FOURTEENTH EUROPEAN ROTORCRAFT FORUM

PAPER NO. 48

A COMPREHENSIVE APPROACH TO COUPLED ROTOR-FUSELAGE DYNAMICS

P T W JUGGINS

WESTLAND HELICOPTERS LIMITED
YEOVIL ENGLAND

20-23 SEPTEMBER 1988
MILANO, ITALY

ASSOCIAZIONE INDUSTRIE AEROSPAZIALI
ASSOCIAZIONE ITALIANA DI AERONAUTICA ED ASTRONAUTICA

A COMPREHENSIVE APPROACH TO COUPLED ROTOR-FUSELAGE DYNAMICS

P T W JUGGINS
WESTLAND HELICOPTERS LTD., YEOVIL, ENGLAND

ABSTRACT

In order to fully describe, by theoretical analysis, the behaviour of a helicopter rotor system, in terms of its dynamic characteristics (natural frequencies and mode shapes), stability and in-flight response and loads, it is necessary to adequately model the additional dynamic systems with which the rotor is coupled. These systems are principally the fuselage (gearbox and engine bodies, isolation and absorber systems and flexible structure), the control circuit (upper controls, swashplate and actuators) and the transmission system (gears, shafts, couplings and engine components).

Previous approaches to this problem have typically calculated the dynamic characteristics of a single blade in a hub-fixed configuration, with perhaps a spring stiffness representation of the control circuit. The natural frequencies and mode shapes have then been used in subsequent calculations of blade stability, rotor-body stability (ground and air resonance), blade loads, fuselage vibration and transmission system dynamics.

This paper describes a comprehensive approach to the representation of these coupled dynamic systems in which the mode shapes and natural frequencies of the complete assembly of systems are calculated, using a rotor model which includes the other component systems as boundary conditions in terms of impedance models. In order to do this, the Coupled Rotor-Fuselage Dynamics (CRFD) model adopts a multiblade transfer-matrix method.

Some initial validation exercises for the CRFD analysis are described. Further work is required to extend the usefulness of the analysis and to complete the definition of a complementary theoretical model for the prediction of coupled rotor-fuselage aeroelastic response.

1 INTRODUCTION

A common method of determining the dynamic characteristics of a helicopter rotor system is to calculate the natural frequencies and mode shapes of a single hub-fixed rotating blade. From the frequency placements and content of the modes, an initial evaluation of a given rotor design may be made against undesirable resonances and couplings, in terms of expected loads and stability. Subsequently, the blade modes may be used as degrees of freedom in calculations of predicted loads, rotor stability and coupled rotor-body stability (ground and air resonance). Fuselage vibration may be predicted by applying the calculated rotor loads to a finite element model of the fuselage, with an inertia representation of the rotor. Such methods for loads, stability and vibration prediction have been well validated against test results (References 1 and 2).

Some important aspects of rotor behaviour may not be adequately described by these methods, however. In order to fully describe the behaviour of the rotor system, it is necessary to adequately model the additional dynamic systems with which the rotor is coupled, in terms of their effects on the rotor as well as (conventionally) the rotor's action on them. Previous comprehensive rotorcraft models such as C81, CAMRAD and GRASP have been reported in literature (Reference 3, 4 and 5, respectively).

A programme of research has been undertaken at Westland (WHL) to develop analytical methods in which the fullest description of the coupled behaviour of the rotor and fuselage systems is obtained, using a comprehensive theoretical model. Applications of such a capability can be specifically identified in the modelling of helicopter manoeuvres, including transition to the hover and limit cases, the optimisation of airframe dynamics, and aeroelastic tailoring of blade design.

The initial stage of the work was to evaluate alternative methods for calculating the natural frequencies and mode shapes of the total coupled system, such that the modes calculated might be used as degrees of freedom in a new response analysis. As a result of that evaluation, an analysis for Coupled Rotor-Fuselage Dynamics (CRFD) has been written, and progress has been made on a corresponding aeroelastic model for calculation of responses and loads, referred to as the Coupled Rotor-Fuselage Aeroelastics (CRFA) analysis. A further phase of this research activity is commencing, to enhance the efficiency of the CRFD computer code as a working design tool, and complete the first version of CRFA code. The activity is being performed under UK Ministry of Defence sponsorship, in collaboration with the Royal Aerospace Establishment, Farnborough.

This paper describes some brief examples of problems to be addressed by a coupled rotor-fuselage approach, and the development and initial validation of the CRFD analysis.

2 EXAMPLES OF COUPLED ROTOR-FUSELAGE EFFECTS

Some examples of coupled system effects can be considered which are not adequately described by separate theoretical analysis of the rotor and fuselage systems.

2.1 Blade Lead-Lag Dynamics

Modifications to the existing WHL rotor blade modes prediction analysis have enabled a transmission system model to be included, using an impedance representation. In Table 1, predicted blade lead-lag frequencies, with and without the transmission included, are tabulated for a Lynx main rotor. The presence of the transmission has a significant effect on the frequencies. In particular, the second blade lead-lag mode has moved from 4.31R to 3.48R. This frequency shift will only occur for collective motion of the blades, when forced at 4R, 8R etc.. Flight test results for Lynx show that 4R mast stresses increase as rotor speed is decreased. This suggests that the effective value of the collective second lead-lag mode is below 4R, as indicated by the prediction.

2.2 Blade Torsion Dynamics

The value of the stiffness of the control circuit seen by a single blade depends on the motion of all the blades of the rotor, producing relative motion in different components of the control circuit. In Table 2, fundamental torsion frequencies calculated for a single Lynx blade are listed, for collective, cyclic (lateral and longitudinal) and reactionless motions of the blades. It can be seen that there are significant differences between the frequencies. In conventional single-blade analysis only one of these modes may be used in subsequent calculations of rotor response loads. A coupled rotor-fuselage analysis allows rotor modes, rather than blade modes, to be included, and a total description of the blade torsion behaviour is made.

2.3 Fuselage Dynamics and Hub Motion

The natural frequencies and mode shapes for the helicopter airframe are typically calculated with an inertia representation of the rotor, at the hub. There are clearly limitations in the accuracy of this type of representation of the rotor.

An assessment of the effect of rotor/fuselage coupling on fuselage vibration predictions was made, using a simplified structural model, in Reference 6. A conclusion of that work was that the magnitude of the resonant response at the fuselage mode frequencies is highly dependent on their proximity to the blade modal frequencies. In Reference 7, the effect of a flexible rotor model on predicted fuselage mode frequencies was not significant. Clearly the significance of coupled rotor-fuselage effects on fuselage modes and response is likely to depend on the characteristics of the rotor-fuselage system considered, and especially on the coalescence or otherwise of rotor and fuselage mode frequencies, and whether fuselage modes are close to resonance at the predominant forcing frequency.

If the presence of the rotor may affect fuselage dynamics, then conversely it might be expected that hub motions may have signficant effects on rotor loads, as suggested in Reference 8.

2.4 Tail Rotor Stability Example

Example calculations for a tail rotor have shown large effects on blade stability margins when transmission system and control circuit impedance models are included in a single blade analysis. Some results from these calculations, performed using the WHL blade modes analysis and single blade stability analysis, are shown in Figure 1, for the first blade lead-lag eigenvalue. The control circuit models, for hydraulics on and hydraulics off cases, were based on two-mass-spring-damper representations of measured dynamic characteristics for collective rotor motions, while the transmission impedance was made up from a ten-mode transmission system model.

The results are given for collective, cyclic and reactionless rotor modes. In the cyclic and reactionless cases no transmission model is included, and the control circuit may be represented by a simple spring stiffness (different for each case). This example serves to illustrate the importance of considering rotor modes, rather than single blade modes.

3 AN INVESTIGATION OF ALTERNATIVE METHODS FOR A COUPLED ROTOR-FUSELAGE ANALYSIS

Alternative methods of analysis identified and subsequently investigated could be classified under four categories:-

(a) Classical impedance matching, using free-free blade modes.

(b) The use of hub-fixed modes superimposed on hub motion.

(c) An imposed blade root condition method, applied to a single blade.

(d) The imposed blade root condition method, using multiblade degrees of freedom.

Also assessed, to be applied within an overall method - strategy, was:-

(e) The use of complex modes.

In all the methods considered, it was assumed that the fuselage systems would be modelled by impedance representations.

Method (a) was considered to be unsuitable due to the necessity for the step of calculating free-free blade modes, which are of little use in themselves. Method (b) has been used successfully in coupled rotor-body stability predictions. It is the approach adopted in the WHL ground resonance analysis (Reference 2) and the AGEM program developed at City University (Reference 7) except that the latter program uses a numerical generation of the equations of motion.

Experience with this approach has shown that the results are sensitive to the number and location of radial points used to describe the blade. As the number of radial points is increased, the advantage (in computation time) of using the hub-fixed modes as degrees of freedom decreases. In the case of the CRFD analysis, it was decided to develop a direct method of deriving the coupled system modes, avoiding the intermediate step of calculating fixed-hub modes.

The basis of Methods (c) and (d) was that the presence of hub motions may be accommodated by modifying the assumed boundary conditions for the rotating blade. This approach had already been applied to the WHL single blade modes analysis for modelling effects of transmission system and control circuit impedances (See Section 2, above). Both of these impedances may be expressed in the rotating system for a single blade with no time-dependant coefficients, provided that the appropriate mode of rotor motion is assumed (ie collective motion for the transmission, collective, cyclic or reactionless motion for the control circuit). A comprehensive analysis must be able to model all coupled system motions, including hub translations and rotations perpendicular to the axis of rotor rotation, and motion in which collective and cyclic rotor modes may be coupled through the hub impedance. A single blade model, including such a capability, inevitably includes time-dependent coefficients. In order to remove this time-dependency, the equations of motion may be transformed into multiblade degrees of freedom.

The approach selected for development of the CRFD analysis was the imposed blade root condition method, using multiblade degrees of freedom (method d).

The retention of velocity terms in the CRFD analysis was considered to give identifiable advantages. Confidence that a full description of the coupling between rotating and fixed systems had been achieved was greater with a complex analysis (method (e)). Linearised Coriolis terms could be retained in CRFD, and hence need not be included in the CRFA response program. In addition, complex modes enabled important effects of the blade lead-lag damper to be included in the mode shapes. At the conclusion of the current phase of CRFD research, the use of real modes remains an option, at least for input to the initial development versions of CRFA.

4 APPLICATION OF ALGEBRAIC COMPUTING

The equations of motion for the CRFD analysis were derived using the REDUCE algebraic computing software, available from the Rand Corporation. After experience had been gained in the application of this software, using simplified examples, the considerable advantages of algebraic computing over hand-derivation could be realised, in terms of shortened time scales and lessened likelihood of errors. A recognised disadvantage of this approach was the reduced visibility of the derivation of the analysis to the dynamicist. This could lead to errors or inefficiencies in analytical technique rather than in the algebraic manipulations.

5 DESCRIPTION OF THE CRFD ANALYSIS

The rotor blade is defined by a continuous beam model similar to that developed for WHL program J146, where it was used to calculate the natural frequencies and mode shapes of a bearingless rotor blade (Reference 2). It also bears some resemblance to the approach of Reference 9.

The derivation of the equations of motion, defined at a point on the blade reference axis, proceeds by application of Hamilton's Principle to expressions for kinetic energy, strain energy and virtual work. Once derived for a single blade, these equations are transformed into multiblade degrees of freedom, to describe the motion of a point in a rotor made up of a number (greater than two) of identical blades.

The fuselage is expressed as a frequency-dependent impedance, derived from natural frequencies, modal damping and hub modal motion for the fuselage alone.

5.1 Definition of the Equations of Motion

The equations of motion for the system are obtained when Hamilton's Principle is applied to the following energy components:

Blade Kinetic Energy
Blade Strain Energy
Virtual Work from (i) Blade Internal Loads
(ii) Blade Distributed External Loads (for the steady state).
(iii) Blade Gravitational Loads (for the steady state).
(iv) Hub Reactions.

The equations of motion for a single (ith) blade plus fuselage (hub motion) may be written in terms of coefficient matrices as:-

$$A_0 \cdot \underline{U}_i' + A_1 \cdot \underline{U}_i + A_2 \cdot \underline{F}_i + \underline{SA}_i = \underline{0}$$

$$B_0 \cdot \underline{F}_i' + B_1 \cdot \underline{F}_i + B_2 \cdot \underline{U}_i + B_3 \cdot \dot{\underline{U}}_i + B_4 \cdot \ddot{\underline{U}}_i + B_{5i(t)}\dot{\underline{H}} + B_{6i(t)}\ddot{\underline{H}} + \underline{SB}_i = \underline{0}$$

where the notation is as follows:

- ′ Differentation with respect to blade radius
- . First time derivative
- .. Second time derivative
- $\underline{U}_i$ Vector of three translations and three rotations at a point on the blade axis of shear centres
- $\underline{F}_i$ Vector of three shear forces and three bending moments at a point on the blade axis of shear centres.

A_0 , A_1 , A_2 , B_0 , B_1 , B_2 , B_3 , B_4 , B_{5i} , B_{6i}

Coefficient matrices, with B_{5i} and B_{6i} functions of blade azimuth and hence time, t.

$\underline{SA}_i$, $\underline{SB}_i$

Vectors of constant terms, from external loads and steady state linearisation correction terms.

$\underline{H}$

Vector of three translations and three rotations at the rotor centre line, due to hub motions.

Note that $\underline{U}_i$ and $\underline{F}_i$ are vectors defined in a rotating frame of reference. $\underline{U}_i$ is defined relative to hub motion $\underline{H}$, which is defined in a fixed frame of reference.

The solution is defined as consisting of steady plus perturbatory components in $\underline{U}$ and $\underline{F}$, and perturbatory components only in $\underline{H}$.

The steady-state solution proceeds for a single blade, hub-fixed condition, using the equations:

$$A_0\underline{U}' + A_1\underline{U} + A_2\underline{F} + \underline{SA} = \underline{0}$$

$$B_0\underline{F}' + B_1\underline{F} + B_2\underline{U} + \underline{SB} = \underline{0}$$

If the time-dependent, periodic, nature of B_{5i} and B_{6i} is stated explicitly as

$$B_{5i} = B_{50} + B_{5C}\cos\Psi_i + B_{5S}\sin\Psi_i$$

$$B_{6i} = B_{60} + B_{6C}\cos\Psi_i + B_{6S}\sin\Psi_i$$

Where Ψ_i is the azimuth angle of the ith blade, the perturbatory (modes) solution can proceed, using the equations:

$$A_0\underline{U}_i' + A_1\underline{U}_i + A_2\underline{F}_i = \underline{0}$$

$$B_0\underline{F}_i' + B_1\underline{F}_i + B_2\underline{U}_i + B_3\underline{\dot{U}}_i + B_4\underline{\ddot{U}}_i$$
$$+ (B_{50} + B_{5C}\cos\Psi_i + B_{5S}\sin\Psi_i) . \underline{\dot{H}}$$
$$+ (B_{60} + B_{6C}\cos\Psi_i + B_{6S}\sin\Psi_i) . \underline{\ddot{H}} = \underline{0}$$

plus the hub equations (see Section 5.1.2, below)

5.1.1 <u>Blade Equations in Multiblade Form</u>

Using transformations into multiblade degrees of freedom, of the form described below, the perturbatory solution equations may be re-written as follows: (where Ω is rotor speed)

Collective equations (3,4 or 5 blades)

$$A_0\underline{U}_0' + A_1\underline{U}_0 + A_2\underline{F}_0 = \underline{0}$$

$$B_0\underline{F}_0' + B_1\underline{F}_0 + B_2\underline{U}_0 + B_3\underline{\dot{U}}_0 + B_4\underline{\ddot{U}}_0 + B_{50}\underline{\dot{H}} + B_{60}\underline{\ddot{H}} = \underline{0}$$

Cyclic (cos) equations (3,4 or 5 blades)

$$A_0\underline{U}_C' + A_1\underline{U}_C + A_2\underline{F}_C = \underline{0}$$

$$B_0\underline{F}_C' + B_1\underline{F}_C + B_2\underline{U}_C + B_3(\underline{\dot{U}}_C + \Omega\underline{U}_S)$$
$$+ B_4(\underline{\ddot{U}}_C + 2\Omega\underline{\dot{U}}_S - \Omega^2\underline{U}_C) + B_{5C}\underline{\dot{H}} + B_{6C}\underline{\ddot{H}} = \underline{0}$$

Cyclic (sin) equations (3,4 or 5 blades)

$$A_0\underline{U}_S' + A_1\underline{U}_S + A_2\underline{F}_S = \underline{0}$$

$$B_0\underline{F}_S' + B_1\underline{F}_S + B_2\underline{U}_S + B_3(\underline{\dot{U}}_S - \Omega\underline{U}_C)$$
$$+ B_4(\underline{\ddot{U}}_S - 2\Omega\underline{\dot{U}}_C - \Omega^2\underline{U}_S) + B_{5S}\underline{\dot{H}} + B_{6S}\underline{\ddot{H}} = \underline{0}$$

Reactionless Equations (4 blades)

$$A_0\underline{U}_R' + A_1\underline{U}_R + A_2\underline{F}_R = \underline{0}$$

$$B_0\underline{F}_R' + B_1\underline{F}_R + B_2\underline{U}_R + B_3\dot{\underline{U}}_R + B_4\ddot{\underline{U}}_R = \underline{0}$$

Reactionless (cos) equations (5 blades)

$$A_0\underline{U}_{2C}' + A_1\underline{U}_{2C} + A_2\underline{F}_{2C} = \underline{0}$$

$$B_0\underline{F}_{2C}' + B_1\underline{F}_{2C} + B_2\underline{U}_{2C} + B_3(\dot{\underline{U}}_{2C} + 2\Omega\underline{U}_{2S})$$

$$+ B_4\,(\ddot{\underline{U}}_{2C} + 4\Omega\dot{\underline{U}}_{2S} - 4\Omega^2\underline{U}_{2C}) = \underline{0}$$

Reactionless (sin) equations (5 blades)

$$A_0\underline{U}_{2S}' + A_1\underline{U}_{2S} + A_2\underline{F}_{2S} = \underline{0}$$

$$B_0\underline{F}_{2S}' + B_1\underline{F}_{2S} + B_2\underline{U}_{2S} + B_3(\dot{\underline{U}}_{2S} - 2\Omega\underline{U}_{2C})$$

$$+ B_4\,(\ddot{\underline{U}}_{2S} - 4\Omega\dot{\underline{U}}_{2C} - 4\Omega^2\underline{U}_{2S}) = \underline{0}$$

Note that reactionless degrees of freedom are included for 4 or 5 blades, which arise from the multiblade transformation and do not couple with hub motions. The multiblade degrees of freedom are, from Reference 10, and expressed for a general blade freedom q_i:

Collective $$q_o = \frac{1}{N}\sum_{i=1}^{N} q_i$$

Cyclic $$q_c = \frac{2}{N}\sum_{i=1}^{N} q_i\cos\Psi_i \qquad q_s = \frac{2}{N}\sum_{i=1}^{N} q_i\sin\Psi_i$$

Reactionless (4 blades)

Reactionless (5 blades)

$$q_R = \frac{1}{N}\sum_{i=1}^{N} q_i(-1)^{i+1}$$

$$q_{2C} = \frac{2}{N}\sum_{i=1}^{N} q_i\cos 2\Psi_i \qquad q_{2S} = \frac{2}{N}\sum_{i=1}^{N} q_i\sin 2\Psi_i$$

Where Ψ_i is the azimuth angle of the ith blade, and N is the total number of blades.

The multiblade degrees of freedom are expressed diagrammatically in Figure 2, for the example of lead-lag blade motion.

5.1.2 The Hub Equations

The hub equations may be written by considering the Virtual Work terms in the variation of hub motions, defined at the rotor centre line. By equating the coefficients of hub motion variations to zero, from Hamilton's Principle, the equations are obtained for the effective force equilibrium conditions at the hub.

The set of hub equations may be written in matrix form as:

$$\left(\frac{1}{N}Z + S\right)\underline{H} = FC \cdot \underline{F}_{C_H} + FS \cdot \underline{F}_{S_H} + FO \cdot \underline{F}_{0_H}$$

Where N is the number of blades.

Z is the fuselage impedance, at the hub.
S is a matrix containing steady blade root forces (linearised terms)
$\underline{H}$ is the vector of hub motions, as before
$\underline{F}_{C_H}$, $\underline{F}_{S_H}$, $\underline{F}_{0_H}$ are blade root forces, defined in multiblade form.

FC,FS,FO are coefficient matrices of the forces.

Motion compatability conditions at the blade root point, defined in multiblade form, are dependent on the hinge configuration of the rotor. For a general rotor (articulated or non-articulated),

$$A\ \underline{U}\ \text{root} + B\ \underline{F}\ \text{root} = \underline{0}$$

where $\underline{U}$ root, $\underline{F}$ root are displacements and loads at the root point, in any multiblade vector, A and B are coefficient matrices, dependent in form on the number of root hinges or flexibilities required.

In order to describe the principle of the solution method, the case of a non-articulated blade will be pursued here. For this configuration, A is a unit matrix and B is a zero matrix.

6 METHOD OF SOLUTION

The method of solution for the coupled rotor-fuselage system is based on the Transfer Matrix approach.

6.1 The Steady-State Solution

The linearised form of the steady-state equations, for the single hub-fixed blade, from Section 5.1, may be re-expressed as

$$\underline{U}' = -A_0^{-1}\,(A_1\underline{U} + A_2\underline{F} + \underline{SA})$$

$$\underline{F}' = -B_0^{-1}\,(B_1\underline{F} + B_2\underline{U} + \underline{SB})$$

These equations may be integrated along the blade, from blade tip to root, to give values for $\underline{U}$ and $\underline{F}$ at any point on the blade, for assumed values of $\underline{U}$ and $\underline{F}$ at the tip. The $\underline{U}$ and $\underline{F}$ values at the blade root may be described in transfer matrix form as follows:

$$\begin{bmatrix} \underline{U} \\ \underline{F} \end{bmatrix}_{root} = \begin{bmatrix} T11 & - \\ T21 & - \end{bmatrix} \begin{bmatrix} \underline{U} \\ \underline{0} \end{bmatrix}_{tip} + \begin{bmatrix} \underline{CD} \\ \underline{CS} \end{bmatrix}$$

where T11, T21 are transfer matrices, and $\underline{CD}$, $\underline{CS}$ are corresponding vectors, which may be evaluated by using appropriate unit or zero tip values for $\underline{U}$, and including or excluding constant terms from the equations of motion. Note that, at the blade tip $\underline{F} = \underline{0}$ and hence it is not necessary to define the full 12 x 12 matrix which contains T11 and T21. For a non-articulated rotor, $\underline{U} = \underline{0}$ at the blade root and consequently the values for $\underline{U}$ at the tip can be obtained from

$$\underline{U}_{tip} = -T11^{-1}\,\underline{CD}$$

Integration along the blade from tip to root, using these values as the starting values, will then yield the full distribution of $\underline{U}$ and $\underline{F}$. An iterative application of the solution method is adopted, to include non-linear terms.

6.2 Solution - Collective and Cyclic Case

In multiblade degrees of freedom, the collective and cyclic equations of motion, given in Section 5.1.1., are solved together. The cyclic equations are coupled together by the cyclic degrees of freedom and by the hub motion. The collective equations may be coupled to the cyclic equations through the hub motion, depending on the form of the hub impedance.

In transfer matrix form, the collective and cyclic equations for the perturbatory solution, linearised about the steady-state, may be written as

COLLECTIVE

$$\begin{bmatrix} Uo \\ \underline{Fo} \end{bmatrix}_{root} = \begin{bmatrix} T11o & - \\ T21o & - \end{bmatrix} \cdot \begin{bmatrix} Uo \\ \underline{0} \end{bmatrix}_{tip} + \begin{bmatrix} TH11o & 0 \\ TH21o & 0 \end{bmatrix} \cdot \begin{bmatrix} \underline{H} \\ - \end{bmatrix}$$

CYCLIC (COS)

$$\begin{bmatrix} Uc \\ \underline{Fc} \end{bmatrix}_{root} = \begin{bmatrix} T11cc & - \\ T21cc & - \end{bmatrix} \cdot \begin{bmatrix} Uc \\ \underline{0} \end{bmatrix}_{tip} + \begin{bmatrix} T11cs & - \\ T21cs & - \end{bmatrix} \cdot \begin{bmatrix} Us \\ \underline{0} \end{bmatrix}_{tip} + \begin{bmatrix} TH11c & 0 \\ TH21c & 0 \end{bmatrix} \cdot \begin{bmatrix} \underline{H} \\ - \end{bmatrix}$$

CYCLIC (SIN)

$$\begin{bmatrix} Us \\ \underline{Fs} \end{bmatrix}_{root} = \begin{bmatrix} T11sc & - \\ T21sc & - \end{bmatrix} \cdot \begin{bmatrix} Uc \\ \underline{0} \end{bmatrix}_{tip} + \begin{bmatrix} T11ss & - \\ T21ss & - \end{bmatrix} \cdot \begin{bmatrix} Us \\ \underline{0} \end{bmatrix}_{tip} + \begin{bmatrix} TH11s & 0 \\ TH21s & 0 \end{bmatrix} \cdot \begin{bmatrix} \underline{H} \\ - \end{bmatrix}$$

The expressions for $\underline{Fo}$ root, $\underline{Fc}$ root and $\underline{Fs}$ root from these equations may be substituted into the hub equations given in Section 5.1.2, and the values $\underline{Uo}$ root = $\underline{Us}$ root = 0 substituted in the remaining transfer equations, for a non-articulated rotor, such that:

$$\left(\frac{1Z}{N} + S\right) \underline{H} = FC.\ (T21cc\ \underline{Uc} + T21cs\ \underline{Us} + TH21c\ \underline{H})$$
$$+ FS.\ (T21sc\ \underline{Uc} + T21ss\ \underline{Us} + TH21s\ \underline{H})$$
$$+ FO.\ (T21o\ \underline{Uo} + TH21o\ \underline{H})$$

$$T11o\ \underline{Uo} + TH11o\underline{H} = \underline{0}$$
$$T11cc\ \underline{Uc} + T11cs\ \underline{Us} + TH11c\ \underline{H} = \underline{0}$$
$$T11sc\ \underline{Uc} + T11ss\ \underline{Us} + TH11s\ \underline{H} = \underline{0}$$

These equations, in which $\underline{Uo}$, $\underline{Uc}$, $\underline{Us}$ are defined to be at the blade tip, may be alternatively expressed in full matrix form to give the condition

$$[D] \cdot \begin{bmatrix} \underline{H} \\ \underline{Uo} \\ \underline{Uc} \\ \underline{Us} \end{bmatrix}_{tip} = \underline{0}, \qquad \text{or } D\ \underline{V} = \underline{0}$$

where D and $\underline{V}$ are complex

In practice, D is evaluated successively for given complex search frequencies in a search routine which determines the value of frequency which gives a zero of the determinant of D. Note that D is a 24 x 24 complex matrix.

Each such frequency found is then a predicted complex mode frequency (eigenvalue or natural frequency) of the coupled rotor-fuselage system. By back-substitution in D, the corresponding complex mode shape is defined. The mode shape is normalised to unity and zero phase of the largest component of $\underline{V}$.

A similar solution for the reactionless case (4 or 5 blades) may be made, but with no hub motion.

7 VALIDATION EXERCISES

Initial validation exercises for the CRFD analysis have been based on comparison with other predictions, for a number of rotor-fuselage configurations. Some examples of results are given in Tables 3 and 4.

In order to check the representation of fundamental couplings between fuselage and rotor motion, a simple model was defined for analysis by the WHL ground resonance program. This consisted of an approximate representation of the Lynx semi-rigid main rotor, and an arbitrary set of rigid body fuselage modes in which the constituent degrees of freedom - three translations, and roll and pitch rotations - were uncoupled. In the ground resonance analysis, pure flap and lag blade modes are used. For comparative purposes, the blade data for the CRFD program was arranged to give no coupling between flap and lag, by removing pre-twist and steady coning. The results from the two analyses, for four modes, are given in Table 3, in terms of frequencies, and magnitude and phase of mode shape components. The exercise of comparison was valuable, and the final results show very good agreement between the two analyses. Note that no aerodynamic terms were included in this exercise, or that of Table 4.

The introduction of coupling between the fuselage degrees of freedom, for arbitrary rigid-body mode shapes, allowed further comparison between predictions from CRFD and the ground resonance program. Table 4 shows results for a predominantly translational mode shape in this case. Agreement is good, with the largest difference appearing in the flap (cosine) component.

A requirement for test data against which to validate the program has been identified, and it is hoped that model rotor tests will take place in the near future.

8 EXAMPLE APPLICATION - BLADE LAG DAMPER

An example of an application of CRFD is given in Figure 3, where shear force and bending moment predictions for the fundamental lead-lag mode are plotted, for an articulated rotor. The force and moment distributions were obtained from CRFD, for a single blade, with and without a lag damper included. For the case with no damper, all velocity terms were omitted from the analysis, to give a "conventional" rotating blade real mode. With the lag damper, all velocity terms were included, and a complex mode was obtained. Large differences between the distributions are apparent, due to the action of the damper. Such a representation of the damper, although linear, provides a basis for modes to be used in a response analysis, in which additional amplitude-dependent effects may be included. The representation may also be used in mode shapes for fitting to flight test measurements, in order to reconstruct hub loads from blade strain gauge data.

9 CURRENT CAPABILITIES AND FURTHER DEVELOPMENT

The current capabilities of the CRFD model, which are the subject of continuing validation exercises, are the ability to model the following elements, in combination:

. Rotor (3,4 or 5 blades)

By multiblade continuous beam model, including non-linear steady-state solution.

. Control Circuit

By secondary load path to earth model, as an impedance calculated from modal data, with definition for collective, cyclic (twice) and reactionless rotor motion.

. General Blade Secondary Load Paths and Point Flexibilities.

. Blade Lag Damper

With transmission of damper root forces to the rotor hub.

. Fuselage and Transmission

By impedance calculated from modal data, including interface with NASTRAN results.

Further development is proceeding, to include the following:

. Improvements to software to reduce computation times.

. Addition of steady state trim and pertubatory aerodynamics models.

. Interface for CRFA program and graphics post-processing.

. Verification of modal orthogonality conditions.

. Addition of multiple flexural load paths, for bearingless rotors.

The CRFA Coupled Rotor-Fuselage Aeroelastics analysis is under parallel development, to include a comprehensive description of a manoeuvring flight wake model, control logic for three dimensional simulation, modelling of engine control response and calculation of rotor structural loads.

10 REFERENCES

1 R.E. Hansford: "Rotor Load Correlation with the British Experimental Rotor Program Blade" AHS Journal, July 1987.

2 P.T.W Juggins: "Substantiation of the Analytical Prediction of Ground and Air Resonance Stability of a Bearingless Rotor, Using Model Scale Tests" presented at the 12th European Rotorcraft Forum, Paper 83, September 1986.

3 J.M. Davis, R.L. Bennett and B.L. Blankenship: "Rotorcraft Flight Simulation with Aeroelastic Rotor and improved Aerodynamic Representation", USAAMRDL TR74-10, June 1974.

4 W. Johnson: "Assessment of Aerodynamic and Dynamic Models in a Comprehensive Analysis for Rotorcraft", Computers and Mathematics with Applications, Vol 12A, (1), January 1986.

5 D.H. Hodges, A.S. Hopkins, D.L. King and H.E. Hinnant: "Introduction to GRASP - General Rotorcraft Aeromechanical Stability Program - A Modern Approach to Rotorcraft Modeling", AHS Journal, April 1987.

6 M.J. Rutkowski: "Assessment of Rotor-Fuselage coupling on Vibration Predictions Using a Simple Finite Element Model", AHS Journal, July 1983.

7 G.T.S. Done, P.T.W. Juggins and M.H Patel: "Further Experience with a New Approach to Helicopter Aeroelasticity" presented at the 13th European Rotorcraft Forum, Paper 6.11, September 1987.

8 T-K Hsu and D.A. Peters: "Coupled Rotor/Airframe Vibration Analysis by a Combined Harmonic-Balance, Impedance-Matching Method", AHS Journal, January 1982.

9 D.H. Hodges and E.H. Dowell: "Nonlinear Equations of Motion for the Elastic Bending and Torsion of Twisted Non-uniform Rotor Blades", NASA TN D-7818, December 1974.

10 W. Johnson: "Helicopter Theory", P.350 et seq., Princeton University Press, 1980.

Table 1.

Predicted Collective Lead-Lag Frequencies, Lynx Rotor

BLADE ONLY	WITH TRANSMISSION MODEL
0.65R	0.99R
	1.44R
	3.05R
4.31R	3.48R

(1R = MAIN ROTOR SPEED)

Table 2.

Predicted Blade Torsion Frequencies, Lynx Rotor

CONTROL DEFLECTION	FREQUENCY
COLLECTIVE	3.8R
CYCLIC (LONG.)	3.8R
CYCLIC (LAT.)	4.5R
REACTIONLESS	6.2R

(1R = MAIN ROTOR SPEED)

Table 3.

Predicted Rotor-fuselage Modes for a Simple Semi-rigid Rotor with Arbitrary Uncoupled Rigid-body Fuselage Modes

CRFD - Coupled Rotor-Fuselage Dynamics Code
WHL G.R. - Westland Ground Resonance Code

MAGNITUDES AND PHASE.

NOTES: (i) Blade angles based on tip deflections
(ii) Real parts of complex frequencies are all very small

3a	CRFD 5.61Hz		WHL G.R. 5.60Hz	
HUB VERTICAL	28.7 IN	0°	28.5 IN	0°
FLAP (COLLECTIVE)	1.0 RAD	0°	1.0 RAD	0°

3b	CRFD 0.546Hz		WHL G.R. 0.547Hz	
HUB ROLL	0.15 RAD	0°	0.14 RAD	0°
HUB PITCH	0.24 RAD	90°	0.24 RAD	90°
FLAP (COSINE)	0.96 RAD	-90°	0.96 RAD	-90°
FLAP (SINE)	1.00 RAD	0°	1.00 RAD	0°

3c	CRFD 1.97Hz		WHL G.R. 1.96Hz	
HUB TRANSLATION (X)	2.78 IN	0°	2.74 IN	0°
HUB TRANSLATION (Y)	113.9 IN	90°	107.8 IN	90°
LAG (COSINE)	0.93 RAD	90°	0.93 RAD	90°
LAG (SINE)	1.00 RAD	90°	1.00 RAD	0°

3d	CFRD 1.80Hz		WHL G.R. 1.80Hz	
HUB ROLL	0.044 RAD	90°	0.044 RAD	90°
HUB PITCH	0.85 RAD	0°	0.84 RAD	0°
FLAP (COSINE)	1.00 RAD	0°	1.00 RAD	0°
FLAP (SINE)	0.34 RAD	90°	0.33 RAD	90°

Westland

J4537/5

Table 4.

CRFD Validation Case

PREDICTED ROTOR-FUSELAGE MODE FOR A SIMPLE SEMI-RIGID ROTOR WITH ARBITRARY COUPLED RIGID-BODY FUSELAGE MODES

	PROGRAM: CRFD REAL	PROGRAM: CRFD IMAGINARY	PROGRAM: WHL G.R. REAL	PROGRAM: WHL G.R. IMAGINARY
FREQUENCY	0.0	14.46	-10^{-7}	14.43 (RAD/S)

MODE SHAPE

	MAGNITUDE	PHASE (°)	MAGNITUDE	PHASE (°)
X	0.38	-90	0.38	-90
Y TRANSLATIONS	75	0	75	0
Z	2.6	-90	2.6	-90
ROLL	0.98	180	0.98	180
PITCH	0.33	90	0.34	90
FLAP (cos.)	0.03	90	0.05	90
FLAP (sin.)	1.0	0	1.0	0
FLAP (coll.)	0.003	-90	0.003	-90
LAG (cos.)	0.31	0	0.32	0
LAG (sin.)	0.37	-90	0.38	-90
LAG (coll.)	(10^{-8})	0	(10^{-8})	0

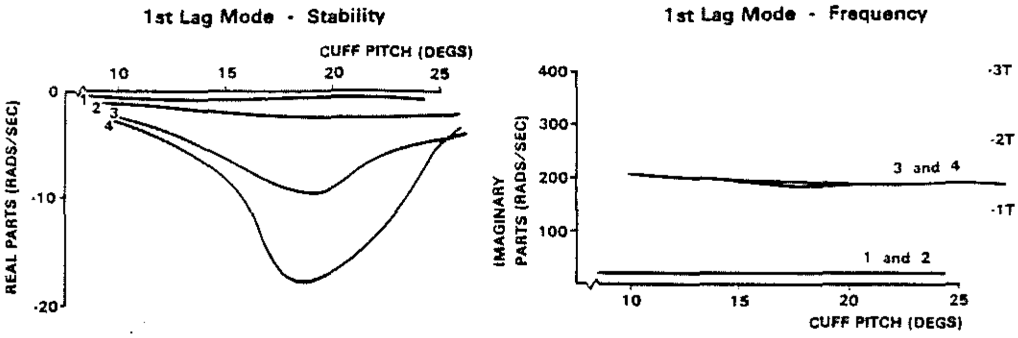
NOTE: X, Y, Z in inch units, all other components in radians.

Westland

J4537/6

Figure 1.

Example of Predicted Tail Rotor Blade Eigenvalues



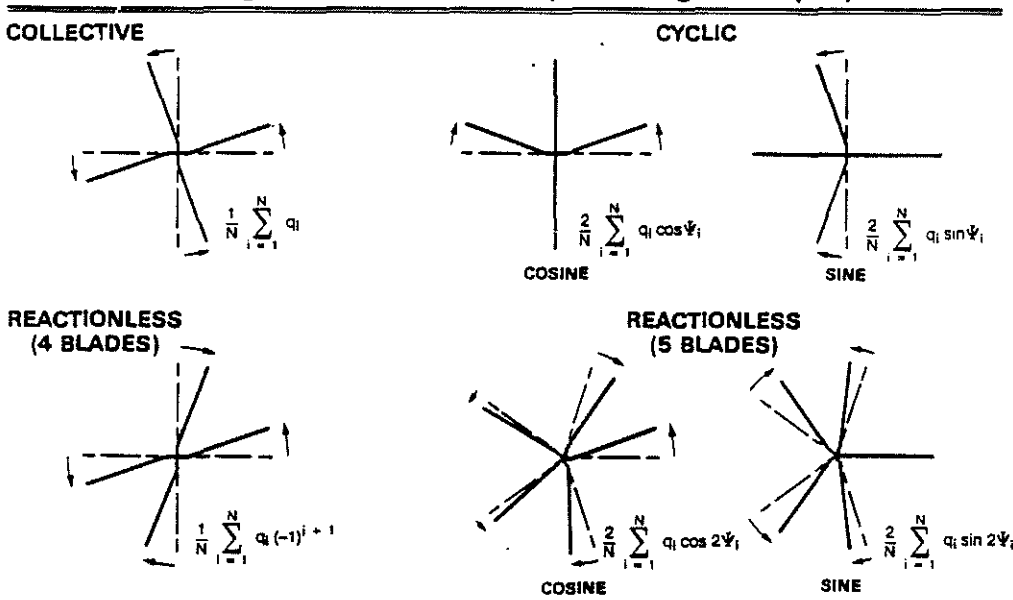
CONTROL CIRCUIT AND TRANSMISSION SYSTEM MODELS	1 SYMMETRIC (HYDRAULICS ON)	3 ASYMMETRIC
	2 SYMMETRIC (HYDRAULICS OFF)	4 REACTIONLESS

Westland

J4537/1

Figure 2.

Multiblade Degrees of Freedom (Lead-lag Example)



Westland

J4537/2

Figure 3.

Articulated Rotor, with Blade Damper Fundamental Blade Lead-Lag Mode Shape