaw axis of the vehicle. The cycloidal rotor can provide thrust in a plane normal to the yaw axis. As such, it can act as an anti-torque device and provide propulsion. To obtain a similar effect using a tail rotor, one would need to tilt the entire rotor. The cycloidal rotor only needs to change the pitch of the blades.

In the second case, Figure 3(b), a cycloidal rotor is mounted on the tail of a conventional helicopter design with the rotor axis along the longitudinal axis of the vehicle. The cycloidal rotor can provide thrust in a plane normal to the longitudinal axis. As such, it can act as an anti-torque device as well as provide an adaptive capability to stabilize and control the aircraft about the pitch axis. In this case, in forward flight the cycloidal rotor may theoretically experience a significant component of airspeed along the blade axis. This configuration is similar to a patent owned by Eurocopter.

In the third case, Figure 3(c), two cycloidal rotors are mounted on both sides of the airframe with the axis of the rotors along the pitch axis of the vehicle. The rotors provide anti-torque and yaw control by antisymmetric thrust along the fore/aft direction, as well as thrust. They can also contribute to lift, through the vertical component of thrust, and to control about the roll axis. This last configuration is similar to the Eurocopter X3 and other aircraft known as gyrodynes. They have the additional advantage of vectoring the thrust. Previous work\cite{7, 8} shown that this latest configuration is by far the most promising configuration. A concept illustration is shown on Fig. 4.

Figure 4: Cycloidal rotor arrangement which was studied in depth.

To allow the study of the rotor, various aerodynamic and aeroelastic models were developed. Overall, the aerodynamic models were four, being an algebraic, a multibody, a twodimensional CFD, and a threedimensional CFD model. These models were presented in previous articles\cite{7, 8}. The results presented here are obtained from a further development of the algebraic model which is explained in detail in the following section.

Finally, this research project is undertaken as part of an European effort for the optimization of cycloidal rotors with the objective of using them in passenger carrying missions. Thus, four universities and two companies from across Europe are part of the Cyclic Rotor Optimized for Propulsion (CROP) consortium\footnote{http://crop-project.eu}.

2. **ANALYTICAL AERODYNAMIC MODEL**

2.1 **Definitions**

The algebraic mathematical model allows one to obtain the pitching schedule necessary for a specific cycloidal rotor to produce the wanted thrust and direction. It can be used in the opposite mode, where the pitching schedule is known and the resulting thrust and direction are to be found. The model also allows the computation of the power required by the rotor. It assumes that the lift of a blade is in agreement with a constant slope lift coefficient $a = C_L/\alpha$ and a constant drag coefficient $C_D_0$. As a result, it does not consider the effect of blade and wake interaction. A solution is presented for different flight regimes. The schematic of the model is shown in Fig. 5 which represents the side view of the cycloidal rotor, and is in the plane of the supporting arms previously shown in Fig. 2. Three different reference systems are used and are explained in Table 1.

![Figure 5: Definition of reference systems.](image_url)
Figure 3: Cycloidal rotor arrangements considered.

<table>
<thead>
<tr>
<th>System name</th>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic reference</td>
<td>o</td>
<td>( Y_o ) points in the direction opposite to gravity. ( X_o ) is positive towards the right.</td>
</tr>
<tr>
<td>Rotating reference</td>
<td>r</td>
<td>( y_r ) is directed radially outward the circular rotor path. ( x_r ) is tangent to the circular path and points backwards.</td>
</tr>
<tr>
<td>Body reference</td>
<td>b</td>
<td>Rotating reference rotated by the pitch angle ( \theta ) which is positive in the clockwise direction.</td>
</tr>
</tbody>
</table>

Thus,

\[ T_o = \left\{ \begin{array}{c} -\sin \beta \\ \cos \beta \end{array} \right\} T. \]  

An inflow velocity \( v_i \) results from the thrust and moves in a direction exactly opposite to the thrust. Thus,

\[ v_o = \left\{ \begin{array}{c} \sin \beta \\ -\cos \beta \end{array} \right\} v_i \]

The free airstream velocity \( U \) has an angle \( \gamma \) with respect to the horizontal \( X_o \) axis. Thus,

\[ U_o = \left\{ \begin{array}{c} \cos \gamma \\ \sin \gamma \end{array} \right\} U \]

Now that the variables \( U, v_i, \beta, T, \) and \( \gamma \) are defined, the definition of a few typical flight regimes is given in Table 2.

The other typical variables are defined as follows. First, the thrust coefficient is,

\[ C_T = \frac{T}{\rho(\Omega R)^2 A}, \]

where \( A \) is the area of the hollow cylinder described by the path of the cyclorotor blades.

\[ A = 2\pi Rb, \]

with \( b \) being the span of the rotor.

Solidity \( \sigma \) is the ratio of the total area covered by the blades to the ring area \( A \)

\[ \sigma = \frac{cbN}{2\pi Rb} = \frac{cN}{2\pi R} \]

with blade chord \( c \) and number of blades \( N \).

Using the simplest momentum theory and referring to Fig. 5 we have a mass flow \( \dot{m} \) through the cycloidal rotor given by

\[ \dot{m} = \rho A_{eff} \sqrt{U^2 + v_i^2 + 2U v_i \sin(\beta - \gamma)} \]

where \( A_{eff} = 2Rb \) is the area of an ideal stream tube that runs across the drum, perpendicular to the drum axis.

And since the thrust \( T \) is equal to the rate of change of the momentum, we obtain that,

\[ T = \dot{m}w = 2m v_i \]

where \( w \) is the velocity at the end of the streamtube defined by the momentum theory applied to the cycloidal rotor in a fashion inspired by Johnson. \[9\]

The induced velocity can be expressed as,

\[ v_i = \frac{T}{(2\rho A_{eff})} \sqrt{U^2 + v_i^2 + 2U v_i \sin(\beta - \gamma)} \]

\[ = \frac{T}{(2\rho A_{eff})} \sqrt{(U \cos(\beta - \gamma))^2 + (U \sin(\beta - \gamma) + v_i)^2} \]
In general we may expect $\theta$ to be small so as a first approximation it is possible to say that

$$\frac{\theta}{\bar{\theta}} \approx 1 - \psi \theta.$$ 

The rotation matrix to transform a vector from the basic to the rotating frame is

$$R_{br} = \begin{bmatrix} \cos \psi & -\sin \psi & v \sin \theta \\ \sin \psi & \cos \psi & v \cos \theta \\ 0 & 0 & 1 \end{bmatrix}$$

2.2 Derivation

To start, it is necessary to compute the component of the air velocity with respect to the airfoil in the body reference frame. The flow velocity, $\mathbf{V}$, as seen from the body reference frame is thus,

$$\mathbf{V} = R_{br} \left( \begin{bmatrix} \Omega R \\ 0 \end{bmatrix} + R_{ro} \left( U \begin{bmatrix} \cos \psi \\ \sin \psi \end{bmatrix} + v_i \begin{bmatrix} \sin \beta \\ -\cos \beta \end{bmatrix} \right) \right)$$

As a consequence,

$$V_x = \Omega R - \cos \psi (U \sin \gamma - v_i \cos \beta)$$

$$+ \sin \psi (U \cos \gamma + v_i \sin \beta) - \theta \left( \cos \psi (U \cos \gamma + v_i \sin \beta) \right)$$

$$+ \sin \psi (U \sin \gamma - v_i \cos \beta)$$

Assuming that $\Omega R \gg U, \Omega R \gg v_i$ and $|\theta| \ll 1$,

$$V_x \approx \Omega R$$

$$V_y \approx \theta \Omega R - v_i \sin (\psi - \beta) + U \cos (\psi - \gamma)$$

The angle of attack of the airfoil is

$$\alpha = \tan \frac{V_y}{V_x} \approx \frac{V_y}{V_x} = (\theta - \lambda \sin (\psi - \beta) + \mu \cos (\psi - \gamma))$$

Using the angle of attack it is possible to compute the lift and drag forces using a simple steady state and linear approximation,

$$D = \frac{1}{2} \rho V^2 c b C_D \alpha$$

$$L = \frac{1}{2} \rho V^2 c b C_L/\alpha \alpha$$
These two force components are in the wind reference frame (i.e. \( D \) parallel to the wind and \( L \) perpendicular), so they must be transformed in force components in the body reference frame,

\[
F_{b_x} = -L \sin \alpha + D \cos \alpha \approx -L \alpha + D \\
F_{b_y} = L \cos \alpha + D \sin \alpha \approx L + D \alpha
\]

Then, these two force components must be transformed in the rotating reference frame,

\[
F_r \approx F_{b_x} + \theta F_{b_y} = -L \alpha + D + \theta L + \theta D \alpha \\
\approx L (\theta - \alpha) + D
\]

So, considering that \( V^2 \approx (\Omega R)^2 \),

\[
F_{rs} \rho (\Omega R)^2 cb/2 = C_{L/\alpha} (\theta - \lambda \sin (\psi - \beta) + \mu \cos (\psi - \gamma)) \\
\approx L (\theta - \alpha) + D \alpha
\]

The imposed pitch angle is expressed as a harmonic series truncated at the second harmonic,

\[
\theta = \theta_0 + \sum_{n=1}^{2} (\theta_{cn} \cos n \psi + \theta_{sn} \sin n \psi)
\]

The two force components can be expressed as truncated Fourier series as well,

\[
F_{rs} = F_{r0} + \sum_{n=1}^{N} (F_{r_{ncn}} \cos n \psi + F_{r_{scn}} \sin n \psi)
\]

\[
F_{rp} = F_{r0} + \sum_{n=1}^{N} (F_{r_{scn}} \cos n \psi + F_{r_{ncn}} \sin n \psi)
\]

Now, the constant part of the force tangential to the cylindrical path, \( F_{r0} \), is computed by integration over a period. It will allow the calculation of the torque and thus the power required by the motor,

\[
\frac{F_{r0}}{\rho (\Omega R)^2 cb/2} = C_D + \frac{1}{2} C_L/\alpha (\mu^2 + 2 \mu \lambda \sin (\beta - \gamma) + \lambda^2) \\
- \frac{1}{2} C_L/\alpha (\mu \cos \gamma + \lambda \sin \beta) \theta_{c1} \\
- \frac{1}{2} C_L/\alpha (\mu \sin \gamma - \lambda \cos \beta) \theta_{s1}
\]

and the torque coefficient is,

\[
\frac{C_Q}{\sigma} = \frac{NF_{r0} R A}{\rho (\Omega R)^2 AR N cb} = \frac{F_{r0}}{2 \rho (\Omega R)^2 cb/2}
\]

or,

\[
\frac{C_Q}{\sigma} = \frac{1}{2} C_D + \frac{1}{4} C_L/\alpha (\mu^2 + 2 \mu \lambda \sin (\beta - \gamma) + \lambda^2) \\
- \frac{1}{4} C_L/\alpha (\mu \cos \gamma + \lambda \sin \beta) \theta_{c1} \\
- \frac{1}{4} C_L/\alpha (\mu \sin \gamma - \lambda \cos \beta) \theta_{s1}
\]

The forces must be translated into the basic reference frame using the definition,

\[
T = R f_{rs} T_r
\]

which gives,

\[
T_x = \sin \psi F_{rs} + \cos \psi F_{rp} \\
T_y = -\cos \psi F_{rs} + \sin \psi F_{rp}
\]

The total average force is given by the constant part of \( T \) times the number of blades \( N \). Recalling the definition of the thrust coefficient, \( C_T \), and of the solidity, \( \sigma \), the following relations are obtained,

\[
\frac{C_T}{\sigma} = \frac{NT_{r0}}{\rho (\Omega R)^2 A N cb} = \frac{T_{r0}}{\rho (\Omega R)^2 cb}
\]

\[
= \frac{C_{L/\alpha}}{8} (2 \theta_{c1} + \lambda (\cos \beta (2 \theta_0 - \theta_{s2}) + \sin \beta (2 - \theta_{s2}))) \\
+ \mu (\cos \gamma (2 - \theta_{s2}) + \sin \gamma (-2 \theta_0 + \theta_{s2})) \\
+ \frac{3 C_D}{4} (\lambda \sin \beta + \mu \cos \gamma)
\]
The force coefficient and direction are given by

\[
\frac{CT}{\sigma} = \frac{NT_m}{\rho(\Omega R)^2 A N c b} = \frac{T_m}{\rho(\Omega R)^2 c b}
\]

\[
= \frac{C_L/\alpha}{8} \left(2 \theta_{\chi 1} + \lambda \left(\cos \beta (-2 + \theta_{\chi 2}) + \sin \beta (2 \theta_0 + \theta_{\chi 2})\right) + \mu (\cos \gamma (2 \theta_0 + \theta_{\chi 2}) + \sin \gamma (2 + \theta_{\chi 2}))\right) + \frac{C_D_0}{4} (-\lambda \cos \beta + \mu \sin \gamma)
\]

The force coefficient and direction are given by

\[
CT = \sqrt{C_{T_x}^2 + C_{T_y}^2}
\]

\[
\beta = -\arctan \left(\frac{C_{T_x}}{C_{T_y}}\right)
\]

The force coefficient components, in wind axes, are

\[
\frac{C_{T_x}}{\sigma} = \frac{C_T}{\sigma} \cos \gamma + \frac{C_r}{\sigma} \sin \gamma
\]

\[
\frac{C_{T_y}}{\sigma} = -\frac{C_T}{\sigma} \sin \gamma + \frac{C_r}{\sigma} \cos \gamma
\]

which yield

(46)

\[
\frac{C_{T_x}}{\sigma} = \frac{C_L/\alpha}{8} \left(2 \theta_{\chi 1} \cos \gamma + 2 \theta_{\chi 1} \sin \gamma + \mu \left(2 + \sin(2\gamma) \theta_{\chi 2} - \cos(2\gamma) \theta_{\chi 2}\right) + \lambda \left(2 \sin(\beta - \gamma) + 2 \cos(\beta - \gamma) \theta_0 - \cos(\beta + \gamma) \theta_{\chi 2} - \sin(\beta + \gamma) \theta_{\chi 2}\right)\right) + \frac{C_D_0}{4} (\lambda \sin(\beta - \gamma) + \mu)
\]

(47)

\[
\frac{C_{T_y}}{\sigma} = \frac{C_L/\alpha}{8} \left(-2 \theta_{\chi 1} \sin \gamma + 2 \theta_{\chi 1} \cos \gamma + \mu (2 \theta_0 + \cos(2\gamma) \theta_{\chi 2} + \sin(2\gamma) \theta_{\chi 2}) + \lambda (-2 \cos(\beta - \gamma) + 2 \sin(\beta - \gamma) \theta_0 + \sin(\beta + \gamma) \theta_{\chi 2} - \cos(\beta + \gamma) \theta_{\chi 2})\right) - \frac{C_D_0}{4} \lambda \cos(\beta - \gamma)
\]

This concludes the definition of the analytical aerodynamic cycloidal rotor model. The following section will present the implicit algebraic solution of these equations.

3. ALGEBRAIC SOLUTION

For the stated purpose of using the cycloidal rotor as a replacement part for the antitorque rotor of the helicopter, the two most important flight regimes that were considered are the Propulsion and Reverse Propulsion scenarios. These are the regimes in which the lateral cycloidal rotors work to provide the antitorque around the main rotor when there is no vertical component to the incoming wind. Furthermore, the cycloidal rotors used in this fashion can provide a net thrust. For both scenarios, the main interest is to know what power is required to obtain the wanted torque and thrust. The wanted thrust of each cycloidal rotor is found by the following procedure, where the original antitorque and forward thrust provided by the Bo-105 tail and main rotor are \(M_t\) and \(T_t\), respectively.

First, the right, \(T_R\) and left, \(T_L\), required thrusts are computed as,

\[
(48) \quad T_R = \frac{M_t + T_t d_L}{d_L + d_R}
\]

and

\[
(49) \quad T_L = T_t - T_R
\]

The power will be obtained by using Eq. (34) derived earlier as the input of,

\[
(50) \quad P = \left| \frac{F_{t_0}}{\Omega R N} \right|
\]

Equation (34) will be solved using the \(\lambda\) and \(C_T\) obtained from the procedure presented in the following two sections. The regime is Propulsion if the requested thrust is positive and Reverse Propulsion if the requested thrust is negative. Both regimes start from the definition of the Forward Flight mode of a cyclogyro. They differ from that regime because the perpendicular thrust, \(C_{T_{Lx}}\), is set to zero. This definition holds for cases where the main helicopter rotor takes all the antigravity forces.

3.1 Solving the Equations in Propulsion

In this configuration, the objective is to produce \(C_{T_t} = 0\) and \(C_{T_r} = -C_{T_r} < 0\), i.e., \(\beta = \pi/2\), with \(\mu > 0, \gamma = 0\). Substituting these values into Eqs (46) and (47) yields

\[
\left\{ \begin{array}{l} -C_{T_r} \\ 0 \end{array} \right\} = \frac{\sigma C_L/\alpha}{4} \left( \mu + \lambda \phi \right) \left\{ \begin{array}{l} 1 \\ \theta_0 \end{array} \right\}
\]

\[
+ \frac{\sigma C_L/\alpha}{4} \left\{ \begin{array}{l} \theta_{\chi 1} \\ \theta_{\chi 1} \end{array} \right\}
\]

\[
+ \frac{\sigma C_L/\alpha}{8} \left( \mu + \lambda \phi \right) \left\{ \begin{array}{l} 0 \\ -1 \\ 0 \end{array} \right\} \left\{ \begin{array}{l} \theta_{\chi 2} \\ \theta_{\chi 2} \end{array} \right\}
\]
The collective pitch \( \theta_0 \) is currently maintained at zero. Thus, a negative cosine one per revolution pitch, \( \theta_1 \), is used to produce the wanted negative horizontal force. This force pushes the rotor towards the left, opposite to the airstream direction. For now, a simple cycloidal rotor which uses only a single harmonic pitching motion is considered. Consequently, we set \( \theta_{s1} = \theta_0 = 0 \). Then, for a null perpendicular thrust and the conditions just stated,

\[
\theta_{s1} = - (\mu + \lambda) \theta_0
\]

which indicates that \( \theta_{s1} \) is null. The inflow coefficient in this case is

\[
\lambda_p = - \frac{\mu}{2} + \sqrt{\left( \frac{\mu}{2} \right)^2 + \frac{\pi C_{TP}}{2}}
\]

The solution of (51) with (53) is thus,

\[
\left( \frac{-4C_{TP}}{\sigma_0^2} - \mu + \kappa^2 - \theta_c - \frac{C_{d}}{\sigma_0} (\mu - \kappa^2) \right)^2
- \left( \frac{\mu}{2} + \frac{C_{TP}}{2} \right) = 0
\]

where \( \sigma \) is \( C_{L/\alpha} \) and \( \kappa \) is a term which integrates flow nonuniformity and tip losses in the equations. This term is inserted as a multiplier of the inflow terms when solving Eq (51). This method is inspired by the empirical factor of the Johnson momentum theory in hover which uses \( \lambda = \lambda \sqrt{C_T/2} \). The current \( \kappa \) is adapted to the cycloidal rotor by an approach similar to Yun et al.\(^{[3]} \) which use an empirical multiplier on one term when solving their implicit inflow equation.

Now, the solution of Eq. (51) yields, when choosing the appropriate root,

\[
\theta_{cP} = \frac{1}{(2a \sigma)} \left( (a \kappa - 2a) \mu \sigma + (C_d \kappa - 2C_d) \mu \right)
- \left( a \kappa \sigma + C_d \kappa \right) \sqrt{\frac{2 \pi C_{TP} + \mu^2 - 8 C_{TP}}{2}}
\]

if \( C_{TP} \) is the known variable. Otherwise, if \( \theta_c \) is the known variable we use,

\[
C_{TP} = \frac{\pi a^2 \kappa^2 \sigma^2}{64} + \frac{a \sigma \theta_c}{64} + \frac{(C_d \kappa - 2C_d) \mu}{4} + \frac{(a \kappa \sigma + C_d \kappa)}{32}
\]

\[
\left( \pi \kappa^2 \mu^2 + 4 \pi \kappa \sigma (a \kappa - 2a) \mu \right) \sigma
\]

which implies that,
\( T = (v_i - U)pA_{cf}/2v_i \)

which when solved algebraically yields

\[
- U = \sqrt{\frac{U^2 + \frac{4T}{2\rho v_i}}{2}}
\]

(61)  \( v_i = \frac{\mu}{2} + \sqrt{\left(\frac{\mu}{2}\right)^2 + \frac{\pi C_{T_{RP}}}{2}} \)

respectively.

This latest equation has a real solution for any positive value of a thrust pointing to the right, which was not the case when the equation was solved for the Propulsion regime.

The positive root of Eq. (61) is kept because \( T \) and \( v_i \) need to have the same sign. That logic comes from the definitions of Fig. 5 and the fact that the rate of change of the momentum is equal to the thrust. Thus, the inflow parameter of the Reverse Propulsion case is,

\[
\lambda_{RP} = \frac{\mu}{2} + \sqrt{\left(\frac{\mu}{2}\right)^2 + \frac{\pi C_{T_{RP}}}{2}}
\]

When setting for a null thrust perpendicular to the wind we obtain,

\[
\begin{align*}
\begin{bmatrix} -C_{T_{RP}} \\ 0 \end{bmatrix} &= \frac{\sigma C_{L/\alpha}}{4} \begin{bmatrix} \mu - \lambda_{RP} \end{bmatrix} \begin{bmatrix} 1 \\ \theta_0 \end{bmatrix} \\
&+ \frac{\sigma C_{L/\alpha}}{4} \begin{bmatrix} \theta_{k1} \\ \theta_{r1} \end{bmatrix} \\
&+ \frac{\sigma C_{L/\alpha}}{8} \begin{bmatrix} \mu - \lambda_{RP} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} \theta_{k2} \\ \theta_{r2} \end{bmatrix} 
\end{align*}
\]

and thus,

(64)  \( \theta_{k1} = (\lambda_{RP} - \mu) \theta_0 \)

the equation to solve is thus,

\[
\begin{align*}
\left(\frac{4C_{T_{RP}}}{\sigma a} - \mu - \kappa \frac{\lambda}{2} - \theta_c - \frac{C_{L/\alpha}}{2} (\mu + \kappa \frac{\lambda}{2})\right)^2 \\
- \left(\frac{\mu}{2} + \pi \frac{C_{T_{RP}}}{2}\right) &= 0
\end{align*}
\]

(65)

where \( \kappa \) is the same empirical term defined in Section 3.2. Finally, solving Eq. (65) yields,

\[
C_{T_{RP}} = \frac{\pi a^2 \kappa^2 \sigma^2}{64} + \frac{\pi C_{L/\alpha}^2}{64} + \frac{a \sigma \theta_{k1}}{4} + \frac{(C_d \kappa + 2C_d)\mu}{8}
\]

\[
+ \frac{(\sigma C_{L/\alpha} + 4(\alpha \kappa + 2a)\mu)\sigma}{32}
\]

\[
- \frac{(\alpha \kappa + C_d \kappa)}{64} \left[ \kappa^2 a^2 \kappa^2 \sigma^2 + \pi^2 C_{L/\alpha}^2 + 32\pi a \sigma \theta_c \right.
\]

\[
+ 16(\pi C_{L/\alpha} + 2\pi C_d)\mu
\]

\[
+ 64 \mu^2 + 2(\pi^2 C_{L/\alpha}^2 + 8(\pi a \kappa + 2\pi a)\mu)\sigma \right]^{1/2}
\]

or, solving for \( \theta_c \), when the pitch function is wanted and the required thrust is known,

\[
\theta_{c_{RP}} = -\frac{1}{2\mu(\sigma)} \left( (\alpha kappa + 2a)\mu + (C_d \kappa + 2C_d)\mu \\
+ (a \kappa \sigma + C_d \kappa) \sqrt{2\pi C_{T_{RP}} + \mu^2 - 8\sigma C_{T_{RP}}} \right)
\]

where \( C_{T_{RP}} = -C_T = C_{T_{R}}. \)

4. CALIBRATION AND VALIDATION OF THE MODEL

Due to the absence of experimental data in forward flight, the Propulsion and Reverse Propulsion flight regime models are validated using hover experimental data. Three experimental dataset are available and are used to calibrate and validate the analytical propulsion models by imposing a null advance ratio. A dataset comes from Yun et al.\cite{5} and another from IAT21,\cite{10} which is a member of the CROP consortium. An experimental campaign was also run by Bosch Aerospace and reported by McNabb.\cite{11} This last one had the particularity of using airfoils with a high drag coefficient which was reported by McNabb to be \( C_{L/\alpha} \approx 0.07 \) and was used as such in the current algebraical model. Another difference between the three experimental setups is that the Bosch rotor transmitted movement to the blades by an apparatus located a midspan of the blades and covered by a cylindrical shell. The IAT21 experiments had a similar setup, but located at the external edges of the blades. The Yun et al. setup was positioned similarly to IAT21 but the arms were uncovered. Another noted difference is that the power measured by Yun et al. was the supplementary power required by the electrical drive when blades are added to the device. The power measured by McNabb is the power measured by a load cell. The power measured by IAT21 is the total motor power, for which they estimated a 5% total loss. The experimental data was thus taken as is from Yun et al. and McNabb and a 5% reduction was applied to the power measured by IAT21. Finally, IAT21 used NACA-0016 airfoils, as opposed to NACA-0012 for McNabb and Yun et al. Even though the blades differed, a slope of the lift coefficient \( a = C_{L/\alpha} = 6.04 \) is used for all three configurations.
The other experimental parameters are briefly described in Table 3. Calibration was done by curve fitting the powers and thrusts obtained algebraically to the ones obtained experimentally. The results are shown in Figs 6 to 8.

Although various weightings between power and thrust were tested, it was chosen to give an equal importance to both of them. Optimizing with a strong weight on power gave an excellent match for power, but yielded a very high κ, which is the only variable that is calibrated by the optimization. The smaller correctors κ required for the IA T21 and McNabb experimental data may be due to the fact that their experimental model had full cylinders which are known to reduce the tree-dimensionality of the flow. To verify this hypothesis, a previously developed three dimensional fluid dynamics model[8] is used to confirm the influence of the use of endplates on the rotor. A short description of the model along with the found effect of the endplates is presented in the following paragraph.

The OpenFOAM CFD toolkit has been relied upon to perform the fluid dynamic calculations. A laminar non-viscous solver is relied on since the main contributors to the thrust are the pressure induced forces. A double mesh interface which allows one to model a fixed angular velocity rotor zone and six embed periodically oscillating zones has been developed. A moving no-slip boundary condition was also developed to constrain the perpendicular velocity of the fluid at the foil to equal to the airfoil velocity and letting the parallel velocity uninfluenced. The timestep used for the simulations is variable and set to follow a Courant number of about 10. The mesh used without endplates has 366k cells while the one with endplates has 926k cells. The reason for such a big discrepancy is the difficulty of the snappyHexMesh meshing software to mesh surfaces close to interfaces and the thus related need to have a highly refined mesh in these zones. The spacing between the endplates and foils is one tenth of the chord length, which is
Table 3: Description of the experimental data.

<table>
<thead>
<tr>
<th>Author</th>
<th>Radius R (m)</th>
<th>Span b (m)</th>
<th>Chord c (m)</th>
<th>Number of blades N</th>
<th>Three-dimensional devices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yun et al. [5]</td>
<td>0.4</td>
<td>0.8</td>
<td>0.15</td>
<td>6</td>
<td>Arms at edges</td>
</tr>
<tr>
<td>IAT21 [10]</td>
<td>0.6</td>
<td>1.2</td>
<td>0.3</td>
<td>6</td>
<td>Cylinders at edges</td>
</tr>
<tr>
<td>Bosch [11]</td>
<td>0.610</td>
<td>1.22</td>
<td>0.301</td>
<td>6</td>
<td>Cylinder at midpoint</td>
</tr>
</tbody>
</table>

Figure 8: Comparison with the Yun et al. [5] experimental data of their baseline model at various pitching function magnitudes. Using \( \kappa = 1.4804 \) and \( C_{D_0} = 0.008 \).

Figure 9: Effect of the presence of endplates on the cycloidal rotor as confirmed by the 3D CFD simulations.

reported by Calderon et al. [12] to have the same effect as if it were attached to the foil.

The CFD results of the IAT21 L3 model at various angular velocities are shown with and without endplates on Fig. 9. They confirm the idea that the experimental apparatus used to transmit power will have an influence on the overall performance. In the experiment IAT21 used endplates. The instantaneous velocity fields and streamlines of the IAT21 L3 case where the pitch function angle magnitude is 37.5° at the angular velocity of 250 RPM are shown on Figs. 10(a) to 10(d). These figures further confirm the influence of the endplates geometry. Also, Fig. 10(d) confirms the fact reported by IAT21 that the flow enters the cycloidal rotor over an arc of roughly 90°. Figure 10(c) agrees well with the findings of Yun et al. [5] that the flow is deviated by the rotor.

5. EFFICIENCY EVALUATION

As was stated in the introduction, of the three helicopter configurations proposed the most promising one is the helicopter with two lateral cycloidal rotors as shown in Figs 3(c) and 4. It has the advantage of reducing the demand on the
main rotor and this aspect will be studied in more depth in the current section. A first step consists of obtaining the performance characteristics of the Bo-105 helicopter.

5.1 Bo-105 Performance Characteristics

The performance characteristics of the original helicopter are shown in Figs 11 to 15. They consider diverse constant velocity advance ratios. They are obtained from a comprehensive aeroelastic helicopter model which takes into account the dynamics of the main rotor, the fuselage, and the tail rotor. That model is fully aeroelastic and provides torque and power figures for both main and tail rotors. It also yields the thrust they generate in magnitude and direction. A more complete description of the model can be found in a paper by Muscarello.\footnote{13} That same Bo-105 model was also used in a wind tunnel mode. There, the rotor was solidly fixed to the ground with a null pitch angle and was subjected to the same advance ratios and weight as the full helicopter. The data thus obtained gives the required power to provide anti-gravity alone and is used in the efficiency evaluation of the proposed design. Subtracting the power required by the main rotor to only provide lift at a null pitch angle from the power required by the full helicopter in trim yields the portion of power that is dedicated to forward propulsion. The resulting distribution of power shown on Fig. 12.

5.2 First Optimization

The power demand of the full original Bo-105 helicopter is compared with that of the modified helicopter for identical advance ratios. This means the power demand of the main and tail rotor for the original helicopter and the power demand of the main and two lateral cycloidal rotors for the modified helicopter. To obtain a first pair of cycloidal rotors, an optimization of their performance characteristics was performed such as to obtain the required torque and thrust at the smallest expense of power.

Table 4 lists the parameters that were allowed to change during the optimization process. The maximum allowed angle of attack was set to 13.06°. Greater angles are no longer valid when using a constant slope lift coefficient. Furthermore, lift generation remains constant until 14.14° and then drops rapidly. This maximum angle of attack is obtained from Eq. (22) with $\gamma = 0$ and $\beta = \pi/2$. It becomes, after subbing in Eq. (31) truncated at the first harmonic term and Eq. (52),

$$
\alpha \approx \theta_o + \theta_c \cos(\Psi) + \theta_s \sin(\Psi) - \lambda \sin(\Psi - \pi/2) + \mu \cos \Psi \\
\approx \theta_o (1 - (\mu + \lambda) \sin \Psi) + (\theta_c + \lambda + \mu) \cos \Psi
$$

where the part which multiplies $\theta_o$ confirms the need for a null collective pitch. Indeed, the maximum and minimum values of the part of the equation multiplied by $\cos(\Psi)$ correspond to $\Psi = 0, \pi$. A non null $\theta_o$ at these values of $\Psi$ can thus only increase that maximum angle of attack.

The maximum allowed distance between the centers of the two cycloidal rotors was 8.25 m and was chosen as such in order to prevent the cycloidal apparatus from extending further than the main rotor. The maximum distance is derived from,

$$
\frac{d_L + D_R}{2} + b = \frac{d_L + D_R}{2} + d_L + D_R - L_W \leq D_{MR}
$$
where $D_{MR} = 9.84$ m and is the main rotor diameter and $L_w$ is the width of the skids. Each rotor uses NACA-0012 airfoils and the rotor radii were limited to avoid any contact with the ground or the main rotor. The maximum tip velocity at maximum forward velocity was limited to Mach 0.85.

The first optimization lead to the data presented in Table 5 and gives the results show on Figs. 16 to 18. The angular velocity of the cycloidal rotors was not allowed to vary when changing advance ratio or flight regime, the pitch was restrained to a single harmonic variation, and the geometries of the two cycloidal rotors were identical.

These figures do consider the larger amount of power which is saved by the main rotor which does not have to pitch anymore. In fact, the main rotor only pitches in order to balance the position offset of its center of mass, but this might be set to be null if the center of mass and the rotors are aligned. Figures 17 and 18 show the provided thrust and anti-torque of cycloidal rotors and of the main or tail rotor, respectively. As required, the resulting torques and thrusts for both the original helicopter and the proposed design are equal.

### 5.3 Variable Angular Velocity

Taking the assumption that the cycloidal rotor could be powered electrically, it is reasonable to assume that the angular velocity of the rotors could be controlled independently and vary with advance ratio. Constraining the algorithm to use the same geometry found before and allowing each rotor to have a different angular velocity at each advance ratio considered yields a negligible increase in efficiency. The resulting power required for this configuration is shown in Fig. 19 and the optimal angular velocities of the rotors are shown in Fig. 20. Although the strange angular velocity distribution is shown, it was noticed that many angular velocity configurations yield similar results and that the limiting factor is the maximum allowed blade pitch an-
Table 4: Cycloidal rotors optimization properties.

<table>
<thead>
<tr>
<th></th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius $R$ (m)</td>
<td>0.1</td>
<td>1.275</td>
</tr>
<tr>
<td>Midspan dist. $d_L + D_R$ (m)</td>
<td>0.5</td>
<td>8.25</td>
</tr>
<tr>
<td>Chord $c$ (m)</td>
<td>0.05</td>
<td>0.5</td>
</tr>
<tr>
<td>N. of blades $N$</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Ang. vel. $\Omega$ (RPM)</td>
<td>20</td>
<td>2000</td>
</tr>
</tbody>
</table>

Table 5: First optimization results.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Span $b$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chord $c$</td>
<td>0.35368 m</td>
<td></td>
</tr>
<tr>
<td>Radius $R$</td>
<td>1.2452 m</td>
<td></td>
</tr>
<tr>
<td>Angular velocity $\Omega$</td>
<td>397.15 RPM</td>
<td></td>
</tr>
<tr>
<td>Center to center distance $d_{lat}$</td>
<td>9.4950 m</td>
<td></td>
</tr>
<tr>
<td>Number of blades $N$</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Figure 16: Total power of the original and new designs.

Figure 17: Propulsive thrust provided by the modified Bo-105 helicopter model.

A small influence of the resulting moment of the cycloidal rotors is expected. With the simple first design, a maximum combined cycloidal rotor moment of 850 Nm is expected. Quick calculations show that this could require at slight increase of main rotor power, reaching at most 10 kW. This is negligible when compared to the savings incurred. It is further reduced, or eliminated, by the reduced requirement for propulsion by the cycloidal rotors. Furthermore, a design where one of the cycloidal rotors rotates in the opposite direction would also reduce the induced moment. A more complete optimization, considering also main rotor trim, must be conducted to find the optimal configurations.

7. CONCLUSION

An algebraic model for the computation of the performance characteristics of a cycloidal rotor has been presented. It was validated against three sets of experimental data. It was used to assess the benefits of using cycloidal rotors as a replacement for the traditional tail rotor of a helicopter. Power savings of up to 50% are found. The dependence of efficiency on several design parameters have been highlighted. This provides useful indications for the design of the overall system. Analyses of increasingly detailed models of the cycloidal rotor concept are under way, focusing on propulsive efficiency, interference effects, and preliminary weight estimation. Future plans also include the use of the three dimensional CFD model to refine the current inflow model.
Figure 18: Anti-torque provided by the modified Bo-105 helicopter model.

Figure 19: Total power of the original and variable angular velocity designs.

8. ACKNOWLEDGMENTS

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References


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