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THE USE OF ADVANCED AERODYNAMIC MODELS IN THE AEROELASTIC  
COMPUTATIONS OF HELICOPTER ROTORS

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Summary

The development of theoretical and semi-empirical methods enables one to take into account 3D, transonic and unsteady stall effects for a prescribed blade motion, but as the dynamics equations of the coupled aeromechanical system cannot be formulated in a simple manner, the calculations of periodic responses and stability analysis are difficult to perform. A procedure of solution by iteration is discussed for the case of periodic responses.

Introduction

The internal forces resulting from the displacements and deformations of a structure depend on the instantaneous motion and deflection.

After a discretization based on appropriate kinematic assumptions, such as modal representation or finite elements,... the structural response to external loads is governed by a set of second order differential equations.

In the case of a helicopter, the geometry of the coupled rotor-fuselage system varies with the blade azimuth, hence the presence of periodic coefficients in the equations. If the analysis is restricted to small displacements (in a fixed frame for the fuselage and in a rotating frame for the blades) the equations can be linearized and it is even possible, with rotors having more than two blades, to use an appropriate set of rotor variables (Coleman variables) which make the coefficients of the equations independent of the azimuth.

However, for the sake of generality, we state that the structural dynamics model of a flexible helicopter is a set of second order non-linear differential equations with periodic coefficients. These coefficients depend on the kinematic assumptions which define the generalized coordinates and on the distribution of structural stiffness, inertia and dissipation characteristics. The solution may be found with a step by step time integration or using the Floquet's theory of differential equations with periodic coefficients.

It is often assumed that the coupled structure-aerodynamic (or aeromechanical) system is governed by a similar set of equations and that the solution can be found in the same manner. But this is true only if one uses a simplified aerodynamic model which enables one to relate the aerodynamic loads and the structure state variables with differential equations. This possibility does not exist with models resulting from advanced researches in the field of unsteady aerodynamics because the coupled aerodynamic forces are depending, in a complex manner, on the time history of the motion of the lifting surfaces.

## 1. Blade unsteady aerodynamics

The flow over an advancing and rotating blade is so complex, compared to the flow around a fixed wing that the adaptation to rotors of the basic methods of Fluid Dynamics which are operational for wings is a formidable task.

### 1.1. Simplified aerodynamic models

The simplest blade aerodynamic model is based on the assumption of two-dimensional quasi-steady flow. This model is generally associated to an assumption of prescribed induced flow (i.e. the induced velocity is distributed on the rotor disc according to a prescribed function of the azimuth and radial coordinates and it is independent of the blade dynamic responses [10]). Then the angle of attack on each blade section is determined by the blades motion and deflection and can be related to the rotor state variables with kinematic equations. The blade profile lift, pitch moment and drag characteristics are used to determine the aerodynamic loads.

The quasi-steady model can be slightly improved with an additional term providing an aerodynamic damping to the torsion and pitch oscillation. Then the equations damping to the torsion and pitch oscillation. Then the equations relating the local lift, pitch moment and velocity components may be written as :

$$\begin{bmatrix} Nc \\ M \end{bmatrix} = -\frac{1}{2} \rho c^2 \begin{bmatrix} K_{Ni} \\ K_{Mi} & K_{M\dot{\theta}} \end{bmatrix} \begin{bmatrix} Vw \\ Vc \dot{\theta} \end{bmatrix} \quad (1)$$

where :

$c$  is the blade section chordwise length

$\rho$  the air density

$N$  the normal lift per unit length

$M$  the pitch moment per unit length at the reference axis

$V$  the chordwise velocity component

$w$  the velocity component normal to the blade surface (upwash)

$\dot{\theta}$  the pitch oscillation velocity

$K_{Ni}$  the normal lift coefficient relative to the angle of attack

$K_{Mi}$  the pitch moment coefficient relative to angle of attack

$K_{M\dot{\theta}}$  the pitch moment coefficient relative to the pitch oscillation velocity.

The coefficients  $K_{Ni}$ ,  $K_{Mi}$  and  $K_{M\dot{\theta}}$  are depending on the local angle of attack  $\frac{w}{V}$ . The velocity components considered here define the motion of the blade section relative to air (i.e. they result from the

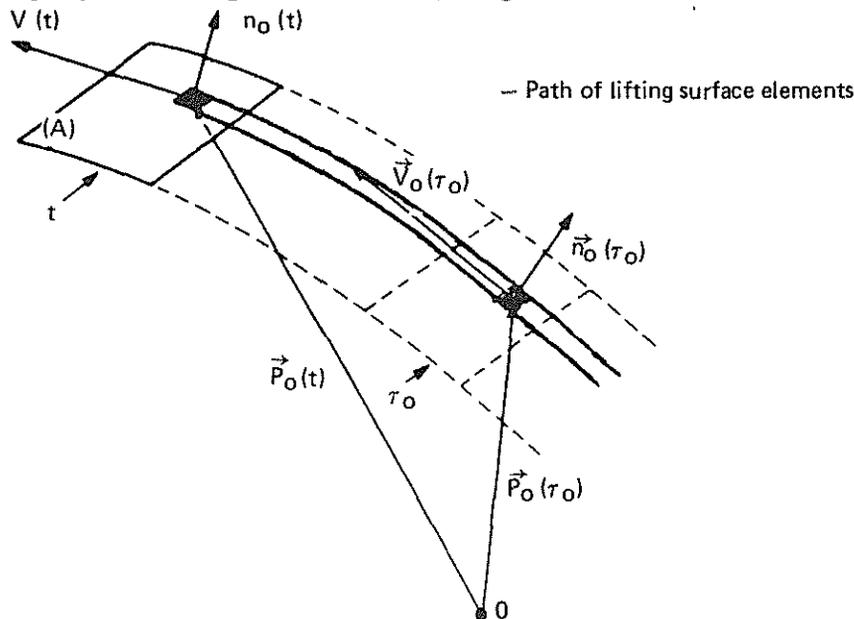
combination of the blade absolute velocity and the fluid induced velocity).

An important feature of this simple model is that the aerodynamic loads depend on the instantaneous state of velocity of the blades. This feature enables one to formulate explicitly the dynamic equations of the coupled aeromechanical system and to solve them either by a time integration procedure or using Floquet's theory of differential equations with periodic coefficients.

The coupled loads predicted by the model are different from coupled structural forces because they do not satisfy the same properties of symmetry (this lack of symmetry is an important feature of aerodynamic coupling which explains certain risks of aeroelastic instabilities). However, the effect of the motion time history which is also an important feature of unsteady flows is not simulated by the quasi-steady model.

### 1.2. The 3D lifting surface theory

The 3D linear lifting surface theory is valid if the angle of attack is small. Then the velocity potential  $\phi$  can be related to the lift  $\Delta p$  by an integral equation, fig. (1). The integration is performed



$$\begin{aligned} \phi(P, t) &= \iint_{(A)} \frac{\Delta p(\bar{P}_0(\tau)) [\bar{P} - \bar{P}_0(\tau)] \cdot \bar{n}_0(\tau) d\sigma_0}{4\pi\rho_\infty c |\bar{P} - \bar{P}_0(\tau)|^2 \left[ 1 - \frac{[\bar{P} - \bar{P}_0(\tau)] \cdot \bar{V}_0(\tau)}{c |\bar{P} - \bar{P}_0(\tau)|} \right]} \\ &+ \iint_{(A)} \int_{-\infty}^t \frac{\Delta p(\bar{P}_0(\tau_0)) [\bar{P} - \bar{P}_0(\tau_0)] \cdot \bar{n}_0(\tau_0)}{4\pi\rho_\infty |\bar{P} - \bar{P}_0(\tau_0)|^3} d\tau_0 d\sigma_0, \\ t - \tau &= \frac{|\bar{P} - \bar{P}_0(\tau)|}{c} \end{aligned}$$

Fig. 1 – Linear lifting surface theory. Integral equation relating the velocity potential to the lift time history of an element of lifting surface performing an arbitrary motion (from ref. [2]).

over the path of the lifting surface elements (prescribed wake) and so depends on the time history of the lift [1]. In order to find the periodic solutions in the case of forward flight, we assume that  $\Delta p$  is a linear combination of prescribed functions of the radial coordinate and azimuth and solve with a collocation method. The procedure gives an aerodynamic matrix which relates the lift coefficients to the values of the normal velocity (or upwash) at collocation points distributed over the rotor disc. This formulation is extremely convenient, as long as the assumption of small angle of attack is valid, to determine the periodic solution in the form of a limited Fourier series [4] and fig. 2.

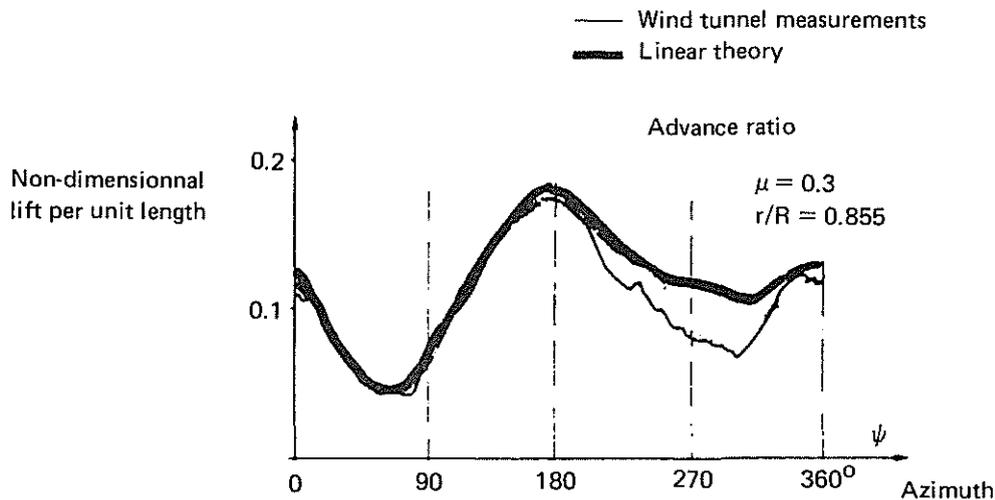


Fig. 2 — Calculation of a blade lift distribution with the 3D linear lifting surface theory (from ref. [2]). The discrepancy between experiment and theory toward azimuth  $270^\circ$  results from the effect of retreating blade unsteady stall.

### 1.3. Semi-empirical two-dimensional model with unsteady stall

The two-dimensional semi-empirical model implemented at ONERA to predict the unsteady aerodynamic loads on the retreating blade has already been described in ref. [5-8]. The model uses a set of differential equations with incidence dependent coefficients to relate the components of blade profile aerodynamic forces and velocity. As the equations contain lift and pitch moment time derivatives, the aerodynamic loads result from a time integration which makes them dependent on the blade motion time history as real unsteady flow are. The non-linear effects result from the variation of the coefficients with angle of attack (fig. (3)).

The model enables one to formulate explicitly the full dynamic equations of the coupled aeromechanical system, but the time derivatives of the aerodynamic forces introduce artificial aerodynamic degrees of freedom (similarly to the augmented states used in ref. [11]) which increase considerably the computing time necessary for the solution of the full equations.

The two-dimensional model can be associated with the linear lifting surface theory to predict the combined effects of unsteady stall and 3D flow (fig. (4)). But when this is done, it becomes impossible to formulate explicitly the equations relating the aerodynamic loads to the generalized coordinates [2, 12].

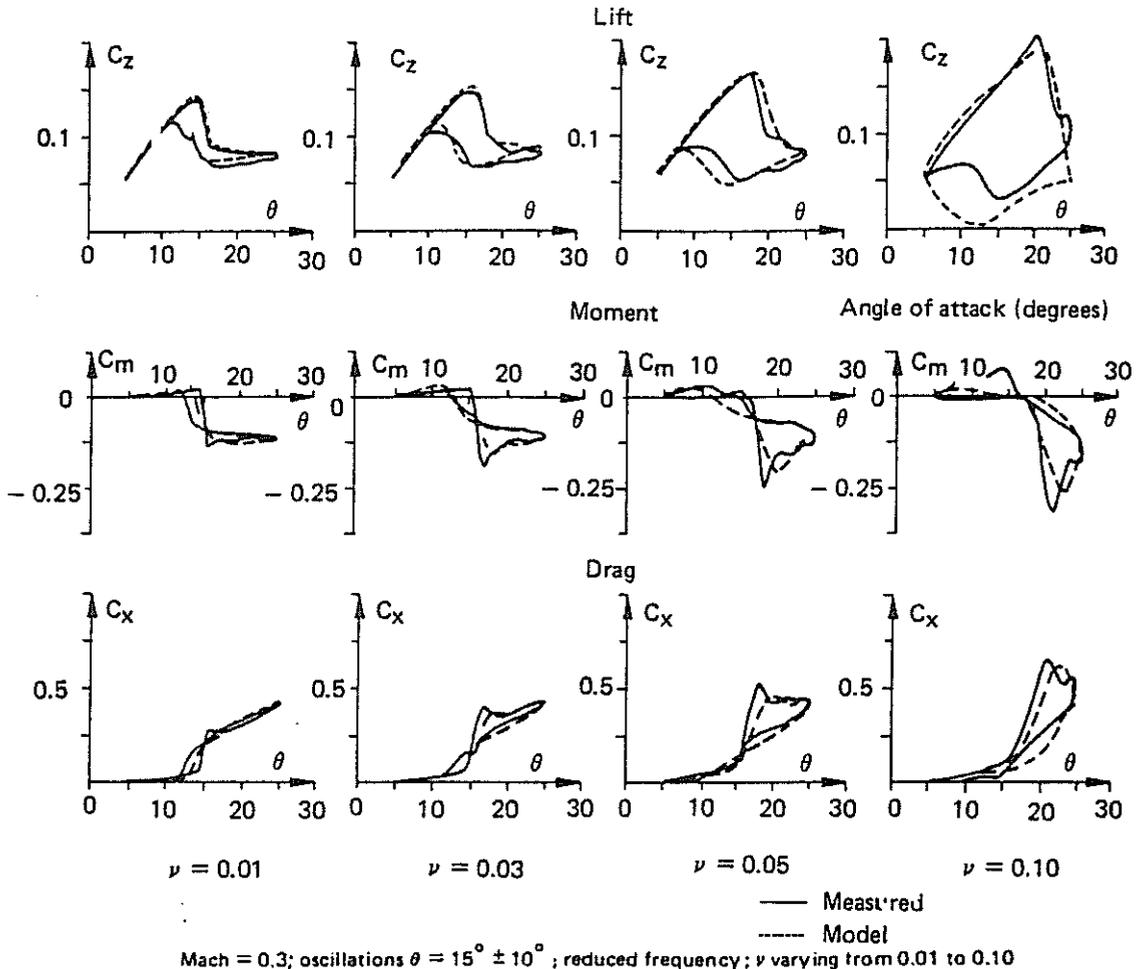


Fig. 3 — Lift, moment and drag-incidence hysteresis loops found on an oscillating NACA 0012 profile with the 2D semi-empirical model based on differential equations with incidence dependant coefficients.

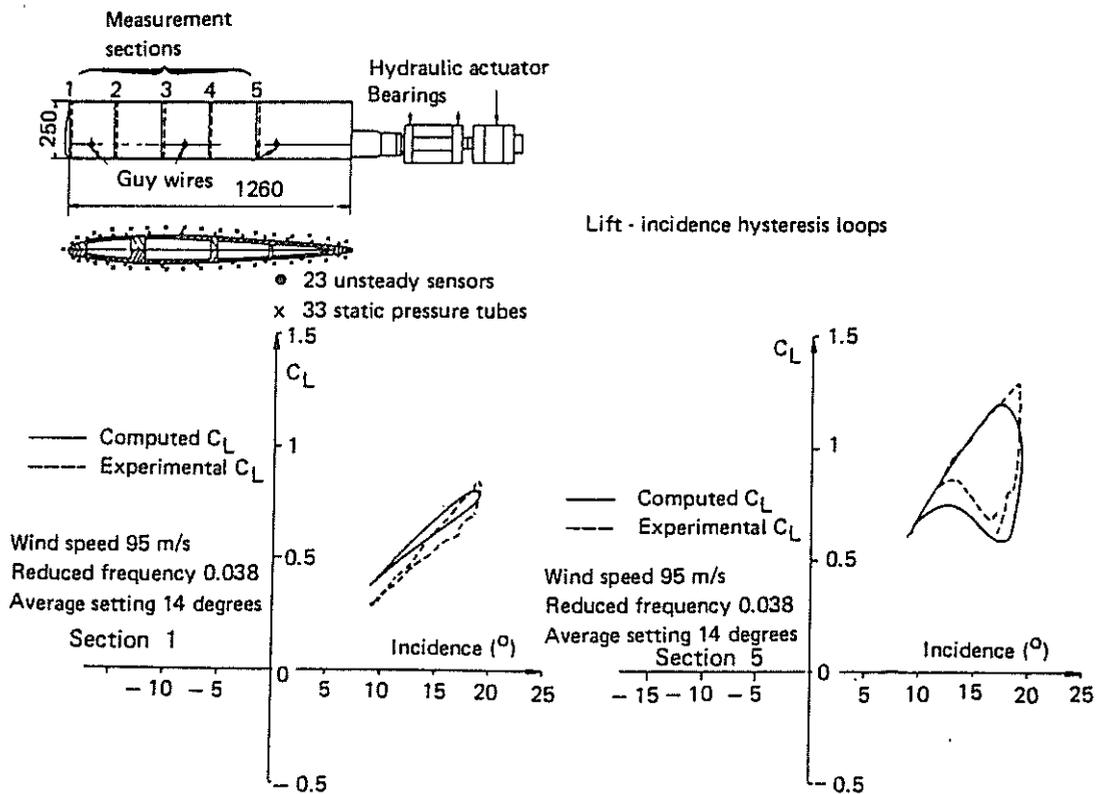


Fig. 4 — Investigation of 3D unsteady stall on a oscillating rectangular wing.

#### 1.4. Transonic effects

The computations of transonic flow on the advancing blade tip are based on the solution of the Transonic Small Perturbation equation (TSP) or the Euler equations. Satisfactory results have been obtained as shown on fig. (5), but in this case, it is also impossible to formulate explicitly the equations relating the generalized aerodynamic forces and the rotor state variables [9].

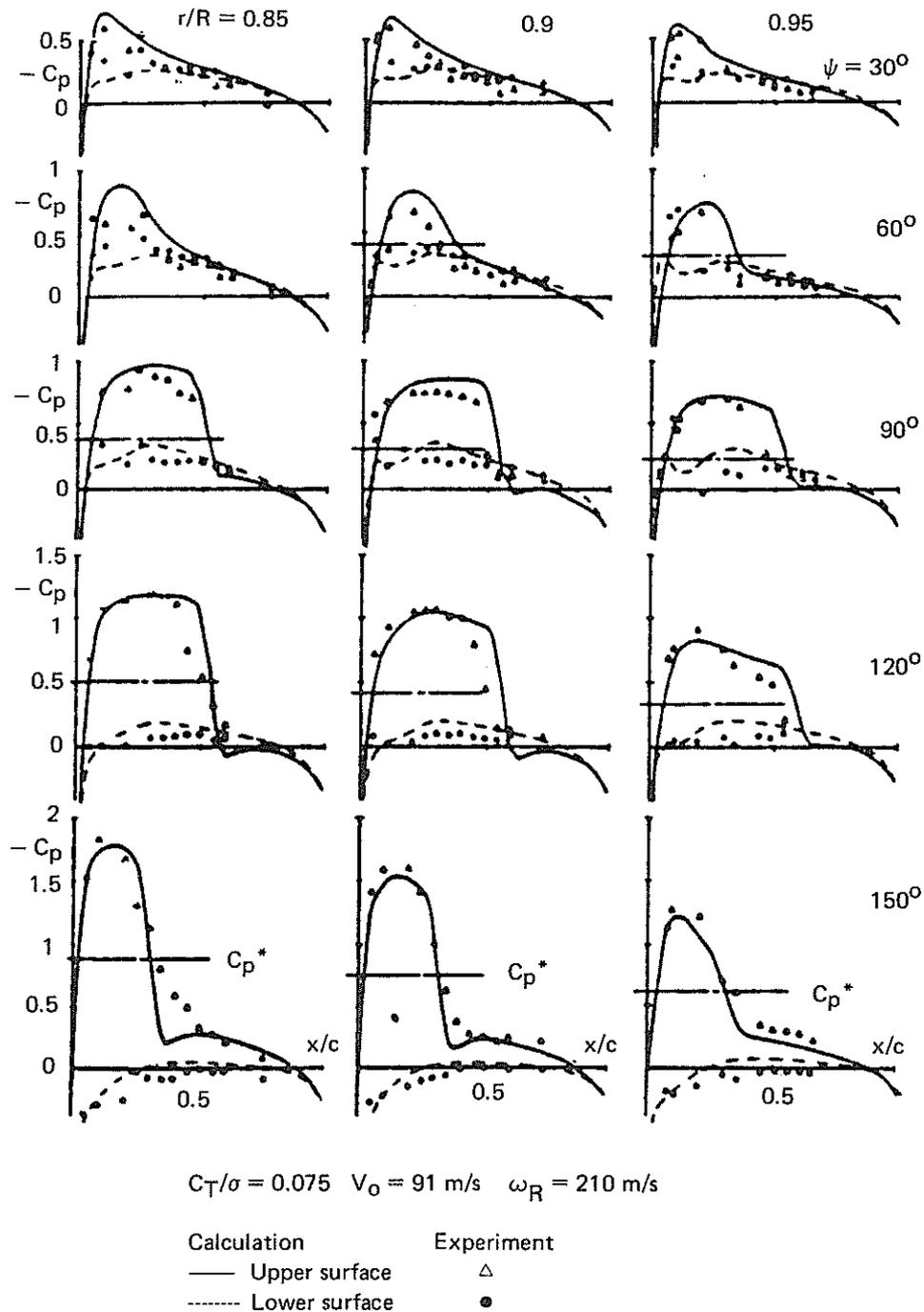


Fig. 5 — Experimental and computed pressure distributions on rectangular blade tips, (ONERA TSP code), from [9].

Consequently, this short discussion shows that the advanced aerodynamic models enables one to predic the 3D, transonic and unsteady stall effects on an advancing rotor, for a given blade periodic motion.

They also could be adapted to the prediction of the aerodynamic loads for a given transient blade motion. However they are not formulated in a manner which is suitable for their incorporation into the dynamic equations of the coupled aeromechanical system.

## 2. Calculation of the coupled aeromechanical system response and stability

The response and stability of the coupled aeromechanical system is often predicted with simplified aerodynamic models, such as two-dimensional quasi-steady, or slightly improved quasi-steady models because these models facilitate the explicit formulation of the full dynamics equations of the coupled system. The solution can be performed with a time integration procedure or using the Floquet's theory for equations with periodic coefficients.

As it has been shown, that possibility does not exist with the more sophisticated models which take into account the flow time history, 3D, transonic and unsteady stall effects. In this case, the full coupled problem can be solved only with an iteration procedure.

For the sake of simplicity, the solution by iteration will be discussed first in the case of the fixed wing aircraft.

### 2.1. Iteration procedure for a fixed wing

The linear equation which determines the frequency response of a flexible aircraft to external forces may be written as :

$$[Z(i\omega) + G(i\omega)] q = Q \quad (2)$$

with  $Z = -\omega^2 \mu + i\omega \beta + \gamma$

Z is the structural impedance matrix

G the aerodynamic transfer function matrix relating the coupled generalized aerodynamic forces to the generalized coordinates

q is the column of generalized coordinates which determines the vibration deflection through kinematic assumptions

Q the column of generalized external forces (e.g. forces due to turbulence or excitation forces provided by shakers in a flight vibration test).

The numerical values of the coefficients  $G_{kl}$  can be computed for given value of  $\omega$ , but their variation with  $\omega^{kl}$  cannot be formulated explicitly.

Even if equation (2) can be solved directly, this simple case is considered because it makes possible a preliminary discussion of the iteration procedure before considering the complex application to helicopters.

Let S be an approached aerodynamic matrix based on a simplifying assumption (e.g. quasi-steady flow). Equation (2) may be written as :

$$[Z + S] q = Q - [G - S] q \quad (3)$$

The solution by iteration can be formulated using the following equations :

$$\Delta\phi_i = S q_i - G q_i \quad (4.1)$$

$$[Z + S] q_{i+1} = Q + \Delta\phi_i \quad (4.2)$$

Equation (4.1) denotes the calculation of an aerodynamic error vector which is the difference between the approached and "exact" aerodynamic forces for the oscillation found at the  $i^{\text{th}}$  iteration step.

Equation (4.2) must be solved to determine  $q_{i+1}$  which defines the oscillation at the  $(i+1)^{\text{th}}$  step.

$q$  being the exact solution of (3), we have :

$$q_{i+1} - q = - [Z + G - (G - S)]^{-1} [G - S] (q_i - q)$$

This equation suggests that the iteration generally converges if the matrix  $G - S$  which has been separated from the full impedance matrix  $Z + G$  is a small part of this last matrix.

Consequently, convergence difficulties may be expected in the following cases :

- a) if the impedance matrix  $Z + G$  is almost singular, a situation which may happen if the frequency  $\omega$  is close to the resonance frequency of a weakly damped mode,
- b) if the structure is light and flexible resulting in small values of the generalized masses and stiffnesses in the impedance matrix.

When the convergence is not satisfactory, the approached aerodynamic model should be adjusted in order to minimize the difference with the "exact" model. This adjustment may be performed with a parameter identification method. It is always possible to consider a particular vibration deflection as a reference and to find the coefficients of the approached model which minimize the difference  $\|S q_R - G q_R\|$ , where  $q_R$  is the column of generalized coordinates corresponding to the reference vibration deflection.

## 2.2. Application to a helicopter rotor

This iteration procedure can be used to predict the periodic loads and deflections on helicopter rotor blades in forward flight. In this application, the iteration is the only procedure which makes possible the calculations with advanced aerodynamic models.

The dynamics equations of a helicopter in forward flight may be written as :

$$\phi_S(q, \dot{q}, \dots, \theta, \dot{\theta}) + \phi_E = 0 \quad (5)$$

$\phi_S$  denotes the structural generalized force vector which can be formulated explicitly as function of the generalized coordinates, pitch angle and their time derivatives.

$q$  is the column of generalized coordinates which determine the

blades motion and deflections through kinematic equations resulting from the assumptions used in the process of discretization (e.g. modal analysis or finite elements)..

$\theta$  is the blade pitch angle (collective + cyclic pitch).  
 $\phi_E$  is the vector "exact" generalized aerodynamic forces.

Generally this last vector is determined by the whole periodic motion of the blades and cannot be formulated as function of  $q, \dot{q}, \dots, \theta$  and  $\dot{\theta}$  like the structural forces, but the variation of  $\phi_E$  with the azimuth  $\psi$  can be determined with computation codes based on advanced aerodynamic theories if the blades motion,  $q(\psi)$  and  $\theta(\psi)$  is given.

If the blades deflections are small, the vectors  $\phi_S$  and  $\phi_E$  depend linearly on the periodic motion. Then it is possible to build the solution as a superposition of several prescribed periodic motions (see § 1.2). But this is not possible in most real cases, when non-linear effects cannot be neglected. Consequently, the solution must be found by iteration.

The iterative procedure described here used an approached aerodynamic model which enables one to relate the aerodynamic forces and the generalized coordinates with a system of differential equations with periodic coefficients. The approached aerodynamic forces may be written as :

$$\phi_A = \phi_A (q, \dot{q}, \dots, \theta, \dot{\theta})$$

If the approached aerodynamic model is the quasi-steady model of § 1.1,  $\phi_A$  is determined by equation (1) and by the kinematic equations relating the velocity components  $V, w$  and  $\theta$  to the generalized coordinates.

Equation (5) may be written as :

$$\phi_S (q, \dot{q}, \dots, \theta, \dot{\theta}) + \phi_A (q, \dot{q}, \dots, \theta, \dot{\theta}) = \phi_A (q, \dot{q}, \dots, \theta, \dot{\theta}) - \phi_E$$

and the iteration process is defined by the two equations :

$$\Delta\phi_i = \phi_A (q_i, \dot{q}_i, \dots, \theta, \dot{\theta}) - \phi_{Ei} \quad (6.1)$$

$$\phi_S (q_{i+1}, \dot{q}_{i+1}, \dots, \theta, \dot{\theta}) + \phi_A (q_{i+1}, \dot{q}_{i+1}, \dots, \theta, \dot{\theta}) = \Delta\phi_i \quad (6.2)$$

Equation (6.1) denotes the calculation of the "error" aerodynamic vector (difference between approached and "exact" aerodynamic forces) for the periodic motion found at the  $i^{\text{th}}$  iteration step.

$q_{i+1}$ , which defines the periodic solution at the  $(i+1)^{\text{th}}$  iteration step, is the solution of (6.2). This equation is a system of differential equations with periodic coefficients with a forcing function  $\Delta\phi_i$ .

Similarly to the fixed wing, the procedure converges only if the relative difference between the "exact" and approached aerodynamic models is small enough.

Convergence difficulties may be expected :

- if a resonance frequency is close to the rotor r.p.m or to a harmonic of it,
- if the rotor is relatively flexible.

The applications show that the presence of blade torsion modes in the modal representation tends to make the convergence difficult, probably because the aerodynamic pitch moment coefficients are difficult to evaluate and because there is a strong asymmetric aerodynamic coupling between torsion and bending modes.

When convergence difficulties are encountered, it is necessary to adjust the approached model in order to minimize the difference with the "exact" model.

Different methods may be used. The method suggested here can be implemented easily.

Using the quasi-steady model of § 1.1 as approached model, the generalized aerodynamic forces (which are resulting from equation (1) and from the kinematic equations relating the velocity components to the generalized coordinates) are depending linearly on the coefficients of the model,  $K_{Ni}$ ,  $K_{Mi}$  and  $K_{M\dot{\theta}}$ .

Then, for a reference periodic motion defined by the generalized coordinates  $q_R$  ( $\psi$ ), the vector  $\phi_A$  can be related to the three coefficients by a matrix equation :

$$\phi_A (\psi) = \begin{bmatrix} M_R (\psi) \end{bmatrix} \begin{bmatrix} K_{Ni} \\ K_{Mi} \\ K_{M\dot{\theta}} \end{bmatrix}$$

An adjusted aerodynamic model can be derived from the initial approached model by replacing the coefficients  $K_{Ni}$ ,  $K_{Mi}$  and  $K_{M\dot{\theta}}$  respectively by  $K_{Ni} + \Delta K_{Ni}$ ,  $K_{Mi} + \Delta K_{Mi}$ ,  $K_{M\dot{\theta}} + \Delta K_{M\dot{\theta}}$ .

If  $\tilde{\phi}_A$  denotes the generalized aerodynamic forces given by the adjusted model, we have :

$$\tilde{\phi}_A = \phi_A + \begin{bmatrix} M_R \end{bmatrix} \begin{bmatrix} \Delta K_{Ni} \\ \Delta K_{Mi} \\ \Delta K_{M\dot{\theta}} \end{bmatrix}$$

The difference between "exact" and "adjusted" aerodynamic forces is given by :

$$\tilde{\phi}_A - \phi_E = \phi_A - \phi_E + \begin{bmatrix} M_R \end{bmatrix} \begin{bmatrix} \Delta K_{Mi} \\ \Delta K_{Mi} \\ \Delta K_{M\dot{\theta}} \end{bmatrix}$$

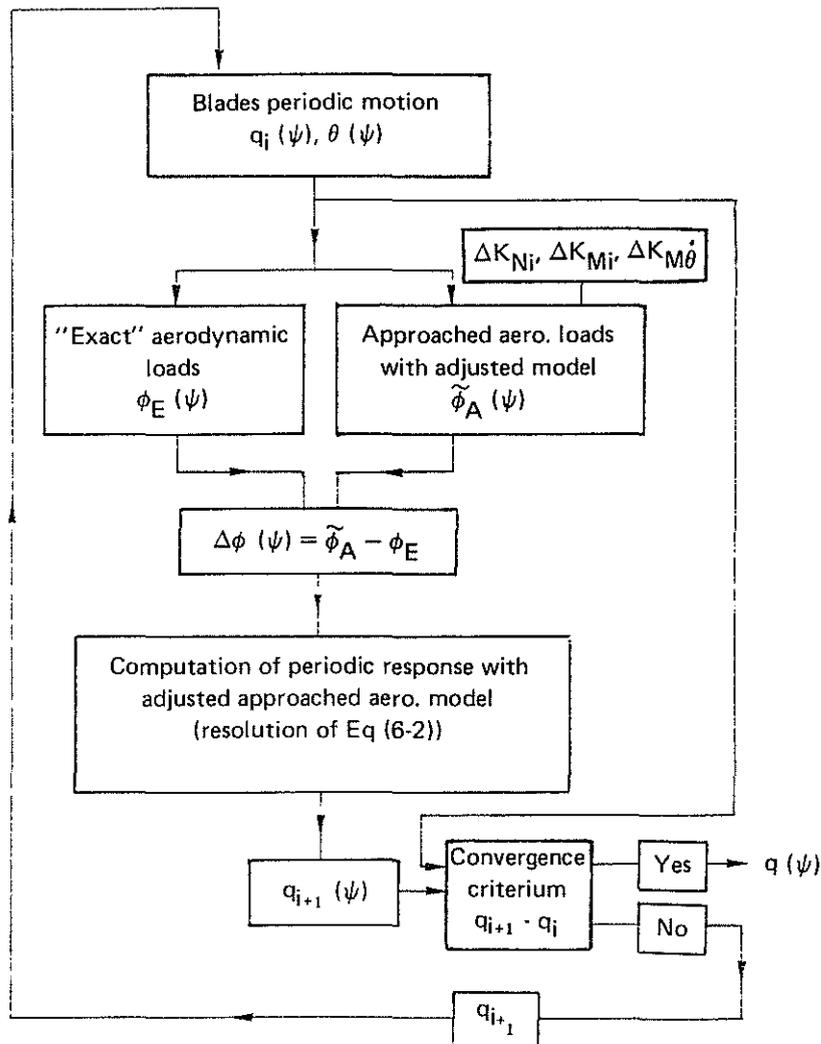
A least square solution may be used to determine the values of the additional coefficients which minimize  $\|\tilde{\phi}_A - \phi_E\|$ .

Since  $\phi_A$ ,  $\phi_E$  and  $M_R$  depend on the azimuth  $\psi$ , the adjustment can be performed at different values of  $\psi$  and so the additional coefficients are functions of  $\psi$ .

The adjustment of the approached aerodynamic model must be considered as an important sequence to make the iteration procedure successful.

The periodic solution given by the approached aerodynamic model (solution of equation (6.2) with  $\Delta\phi_i = 0$ ) may be used as reference periodic motion for this adjustment.

In the block diagram fig. (6), the computation of the periodic response with approached aerodynamic forces denotes the solution of the differential equations with periodic coefficients (6.2). This solution may be carried out with a classical method: step by step time integration or application of Floquet's theory.



In the same manner, the iteration enables one to use any computation code resulting from advanced researches in the field of unsteady aerodynamics to compute the "exact" aerodynamic forces.

The iteration procedure is in the process of development at ONERA. Figure (7) illustrates results obtained so far. In this calculation, the "exact" aerodynamic forces were computed with the two-dimensional unsteady model of § 1.3 and the approached forces were given by the quasi-steady model. No convergence difficulty was found in that application which is at a relatively low advance ratio.

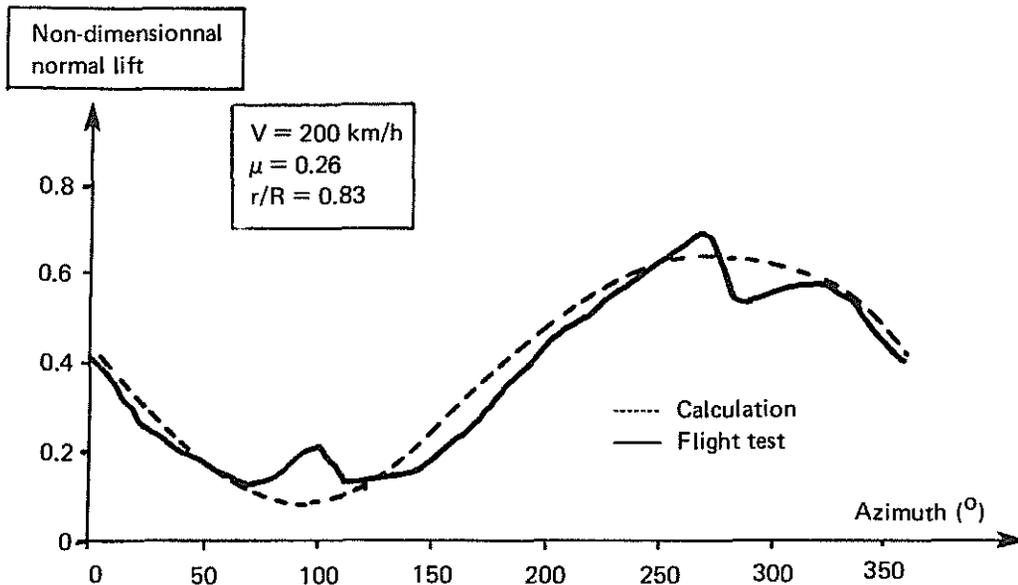


Fig. 7 - Application of the iteration procedure to the calculation of the periodic loads and deflections of the A 349 helicopter.

As already mentioned, the two-dimensional unsteady model introduces artificial aerodynamic degrees of freedom which make the direct solution of the full dynamic equations difficult and increase the computing cost. This difficulty is not found with the iteration procedure and so the computing time is much smaller.

The calculations performed so far show that convergence difficulties are met at high advance velocity when the blade torsion modes are included in the modal representation.

### 2.3. Application to stability investigations

The stability analyses are often performed with simplified aerodynamics models.

The extension of the iteration procedure discussed above into stability investigations implies that transient motions are considered instead of periodic motions. This is possible, in principle, but extremely difficult to implement.

Another possibility consists of using a simplified aerodynamic model whose coefficients can be "identified" at each iteration step with the "exact" aerodynamic forces.

But obviously, the implementation of advanced aerodynamic models in stability analysis remains a difficult problem [11].

#### Concluding remarks

The prediction of helicopter dynamics and vibration responses is still very difficult.

Some difficulties are common to helicopters and airplanes, but the major ones are specific to rotorcraft.

In the field of unsteady aerodynamics, the complexity of the flow due to the combination of the blade rotation and translation motions is such that there is always a considerable delay between the development of new calculation techniques for the airplanes and their application to helicopter blades.

It has also been shown that the advanced aerodynamic models cannot be coupled with the structural dynamic equations in a simple manner. The fundamental reason for that is that the unsteady aerodynamic forces depend on the time history of the blades motion. As a consequence of that complexity, the use of an advanced aerodynamic model implies a solution by iteration of the full coupled aeromechanical problem.

An iteration algorithm is in the process of development at ONERA. This development has been found necessary to implement modern methods of unsteady aerodynamics in the calculations of helicopter performance and vibration.

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