MULTIVARIABLE METHODS FOR HELICOPTER FLIGHT CONTROL LAW DESIGN : A REVIEW

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Abstract

The design of flight control systems for actively controlled helicopters presents problems which are associated not only with the complex nature of the dynamics of the vehicle itself but also with the range of design objectives which must be satisfied. The techniques of multivariable control system design provide tools of potential value for helicopter flight control applications. The paper describes an approach which has been developed for the comparison of multivariable design methods using control law design criteria which can be related to handling quality requirements, robustness, noise rejection properties and insensitivity to atmospheric disturbances. The definition of the flight control task for these comparative investigations involves a reduced design problem with performance assessments based on linearised models.

1. Introduction

A review of published accounts of helicopter flight control law design shows that the approaches most widely used have involved the application of single-input single-output techniques to each control loop individually. Enns [Ref. 1] provides an interesting and recent account of this classical type of approach for the Apache AV05 YAH64 flight control system and Tischler [Ref. 2] has given a further example of classical methods in a highly detailed assessment of flight control system design and implementation considerations for the Advanced Digital Optical Control System (ADOCS) demonstrator.

Although classical design methods are of very great practical value, and are likely to remain important tools for control system designers for the foreseeable future, careful consideration must also be given to the techniques of multivariable system design which have been developed during the past two or three decades. These techniques provide an integrated approach which has been found to have benefits in applications involving other highly-coupled multi-input multi-output systems and must therefore be given careful consideration in the case of the actively controlled helicopter.

In control systems theory a multivariable system is one in which there is more than one input and more than one output. The linear time-invariant state-space representation for the system to which control is to be applied is given by the equations

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad (1)$$

$$\underline{y} = C\underline{x} + D\underline{u} \qquad (2)$$

The vector $\underline{u}$ represents the inputs to the system, the vector $\underline{x}$ represents the state variables and the vector $\underline{y}$ represents the outputs. Taking Laplace transform of variables in eqns. (1) and (2) and ignoring initial conditions we obtain the equations

$$s\underline{X}(s) = A\underline{X}(s) + B\underline{u}(s) \qquad (3)$$

$$\underline{Y}(s) = C\underline{X}(s) + D\underline{U}(s) \qquad (4)$$

Re-arranging equation (3) to obtain $\underline{X}(s)$ in terms of the input vector $\underline{U}(s)$ gives

$$\underline{X}(s) = [sI - A]^{-1} B\underline{U}(s) \qquad (5)$$

and thus

$$\underline{Y}(s) = [C(sI - A)^{-1} B + D]\underline{U}(s) \qquad (6)$$

For any given system there may be many different state-space representations and the matrices A, B, C and D are therefore not unique. The overall transfer function matrix P(s) which relates the output $\underline{Y}(s)$ to the input $\underline{U}(s)$ is, however, unique and is given, from eqn. (6) by

$$P(s) = C(sI - A)^{-1} B + D \qquad (7)$$

The helicopter provides a classic example of a system which is multivariable in form having four control inputs in terms of the conventional collective, longitudinal cyclic, lateral cyclic and tail rotor controls. The helicopter also tends to be highly cross-coupled in its characteristics.

In general for a practical multivariable control design problem the characteristics of the system to be controlled, P(s), are only partly known. The objective is to determine the characteristics of the controller which will yield a desired relationship between the input and the output.

Many synthesis techniques exist for multivariable control systems. The suitability, or otherwise, of each of these methods for helicopter flight control system design is by no means obvious and a collaborative programme of research has been initiated between the Royal Aerospace Establishment (Bedford) and the University of Glasgow to assess the potential of a number of frequency-domain and time-domain approaches in the helicopter application.

The study has three distinct aspects. First, as outlined above, a number of different design methodologies are being reviewed. Techniques being considered include time-domain techniques such as linear quadratic regulator/Gaussian methods which have been quite widely used in helicopter applications [Refs. 3,4]; eigenstructure assignment methods which have been described in papers by Parry and Murray-Smith [Ref. 5], Garrard and Liebst [Ref. 6] and Innocenti and Stanziola [Ref. 7]; and singular perturbation methods which have been considered for other aerospace applications [Ref. 8]. In the frequency domain approaches being considered include the H_∞ method which is the subject of a recent paper by Yue and Postlethwaite [Ref. 9] quantitative feedback theory [Ref. 10], the Nyquist array [Ref. 11] and characteristic locus methods [Ref. 12].

The second objective of the study is to gather together a comprehensive set of control law design criteria appropriate for multivariable design methods. The rotorcraft handling quality requirements [Ref. 13] provide the basis of these criteria with specifications of response-types, including the choice of controlled variables, and dynamic response measures such as bandwidth, phase delay and damping ratios. However, the handling qualities are not sufficient in themselves to provide a good design and other factors relating to robustness, sensor noise rejection and insensitivity to atmospheric disturbances must also be taken into account.

The third objective of the study is to conduct more detailed design investigations using selected design methods to produce control laws which can be evaluated through piloted simulation.

The assessment and comparison of control law design methods involves not only a theoretical review but also the completion of a limited design example for each approach considered. The definition of the flight control problem for these comparative investigations involves a reduced problem with performance assessments based on linearised models. Efforts have however been made to include most of the features peculiar to rotorcraft flight control problems.

The purpose of the initial design task is to facilitate the selection of candidate design methods for use in the third phase of the study. In the preliminary analysis questions concerning available design freedoms are being posed to determine which techniques are suitable for rotorcraft applications. Specifically, the design freedoms are evaluated in terms of their physical significance and whether they can be used to improve handling qualities or the overall robustness of the system.

The paper addresses the general problem of comparing control system design methods and describes the approaches which have been used. More detailed consideration of linear quadratic optimisation and eigenstructure assignment methods may be found in two other papers being presented at this Forum [Refs. 14, 15].

2. The Influence of Rotorcraft Dynamics on Control Law Design

Many of the problems of applying multivariable control system design methods to rotorcraft are associated with the dynamics of the vehicle. The rigid-body characteristics are of principal interest in the context of flight control law design in the sense that state variables of the rigid body equations are the controlled quantities of the closed-loop system. The rigid-body dynamics with a quasi-steady rotor representation are characterised by strong cross-coupling effects, non-minimum phase zeros and significant nonlinearities. The main source of cross-coupling and nonlinearity is associated with the fact that the main rotor is used to generate both thrust and control moments. Any change of tip-path-plane orientation which is intended to produce a change of flight condition will give rise to precessional effects which will influence all of the forces and moments applied to the rotor hub.

The approach most generally adopted for the design of helicopter flight control laws involves tailoring the response of the low-order rigid-body dynamics to satisfy appropriate design objectives in the presence of higher-order dynamics associated with the engines, the actuators and the main rotor. The higher-order effects involving the actuators and rotor are normally found in the upper part of the frequency range of interest for flight control system operation.

The characteristics of turbine engines introduce both additional lags and a form of coupling between yawing motion of the fuselage and collective pitch changes [Ref. 16]. This coupling arises because the engine governor senses yawing motion as a change of shaft speed and the resulting action of the governor produces a change of rotor thrust.

The dynamics of the actuators used to control blade pitch may be represented by first-order lag transfer functions with additional rate limits and authority limits. In order to avoid these actuator nonlinearities, particularly the authority limits, input signal amplitudes at the actuators should be kept small and this imposes constraints on the range of controller gains which is acceptable.

Rotor flapping dynamics, lead-lag dynamics and inflow dynamics are all believed to be significant factors which must be taken into account in flight control system design. The essential problem here is that, as controller gains are increased, non-minimum phase zeros attract poles of the rotor model, such as those associated with the regressing flapping mode, towards the right half plane [Ref. 17]. This imposes a further constraint on the values used for feedback gains in order to avoid problems of instability. It should be noted that rotor state information cannot, at present, be used within control laws due to problems associated with the integrity of signals derived from the rotor. The use of feedback to stabilise modes associated with the rotor is therefore not believed to be feasible in the foreseeable future.

Although actuator and rotor dynamics generally affect the upper part of the frequency range of interest for flight control system design the control system bandwidth inevitably includes frequencies for which these unmeasurable elements of the system are active. Controllers must be based upon output feedback rather than full state feedback. Lack of detailed information relating to high frequency modes also limits the effectiveness of the flight control system at high frequencies due to unmodelled dynamics.

3. Flight Control Law Design Objectives

Control law design objectives are closely linked to mission-oriented handling qualities requirements. However for a helicopter equipped with a flight control system it must be noted that while good handling qualities are necessary they are not sufficient to guarantee acceptable performance in that system. A good flight control law should also provide low sensitivity to atmospheric disturbances, should reject sensor noise and should be robust both to variations of flight condition and to unmodelled dynamics of the fuselage or rotor.

In considering multivariable control law design methods the design objectives should also be considered in a multivariable context since a range of design requirements must be satisfied simultaneously. The handling qualities specified in [Ref. 13] involve classical time domain and frequency domain parameters such as time constants, damping ratios, bandwidths and phase delays. Transfer functions are used to describe the characteristics of each control channel between a pilot input and the corresponding output variable. This single-input single-output perspective and the combination of time-domain and frequency-domain measures leads to some difficulties in integrating the handling quality requirements within a multivariable control law design process. However it is necessary to use the handling qualities criteria in the evaluation of multivariable designs and the subset of the requirements which involve required response type and dynamic response criteria can be used more explicitly as design objectives for multivariable control law development [Ref. 18].

4. A Generic Flight Control Law Structure

One of the main objectives in the assessment of multivariable control system design methods has been to compare methods of approach and to analyse the relative merits of resulting designs. In order to evaluate designs in this way using computer simulations it has been necessary to develop a generic control law structure which would allow controllers to be changed with relative ease. Such a structure also allows controllers to be compared in terms of the complexity of their implementations. For example, one technique may require dynamic compensation elements where another does not. By forcing all of the control laws to be described in terms of the same generic structure such comparisons can be accomplished with relative ease.

If one considers only linear control laws the most general form of equation describing the controller, using the state space formulation of equations (1) and (2), may be written as

$$\underline{x}_{GCL} = A_{GCL}\,\underline{x}_{GCL} + B_{GCL}\,\underline{u}_{GCL} \qquad (8)$$

$$\underline{y}_{GCL} = C_{GCL}\,\underline{x}_{GCL} + D_{GCL}\,\underline{u}_{GCL} \qquad (9)$$

where the subscript GCL denotes a generic control law entity. As the output vector of the controller is the input vector for the vehicle state equation (1) it follows that

$$\underline{u} = \underline{y}_{GCL} \qquad (10)$$

In addition it should be noted that since feedback matrices are a necessary feature of the generic structure it is necessary to define the generic control law input vector $\underline{u}_{GCL}$ as

$$\underline{u}_{GCL} = \begin{bmatrix} \underline{r} \\ \underline{y} \end{bmatrix} \qquad (11)$$

where $\underline{r}$ is the pilot input vector and $\underline{y}$ is the overall system output.

All linear controllers can be manipulated into the above generic form using block diagram algebra and software has been written to perform this operation using the facilities of the MATLAB matrix operations package [Ref. 19]. This generic structure has been implemented within a helicopter simulation model.

For nonlinear control laws there appears to be no simple generic structure available. Nonlinear controllers must be dealt with on an individual basis.

5. Definition of the Control Law Design Task for Initial Testing

The purpose of defining a controller design task for initial testing of control law design methods is to identify difficulties associated with the application of each technique to the helicopter flight control problem. Testing of the resulting controllers should provide sufficient information to allow techniques to be selected for further more detailed design and evaluation studies.

For the initial testing task it is necessary to constrain the design problem in such a way that the description of the system being controlled and the design objectives are similar for each design method considered. This requires separate consideration of the design issues raised by system dynamics and the design issues raised by the control law objectives.

5.1 Design Issues Associated with System Dynamics

Characteristics of the helicopter which restrict the way in which particular design methodologies may be applied must be considered very carefully for each design technique. Factors relating to the form of rigid body dynamics, rotor dynamic actuator dynamics and the available measurements can have an important bearing on the applicability of a particular design method.

5.1.1 Choice of Simulation Model

The choice of which simulation model to use in the initial testing of controllers affects both the time involved in the study and the amount of detail which can be used in comparing results. If one were to use a full nonlinear helicopter model, then it would be possible to compare techniques in terms of both the aerodynamic nonlinearities due to flight condition and the hard nonlinearities associated with authority and rate limits on the actuators. However, it was decided in the case of the present study that the time needed to develop controllers using a large number of techniques with a nonlinear simulation would be prohibitive. Therefore, linear models of the Lynx derived from the HELISTAB package [Ref. 20] have been used for initial testing. The linear representations of the system's dynamics can be either the state space description of MATLAB [Ref. 19] or a TSIM model [Ref. 21]. The development of control laws make use of the facilities of both MATLAB and TSIM. Software has been written to ease the transfer of data from HELISTAB to MATLAB and a TSIM linear helicopter model HELIGEN.

5.1.2 Choice of Flight Condition

Having decided that linear models are to be used in the study, the question of what flight condition to use is immediately raised. The dynamics of rotorcraft are significantly different in the hover than in forward flight. In order to limit the initial testing it has been necessary to restrict the study to one of these flight regimes. The choice of using a linear model based on a forward flight condition has been made because:

i) the dynamics change less rapidly than in the neighbourhood of the hover.

ii) HELISTAB is known to be more accurate in this flight regime.

iii) the research group at Glasgow University had previous experience of forward flight control law development.

The statement indicating that HELISTAB is more accurate in forward flight is based on the fact that inflow significantly affects both short term and long term hover dynamics. The currently available HELISTAB model does not incorporate inflow dynamics. The flight condition for controller design will be 80 knots level flight because it is near to the minimum power point, at which control margins are a maximum. In addition, the rate of change of system dynamics with flight condition is not rapid near 80 knots.

5.1.3 Representation of Rigid Body and Rotor Dynamics

In terms of the vehicle's rigid body

dynamics, the main design issue involved in the use of a linear model is whether or not the heading angle, Ψ, should be included as an element of the state vector. The heading angle, Ψ, is not fully coupled with the other states and the inclusion of Ψ causes the linear model's system matrix, A, to have an eigenvalue at the origin. Thus, including Ψ as a state precludes the use of any design method which requires the inversion of A. Ignoring the presence of Ψ causes difficulties in the implementation of heading hold functions which are required by handling quality considerations. For the initial testing of design methods, it has been decided that Ψ should be included as a state variable if possible. However, the analysis of the controlled system should discuss whether or not Ψ is included and its effects on the stability of the controlled system. This may involve the development of controllers with and without Ψ as a state variable.

Rotor dynamics will restrict the level of feedback gains in a system and hence in a detailed design study they must be considered. However, it has been deemed inappropriate to include rotor dynamics in the initial testing because of the complications they would introduce and the limited amount of time which is available. Therefore, the quasi-static approximation will be used in lieu of rotor flapping degrees of freedom.

5.1.4 Representation of Actuators

Actuators are commonly represented by first order lag networks with authority and rate limits. The phase lag introduced by actuators is noticed by the pilot if it is too large. Consequently, as total system delay is limited by handling quality considerations, the actuation systems affect the design of the controller. Unfortunately, the actuator states cannot be measured directly which causes difficulties in the implementation of some controller design methodologies. In order to test the effects of actuators on the controlled system, the designer will be permitted to make use of the following three representations of the combined actuator-rigid body system:

i) rigid body model (rigid body state feedback)

ii) rigid body model with four actuator states (rigid body state feedback)

iii) rigid body model with four actuator states (full state feedback including actuators)

During initial testing, these three models are available for use so that the effects of actuators may be considered. For the first two options, the controller is designed on the basis of the rigid body dynamics only, while in the third option the actuators will be assumed to be elements of the state vector. Thus in the second option, the actuators represent unmodelled dynamics. It is noted that the third option is included to ease the academic exercise of comparing techniques. Furthermore, the selection of which of these models to use in the controllers' development is left to the discretion of the designer, but the decision should be based on the theory being evaluated. In assessing results, consideration must be given to any problems caused by the actuators in the development of the control law. For instance, can actuation dynamics be included in the description of the plant? Alternatively, how do actuators rearrange the system's poles when they are included in the design? Do the actuators increase the levels of coupling?

5.1.5 Available Measurements and Output Feedback

One of the problems which will eventually face all of the techniques being studied is the finite amount of information which can be measured on the system and made available to the flight controllers. Eventually, control laws will be required to operate in an output feedback mode where some states of the system are either unmeasurable or even unknown. To give consideration to this problem at an early stage, control laws must be developed for as many of the three models discussed in Section 5.1.4 as possible. That is, actuators are used in the role of unknown dynamics. Therefore, in the analysis of the controllers developed for these three models it will be necessary to carefully distinguish output feedback characteristics which will be unique to actuators from those which will be true for other high-order dynamics.

5.2 Design Issues Raised by Control Law Objectives

The development of control laws will also be constrained by the design objectives. The control law objectives which must be considered in the initial testing task include; stability, command tracking, handling qualities, robustness to unmodelled dynamics, insensitivity to atmospheric disturbances and control effort. For each of these design issues the following sections will describe the relevant criteria and methods used for evaluation.

5.2.1. Stability and Command Tracking

It goes without saying that the controlled system must be stable. That is, all of the eigenvalues of the controlled system must be in the left half plane or, in certain cases, at the origin.

In flying a helicopter the pilot will need to change operating points in the flight envelope in order to manoeuvre the aircraft. Therefore, flight control laws which are implemented will be solutions of a servo problem as opposed to a regulator problem. To allow tracking of the pilot's input commands by particular system outputs the controller may need a more complicated architecture than simply a constant gain feedback matrix. Some control techniques achieve tracking by closing a single feedback loop around a dynamic cascade compensator and the plant, while other methods require separate stability and command feedback loops. A further alternative which is popular in aerospace applications is the use of model following techniques in which a regulating controller is designed for the system's error dynamics. The deciding factor in terms of whether dynamic compensation must be used is the choice of outputs which must track pilot inputs. The list of outputs to be tracked are discussed in Section 5.2.3.

Having identified command tracking as a design issue it is necessary to consider how to deal with the problem during the initial phase of control law testing. Some of the methods under examination can be adapted to generate both regulators and servos while others are suitable for one problem and not the other. At the present time, techniques are still being evaluated in terms of what they can and cannot achieve. Therefore, if a technique can be used to generate a tracking system it should be so used. However, if the method is suitable for the design of regulators only, then it should be judged on the characteristics of the stability loop. Consequently, during the initial testing, regulating techniques will be used to develop stabilizing feedback loops which will be judged against other regulators. Similarly, tracking controllers will be developed using appropriate techniques and judged against other command following systems. It must be remembered that few of the design methodologies are sufficiently comprehensive to be employed on their own: most flight control laws will make use of two or more techniques in order to achieve both stability and command following.

5.2.2 Handling Qualities

The handling quality criteria [Ref. 13] are extensively used during flight trials for the evaluation of the system's performance. Therefore, it makes sense to utilize handling quality criteria as design objectives for the control law development process as much as possible. However, the scope for considering handling quality requirements is severely restricted by the limited validity of the linear models which will be employed during initial testing. Nevertheless, there are two major topics addressed by the handling quality specifications which are relevant, and they are the required response-type and the dynamic response criteria of *Sections 3.2, 3.3 and 3.4* of [Ref. 13]. As the initial testing of control laws will be based on plant models for the Lynx at 80 knots, the Low Speed/Hover dynamic response requirements of *Section 3.3*, [Ref. 13], are not needed. Furthermore, it is assumed that the "Air Combat" requirements in the handling quality specifications are to be used since they are the most stringent in terms of aircraft performance.

For each control law, every effort is made to evaluate it against the requirements listed above. However, the criteria have been written assuming that the control law performs both stabilizing and command tracking functions. When it comes to evaluating the bandwidths of regulators one must take into account the fact that the regulator will be driven by another component of the control law such as a model-following block. In a recent paper, Tischler [Ref. 22] indicates that inner feedback loops should have crossover frequencies between 4 and 6 rads/sec. when a model following configuration is adopted for the controller. Thus, when looking at stability loops which may be used with model-following front-ends, pitch, roll and yaw bandwidths should exceed the 3.5 rad/sec. Level 1 (Air Combat) limits in the MIL-H-8501A update [Ref. 13] as much as possible.

5.2.3 Selection of Tracking Outputs

In some flight control law design methodologies it is necessary to select a set of outputs which will be controlled such that they track the pilot's input commands. The number of tracking outputs will be equal to the number of pilot inceptors. Therefore, in helicopter applications, four tracking variables must be selected and associated with particular pilot inceptors. The following table lists possible tracking variables for each inceptor which appear to be compatible with forward flight handling quality requirements and the results of a Royal Aerospace Establishment piloted simulation study [Ref. 23].

There are 8 possible combinations of these outputs which yield a system configuration which can be flown.

An examination of the dynamics of the plant model has been carried out for each of the following sets used as plant output variables

1) $S_1 = \{\mathring{h}, q, \Omega, \beta\}$
2) $S_2 = \{\mathring{h}, q, p, \beta\}$
3) $S_3 = \{\Gamma, q, \Omega, \beta\}$
4) $S_4 = \{\Gamma, q, p, \beta\}$
5) $S_5 = \{\mathring{h}, \theta, \Omega, \beta\}$
6) $S_6 = \{\mathring{h}, \theta, p, \beta\}$

7) $S_7 = \{\Gamma, \theta, \Omega, \beta\}$
8) $S_8 = \{\Gamma, \theta, p, \beta\}$

Table 1: Inceptor - Tracking Output Possibilities

Vertical	$\dot{h}$ - height rate Γ - flight path angle
Longitudinal	q - pitch rate θ - pitch attitude
Lateral	Ω - turn rate p - roll rate
Pedals	β - sideslip

Specifically, the algorithms of MacFarlane and Karcanias [Ref. 24] were used to determine the location of the invariant zeros for the eight system configurations. Nonminimum phase zeros can cause difficulties for control law design because as one increases feedback gains, a subset of the poles of the system migrate to the invariant zeros. Thus, if these zeros are, in the right half plane, increasing feedback will eventually lead to instability.

For a ninth order HELISTAB [Ref. 19] model of the Lynx in a level 80 flight condition all sets of outputs gave a zero, associated with the heading angle, within a radius of 10^{-14} of the origin in the s-plane. In addition, when q is used in preference to θ, as in the first four output sets, an additional zero appears within the 10^{-14} radius of the origin. Due to computational inaccuracies, it is possible to say whether these zeros are minimum phase or not. It was concluded that the analysis of the systems zeros did not preclude any of the sets from consideration.

For the purposes of initial testing of the design techniques, it is necessary to minimise the work by concentrating on one set of outputs and to leave a comprehensive analysis of the relative merits of the different output configurations until a later stage in the work. In choosing between the sets, it is noted that the handling quality criteria [Ref. 13] have requirements on the system's $\dot{h}$ and Ω responses, and additionally call for a rate command/attitude hold response-type in forward flight implying the use of q in preference to θ. Therefore, all tracking systems will initially have output vectors given by S_1.

$$y_t = \begin{bmatrix} \dot{h} \\ q \\ \Omega \\ \beta \end{bmatrix}$$

5.2.4 Robustness to Flight Condition and Unmodelled Dynamics

Because the helicopter is nonlinear with respect to flight conditions, it is necessary to control the system with a nonlinear strategy. One approach is to design linear controllers at discrete intervals in the flight envelope and then to use gain scheduling techniques to adjust the control law parameters. The number of design points which must be examined is a function of the robustness of the techniques being used. A good technique might require linear designs at intervals of 20 knots while a less robust control law would force the engineer to use much smaller intervals. For example, in order to gain insight concerning the problem of controller integrity with changing flight conditions, the control law designed for 80 knots could be evaluated with plant matrices for other forward speeds. The forward velocity could be changed from 60 to 100 knots in 10 knot increments and root loci may be plotted showing the migration of the rigid body poles. Although this test is designed to evaluate the control law's performance with changes in flight conditions, it can also be seen to help in the analysis of performance with respect to unmodelled dynamics if one interprets the changes in system matrices to be the result of high-order modes.

In order to limit the amount of testing which must be performed, the robustness of the control law to unmodelled dynamics may be analyzed in terms of two previously described tests. First, one can assume that changes in flight condition represent unmodelled dynamics as discussed above. Thus the root loci can be examined in terms of the direction and amount of movement of the system's rigid body poles with flight condition. The second test involves studying the performance of each control law with respect to the three plant descriptions of Section 5.1.4. That is, actuators play the role of unmodelled dynamics.

5.2.5 Noise Rejection

Because the sensors used to measure the vehicle's flight condition are prone to picking up the n/rev. vibrations of the airframe, it is necessary to consider the integrity of the controller with respect to noise. In discussing n/rev. vibrations, one is referring to those vibrations which are generated by the main rotor and transmitted via the hub to the fuselage. As the rotor frequency is known, the control law will be excited by the superposition of sinusoidal noise, of the rotor frequency and its harmonics, onto the states. The control law should act as a low-pass filter in rejecting the sensor noise.

5.2.6 Insensitivity to Atmospheric Disturbances

Atmospheric disturbances such as gusts, turbulence and wind shear are typically active over the same frequency band as the rigid body modes. The perturbations to the vehicle's flight path cause by these disturbances are evident on measurements of the rigid body's linear and angular velocities which are fed back to control laws. In response to the need to consider atmospheric disturbances, two approaches are being considered. The first option is to use a validated wind model such as [Ref. 25]. The alternative is to consider the transfer function of the disturbance inputs to either the actuation system or the vehicle's states as has been suggested by Baillie and Morgan [Ref. 26]. The control law will be required to actively suppress the motions initiated by atmospheric disturbances.

5.2.7 Control Effort

In order to assess whether actuator authority or rate limits will be reached, it is necessary to consider the control effort used by different systems. For example, if one control law design methodology requires a high level of actuator activity, with high frequency signals of large amplitude, then it should be rejected in favour of a second design technique which drives the actuators with smaller signals of a lower bandwidth. Simple approaches to testing control laws in this regard could be based on comparing the magnitudes of controller gains or the levels of actuator activity in standard manoeuvres. A more sophisticated test could involve an analysis of the variation of maximum singular values with frequency for transfer functions from the states to the actuators or from the inputs to the actuators.

6. Multivariable Control Law Design Methods

The flight control law design problem described in Section 5 is being used to evaluate several multivariable control law design techniques. As this aspect of the study is still in progress, this section of the paper will briefly describe the methods which are being examined and will outline the results which have been obtained to date.

In order to provide a structure to the review, each method has been assigned to one of the four groups: i) Time Domain Methods, ii) Frequency Domain Methods, iii) Model Reference Techniques, and iv) Output Feedback and techniques involving State Estimators. While these categories are not the only ones possible they have been useful in terms of identifying common threads of ideas and procedures between methods which are superficially distinct.

6.1 Time Domain Methods

The time domain methods under consideration are the following:

i) Linear Quadratic Regulator/Linear Quadratic Gaussian Method
ii) Eigenstructure Assignment
iii) Multivariable Root Loci
iv) The Salford Singular Perturbation Method
v) Sliding Mode Control

These techniques are classified as time domain approaches because the designer attempts to manipulate the available design freedoms to optimise time response characteristics such as rise time, settling time, and damping ratio. Eigenstructure assignment, multivariable root loci, and the Salford Singular Perturbation Method are similar in that one attempts to find optimum pole positions.

6.1.1 Linear Quadratic Regulator (LQR)/Linear Quadratic Gaussian Method

The LQR method leads to a full state feedback law for nominal system and in this case enjoys excellent robustness properties [Ref. 27]. In the more realistic situation of output feedback a state estimator may be used or the design may be based on a reduced order model. In these cases the original robustness properties do not apply and it may be appropriate to use loop transfer recovery methods (see section 6.4.2).

The design freedoms in the LQR method are two matrices which penalise excursions of state and input vectors from desired values. Traditionally the LQR method has been seen as lacking 'visibility' in the sense that it was not obvious how to choose these matrices to achieve desired loop properties. In fact, LQR design can now be guided by eigenstructure assignment [Ref. 28] or loop shaping [Ref. 27] considerations and this criticism is no longer completely valid.

The review of the LQR technique is nearing completion and a discussion of some of the findings are presented in a companion paper [Ref. 14].

6.1.2 Eigenstructure Assignment

Eigenstructure assignment gives the designer some degree of insight into the dynamics of the controlled system by virtue of the relationships which exist between eigenvalue locations and the bandwidths of classical single-input single-output systems. The key to a successful eigenstructure design, however, is an appropriate manipulation of the closed-loop eigenvectors. Indeed, the particular eigenstructure assignment algorithm discussed in the companion paper (Reference (15)) has been used with great effect to satisfy the design objectives. As with the review of linear quadratic regulator theory, the study of eigenstructure assignment in terms of the initial testing task is nearing completion.

6.1.3 Multivariable Root Loci

Multivariable root loci are more of an analysis tool than a controller synthesis procedure. The root loci plots show asymptotic pole migrations for multivariable systems as a characteristic closed-loop gain is increased without bounds. Multivariable root loci are used in singular perturbation methods [Ref. 8] to assist in the optimization of high gain error activated controllers. For this reason, a separate analysis of the use of multivariable root loci in the context of the initial design task has not been conducted.

6.1.4 The Salford Singular Perturbation Method

Unlike linear quadratic regulator theory or eigenstructure assignment, which are normally combined with a precompensator or model following section, a Singular Perturbation Method [Ref. 8] which was developed at the University of Salford, allows the direct synthesis of regulated tracking systems using a single technique. By introducing multivariable proportional and integral controller matrices as well as a scalar loop gain parameter, the Salford Singular Perturbation Method allows the selection of pole and zero positions which lead to decoupled control channels between the inputs and the outputs. In order to achieve the decoupled tracking objective, the scalar loop gain, g, must be allowed to increase to high levels at which the systems becomes singularly perturbed with dynamics which are active over two distinct frequency regions. Unfortunately, the method is heavily reliant on benign transmission zero locations and the availability of sensors which can measure the dynamics of the singularly perturbed "fast" subsystem. On helicopters, the presence of nonminimum phase zeros and unmeasurable rotor and actuator states corresponding to the fast subsystem precludes the use of the method. It should also be noted that on examples for which the states of the fast subsystem are measurable, better results in terms of decoupled tracking have been achieved using the eigenstructure assignment method of [Ref. 15] which implements a low gain approach.

6.1.5 Sliding Mode Control

Sliding mode control is a technique for the design of nonlinear regulators [Ref. 29]. The first step in the two part synthesis procedure is to specify a desired sliding subspace. This involves using a regulation technique such as linear quadratic regulation or eigenstructure assignment to stabilize a reduced order system. A nonlinear controller is then developed in the second step to asymptotically drive the system towards the regulated subsystem (sliding subspace). While sliding mode control is known to have robustness attributes [Ref. 30] and can be used in a model reference scheme, there appears to be little guidance on how to design the sliding subspace. Attempts are currently being made to use this design technique to solve the control law problem previously discussed.

6.2 Frequency Domain Methods

The frequency domain methods being considered include the following:

i) Nyquist Array Methods
ii) Characteristic Locus Methods
iii) The H^∞ Approach
iv) Quantitative Feedback Theory

Although they differ considerably in terms of the methods of approach which must be used, all of the above techniques are similar in that the design of the control law is based on a transfer function matrix representation of the system and they all involve frequency-domain performance specifications such as bandwidth.

6.2.1 Nyquist Array Methods

Classical frequency-domain techniques involving the application of Bode diagrams, Nyquist plots and Nichols charts are well established in the design of classical single-input single-output systems. The frequency response approach has been extended by Rosenbrock [Ref. 11] and others [Refs. 31, 32] to allow the Nyquist criterion to be applied to multivariable systems.

The design approach developed by Rosenbrock is based on inverse Nyquist arrays and involves two stages. The first step is to determine a compensator matrix which makes the compensated system diagonally dominant. The resulting diagonal matrix is then used as a basis for single-loop compensation in order to satisfy the overall design specifications.

A second approach involves the use of the direct Nyquist array, through the so-called 'Pseudo-Diagonalisation' procedure proposed by Ford and Daly [Ref. 31]. Essentially, this procedure can improve dominance over a range of frequencies and introduces the possibility of designing dynamic precompensators. This pseudo-decoupling approach is again based upon a two stage process involving initial compensation to achieve an adequate level of dominance followed by the design of individual controllers using classical methods.

Problems identified with the use of these techniques include possible difficulties in achieving a satisfactory design when the system being controlled has highly under-damped resonances. In such cases the pseudo-decoupling approach may give rise to compensator zeros which exactly cancel the resonant poles. This is undesirable in the helicopter application because the resonant poles are included in the closed-loop system and atmospheric disturbances will still excite the resonance. Williams [Ref. 33] has suggested that this problem may be overcome in part by forcing the pre-compensator elements to be of lower order but this will tend to degrade the dominance. The pseudo-decoupling approach has been found to be capable of producing good designs in terms of performance and robustness for other aerospace problems [Ref. 33] and this method is currently receiving attention in terms of the helicopter problem. Software for multivariable control system design using Inverse and Direct Nyquist-arrays is commercially available [Ref. 34].

6.2.2. Characteristic Locus Methods

The Characteristic Locus design method [Ref. 35] is similar to Nyquist Array methods in the sense that it generalises the classical concepts of frequency response techniques to feedback system design in the multivariable case. The technique involves consideration of the eigenvalues of the loop transfer function matrix. These frequency dependent eigenvalues are called the "characteristic gain functions" and the corresponding eigenvectors are known as the "characteristic directions" of the matrix. The characteristic gain functions give rise to open loop characteristic loci which may be regarded as if they were conventional Nyquist plots. A Generalised Nyquist Stability Criterion [Ref. 36] can be applied and a well established design procedure exists for which commercial software is available [Ref. 34].

Characteristic Locus design methods have been applied with success to helicopter flight control law design by Brinson [Ref. 37] in a study involving the use of rotor state feedback. One of Brinson's conclusions concerning this design approach is that the visibility of the process allows the engineer to influence the final form of the control law through heuristic arguments and to make full use of previous experience with single-input single-output design problems.

6.2.3 The H^∞ Approach

There has been a rapid growth of interest in recent years in the use of the H^∞, approach to control system design. This stems from the results of work published by Zames [Ref. 38] in 1981. The method has shown advantages over linear quadratic optimisation techniques in terms of characterising robustness to plant variations [Ref. 39] and has allowed classical analysis and design concepts of proven value for single-input single-output systems to be generalised for the multivariable case.

The approach is based upon the use of the H^∞ norm which is defined, for a transfer function matrix, as the maximum over all frequencies of the largest singular value of that matrix. Singular values provide information concerning guaranteed bounds on system performance and the H^∞ norm can place an upper bound on the uncertainty level in a given system which is to be controlled. Problems of control system design can be formulated in terms of the minimisation of the H^∞ norm of an appropriately weighted closed-loop transfer function matrix. The MATLAB Robust-Control Toolbox [Ref. 40] provides routines for the H^∞ control synthesis problem.

Yue, Postlethwaite and Padfield [Ref. 41] have described the application of H^∞ design methods to the determination of feedback control laws for improving the handling qualities of a combat helicopter. The objectives of that work involved the design of control laws for precise control of pitch and roll attitude, yaw rate and heave velocity for a hover condition. In order to satisfy requirements in terms of both performance and robustness a two-degree of freedom control system structure was adopted. A feedback compensator was designed to have suitable robustness properties against model uncertainty and disturbances and a pre-compensator was found to achieve desired performance objectives in terms of tracking accuracy and speed of response. Further details of this work may be found in [Ref. 9].

6.2.4 Quantitative Feedback Theory

Quantitative Feedback Theory is a control synthesis technique which involves shaping the loop transmission to meet bounds placed upon it by performance specifications in terms of desired system responses and disturbance rejection levels. The approach was developed initially by Horowitz [Ref. 10] for single-input single-output systems and extended later to the multivariable case [Ref. 42].

Quantitative Feedback Theory is currently being reviewed in the context of the helicopter flight control design problem outlined in Section 5. One difficulty encountered with the application of this method is that is essentially a manual approach to design and the highly interactive nature of the design process presents problems in terms of an efficient computer-based implementation.

6.3 Model Reference Techniques

Model reference techniques are those synthesis procedures which can be used to design feedforward controllers. The four methods which are included in the current study are:

i) Model Following Using Linear Quadratic Regulator Theory
ii) Integral Inverse Model Following
iii) Broussard Command Generator Tracking
iv) Controllers Using Nonlinear System Inverses

In the first two techniques a regulator is designed to minimize the error transients between the responses of the system being controlled and a model which describes dynamics. The last two techniques are essentially procedures for the inversion of the system such that each input is linked with an output.

6.3.1 Model Following Using Linear Quadratic Regulator Theory

The premise of this technique is that by defining the error dynamics of a system as the difference between the actual system dynamics and those of a "desired" model, the task of designing a feedforward controller can be formulated in terms of a linear quadratic regulator problem. As with some other model reference techniques, the design can be based on either explicit model following, in which the model of the system appears as an element of the controller's block diagram, or implicit model following, for which the model description is only used in design [Ref. 43].

6.3.2 Integral Inverse Model Following

Integral inverse model following is an explicit model following technique in which the model is seen to run in parallel with the actual system. Regulating controllers are then developed to modify the system's inputs on the basis of the errors of the system's response in relation to the model response. The inverse aspects of this technique appear in the method used to generate the feedforward controller which drives both the system and the model. This technique has been used on the variable-stability BO105 of the DLR [Ref. 44].

6.3.3 Broussard Command Generator Tracking

The Broussard command generator is a linear feedforward controller which maps the transfer function from the inputs of the controlled system to its outputs onto a reference model. In a large number of applications, the reference model is just an identity matrix. This in turn yields a system whose outputs track its inputs. The method works well if the system does not have transmission zeros near the origin. Restricted forms of the Broussard command generator have been used with success in preliminary helicopter flight control law designs and efforts are continuing with respect to the application of the general theory to the initial design task presented in Section 5.

6.3.4 Controllers Using Nonlinear System Inverses

By using a nonlinear system inverse, it is possible to linearize a nonlinear system with a feedforward controller. The design of the feedback loop which is then closed around the nonlinear inverse and the system is simplified because variations of system dynamics with flight condition are taken care of by the feedforward element. Thus, the performance of the system is less reliant on the feedback controller. The difficulty with the use of this technique is the underlying theory which is based on aspects of differential geometry. However, once these ideas are understood, the method allows an analytic approach to the design of a full flight envelope controller. A controller using a nonlinear system inverse has been flown on a Bell UH-1H helicopter by NASA [Ref. 45]

6.4 Output Feedback and techniques involving State Estimators

The methods of classical frequency-domain control system design are most widely applied to single-input single-output systems in which output feedback, with addition cascade or feedback compensation, is the principal option available. This can be seen as an advantage for such systems since the design technique generates control laws which use only those measurements which are already available in practice.

However the helicopter, which is a strongly coupled multi-input multi-output system, is more naturally treated in terms of state space descriptions. Unfortunately the use of the state vector to describe the system tends to lead to control laws which require knowledge of all the components of the state vector irrespective of the measurements that are available in practice. Two possible approaches to overcoming problems inherent in the use of output feedback involve a) the application of state estimators such as the Kalman filter and Luenberger observer or b) loop transfer recovery techniques.

6.4.1 State Estimator Techniques

State estimation techniques such as the Kalman filter and Luenberger observer provide a well established means of generating estimated state variables for feedback from available measurements. However, as has been pointed out by Bryson [Ref. 46] the use of these methods for estimated state feedback can create problems for the designer in that the resulting control laws are not, in most cases, robust to uncertainties or variations in the plant.

6.4.2 Loop Transfer Recovery Methods

The Loop Transfer Recovery problem is concerned with the design of an output feedback law which approaches, in terms of its effects, a desirable full state feedback law. Much attention in the past has been given to one particular approach to the Loop Transfer Recovery (LTR) problem, namely the Linear Quadratic Gaussian Loop Transfer Recovery procedure (LQG/LTR) [Ref. 47]. Previous applications of this approach to rotorcraft include the work of Rodriguez and Athans [Ref. 48]. Loop Transfer Recovery is not limited in its applicability to Linear Quadratic Gaussian design methods. Kazerooni and Houpt [Ref. 49] have considered a more general approach based on eigenstructure assignment of an observer and Garrard and Liebst [Ref. 6] have considered the use of this method in a helicopter flight control context. Prasad et al. [Ref. 50] also provide a review of LTR methods in the context of helicopter applications including an approach which has been developed at the Georgia Institute of Technology and which involves optimisation of a fixed order dynamic compensator.

One important problem with LTR methods is that they are subject to limitations in the case of non-minimum phase plants. This can create difficulties for some types of application. Loop Transfer Recovery problems are being considered as part of the current study of helicopter flight control system design methodologies.

7. Conclusions

The increased reliance of modern combat rotorcraft on active control technology has provided motivation for a review of multivariable flight control law design techniques. At present helicopter flight control laws are generally developed using classical single-input single-output analysis and design methods. The drawback of this approach is that one is forced to treat cross-couplings, which are inherent in rotocraft dynamics, largely as an afterthought. Multivariable techniques promise a more integrated approach to the design of high perfromance flight control laws.

Several multivariable control law design methodologies show interesting possibilities in the helicopter context using control law design criteria which can be related to handling quality requirements and other issues relating to robustness, noise rejection and insensitivity to atmospheric disturbances.

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