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UH-60A HELICOPTER STABILITY AUGMENTATION STUDY

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Abstract

The current UH-60A BLACK HAWK stability augmentation system (SAS) is examined using linear analysis techniques on a high order linear helicopter model that includes: rigid body, rotor blade, inflow, rotor speed, engine/fuel control and control system/SAS dynamics. Derived from the flight test validated, non-linear time domain Sikorsky GEN HEL handling qualities simulation, this linear model provides physical insight into the interaction between the dynamic components of the helicopter system in the frequency domain that is not easily obtained from a non-linear simulation. The complexity of the linear model is increased in stages showing the effect of each refinement on helicopter handling qualities.

Emphasis is placed on understanding SAS roll rate gain limitations historically observed during flight test. A simple rigid body model of the helicopter indicates no limit, but with the inclusion of blade flap and lag and rotor inflow dynamics, high roll rate gains are shown to destabilize blade lag motion. However, adding the servo dynamics, filters and lags present in the flight control system/SAS results in a blade flapping instability. Including engine dynamics does not change the roll rate gain limitation, but does affect helicopter dynamics. Modified SASs that improve the aircraft's handling qualities are explored using both the linear model and the non-linear simulator.

Notation

A	System state matrix
a0f	Rotor average flap angle
a0l	Rotor average lag angle
a1f	Rotor longitudinal tip path plane tilt
a1l	Rotor longitudinal tip path plane skew
A1s	Rotor lateral cyclic control
a2f	Rotor tip path plane weaving term
a2l	Rotor tip path plane skew weaving term
B	System control matrix
b1f	Rotor lateral tip path plane tilt
b1l	Rotor lateral tip path plane skew
B1s	rotor longitudinal cyclic control
C_{LHA}	Rotor aerodynamic roll moment coefficient
C_{MHA}	Rotor aerodynamic pitch moment coefficient
C_{TA}	Rotor thrust coefficient
D_{W0}	Downwash velocity
IB	Blade inertia about flap hinge
Jrest	Inertia downstream of clutch less blades
Jtote	Inertia upstream of clutch
K_{CM}	Cosine harmonic inflow gain
K_{CT}	Uniform inflow gain
K_{RR}	Roll rate feedback gain
K_{SM}	Sine harmonic inflow gain
L_p	*Helicopter roll damping*
nb	Number of rotor blades
N_g	Gas generator speed
N_p	Rotor speed error
p	Helicopter roll rate - body axis
q	Helicopter pitch rate- body axis
Q_{CM}	Compressor torque
Q_e	Turboshaft engine torque
Q_{GT}	Gas generator torque
Qreq	Torque load on rotor
r	Helicopter yaw rate - body axis
T_{DW0}	Inflow time constant
T_{DWC}	Cosine harmonic inflow time constant
T_{DWS}	Sine harmonic inflow time constant
U	Control vector
u	Helicopter longitudinal velocity - body axis
U_{TOT}	*Total rotor inflow velocity*
v	Helicopter lateral velocity - body axis
w	Helicopter vertical velocity - body axis
Wfc	Turboshaft engine fuel flow
X	State vector
$\beta(I)$	Flap angle for for blade I
$\delta(I)$	Lag angle for blade I
ϕ	Helicopter roll attitude - earth axis
θ	Helicopter pitch attitude -earth axis
θmr	Rotor collective(thrust) control
θtr	Tail rotor (directional) control
τ	System time constant
Ω	Rotor speed
ω_d	Damped frequency
ψ	Helicopter yaw attitude - earth axis
$\Psi(I)$	Azimuth position for blade I
ζ	Damping ratio
$\cdot$	First time derivative
$\cdot\cdot$	Second time derivative

1. Introduction

The initial performance of a feedback control system is, to a large extent, dependent on the mathematical models used in the design process. High gain feedback control systems designed with low fidelity models (models that reproduce only the low frequency dynamics of a plant) will normally not achieve expected performance levels when applied to the physical plant. Plant dynamics not present in the low fidelity mathematical model may interact with the feedback system, degrading performance and possibly destabilizing the system.

Helicopters with articulated rotors are complex dynamic systems. In addition to the standard 6 rigid body degrees of freedom(DOF), helicopters have a high number of rotor/blade degrees of freedom. Each rotor blade is permitted to rotate in flatwise (flap) and edgewise (lag) directions, and controls are provided to vary the pitch on each blade. The rotor dynamics are of significantly higher frequency than the basic rigid body dynamics and have only recently been given more detailed attention in linear feedback gain studies. Many studies showing the relationship between blade flap dynamics and feedback systems have been performed, for example Chen and Hindson [ref. 1], but only a few of these studies used linear rotor models which included the lag degree of freedom. Curtiss [ref. 2] used a simple linear model of the rotor system to show that blade lag degree of freedom, as often suspected, could limit the level of helicopter roll rate feedback. Miller and White [ref. 3] extended this analysis to show the importance of control system time delays.

Much work has been done developing comprehensive non-linear helicopter math models that accurately reproduce the flight dynamics of a helicopter. These complex models are used for handling qualities analysis, control system/stability augmentation system design, and pilot in the loop analysis. In the case of the Sikorsky GEN HEL UH-60A BLACK HAWK Engineering Simulation Model [ref. 4], extensive correlation with flight test data was performed to assure the validity of the math model [ref. 5]. However, the literature does not show many cases where these comprehensive simulations have been used to develop high fidelity linear models for use in analysis and control system design.

The reasons for this are partially historical. Helicopter dynamics rarely decouple into simple longitudinal and lateral - directional degrees of freedom, and therefore may not be analyzed using the conventional stability and control techniques available to the fixed wing analyst. The resulting complex computations needed when applying linear analysis techniques often became prohibitively time consuming. To reduce the computational burden, the early linear models did not include the rotor/blade degrees of freedom and many of the feedback systems designed using these models did not perform as expected when tested on the actual helicopters, leaving the designer questioning the value of linear modeling. With the advent of high speed computers and powerful linear control system software packages, very large linear systems may now be efficiently analyzed.

Comprehensive non-linear models are necessary to accurately reproduce the overall flight dynamics of a helicopter, but they are not necessarily the most convenient tool for obtaining physical insight. When stability augmentation systems are added to these models, the gains were often set by increasing the gain until an instability occurs and then backing off by some specified amount (for example 50%). The instability may of course be isolated with the non-linear model, but this can turn out to be a very time consuming task, requiring many experiments. A high fidelity linear approximation to the helicopter can be very helpful in providing an initial direction to the analyst, thereby reducing his workload.

High order linear models along with comprehensive non-linear ones provide a more complete data base for the control system analyst. Almost all of the current control system analysis and design tools are limited to use with linear systems. By applying these tools to the comprehensive linear models, the designer can gain insight into the interaction between the helicopter dynamics and the control system dynamics and may also be able to find new methods for improving overall helicopter performance.

This investigation covers the derivation and analysis of a high fidelity linear model from the correlated GEN HEL UH-60A BLACK HAWK non-linear engineering mathematical model. A simple rigid body linear model of the helicopter is presented first and then augmented in several stages with: rotor dynamics consisting of blade flap and lag dynamics and inflow dynamics, control system dynamics, and engine/fuel control dynamics. The potential importance of each addition to the linear model is illustrated by examining its effect on roll rate feedback gain limits, a condition of particular importance to the helicopter control system designer. The hovering flight condition is examined in detail along with a limited analysis at a forward speed of 100 knots.

2. Linear Modeling

The linear models presented here are derived directly from the non-linear GEN HEL simulation using perturbation techniques. The non-linear math model is trimmed to a defined flight condition and the linear derivatives are generated about the equilibrium point. This method insures consistency between the validated non-linear model and the resulting linear model. The following briefly describes the techniques used to generate linear models of various fidelity levels from the non-linear GEN HEL simulation.

3. Rigid Body Linear Model

To produce a 9 state rigid body linear model from the non-linear GEN HEL mathematical model, all body acceleration integrations are inhibited, and the state variables and control input variables are perturbed in turn about the trim point of interest to calculate the change in the state variable time derivatives. Using this method, the rotor degrees of freedom (blade flap, lag and inflow) continue to be integrated after a perturbation and are allowed to settle before calculating each derivative. In this way, the rigid body derivatives include the steady state effects of each perturbation on the rotor. The resulting linear model has what may be thought of as a quasistatic rotor; changes in rotor forces and moments are an implicit part of the 9 state system, but since the changes are not modeled dynamically, they occur instantaneously. The derivatives are used to produce a state space system in the form:

$$\dot{X} = [A]\,X + [B]\,U$$

$$X = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \\ \psi \end{bmatrix} \qquad U = \begin{bmatrix} A1s \\ B1s \\ \theta mr \\ \theta tr \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$[A] = \begin{bmatrix} \frac{\partial(\dot{u})}{\partial u} & \frac{\partial(\dot{u})}{\partial v} & \cdots & \frac{\partial(\dot{u})}{\partial \psi} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial(\dot{\psi})}{\partial u} & \frac{\partial(\dot{\psi})}{\partial v} & \cdots & \frac{\partial(\dot{\psi})}{\partial \psi} \end{bmatrix}$$

$$[B] = \begin{bmatrix} \frac{\partial(\dot{u})}{\partial A1s} & \frac{\partial(\dot{u})}{\partial B1s} & \frac{\partial(\dot{u})}{\partial \theta mr} & \frac{\partial(\dot{u})}{\partial \theta tr} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial(\dot{\psi})}{\partial A1s} & \frac{\partial(\dot{\psi})}{\partial B1s} & \frac{\partial(\dot{\psi})}{\partial \theta mr} & \frac{\partial(\dot{\psi})}{\partial \theta tr} \end{bmatrix}$$

where: X = standard aircraft velocities, rates and attitudes,
U = helicopter controls,
[A] = system state matrix,
[B] = system control matrix

$\dot{X}$ = time derivative of X

Since the system is now expressed in linear form, a multitude of analysis and control system design tools are applicable. However, as will be shown later, this model's low level of fidelity makes it unacceptable for use in designing high performance, high bandwidth feedback systems.

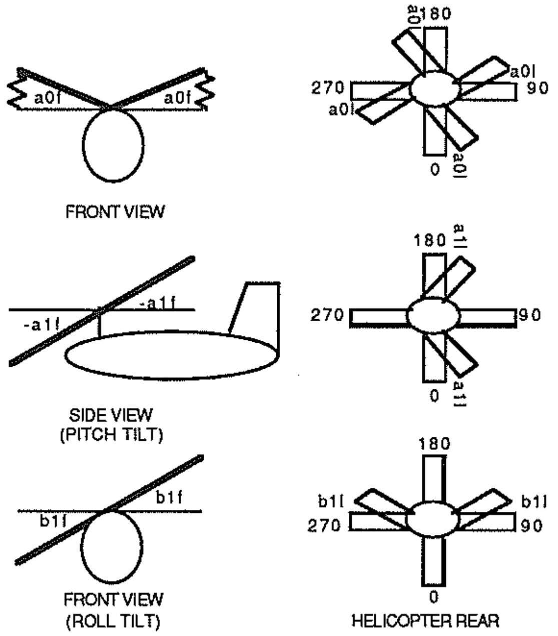


Fig. 1: Blade Flap and Lag State Variable Descriptions

4. Blade Flap and Lag Degrees of Freedom

A multiblade coordinate system [ref. 6] is used to represent blade flap and lag motion. In multiblade coordinates, individual blade flap and lag motion is seen as motion of a rotor disk defined by the tips of each blade in the shaft axis system (figure 1).

The flap motion of each blade, $\beta(I)$, is expressed as follows for a four bladed rotor:

$\beta(I) = a0f - a1f\cos\Psi(I) - b1f\sin\Psi(I) + a2f$

where

$$a0f = \frac{1}{nb}\sum_{I=1}^{nb} \beta(I)$$

$$a1f = \frac{-2}{nb}\sum_{I=1}^{nb} \beta(I)\cos\Psi(I)$$

$$b1f = \frac{-2}{nb}\sum_{I=1}^{nb} \beta(I)\sin\Psi(I)$$

$$a2f = \frac{1}{nb}\sum_{I=1}^{nb} \beta(I)(-1)^{I}$$

nb = number of rotor blades

where a0f represents average coning of the rotor (i.e. all four blades flapped up or down together), a1f represents a longitudinal tilting of the rotor tip path plane and b1f represents a lateral tilting of the rotor tip plane. These coordinates represent changes in rotor loads that are summed at the hub and directly affect flight dynamics. a2f represents a reactionless motion since its forces and moments are not transmitted through the hub [ref. 6].

Blade lag motion, $\delta(I)$, is expressed using the same technique; a0l, a1l, and b1l are parallels to the equivalent flapping variables. a0l represents an average lag angle and a1l and b1l represent longitudinal and lateral skewing of the tip path plane disc, respectively and may also be visualized as a longitudinal and lateral rotor center of mass motion.

In transferring between blade coordinates and multiblade coordinates the number of states is preserved. Therefore the transformation for a rotor with 7 blades retains the first three coordinates with the same meaning but an additional four coordinates are also defined. These coordinates, as in the case of a2f do not directly influence hub forces or moments, but under certain conditions these higher harmonic motions may interact with the first order harmonic tilting and coning modes and consequently affect hub forces and moments. In general, neglecting the higher harmonic terms will reduce the accuracy of the linear model for flight conditions other than hover; however, since these higher order terms result in motion that is an order of magnitude smaller than what the first three modes produce, they are neglected in this investigation.

The coordinates a0f, a1f, and b1f and their time derivatives are chosen as state variables to represent the blade flap motion of the rotor. Similarly a0l, a1l, and b1l and their time derivatives are chosen to represent blade lag motion. To include blade flap and lag as state variables, the integration of flapping acceleration and flap and lag rate are disabled within the non-linear GEN HEL simulation and the flap and lag angle of each blade is calculated as a function of a0f, a1f, b1f, a0l, a1l, b1l-about the trim point of interest. Each state variable, a0f, a0l, etc.., is then perturbed individually and the partial derivatives for each state variable time derivative is calculated.

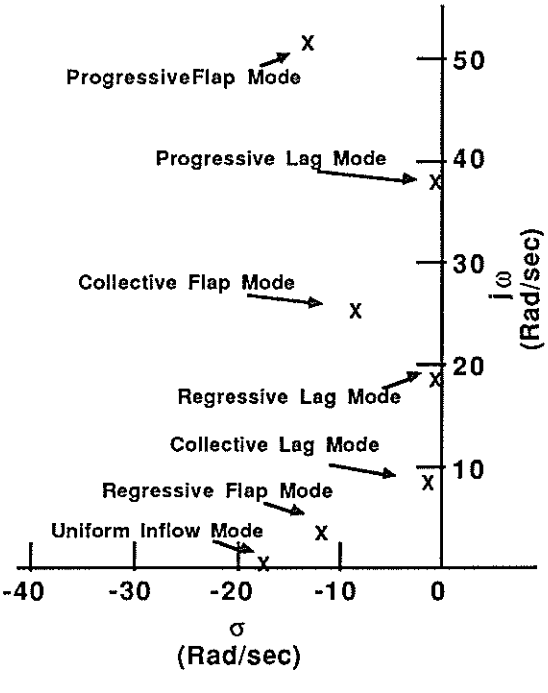


Fig. 2: High Frequency Rotor Modes

5. Uniform Inflow Degree of Freedom

The BLACK HAWK model described in reference 4 includes inflow effects as a function of total rotor thrust only. An average induced velocity is calculated as a function of thrust and then lagged to simulate the time necessary to accelerate air through the rotor. The inflow or downwash, Dwo, is defined as follows:

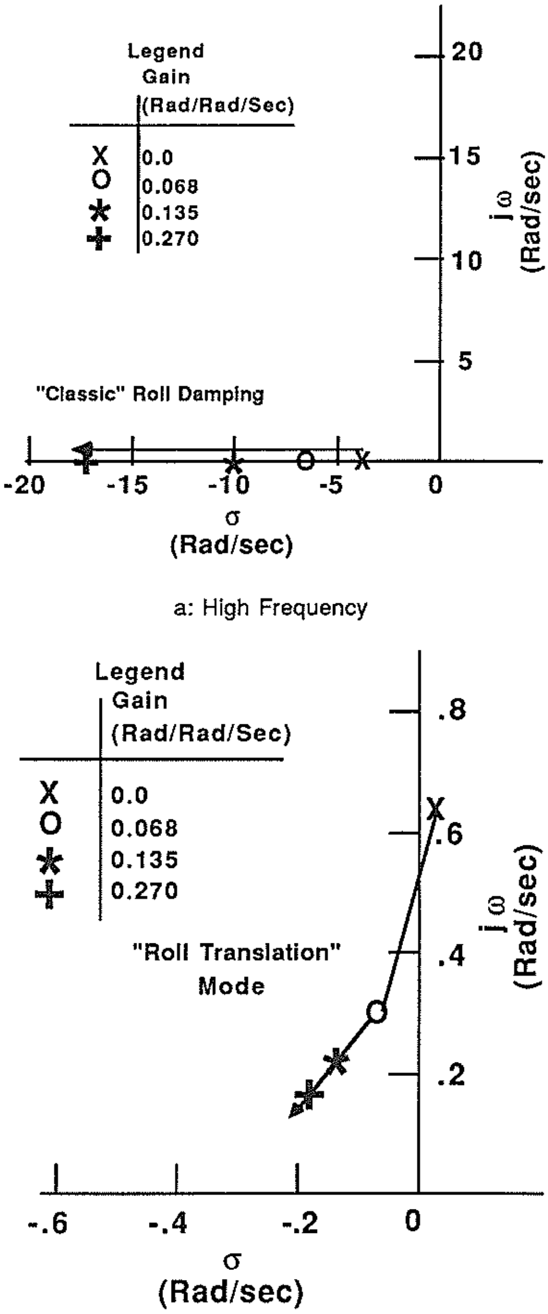
$$D_{WO}=\frac{K_{CT}\,C_{TA}}{2\,U_{TOT}}\left(\frac{1}{1+\left(\frac{T_{DWO}}{U_{TOT}}\right)s}\right)$$

where K_{CT} is an empirically determined gain, U_{TOT} is the normalized total velocity through the rotor, C_{TA} is the rotor thrust coefficient and T_{DWO} is an empirically determined time constant.

Linearizing the uniform portion of the rotor inflow model consists of disabling the lag integration and calculating the partial derivatives for the time rate of change of downwash. During the perturbation process, the calculation for D_{WO} is deactivated and D_{WO}, like all other state variables, is perturbed about its trim value.

The state vector for the 22 state system including blade flap, lag and uniform inflow degrees of freedom is defined as:

$$X=\begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \\ \psi \\ a0f \\ \dot{a0f} \\ a1f \\ \dot{a1f} \\ b1f \\ \dot{b1f} \\ a0l \\ \dot{a0l} \\ a1l \\ \dot{a1l} \\ b1l \\ \dot{b1l} \\ D_{WO} \end{bmatrix}$$



a: High Frequency

b: Low Frequency

Fig. 3: Root Locus - p to A1s, Rigid Body Dynamics Only

As shown in figure 2, the addition of blade flap and lag, and inflow states result in six complex pairs of roots and one real root: collective flap mode, regressive flap mode, progressive flap mode, collective lag mode, progressive lag mode, regressive lag mode and a uniform inflow mode. The appellations collective, progressive and regressive are used here for convenience when describing the modes of the coupled helicopter system. In truth, these modes represent motion in many dimensions as may be seen by examining the eigenvectors of the linear system. For example, the collective flap mode may be more accurately thought of as a collective flap/inflow/vertical mode and the regressive flap mode, as will be illustrated later, is actually a regressive flap/body mode with roll motion as the dominant rigid body contribution.

6. Roll Rate Gain limitations

Historically, flight tests have shown limits to the level of roll damping obtainable with roll rate feedback for an articulated rotor helicopter. BLACK HAWK test pilots have noted an unacceptable oscillation in hover at approximately 2 Hz when the roll rate feedback gain is increased significantly above standard values.

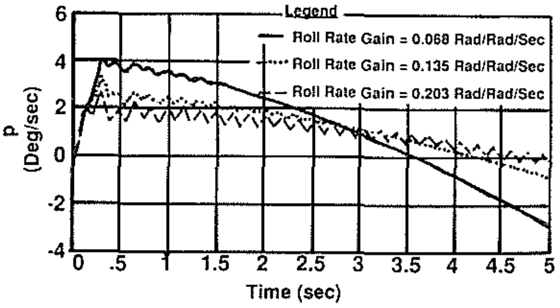


Fig. 4: Effect of Increasing Roll Rate Gain Using the Non-Linear GEN HEL Simulation No Control System or SAS Dynamics

A 6 DOF (9 state) model does not predict this oscillation and in fact does not predict any oscillatory instability as roll rate gain increases. Figure 3 shows the root locus for this model, in hover, as a function of roll rate feedback through A1s, lateral cyclic control. The "Roll Translation" mode is stabilized and the helicopter gives a satisfactory response at a gain of ≈ .1 rad/rad/sec. Increasing the roll rate gain beyond this value increases the "Classic" roll damping, Lp, but does not yield any instabilities. However, the non-linear GEN HEL simulation shows that increasing the roll rate feedback drives a high frequency mode toward instability (figure 4). For this case, the GEN HEL simulation incorporates a simple rate feedback and does not include any of the servodynamics or filters present in the helicopter's control system or stability augmentation system (SAS).

With the addition of the rotor dynamics to the linear model, the cause of the oscillation is understood. The progressive lag mode crosses the imaginary axis at a roll rate gain of ≈.2 rad/rad/sec (figure 5). This high frequency instability agrees with Curtiss' results [ref. 2]; however, it does not agree with the much lower frequency oscillation reported by the Sikorsky project pilot. Control system/SAS dynamics, ignored up to this point, are considered in the next section in an attempt to better understand this discrepancy.

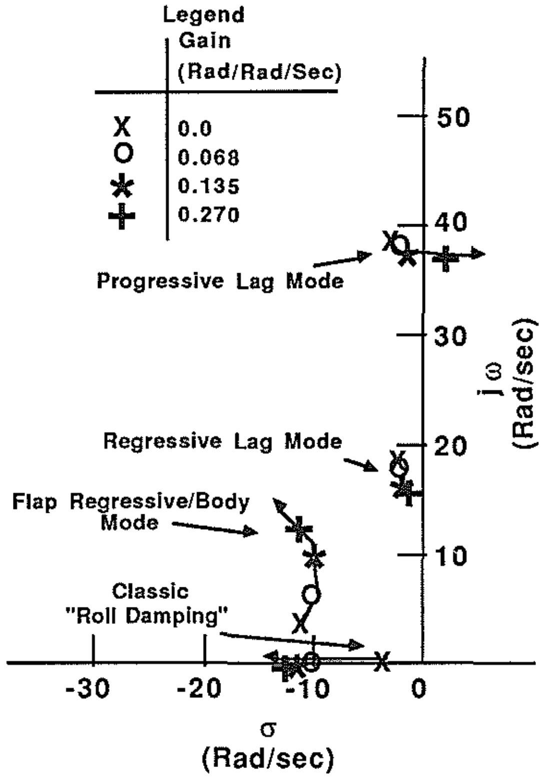


Fig. 5: Root Locus p to A1s - Includes Flap, Lag and Uniform Inflow DOF

7. Control System/SAS Dynamics

Including the control system/SAS for the BLACK HAWK adds dynamic elements to the helicopter system. The relatively high frequency filters and servos that are ignored when analyzing low fidelity 6 DOF helicopter models must now be considered because of possible interaction with "fast" rotor dynamics. Figure 6 depicts the control system/SAS for the BLACK HAWK in hover for the roll and pitch axes. These systems, derived from reference 4 are not complete, but do retain all the important dynamic components necessary for a valid qualitative analysis.

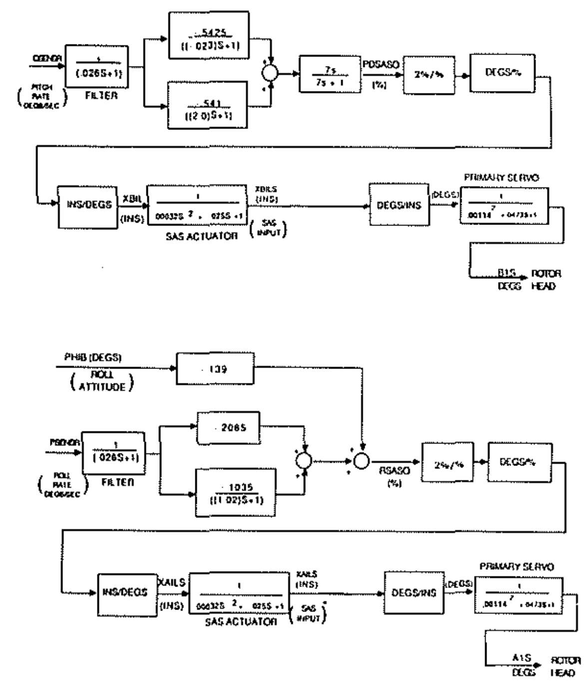


Fig. 6: Simplified Pitch and Roll Control System/SAS [Ref. 4]

Including the control system/SAS dynamics has a profound effect on the total aircraft system stability. Figure 7 shows the root locus for roll rate feedback through A1s (lateral control) with the remainder of the roll control systems/SAS active. It is seen that the lag modes are essentially unaffected, but the coupled regressive flap/body mode is destabilized as roll rate feedback increases. One effect of adding the control system/SAS dynamics is to introduce a time delay between the measurement of roll rate and the eventual movement of the rotor blades to compensate for the error in commanded rate. Essentially, the rotor receives old information. This is not an important effect at low levels of roll rate gain, since the system is required to respond at a moderate rate and the delay is not critical. However attempts at obtaining a very responsive, "fast", system through high levels of roll rate gain makes even small delays important. As the roll rate gain increases the phase delays become large enough to drive the roll control out of phase with the roll response. Instead of providing damping the feedback scheme actually destabilizes the system. By approximating the control system/SAS with a simple time delay Miller and White [ref. 3] have shown a similar effect for the CH47 helicopter.

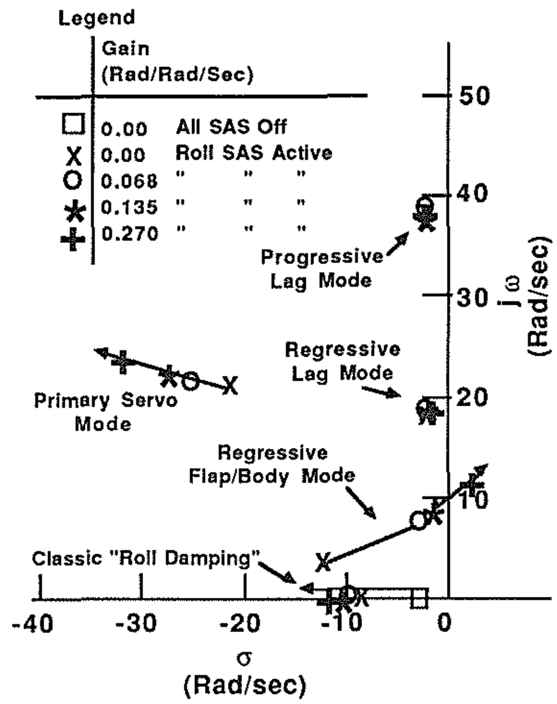


Fig. 7: Root Locus p to A1s - Includes Flap, Lag and Uniform Inflow DOF and Roll Control System/SAS Dynamics

The addition of servodynamics and filters results in more than a time delay. A time delay yields only a change in system phase but the control system/SAS dynamics will also change the magnitude of the system response. This effect is best illustrated with a Bode diagram. Figure 8 shows the gain and phase plots for the transfer function of lateral rotor control to roll rate (all loops open). The phase lag introduced by the control system/SAS dynamics shifts the area of critical gain margin from the open loop lag frequency, 38.8 Rad/sec, to the open loop regressive flap/body frequency, 12.4 Rad/sec, as expected, but the control system/SAS dynamics attenuates the magnitude response at high frequency, essentially filtering the lag dynamics (i.e. lowering the magnitude response). This is also seen in the root locus where the higher frequency lag modes are basically unaffected by increasing roll rate gain (figure 7).

The linear 22 state model linked with the control system/SAS model shows a crossover for instability at approximately 10 rads/sec (≈1.6 Hz) which compares reasonably well to the frequency experienced during flight test. An examination of the validated non-linear GEN HEL time response (figure 9), for 2 x the baseline roll rate gain with control system/SAS dynamics included, shows a lightly damped oscillation with a frequency (≈1.5 Hz) which corresponds to the frequency predicted by the root locus.

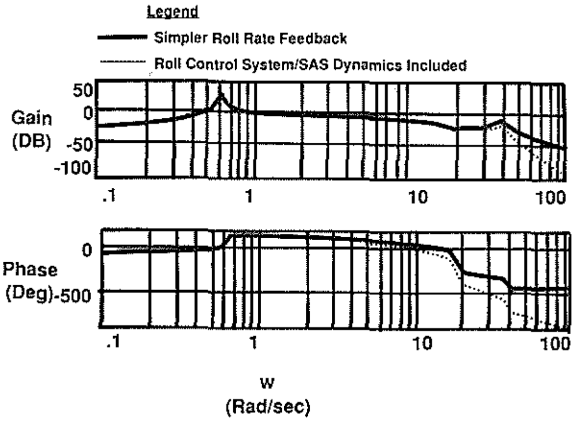


Fig. 8: Bode for p to A1s, All other SAS Pitch and Roll loops open - Includes Flap, Lag and Uniform Inflow DOF

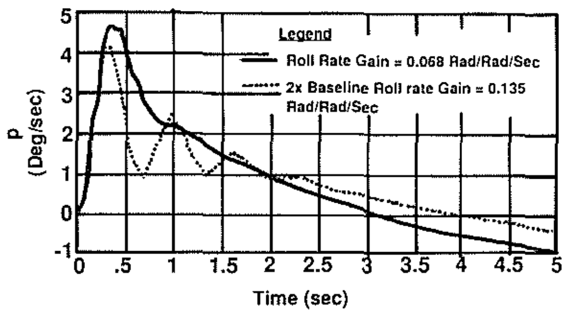


Fig. 9: Effect of Increasing Roll Rate Gain Using the Non-Linear GEN HEL Simulation. Pitch and Roll Control System/SAS Dynamics Included

To summarize up to this point, a low order (9 state) linear model can not represent the helicopter's high frequency dynamics. A higher order model (22 states) that includes rotor and inflow degrees of freedom is necessary for a good approximation. In investigating the limits of roll rate feedback in hover, a rigid body linear model predicts no limit. The 22 state model predicts a limit associated with the blade progressive lag mode. However, the predicted characteristics at this limit do not match flight test experience. Adding the control system/SAS dynamic states yield instability characteristics that do correlate with test. But the cause of the instability is now quite different, involving blade flap dynamics interacting with the control system/SAS. It is precisely this type of behavior that illustrates why high order linear models are needed when analyzing complex systems.

It is clear that applying roll rate feedback beyond a certain level is counter-productive. The addition of rotor and flight control system/SAS dynamics have changed the complexion of the roll response making it very different from the first order system that a simple linear rigid body model predicts. With this in mind, methods for improving the aircraft's roll characteristics are reviewed in the next section.

8. Roll Rate Response Requirements

Traditionally, the short period or fast roll response of the helicopter has been modeled as a first order system between acceleration and roll rate:

$$\dot{p} + Lp\ p = K\ A1s$$

where Lp is a function of the rotor damping, blade inertia about the flap hinge, and body roll inertia, and K A1s is the moment applied by the rotor at the C.G. [ref. 2]. 1/Lp is the time constant of the system and equals the time necessary to obtain 63.2% of the steady state roll rate, p [ref. 7].

The pilot desires a quick, snappy response to roll rate commands, especially in the air to air combat scenario. His objective is to obtain a constant roll rate in a shorter time than is possible with an unaugmented helicopter. For this simple first order system, an improvement in the response is obtained by feeding back sensed roll rate to speed up the system time constant. The first order system becomes:

$$\dot{p} + (Lp + KRR)\ p = K\ A1s$$

Where KRR is the roll rate feedback gain. For this simple system, increasing KRR decreases the system time constant (1/(Lp +KRR)) and gives a faster response.

With rotor dynamics included in the helicopter model, the roll response of the helicopter may be more correctly defined. Instead of a first order approximation, Curtiss [ref. 2] has shown that the helicopter roll rate response is more closely approximated by a third order system where the regressive flap mode is coupled with the classic roll damping Lp, producing both a second order regressive flap/body mode and a first order body/rotor mode. These modes are seen in the root locus of the 22 state linear model with control system/SAS dynamics(figure 7). The roots shown are associated with complex motion including many degrees of freedom, but the flap/body roll motion dominates the motion associated with these modes. For low values of roll rate gain the slow real root, which for convenience retains the name classic roll damping, dominates and the first order system is an acceptable approximation. As the roll rate gain is increased, the first order root speeds up and becomes significantly less important and the roll response exhibits the motion associated with the complex regressive flap/body mode. A second order approximation is now needed to describe the roll time response:

$$p = e^{-t/\tau}(a\cos(\omega_d t) + b\sin(\omega_d t))$$

where τ is the time constant, ω_d is the damped natural frequency. The initial response of this system is dominated by the damped frequency and the long term envelope is defined by the time constant. The second order response in roll rate is clearly seen in figure 9 for the case with 2 x baseline roll rate gain.

Increasing the damping of the second order system will not give the most desirable roll response. A feedback scheme should be devised that will alter the damped frequency and the time constant of the regressive flap/body mode so that the roll response is initially as fast as possible while still avoiding unacceptable overshoot. In the complex plane this corresponds to a location as far away from the origin as possible while lying in a region between damping ratio lines of .5 and .7. With this in mind, various feedback modifications are now considered.

9. Feedback Studies

The effect of roll rate feedback on the regressive flap/body mode is relatively insensitive to increases in the BLACK HAWK control system/SAS bandwidth. Bandwidth is a measure of the speed of the control system response (i.e. an infinite bandwidth system responds instantaneously with no loss in magnitude at any frequency). Figure 10 gives the closed loop system roots for a constant roll rate gain of 2 x the baseline, for four sets of control system/SAS dynamics: standard, 2 x, 4 x and infinite bandwidth. Doubling the bandwidth of each control system element does dampen out the oscillatory response, but has an insignificant effect on the initial response, figure 11. As discussed earlier, in certain situations, the pilot desires a quick acceleration to a constant roll rate. Increasing the bandwidth only provides for a more constant roll rate in that it dampens out the roll rate oscillation; It does not significantly decrease the time needed to obtain the desired roll rate.

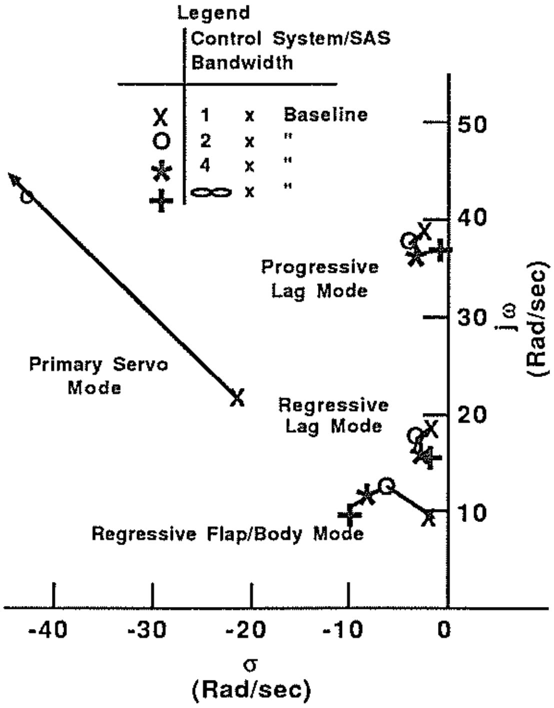


Fig. 10: System Roots With 2 x Baseline Roll Rate Gain, Varying Control System/SAS Bandwidth. Includes Flap, Lag and Uniform Inflow DOF And Roll Control System/SAS Dynamics

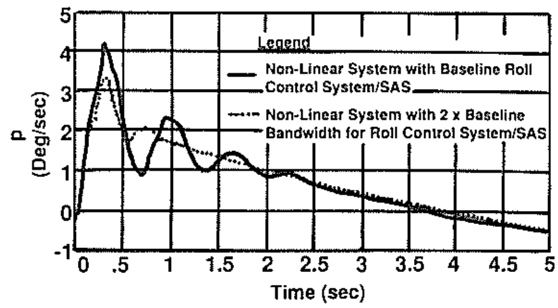


Fig. 11: Effect of Control System/SAS Bandwidth - Hover
Roll Rate Gain= 2 x Baseline Value

Since the rotor dynamics are coupled with the body dynamics, an attempt is made to speed up the helicopter roll response by feeding back rotor states. Starting with the baseline roll rate feedback, a combination of lateral flap tilt, b1f and lateral flap tilt rate, d(b1f)/dt, feedback is applied to the lateral control, A1s. Figure 12 shows that the addition of this combination of flap feedback dampens the regressive flap/body mode, but tends to destabilize the primary servo mode. In the time domain the effect of a small amount of flap feedback is clearly seen. Figure 13 shows a decrease in the roll rate oscillation with the addition of flap feedback. This provides for a more constant roll rate, but results in an insignificant change in the time to maximum roll rate.

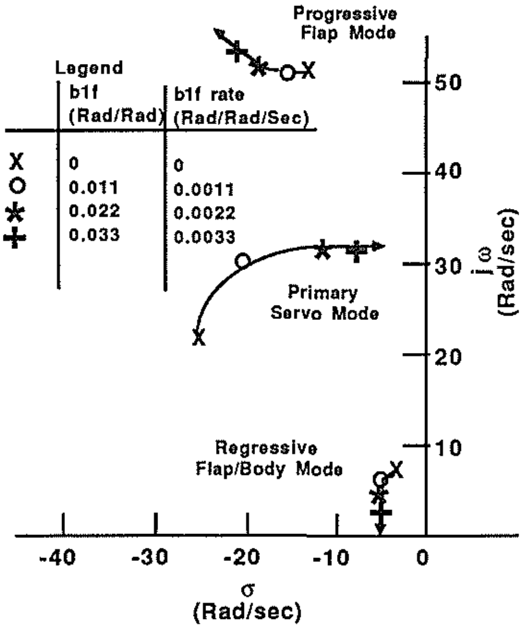


Fig. 12: Root Locus Varying Flap Feedback to A1s with Baseline Roll Rate Gain. Includes Flap, Lag and Uniform Inflow DOF and Roll Control System/SAS Dynamics

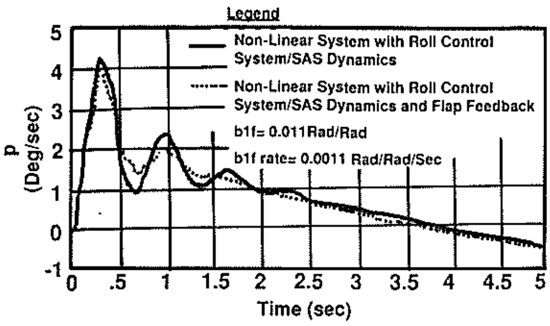


Fig. 13: Effect of Flap Feedback - Hover Roll Rate Gain= 2 x Baseline Value

As a next step, flap feedback is investigated in the presence of a "faster", higher bandwidth, control system/SAS. Figure 14 shows the root locus for the flap feedback scheme as defined in figure 12, except now a control system/SAS with double the bandwidth for each component is used along with 2 x the baseline roll rate feedback gain. The regressive flap/body mode's damping ratio increases as before, but now the progressive flap mode tends toward instability instead of the primary servo mode. Doubling the bandwidth of the primary servo reduces its direct interaction with flap feedback. The progressive flap mode is now affected since the higher bandwidth control system does not filter the flap feedback signal as strongly.

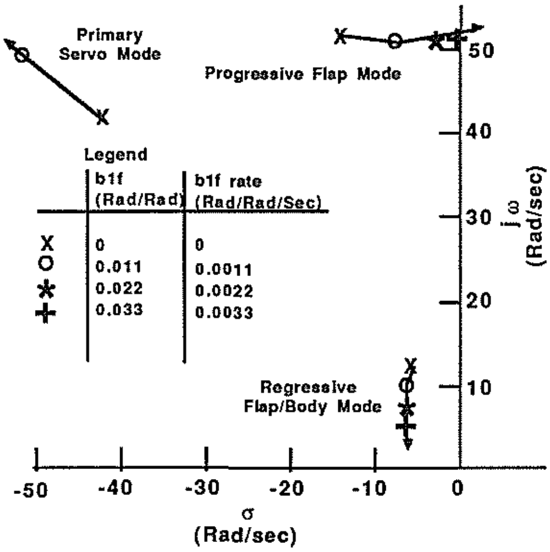


Fig. 14: Root Locus Varying Flap Feedback to A1s. 2 x Baseline Roll Rate Gain and 2 x Baseline Control System/SAS Bandwidth. Includes Flap, Lag and Uniform Inflow DOF And Roll Control System/SAS Dynamics

Rotor flap damping may be increased through the use of a Delta 3 coupling (DEL3). This coupler decreases blade pitch as the blade flaps up, effectively producing a collective flap (a0f) feedback. This type of coupling results in reduced control power and is not used on the UH-60A BLACK HAWK main rotor. However, since the regressive flap/body mode loses damping when roll rate feedback is applied, this mechanical system is explored here for its possible use in reversing this effect. Figure 15 shows the roots for the helicopter with and without DEL3. Adding DEL3 critically damps the regressive flap/body mode turning it into two real roots. However, increasing the roll rate gain gives the same result as seen earlier; the regressive flap/body mode is destabilized at approximately the same gain.

Attempts to improve the roll characteristics of the UH-60A BLACK HAWK by varying control system/SAS bandwidth and applying rotor flap feedback have not yielded any improved designs that may be added to the existing aircraft, however; the trade-offs are now better understood thanks to the added insight provided by the high order linear model.

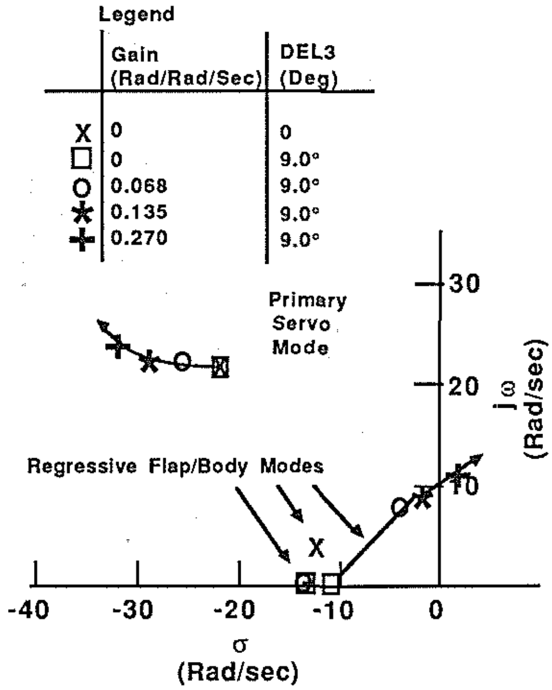


Fig. 15: Root Locus Roll Rate feedback to Lateral Control (A1s), Hover DELTA3=9.0 Includes Flap, Lag and Uniform Inflow DOF and Roll Control System/SAS Dynamics

It has been shown that the inclusion of higher order dynamics in the linear model is necessary to achieve reasonably good correlation with the non-linear model and test data. Next, rotor speed and engine/fuel control dynamics and harmonic inflow are added to the linear model and their effects are assessed in general and with respect to roll rate gain limitations.

10. Rotor Speed and Engine Degrees of Freedom

The rotor speed and engine speed degrees of freedom are added to the linear model by the previously described perturbation method. The non-linear GEN HEL simulation is reconfigured with gearbox and engine dynamics providing the additional degrees of freedom [ref. 4]. The rotor speed, Ω, degree of freedom is added to the overall system through an additional coupled first order system:

$$\dot{\Omega} = \frac{Qe - Qreq}{Jtote + Jrest} \qquad \Omega = \Omega o + \int \dot{\Omega} dt$$

where Qe is the engine torque, Qreq is the torque required by the rotor and Jtote and Jrest are system inertias. The BLACK HAWK model includes 2 General Electric T700T-GE-700 turboshaft engine models and a fuel control system modeled similar to that of reference 8. The engine dynamics are included through an additional first order system for the gas generator dynamics coupled with the full model. The change in gas generator speed is defined by:

$$\dot{Ng} = f(Q_{GT} - Q_{CM}) \qquad Ng = Ngo + \int \dot{Ng} dt$$

where Q_{GT} is the gas generator torque and Q_{CM} is the compressor torque. With the addition of the rotor speed and gas generator speed degree of freedom, the system state vector increases to 24 elements.

11. Engine Fuel Control

To complete the rotor speed degree of freedom system model, an engine fuel control system is added. This simplified system uses rotor speed error, Np, and gas generator speed error, Ng, to modulate the fuel flow to the gas generator. The fuel flow, Wfc, is added to the linear control vector, U, and partial derivatives with respect to fuel flow are included in the linear control matrix, [B]:

$$U = \begin{bmatrix} A1s \\ B1s \\ \theta mr \\ \theta tr \\ Wfc \end{bmatrix} \quad \text{helicopter controls}$$

$$[B] = \begin{bmatrix} \frac{\partial(\dot{u})}{\partial A1s} & \frac{\partial(\dot{u})}{\partial B1s} & \frac{\partial(\dot{u})}{\partial \theta mr} & \frac{\partial(\dot{u})}{\partial \theta tr} & \frac{\partial(\dot{u})}{\partial Wfc} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial(\dot{Ng})}{\partial A1s} & \frac{\partial(\dot{Ng})}{\partial B1s} & \frac{\partial(\dot{Ng})}{\partial \theta mr} & \frac{\partial(\dot{Ng})}{\partial \theta tr} & \frac{\partial(\dot{Ng})}{\partial Wfc} \end{bmatrix}$$

system control matrix

Figure 16 shows the effect of adding the rotor speed and gas generator degrees of freedom and the simplified 6 state engine fuel control model. The rotor speed degree of freedom adds a real root indicative of the rotor and gearbox inertia and also couples with the collective lag mode resulting in a higher frequency, more highly damped mode, sometimes referred to as the rotor first torsional mode. The first fuel control cross over mode associated with fuel control and gas generator dynamics is now seen.

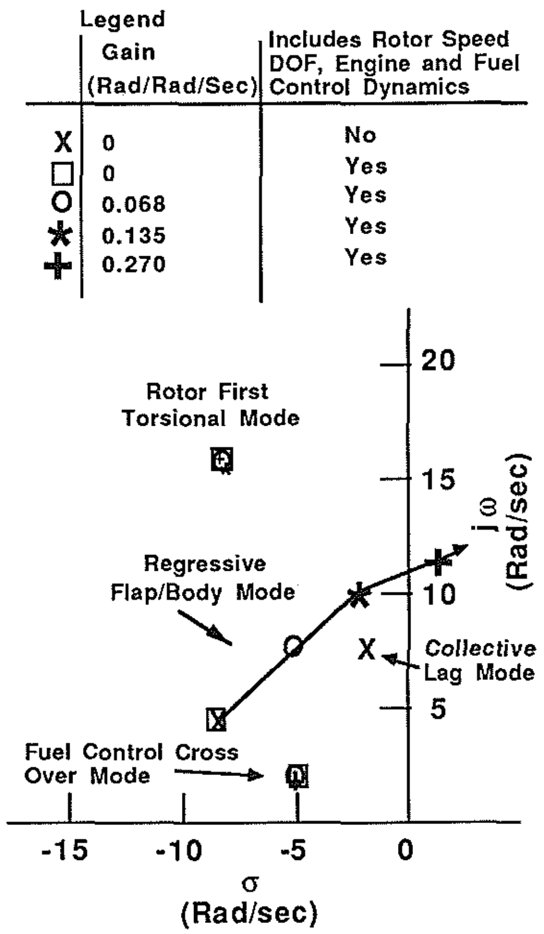


Fig. 16: Root Locus p to A1s.Includes Flap, Lag and Uniform Inflow DOF, Rotor Speed DOF and Engine/Fuel Control Dynamics and Roll and Pitch Control System/SAS Dynamics Hover

With the addition of the engine and rotor speed dynamics to the linear model, the roll rate gain limitation previously determined may be reevaluated. The 24 state model in hover with the pitch and roll control system/SAS (figure 16) and the fuel control system dynamics is used for this analysis. The engine/rotor speed dynamics do not affect the roll rate limitations, and the regressive flap/body mode is destabilized with increasing roll rate gain as shown earlier. Note that with the pitch control system/SAS active, the regressive flap/body mode is initially less damped due to coupling with the body pitch dynamics, but crosses the imaginary axis at approximately the same gain as before. At a forward speed of *100 Kts, the same trend is* observed (figure 17).

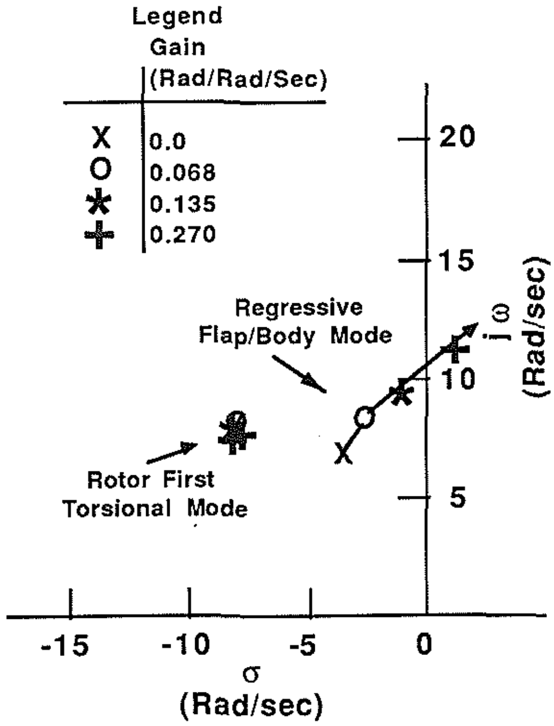


Fig. 17: Root Locus p to A1s.Includes Flap, Lag and Uniform Inflow DOF, Rotor Speed DOF and Engine/Fuel Control Dynamics and Roll and Pitch Control System/SAS Dynamics 100 Kts

12. <u>Harmonic Inflow</u>

Both the non-linear GEN HEL simulation and the linear model only include the effects of uniform inflow. As described earlier, the uniform inflow model predicts the induced velocity of air through the rotor as a function of thrust. Recently published work [ref. 2] shows that the induced velocities resulting from rotor pitch and roll moment are significant and should not be ignored for an articulated rotor. The roll and pitch moments are the result of first harmonic lift distributions across the rotor disk, and consequently, produce first harmonic inflow. The cosine, D_{WC} and sine, D_{WS} contributions to harmonic inflow are defined as functions of pitch and roll moment respectively [ref. 4]:

$$D_{WC} = \frac{K_{CM}\, C_{MHA}}{U_{TOT}} \left(\frac{1}{1 + \left(\frac{T_{DWC}}{U_{TOT}} \right) s} \right)$$

$$D_{WS} = \frac{K_{SM}\, C_{LHA}}{U_{TOT}} \left(\frac{1}{1 + \left(\frac{T_{DWS}}{U_{TOT}} \right) s} \right)$$

where K_{CM} and K_{SM} are empirically determined gains, C_{MHA} and C_{LHA} are the pitch and roll moment coefficients respectively, and T_{DWC} and T_{DWS} are empirically determined time constants. As with uniform inflow, the change in induced velocity is passed through a first order lag that has a time constant dependent on the rate of air flowing through the rotor.

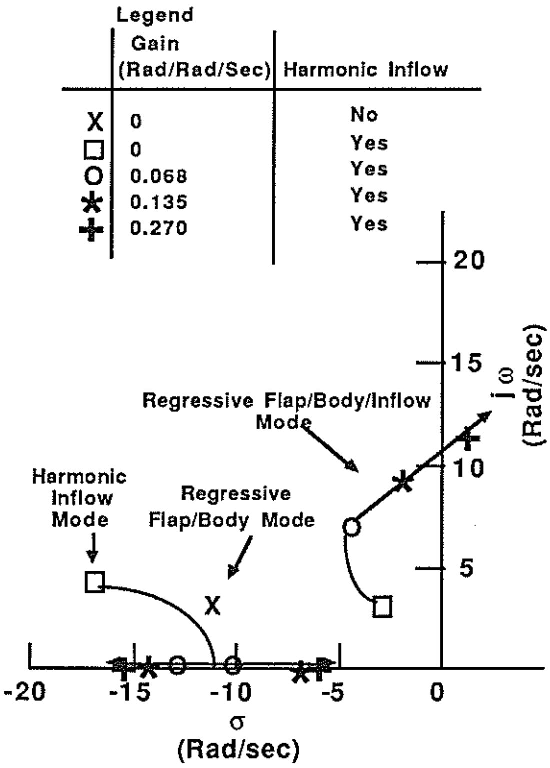


Fig. 18: Root Locus p to A1s. Includes Flap, Lag and Uniform Inflow DOF, Roll and Pitch Control System/SAS Dynamics and Harmonic Inflow
$K_{CM}=K_{SM}=1.0$,
$T_{DWC}=T_{DWS}=.01038$

Most previous BLACK HAWK analyses have ignored the effect of harmonic inflow, with the assumption that the rotor's aerodynamic moments are not large enough to produce a significant change in rotor induced velocities. Curtiss and Xin [ref. 9] show that with the addition of properly defined harmonic inflow to the articulated rotor model, correlation with flight test data is significantly improved.

With the addition of harmonic inflow to the linear model [no engine or rotor speed degrees of freedom], the regressive flap/body mode is replaced by two pairs of complex roots (figure 18). One pair is highly damped and essentially describes the relatively fast motion of accelerating the air through the rotor, and may be thought of as the harmonic inflow mode. The addition of harmonic inflow produces an essential reduction in the lift curve slope of the rotor blades and therefore a reduction in rotor/body damping. The rotor and body mode couple with the harmonic inflow and produce the lightly damped regressive flap/body/inflow mode(figure 18). Dynamic harmonic inflow results in two effects. The basic body/rotor motion is slowed due to a decrease in rotor damping and this slowing is delayed by lagging the harmonically induced velocities.

The change in rotor/body dynamics with harmonic inflow is significant, but it does not change the roll rate feedback gain limit. Figure 18 gives the root locus for the helicopter model with harmonic inflow and the roll control system/SAS dynamics as a function of roll rate feedback. The regressive flap/body inflow mode is destabilized and crosses the imaginary axis at approximately the same gain value as seen for the regressive flap/body mode without harmonic inflow [figure 7].

13. Conclusions

1. High fidelity linear models are essential for the application of the large number of linear analysis techniques, now available, to yield a better understanding of helicopter dynamics and should be included in the array of tools available to the helicopter control law designer.

2. In hover, at low levels of roll rate feedback, the UH-60A BLACK HAWK roll dynamics may be approximated by a first order system, but with gain increasing to practical levels the fuselage couples with the rotor flap dynamics requiring a higher order system representation.

3. The roll rate gain feedback limitation for the BLACK HAWK is a function a body/flap dynamics and control system bandwidth.

4. Initial attempts to improve the helicopter's roll rate response have not provided any realistic solutions, but have provided important insight.

5. Engine/Rotor speed dynamics affect overall helicopter dynamics, but do not affect the roll rate gain limitation.

6. Harmonic inflow has a significant effect on helicopter transient response, but does not affect the roll rate gain limitation.

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