Revisiting the Effects of Blade Geometry on Rotor Behaviour in Descending Flight

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Abstract
A set of simple, testable hypotheses for the behaviour of the rotor flow in axial flight is derived from a fusion of experimental, numerical and theoretical ideas. Although it is unlikely that all of the results described in this paper will stand up in the face of new information as it becomes available, the series of conundrums, paradoxes (and, simply, questions to be answered) that has been generated by this approach has proved invaluable in focusing research and helping to guide our progress in understanding the complexities of the rotor’s aerodynamics - particularly within the vortex ring state.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>VTM</td>
<td>Vorticity Transport Model</td>
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<tr>
<td>A</td>
<td>Rotor Swept Disk Area</td>
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<tr>
<td>( p_u )</td>
<td>Time Average Pressure Above Rotor Disk</td>
</tr>
<tr>
<td>( p_l )</td>
<td>Time Average Pressure Below Rotor Disk</td>
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<tr>
<td>( p_\infty )</td>
<td>Ambient Pressure far from Rotor</td>
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<tr>
<td>R</td>
<td>Rotor radius</td>
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<tr>
<td>( v_i )</td>
<td>Induced Velocity at Plane of Disk</td>
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<tr>
<td>( v_{i,HOV} )</td>
<td>Equivalent Hover Induced Velocity</td>
</tr>
<tr>
<td>( k v_i )</td>
<td>Induced Velocity in Fully Developed Wake</td>
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<tr>
<td>( V_i )</td>
<td>Free Stream Axial Velocity</td>
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<tr>
<td>( \rho )</td>
<td>Air Density</td>
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<tr>
<td>( \lambda_i )</td>
<td>Induced Velocity at Plane of Disk Normalised by Tip Speed (+ve up)</td>
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<tr>
<td>( \overline{\lambda}_i )</td>
<td>Induced Velocity at Plane of Disk Normalised by Equivalent Hover Induced Velocity</td>
</tr>
<tr>
<td>( \lambda_{i,HOV} )</td>
<td>Equivalent Hover Induced Velocity Normalised by Tip Speed (+ve up)</td>
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<tr>
<td>( \mu_Z )</td>
<td>Free Stream Axial Velocity Normalised by Tip Speed (+ve up)</td>
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<tr>
<td>( \overline{\mu}_Z )</td>
<td>Free Stream Axial Velocity Normalised by Equivalent Hover Induced Velocity</td>
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<tr>
<td>( \Omega )</td>
<td>Rotor Rotational Speed</td>
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<tr>
<td>( \Omega_R )</td>
<td>Rotor Tip Speed</td>
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Introduction
The authors are engaged in a programme of investigation into the effects of rotor geometry on the onset and intensity of the vortex ring instability in descending flight at low forward speeds using a combination of detailed analysis using the Vorticity Transport Model (VTM) [Ref 1] and model rotor experiments using PIV [Ref 2]. The preparatory work has been completed and the generation of data is about to commence and will produce considerable volumes of integrated force and flow field data, numerical and experimental, all varying with time. Whilst the data sets themselves will form a useful reference benchmark, to add insight to the aircraft design field, they will need to be inspected for trends which will be generalised in a way that engineers can understand and reflect in their rotor designs.

It is often the case that a generalisation of a complicated problem can take the form of simplified theory with necessary assumptions. Often the assumptions contain and express the essential understanding of the problem. Whilst such simplification can be quite sweeping, if done correctly, the essence of the problem is revealed and made quite visible to the designer. With this insight, the designer is allowed to draw conclusions, whilst remembering to perform the appropriate checks that the assumptions remain accurate enough.

The authors have all worked in the field of rotorcraft design and have learnt to recognise the usefulness of such generalisations or simplifying theories. In addition, experience has shown that in approaching the collection or production of a large body of data, it is useful to start with a set of hypotheses or expectations in order to review the validity of the data as it is collected and to ensure that the activity remains focused and rooted in fundamentally correct physics. This point is extremely important when dealing with a flow situation as manifestly complex as vortex ring. This paper presents some applications of simplified theoretical approaches to specific
flows associated with vortex ring conditions. This is aimed at allowing reasonable hypotheses, concerning flow topology, to be generated which will then be confirmed or otherwise by subsequent detailed calculations and tests.

The discussion focuses on three basic problem areas.

1. Validity of Momentum Climb Solution in Descent
2. Topology of Flow near Minimum Windmill Brake Descent Speed
3. Mean Flow Through Vortex Ring Conditions

Discussion

Simplified Models and Flow Topology as Tools in Understanding Complex Unsteady Flows

One region appropriate to operation of a rotor in vortex ring conditions is that of the axial descent of the rotor prior to the onset of the breakdown of the wake pattern. Applying the simplest theoretical approach, namely actuator disc momentum analysis, it is possible to derive a criterion for the breakdown of the wake structure and to show that the detailed physics of the wake instability is mirrored in the parameters of the simple theory. [Ref 3]

The problem is that momentum theory is usually thought to fail as soon as the rotor begins to descend, due to violations of flow continuity, which effectively amounts to a topology problem. However, if this situation is considered in more detail it yields a useful insight into the flow topology and confirms the simple momentum theory approach.

Problem 1 – Validity of Momentum Climb Solution in Descent

The simplest approach to modelling the flow through a rotor is that of disc momentum. It is assumed that the flow is everywhere uniform across the disc. Because momentum theory is a control volume analysis, the topology of the flow must be described. It is usual to assume that the flow forms a streamtube originating far above the rotor disc, passing through the disc and continuing far below the rotor. (Fig 1) The control volume analysis is to be applied to this assumed flow, resulting in standard momentum theory. [Appendix 1]
flow pattern must involve a rapid lateral contraction, implying that the far upstream inlet plane where the axial velocity is zero is much closer to the rotor in practice than implied by figure 1. However, the topology of the flow remains compatible with that assumed to set up the momentum theory problem. When a climb velocity is added, the flow topology continues to conform to the traditional assumptions of momentum theory.

The principal difficulty is experienced if the rotor begins to descend, whereupon, the situation changes.

Consider the streamline passing through the rotor centre in axial flight at low descent rate. The following points need attention.

- Far above the rotor, on this streamline, the velocity must be that of the free stream with an upward direction relative to the rotor.

- At the rotor disc itself, since the descent rate is small, the velocity must be downward relative to the rotor in order to produce an upward thrust.

At some point on the centre streamline there must be a point of zero velocity where the flow changes direction. This situation is incompatible with the streamtube concept assumed to derive the momentum theory, since continuity is not satisfied in the assumed streamtube above the rotor. On this basis, it is often said that momentum theory may not be applied to descending flight conditions.

Nevertheless, if momentum theory is applied to the descent condition, it seems to give a fair approximation to measured test trends. (Fig 3) The figure shows that momentum theory describes the lower limit of the test data fairly well and that a trend line based on the momentum climb root equation but modified by empirical factors can adequately describe the mean.

Figure 3 - Comparison of Momentum Theory with Test in Descent

To explain the situation illustrated in figure 3, it is necessary to consider the pattern of the flow above the rotor in more detail. As the rotor begins to descend, there must be some point on the streamline passing through the rotor centreline where the axial velocity reduces to zero. Flow proceeds in the axial direction away from this point both upward and downward. In order to support this feature of the flow, continuity demands that flow must approach from a perpendicular direction (otherwise there must be a classical flow source at the zero velocity point). Symmetry demands that the perpendicular flow must move inward in a radial direction. (Up to this point it has been possible to ignore radial flow in the control volume analysis). The radial flow must have its origin in the free stream and a flow pattern that satisfies this requirement is shown. (Fig 4)

Figure 4 – Flow Pattern in Descent with Saddle Point on Centreline
It is possible to define a control volume for this new flow pattern and to derive a momentum analysis analogous to the original momentum analysis for hover and climb. [Appendix 2] Just as in the original theory, the inlet and outlet plane dimensions drop out of the analysis, leaving the rotor swept disc area as the only explicitly defined geometry. Appendix 2 shows that with the appropriate substitutions, the resulting equations become identical with the original momentum theory. Therefore, momentum theory for the hover and climb may be extended directly to conditions at low descent rate. However, the topology of the corresponding inlet flow is very different from that originally assumed. The topology of the wake flow below the rotor remains qualitatively the same. Since the model for the onset of the wake breakdown into vortex ring conditions [Ref 3] was based upon the wake portion of the flow, the application of the momentum theory appears to remain justified.

To recap: the original problem of applying momentum theory to descending flight was the imposition of an inappropriate flow pattern. A new flow pattern has been deduced that must be physically possible, but there is no guarantee that this new flow pattern is the only one that can exist in descent. In order to confirm its likelihood some other information or theoretical modelling is desirable. Appendix 3 outlines a method of calculating flow through the rotor based a simplified wake geometry using vortex elements prescribed arbitrarily. The method uses ring vortices as elements because an exact solution for the velocity induced by such an element is analytically available. Also the ring vortex is the largest simplest element that can represent curvature effects accurately. Increasing the number of rings allows the ideal of a vortex tube wake boundary to be represented as shown. [Appendix 3] Arranging the wake geometry to represent the shape of a typical hovering rotor results in a streamline flow pattern as shown in figure 5.

This pattern is compatible with the flow topology assumed for the original momentum theory climb solution. (Fig 1)

Figure 5 - Flow Streamlines for Hovering Ring Vortex Wake

If the rotor begins to descend from the hover condition at a low rate, it is reasonable to assume that the wake geometry remains largely unmodified. Flow pattern calculations have been repeated using this wake assumption for a slow descent condition and the streamline pattern has been derived (Fig 6).
Figure 6 - Flow Streamlines for Ring Vortex Wake at Very Low Descent Rate

The flow pattern can be seen to contain a saddle point where the flow comes to a stop as deduced for the revised momentum theory for descent. The assumed pattern is confirmed as the most likely for this operating condition therefore, the revised momentum theory and the deductions based on it are also likely to remain valid.

A further theoretical comparison using the full VTM (Ref 1) analysis is shown to support the above conclusions.

Figure 7 – Vorticity Contours & Averaged Flow Streaklines for Hovering Rotor - VTM Wake

The VTM is a CFD model with the capability of modelling the full unsteady rotor wake flow without the usual problem of numerical decay of vorticity because of its unique formulation. In this case it is used simply to remove the arbitrary assumptions of the prescribed wake model in terms of vortex element strength and position in order to confirm that flows of the same topology result in hover and descending flight prior to the onset of vortex ring wake breakdown. The VTM model gives a fully unsteady representation of the flow, so for some cases like that of the hover where unsteadiness is produced by the unstable vortex wake downstream of the rotor, a comparative steady flow pattern is produced by means of a set of streaklines averaged over time. This averaging process tends to smear and reduce the strength of any moving vortex, so the flow pattern where persistent vortex motion exists should be treated with caution. A typical snapshot of the wake vorticity distribution shows that the far wake expands far less than the averaging process suggests. Its form tends to remain a contracted tube that wanders somewhat in direction with time. (Fig 7) In both figures, the shading illustrates a contour map of the mean flow vorticity.

To confirm the flow topology deduced for low descent rates by simpler means, a small descent rate is added to the VTM hover case. (Fig 8) A saddle point of zero axial velocity emerges in the flow directly, confirming the previous deductions.

Figure 8 – Streaklines for VTM Wake in Low Descent Rate showing Stagnation Point

In order to move on to the second problem figure 9 shows the VTM results for a low rate of descent at the high-speed end of the so-called vortex ring state that is found between conditions of moderate descent rate and the situation of the rotor moving towards the
windmill brake state. Again the vorticity contours and mean streaklines are shown.

Figure 9 - Instantaneous Vorticity Contours and Mean Flow Streaklines for VTM Wake in Low Descent Rate

Problem 2 – Topology of Flow Near Minimum Windmill Brake Descent Speed

Another idealised, but important, case occurs in axial flight when the rotor is operating in the windmill brake state. A theoretical model based on disc momentum considerations may be derived [Appendix 4] based on a streamtube flow topology. (Fig 9)

As for the climb situation, the control volume analysis depends explicitly only on the rotor swept disc area. Far below the rotor, the flow in the streamtube approaches the freestream velocity. Far above the rotor, the flow velocity is the sum of the freestream plus the induced velocity of the fully developed wake. Since the induced velocity must oppose the free stream velocity for an upward thrust, the total velocity at the exit plane of the control volume must be lower than the freestream. As a result, the streamtube, defined by the vortex wake in the corresponding vortex model, must increase in area due to continuity.

As the freestream velocity is reduced, the momentum solution for the windmill brake condition shows that the induced velocity increases until it reaches a value equal to that of the equivalent hover induced velocity. This point occurs at a referred freestream velocity of twice the equivalent hover induced velocity. There is no mathematical solution to the momentum equations at lower freestream speeds.

Because the induced velocity doubles between the rotor disc and the far wake position, the net velocity in the far wake reaches zero.

Translating these observations into flow topology, the streamtube area far upstream (below) the rotor must be the order of half the rotor disc area and the wake must eventually become infinitely wide to accommodate the mass flow passing through the plane of the disc. The shape of the streamtube is similar to that of the hover case except that the flow has changed direction. If the exit flow direction were purely radial, the momentum requirement would be satisfied. Obviously this pattern will not be achieved in practice since the wake becomes infinitely wide. (Fig 11)

At slightly higher descent rates, the wake will be wide but finite in width. For the same reasons that the wake in hover contracts rapidly within the first blade passage, the windmill brake wake is likely to expand to most...
of its final width during the interval of the first blade passage. Since in the equivalent vortex representation of the flow the vortex transport rate is approximately the average of the flow inside and outside of the wake, the transport velocity will reduce and the individual vortex filaments trailed from the blade tips are likely to come closer together and begin to influence one another leading directly to wake breakdown, in a manner analogous to that proposed for the breakdown of the climb solution. [Ref 3]

To gain some insight into these topology deductions, a ring vortex model case has been calculated with a topology appropriate to the minimum theoretical windmill brake speed condition. (Fig 12) Another case at a lower descent rate with the ring elements concentrated so that a vortex ring pattern is produced is shown for comparison. The result of assuming an arbitrary wake geometry is that there is some flow through the vortex sheet formed by the ring elements. A truly free wake moving with the flow would lie on a streamline with zero flow penetration, but for most cases, the ring vortex model lies very close to the streamlines of the resulting flow. This is true for the lower descent rate case shown in fig 12 but not true for the geometry chosen to represent the topology of the flow at the minimum windmill brake descent speed. There appears to be a problem with this flow topology, which tends to confirm the deductions made on the basis of momentum theory for this flight condition.

On the other hand, the VTM analysis appears to have no trouble in producing a solution that has the topology of windmill brake flow at descent rates lower than the minimum windmill brake speed. (Fig 13) Surprisingly, the outer boundaries of the wake are smooth with the major unsteady activity associated with the vorticity originating at the rotor blade roots. There is also very little wake expansion. The VTM is producing a flow topology very unlike that suggested by the momentum theory. Why this situation should be the case is not clear, but the VTM models the details of blade geometry including root cut out and these geometry details may affect the overall flow.
The windmill brake flow appears to be similar to the wake off a bluff body. If the analogy holds, it might be expected that the rotor flow would be susceptible to ventilation of the wake by means of rotor geometry – root cut out, twist/stall, the presence of a spinner etc. – as is the wake of a bluff body. It may be that differences in rotor behaviour due to blade geometry are most pronounced near the idealised position of the breakdown of windmill brake flow and that in the experimental programme, close attention should be paid to the topology of the flows in this vicinity.

Problem 3 – Mean Flow Through Vortex Ring Conditions

Although the apparatus for the detailed experimental investigation of the effects of geometry on vortex ring behaviour is only just being commissioned, several simpler rotor rigs have been built and tested as student projects in support of the final rig design. Although of limited scope, these rigs have produced force and flowfield measurements.

It is interesting to make some speculative observations on the nature of the flow field of a rotor passing through the vortex ring state on the basis of the parameters required to fit a simplified model to the experimental results.

The time averaged thrust produced by a small rotor rig is shown as a function of referred descent rate. (Fig 14) The thrust variation using the prescribed ring vortex wake model, for this case, is also plotted. The rotor was of fixed pitch and constant chord and twist rate (Fig 15).

- Solidity = 0.06
- Twist = 35°
- Pitch @ 75% R = 9°
- Collective Pitch = 35.25°

Although the theory indicates that fairly high incidence angles are reached at inboard stations, it is assumed that the rotor aerofoil sections experience the same sort of three dimensional vortical flows as has been reported on windmills [Ref 4], so that the local lift curve slope is not affected by conventional stall.
Making the assumption that the mean thrust is associated with a mean flow pattern and not the mean of several different flow patterns, the vortex model wake shape is adjusted in an attempt to match the thrust/descent rate history. A series of inferred flow patterns are shown in (Fig 16). The equivalent VTM flow patterns (snapshot and mean streakline) are shown in figure 17.

As the rotor commences descent, the thrust increases. This is interpreted to be the effect of the upflow without any real influence from the vortex wake. The positions of the vortex rings are assumed to be close to those in the hover. With an increase in descent rate the wake begins to adjust and move upwards towards the rotor disc. This has the effect of making the rotor thrust fall off to the characteristic dip. Particular discussion is necessary about the dip in the mean thrust at the descent rate close to the maximum unsteady vortex ring activity ($\bar{\mu}_2 \approx 1.0$). The ring vortex strengths are determined by the flow through the rotor in hover. In this situation, the vortex rings are well spread. As the ring vortices move up to the rotor disc
plane, the vortex ring strength is effectively concentrated. In order to obtain a dip of the correct magnitude, the ring vortex strengths are reduced – parabolically with respect to the axial velocity. This can be interpreted as a stretching due to the lateral spreading of the ring vortex elements and any mutual interaction.

As the descent rate increases further, the thrust recovers and the theory accurately predicts the slope. However, there is a pause in the thrust behaviour as the wake moves above the rotor which indicates that the movement of the vortex rings in this condition needs careful study. As the rotor becomes immersed in the windmill brake state, the theory predicts a slow fall off of the thrust. The experimental data shows a fairly rapid fall off in rotor thrust at a referred descent speed of 2 with a subsequent recovery.

Conclusions

1. Simple models are useful and necessary for aircraft designers.

2. The extension of momentum theory to descending flight has been justified.

3. The theoretical minimum descent speed for the windmill brake state has been shown to be very important with indications that rotor blade geometry details can affect the overall flow topology in this region. Careful attention to this flight case will be paid in future tests and detailed calculations.

The progression of a ring vortex model through the axial speed range has permitted a reasonable match of thrust versus axial speed to be achieved with reasonable assumptions about wake configuration.

The results of his study will be used as a guide in further detailed calculations and experiments.

References


Acknowledgement

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Appendix 1

Appendix 1: Rotor Disc Momentum Theory - Derivation of Climb Root

For the climb case, disc momentum theory is usually developed assuming the geometry shown in figure 1. The relationships are expressed in terms of convenient vectors with positive directions as indicated.

The analysis applies to steady flow through a control volume which conserves mass, momentum and energy and produces a net thrust on the control volume. As velocity changes, the assumed streamtube changes cross-sectional area. Lateral velocity components are assumed small in relation to the axial components and they are not taken into account explicitly.

First consider the total pressure in the fluid stream. There is a mean pressure jump at the rotor associated with the rotor thrust. This is the pressure change that would be apparent as a time average to an observer with a fixed position in the non-rotating axis system of the rotor. Above and below the disc, invoking Bernoulli, the total pressure in the streamtube is constant, i.e.

\[ p_\infty + \frac{1}{2} \rho V_1^2 = p_u + \frac{1}{2} \rho (V_1 + v_i)^2 \]

\[ p_\infty + \frac{1}{2} \rho (V_1 + k v_i)^2 = p_i + \frac{1}{2} \rho (V_1 + v_i)^2 \]

\[ T = A (p_i - p_u) \]

\[ = \left( \frac{1}{2} \rho (V_1 + k v_i)^2 - \frac{1}{2} \rho V_1^2 \right) A \]

The thrust is also related to the rate of change of momentum through the control volume:

\[ T = \rho A (V_1 + v_i) \cdot k v_i \]

Where the mass flow through the control volume is that through the disc:

\[ \rho A (V_1 + v_i) \]

and the velocity change is

\[ k v_i \]

Equating thrust expressions, the value of \( k \) is deduced:

\[ k = 2 \]

Substituting into the momentum equation:

\[ \rho A (V_1 + v_i) \cdot 2 v_i = T \]

\[ v_i^2 + V_1 v_i - \frac{T}{2 \rho A} = 0 \]

Re-expressing in usual helicopter notation, with velocities normalised by tip rotor tip speed and with all upward velocities positive:

\[ v_i = -\frac{\lambda_i}{\Omega R} \]

\[ V_1 = -\frac{\mu Z}{\Omega R} \]

\[ \lambda_i^2 + \mu Z \lambda_i - \frac{T}{2 \rho A \Omega^2} = 0 \]

\[ \lambda_i = -\frac{\mu Z}{2} - \sqrt{\left(\frac{\mu Z}{2}\right)^2 + \frac{T}{2 \rho A \Omega^2}} \]

Normalizing further with respect to the equivalent hover induced velocity:

\[ \lambda_{i,HOV} = -\sqrt{\frac{T}{2 \rho A \Omega^2}} \]

\[ \mu Z = \frac{\mu Z}{\lambda_{i,HOV}} \]

\[ \lambda_{i,HOV} = -\frac{\lambda_i}{\lambda_{i,HOV}} = -\frac{\mu Z}{2} + \sqrt{\left(\frac{\mu Z}{2}\right)^2 + 1} \]

Appendix 2 - Rotor Disc Momentum Theory - Derivation of Climb Root in Descent

In axial flight, at low descent rates, the velocity in the far field above the rotor must match the freestream velocity, travelling away from the rotor. Close to the rotor, the flow must be in the opposite direction if the rotor is to produce an upward thrust. Clearly, this requirement is not compatible with the streamtube form of flow usually assumed to derive the climb solution, if the continuity requirement is to be enforced.

The continuity problem is sometimes taken as proof that the climb root predicted by momentum theory is inapplicable to descending rotors. However, it should be remembered that the flow geometry is based on an assumption. In many cases, more than

11 - 11
one pattern of flow may satisfy the differential equations that control the flow. An alternative possibility for the flow topology is shown in figure 3 where the flow is expressed in terms of convenient dimensional vectors to set up the control volume equations.

Considering the total pressure in the stream tube above and below the rotor disc:

\[
p_\infty + \frac{1}{2}\rho V_{1,1}^2 = p_i + \frac{1}{2}\rho(v_i - V_1')^2
\]

\[
p_\infty + \frac{1}{2}\rho(kv_i - V_1')^2 = p_i + \frac{1}{2}\rho(v_i - V_1')^2
\]

\[
T = A(p_i - p_\infty)
\]

\[
= \left(\frac{1}{2}\rho(kv_i - V_1')^2 - \frac{1}{2}\rho V_i'^2\right)A
\]

The thrust is also related to the rate of change of momentum through the control volume:

\[
T = \rho A(v_i - V_1') \cdot kV_i
\]

Re-expressing the velocities in conventional rotor terms:

\[
V_1' = \mu_2\Omega R = -V_1
\]

Substituting into the equations for thrust, relationships identical to those derived for the climb in Appendix 1 are obtained:

\[
T = A(p_i - p_\infty)
\]

\[
= \left(\frac{1}{2}\rho(V_1 + kV_i)^2 - \frac{1}{2}\rho V_i'^2\right)A
\]

\[
T = \rho A(V_i + v_i) \cdot kV_i
\]

Therefore, the climb root solution is applicable to the descending flight case without violating continuity. However, the topology of the flow differs considerably from that usually assumed to derive the climb root of disc momentum theory.

Appendix 3 – Linking Wake Modelling using Momentum Theory to that using Vortex Rings

The previous discussion has shown that momentum theory can go some way towards understanding the behaviour of a helicopter rotor in axial flight, particularly descent. The emphasis on the results is centred on the decision of which mathematical solution should be appropriate. This situation is well seen in figure A.1.

Figure A.1 - Momentum Theory in Axial Flight

Here the passage from a climb and hover methodology has to change to a descent strategy, but it requires a discontinuous jump between solutions to be made. Indeed this decision is forced in some circumstances but is a choice in others. To complicate this situation more is the consideration that the climb solution is required for a rotor moving into slow descent from a hovering condition. The vertical component of flow velocity will be downwards at the rotor disc but upwards far above the rotor. As discussed in the main text, by continuity considerations, this means that a stagnation point will occur above the rotor centre. This is obviously at odds with the momentum based analysis. In order to pursue this problem, due regard is paid to the amount of experimental work which has been conducted into these flows and all show a condition containing a significant amount of vortical type behaviour. In order to move from a momentum theory to one more typical of the experimental results, a bridge to a vortex model is needed. This bridge is relatively easily found in the shape of a vortex tube.

Figure A.2 shows the induced velocity of a single vortex ring. The downflow directly under the centre of the vortex ring shows a substantial radial variation which differs with the actuator disc approach. However, the actuator disc can be viewed as a limiting case whereby the number of blades increases whilst the lift from each decreases in an inverse sense. The wake of such a rotor will change from a set of discrete vortices – which can be simplified to a set of vortex rings – to a continuous distribution of vorticity around a right cylinder based on the rotor periphery. As
an example of this situation, figure A.3 shows the downwash variation for a closely spaced stack of 50,000 vortex rings. The uniformity of the downwash distribution can be seen to link directly to the actuator disc via the vortex tube.

Figure A.2 - Induced Velocity of a Single Vortex Ring

Figure A.3 - Induced Velocity of a Stack of Closely Spaced Vortex Rings

Appendix 4 - Rotor Disc Momentum Theory - Derivation of Windmill Brake Root

In descending flight, another possible topology for the flow involves flow upward everywhere including through the rotor disc, as shown in figure 6. Since thrust is upward, the induced velocity opposes the direction of the free stream flow but is of insufficient magnitude to overcome the free stream velocity. Convenient vector directions are used to set up the problem before transforming into conventional rotor notation. Since the total velocity in the wake is lower than the free stream, the wake expands in cross section. In the corresponding vortex model of the rotor, a sheet made of trailed vortices defines the wake boundary above the rotor disc as shown.

As for the climb root derivations, the total pressure and momentum equations yield expressions for the thrust that allow the unknown induced velocity in the fully developed wake downstream of the rotor to expressed in terms of the induced velocity in the disc plane:

\[ p_\infty + \frac{1}{2} \rho V'_1^2 = p_i + \frac{1}{2} \rho (V'_1 - v_i)^2 \]

\[ p_\infty + \frac{1}{2} \rho (V'_i - k v_i)^2 = p_s + \frac{1}{2} \rho (V'_1 - v_i)^2 \]

\[ T = A (p_i - p_s) \cdot \left( \frac{1}{2} \rho V'_1^2 - \frac{1}{2} \rho (V'_1 - k v_i)^2 \right) \]

Again the thrust is also related to the rate of change of momentum through the control volume:

\[ T = \rho A (V'_i - v_i) \cdot k v_i \]

Where the mass flow through the control volume is that through the disc:

\[ \rho A (V'_i - v_i) \]

and the velocity change is:

\[ k v_i \]

As for the climb root:

\[ k = 2 \]

Substituting into the momentum equation:

\[ \rho A (V'_i - v_i) \cdot 2 v_i = T \]

\[ v_i^2 - V'_1 v_i + \frac{T}{2 \rho A} = 0 \]

Re-expressing in usual helicopter notation, with velocities normalised by tip rotor tip speed and with all upward velocities positive:

\[ v_i = -\lambda \Omega R \]

\[ V'_1 = \mu z \Omega R \]
\[ \lambda_i^2 + \mu_Z \lambda_i + \frac{T}{2\rho A \Omega R} = 0 \]

\[ \lambda_i = \frac{-\mu_Z}{2} + \sqrt{\left(\frac{\mu_Z}{2}\right)^2 - \frac{T}{2\rho A \Omega R}} \]

The expression exhibits real solutions only for high values of \( \mu_Z \) and cannot exist beyond some point as the rotor slows to the hover. Normalizing further with respect to the equivalent hover induced velocity, defined by the climb root solution:

\[ \lambda_{i,HOV} = \sqrt{\frac{T}{2\rho A \Omega R}} \]

\[ \overline{\mu}_Z = \frac{\mu_Z}{\lambda_{i,HOV}} \]

\[ \overline{\lambda}_i = \frac{\lambda_i}{\lambda_{i,HOV}} = -\frac{\overline{\mu}_Z}{2} - \left(\frac{\overline{\mu}_Z}{2}\right)^2 - 1 \]