STATIC STABILITY AND CONTROLLABILITY OF HELICOPTER WITH AN EXTERNAL LOAD

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Abstract. Standard factors of static stability and controllability for helicopters carrying typical external loads suspended from a two-link cargo sling are analyzed through the techniques of math modeling.
The helicopter dynamics is described by conventional system of linearized equations, where the Dalamber principle is introduced to account for the external load effect, while the tension of the elastic sling is calculated by an original algorithm. The cargo is supposed to be a parallelepiped with assigned aerodynamic properties.
The authors study the longitudinal static stability in speed and angle of attack, its lateral side-slipping stability, as well as longitudinal and lateral control effectiveness and response in hovering and horizontal flying of a helicopter with an external load.

Most modern helicopters nowadays are capable of external sling operations, which enhance their operational capabilities significantly and makes them extensively applicable for transportation of bulky loads and other challenging aviation works, fire extinguishing, etc.

Piloting a helicopter with an external load is however a peculiar and more complicated flight mission as compared to conventional freight service and requires special skills of the aircrew. Yet neither Russian nor international publications have so far offered a well-defined solution to the problem of the effect of the external-sling load on the helicopter basic dynamic performance such as controllability and stability.

According to the current classification [1] let us analyze helicopter static controllability and stability under the external-sling load, namely:

- longitudinal trim at the level straight-line flight;
- pitch control effectiveness and helicopter stability with speed and angle of attack;
- roll control effectiveness and lateral stability in sideslip.

Let's consider a helicopter carrying a double-link sling with a load m at the level straight-line flight at a speed V (figure 1).

The external sling is attached to a universal joint at a distance l below the helicopter centre of gravity. At this flight condition it is deflected at an angle φ_x from the vertical body axis $0y$, which coincides with the main rotor shaft axis ($\varphi_x < 0$ if the load drops behind).

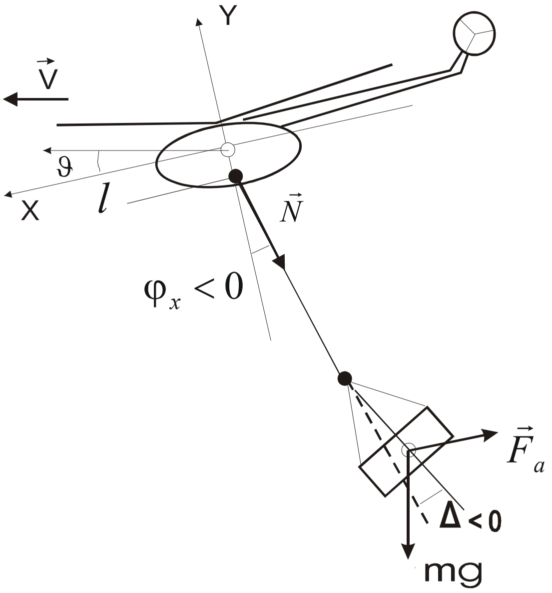

Figure 1. External-sling operation diagram

The load experiences the force of gravity $m\vec{g}$, aerodynamic force $\vec{F}_a$, and cable tension ($-\vec{N}$), while the helicopter is exerted upon by the sling tension $\vec{N}$ which creates a longitudinal moment (pitching down if the load drops behind) due to the displacement of the sling attachment from the helicopter centre of gravity.

$$\Delta M_z = N \cdot l \sin \varphi_x \tag{1}$$

This moment reduces forward deflection of the swash plate assembly required for trimming the helicopter.

The equations of linearized short-period helicopter motion in level flight [2] enable approximate analytic expressions for the load effect upon the helicopter trim in straight-line level flight.

Taking into consideration that in level flight $\vartheta_0 = \alpha_0$,

$$\frac{|Y^\delta|}{m_H} >> \frac{|Y^\alpha|}{m_H} \sim g \cdot \sin \alpha_0, \qquad M_z^\delta >> M_z^\alpha,$$

let's keep only main terms in the above equations thus omitting the variation sign.

$$\begin{aligned} m_H \cdot \dot{V}_y &= Y^\delta \cdot \delta + Y^\varphi \cdot \varphi + \Delta Y \\ J_z \cdot \dot{\omega}_z &= M_z^{\omega_z} \cdot \omega_z + M_z^\delta \cdot \delta + \Delta Mz \\ \dot{\vartheta} &= \omega_z \end{aligned} \quad , \tag{2}$$

where

m_H and Jz are the helicopter weight and moment of inertia,
ΔY and ΔMz are the sling-induced force and moment accordingly,
δ is the longitudinal deflection of the swash plate assembly, and
φ is the main rotor pitch.

Assume that the helicopter with no load suspended from the external sling is trimmed in straight-line level flight at a height of Ho and speed of Vo with certain values of δ_0 and φ_0, then additional deflections of flight controls to trim the helicopter carrying an external load $\Delta\delta$ and $\Delta\varphi$ are derived from (2) at $\omega_z = \dot{\omega}_z = \dot{V}_y = 0$:

$$\Delta\delta = -\frac{\Delta Mz}{M_z^{\delta}} \tag{3}$$

Insertion of ΔMz from (1) (remember that at low ϑ $N \approx G/\cos\varphi_x$, where $G=mg$ is the load weight) gives

$$\Delta\delta \approx -\frac{G \cdot l}{\mathrm{M}_z^{\delta}} \cdot tg\varphi_x \tag{4}$$

$$\Delta\varphi = -\frac{\Delta Y + Y^{\delta} \cdot \delta}{Y^{\varphi}} \approx \frac{G}{Y^{\varphi}} \cdot \left(1 + \frac{Y^{\delta}}{M_z^{\delta}} \cdot l \cdot tg\varphi_x\right) = \frac{G}{Y^{\varphi}}\left(1 - \frac{Y^{\delta} \cdot \Delta\delta}{G}\right) \tag{5}$$

From equations (4) and (5) it follows that:

- external sling load reduces the forward trimming deflection of the swash plate assembly, the more φ_x, the less deflection (this corresponds to higher speed of load transportation or higher drag of the load), and hence, improves the forward margin of control;
- the collective pitch increment to make up for the load weight is not as high in level flight as in hovering.

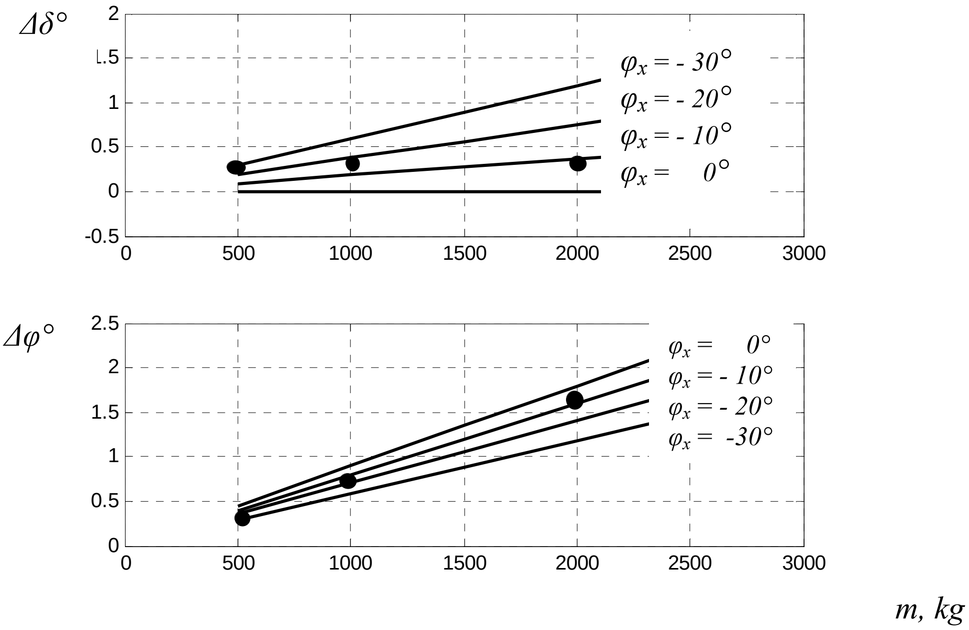

Figure 2. Dependence of the load weight on helicopter longitudinal trim

Figure 2 shows dependence of *Δδ* and *Δφ* on the weight of the external-sling load for the Mi-8 family in level flight at a speed of 100 km/h, these dependence being derived from (4) and (5). It is obvious that within the area of sling deflections ~10÷30º (typical of most sling operations) additional trimming application of flight controls come up to:
~0.3÷0.5º pitching up application of the swash plate assembly;
~0.5÷2º increase in the collective pitch.

The linearized model of helicopter motion in level flight [2] was taken for the disturbed motion math model, where a complete set of solid body equations with relevant weight & inertia and aerodynamic properties governed the load motion (in the shape of a parallelepiped).

The terms on the right-hand side of the force and moment differential equations in the helicopter - load math model were enhanced with the external sling forces and moments taken from [3] and [4]. Here the sling tension was derived from the empiric type law (a modified Hooke's law)

$$N = k_l \cdot \Delta L + k_{\dot{l}} \cdot \dot{L}, \tag{6}$$

where ΔL and $\dot{L}$ are accordingly the sling elongation and rate of elongation to be determined geometrically by joint integration of motion equations for the helicopter and properly load.
Figure 3 presents data obtained from computations of the additional trimming deflections of flight controls in level flight with an external-sling load. These data are the results of math modeling for the Mi-8 external-sling operations with a container 4x2x3 m in dimensions and variable weight of 500, 1000, and 2000 kg.

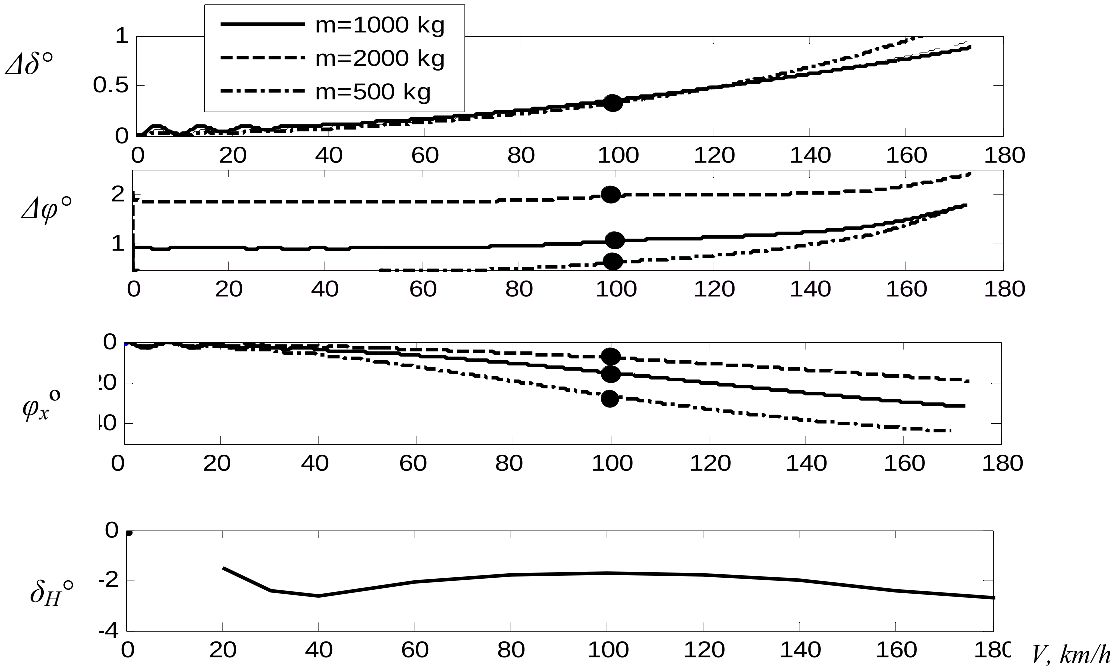

Figure 3. Level flight trim of a helicopter with the external-sling load

The *Δδ* and *Δφ* values derived from formulae (4), (5) (dots in figure 3) correspond well to the math modeling data.

Figure 3 makes it clear that for the load whose horizontal component of the aerodynamic force is much higher than the vertical one the additional swash plate deflection required to trim the helicopter in steady level flight *Δδ(V)* does not actually depend on the weight of the external load.

In this case $tg\varphi_x \approx -\frac{X}{G}$, then it follows from (4) that $\Delta\delta \approx \frac{X \cdot l}{\mathrm{M}_z^\delta}$, i.e. the additional load trimming deflections of flight controls *Δδ* depend only on the external load drag, while the trimming values of the main rotor pitch goes up with the load weight.

Let us now consider the influence of the external-sling operations on the helicopter longitudinal control effectiveness which is known to depend on the control moment increment due to a unit application of the control body (the swash plate assembly in our case).

It follows from (1) that at positive (pitching up) deflection of the swash plate the load will definitely prevent the helicopter from pitching up due to the growing φ_x , which makes the pilot believe that his rotorcraft has lost effectiveness of the longitudinal control. Noteworthy, that φ_x depends both on angular motion of the helicopter and on the external load motion about the helicopter itself.

Let's derive quantitative characteristics for the external load effect on the effectiveness of longitudinal control by comparison between the gain in the free helicopter angular acceleration $\dot{\omega}_{z\,\max}$ due to estimated disturbance (δ=1° during 1 s) and that with the above-mentioned external-sling load-container.

Results of calculations are shown in figure 4, where $\Delta\overline{M}_z^\delta \approx \frac{\Delta\dot{\omega}_{z\,\max}}{\dot{\omega}_{zH\,\max}}$,

$\Delta\dot{\omega}_{z\,\max}$ is the difference between the disturbance-induced maximum angular accelerations of the helicopter with an external-sling and free helicopter,

$\dot{\omega}_{zH\,\max}$ is the maximum angular acceleration acquired by the free helicopter.

These calculations have proven that loss of the longitudinal control effectiveness still worsens with the speed of transportation and weight of the load.

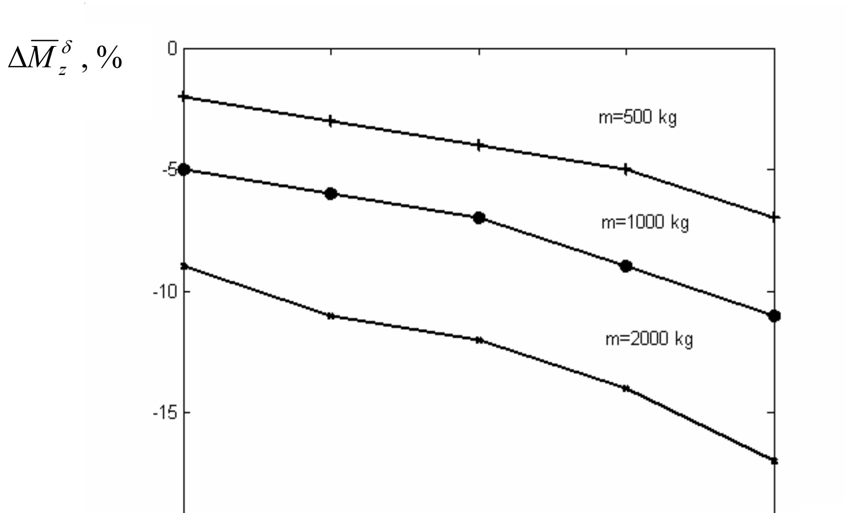

Figure 4 The external load effect on effectiveness of helicopter longitudinal control

Let's now dwell upon the load effect on the helicopter static stability in speed. The qualitative assessment will be derived from the analysis of forces and moments acting on the helicopter in disturbed motion.

Assume that the helicopter penetrates a short-run wind gust W_X, which implies a gain in airspeed V by the said value (figure 5).

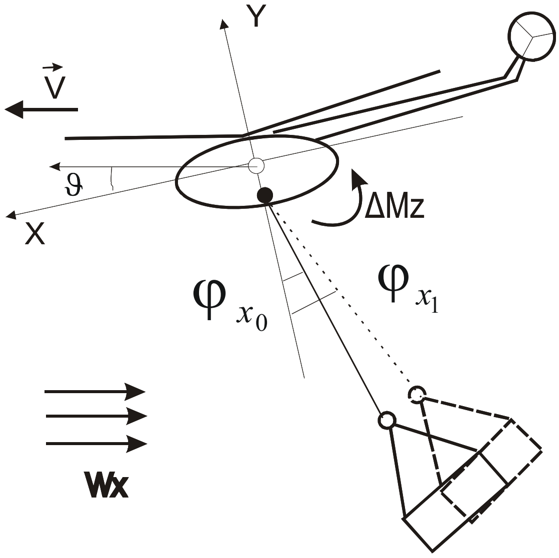

Figure 5. Air disturbances – helicopter and external load force diagram in the longitudinal plane

Inertia enables the helicopter under a wind gust to maintain its ground speed at the first moment, yet the comparatively light external load attached to the universal joint drops behind from its former angular attitude φ_{x0} to the new attitude φ_{x1} with the helicopter experiencing an additional nose down moment $\Delta M_z = l\ (N_1 \sin \varphi_{x1} - N_0 \sin \varphi_{x0}) < 0$ thus causing a decrease in the pitch angle and further airspeed gain, i.e. $\Delta M_z^V < 0$ (the symbol Δ here emphasizes the additive nature of this stability criterion due to the external load).

Modern helicopters are reputed to enjoy static longitudinal stability in speed in most operational conditions ($M^V_z > 0$); however an external-sling load reduces stability margin. As this takes place, the more windage of the external load, the higher effect it has.

With this effect of the external-sling load on the trim curves and effectiveness of helicopter control we can make a quantitative analysis of the load influence on helicopter stability in speed, whose derivative is $\left.\frac{dM_z}{dV}\right|_{n_y=const}$, specifically for the level flight

$$\frac{dM_z}{dV} = -M^{\delta}_{z\,|\delta=\delta_{bal}} \cdot \frac{\partial \delta_{bal}}{\partial V}, \tag{7}$$

i.e. the degree of helicopter longitudinal stability in speed depends on the control effectiveness and trim curve derivative in speed $\delta_{bal}(V)$ for the level flight condition [2].

With the understanding that an external-sling load has but a slight effect on the helicopter static stability in speed, let's take an increment due to the external-sling load $\frac{dM_z}{dV}$ as

$$\Delta\left(\frac{dM_z}{dV}\right) = -\left(\Delta M_z^{\delta} \cdot \frac{\partial \delta_{bal}}{\partial V} + M_z^{\delta} \cdot \Delta\delta_{bal}^{V}\right). \tag{8}$$

The load relative effect on helicopter stability in speed is therewith found from

$\Delta\overline{M}_z^V = \Delta\left(\frac{dM_z}{dV}\right) / \frac{dM_z}{dV}$ it follows from (8):

$$\Delta\overline{M}_z^V = \Delta M_z^{\delta} / M_z^{\delta} + \Delta\delta_{bal}^{V} / \delta_{bal}^{V} \tag{9}$$

Figure 6 illustrates this relative effect of the external-sling load on helicopter static stability in speed.

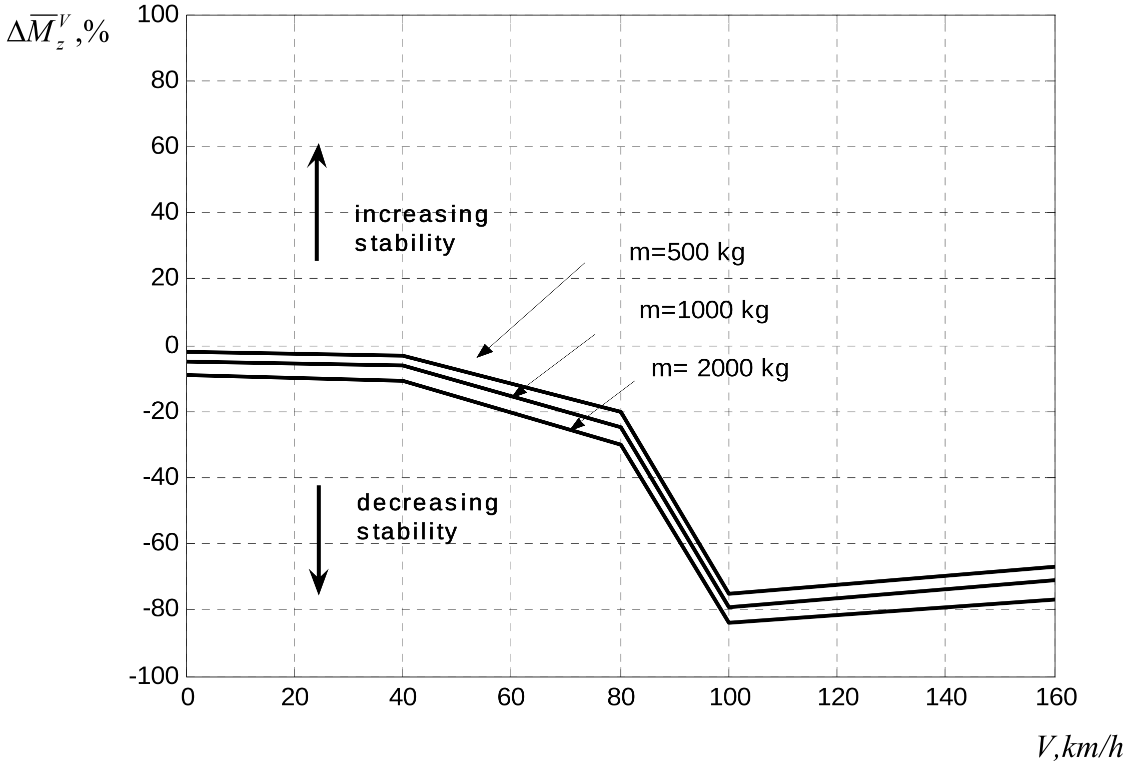

Figure 6. External-sling load effect on helicopter static stability in speed

At a low transportation speed (below ~ 40 km/h) the external-sling load exerts but a slight relative effect: the longitudinal stability in speed reduces by some 5-10 %, however this effect becomes greater with higher speeds of transportation, and at *V > 100* km/h the helicopter with an external-sling load has nearly neutral stability in speed.

It becomes obvious from the above-considered wind gust impact on the helicopter with an external-sling load that it must as well have an effect on the helicopter stability in angle of attack. The said effect is however not as well-defined as on static stability in speed.

Assume that the helicopter carrying an external load penetrates a short-run vertical wind gust *Wy*, which implies a gain in the angle of attack of the helicopter and the external-sling load by $\Delta\alpha_w = \arcsin(Wy / V)$. The additional moment from the sling will be determined by the relationship between the angular and linear accelerations of the helicopter and load accordingly due to the wind gust. It can either counteract or stimulate the increase in the helicopter angle of attack and as a result it either improves or reduces its static stability margin in angle of attack (if any).

For the quantitative analysis of the impact on the helicopter static stability in angle of attack, which is determined by M_z^α, we can take the variation of helicopter angular acceleration $\dot{\omega}_z$ resulted from the angle of attack disturbance, namely from the 1 second long wind gust equal to $\Delta\alpha_W = 5°$. This variation was obtained from math modeling of the helicopter external-sling operation.

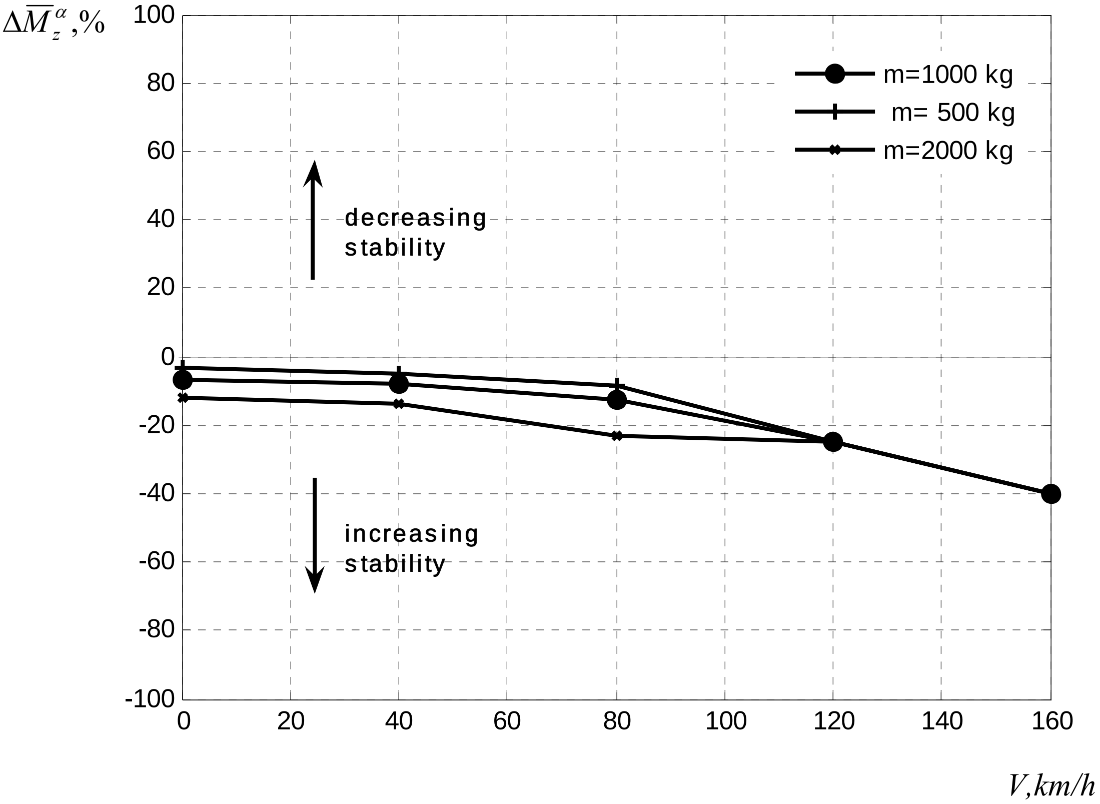

Fig. 7. The external load effect on the helicopter static stability in angle of attack

Figure 7 represents data obtained from calculation of the external load (a typical rectangular container) relative effect on the Mi-8 static stability in angle of attack, where

$$\Delta\overline{M}_z^\alpha \equiv \frac{\Delta M_z^\alpha}{M_z^\alpha} \approx \frac{\Delta\dot{\omega}_{zW}}{\dot{\omega}_{zHW}} ,$$

$\Delta\dot{\omega}_{zW}$ is the difference between maximum disturbance-induced angular accelerations of the free helicopter and that with an external-sling load,
$\dot{\omega}_{zHW}$ is the maximum wind gust-induced angular acceleration of the free helicopter.

It is obviously seen that the external-sling load entails greater stability margins in angle of attack with higher transportation speeds. Noteworthy, that the heavier load the more significant is this effect (it should be also noted here that this effect of the external-sling load – helicopter stability in angle of attack is typical of loads with $|Fa| << mg$).

We shall highlight that proportional effect of the container weight on helicopter stability in angle of attack becomes slighter with the speed of transportation, which results from a significant $\sim \frac{1}{\cos \varphi_x}$ increase in the sling tension in case of lighter external loads due to their higher drag.

Let us consider the impact of the external-sling operations on the helicopter lateral stability and controllability, namely on the roll control effectiveness M_x^η (η- the swash plate deflection in roll) and the lateral sideslip stability M_x^β.

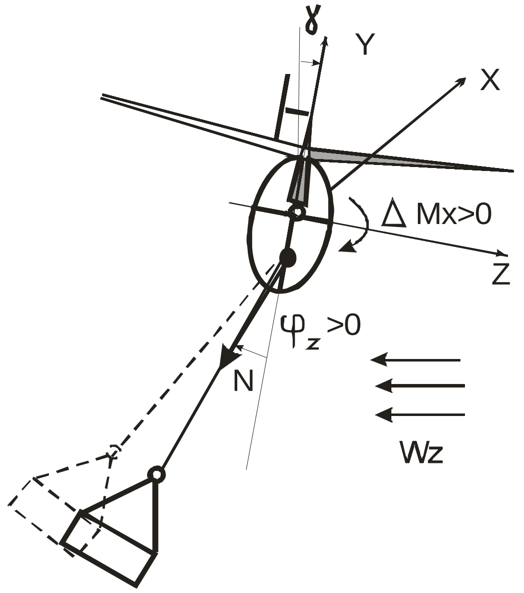

Figure 8. The wind gust effect diagram in lateral plane

With the external-sling load deflected in the longitudinal plane by an angle of φ_x, and by φ_z in the lateral plane (figure 8), the helicopter experiences an additional lateral moment due to the sling tension

$$\Delta M_x = N \cdot l \cos \varphi_x \cdot \sin \varphi_z, \tag{10}$$

which worsens the lateral control effectiveness and also affects lateral stability of the helicopter.

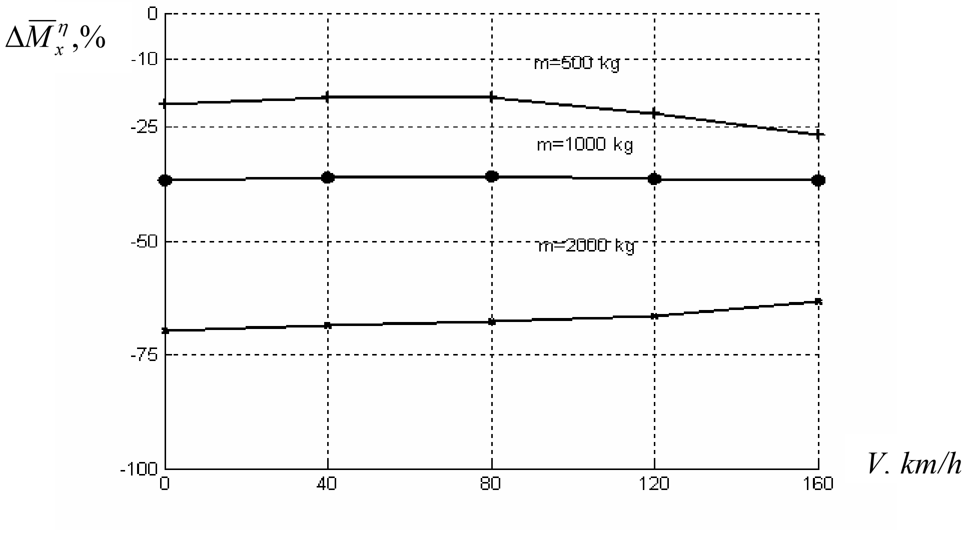

Figure 9. The external-sling load effect on helicopter lateral control effectiveness

We can estimate the lateral control effect by comparing the load-induced angular accelerations $\dot{\omega}_{x\,\max}$ due to the assumed 1 second-long disturbance (*$\eta=1^{o}$*) for the free helicopter and that with an external-sling load (see figure 9), where $\Delta\overline{M}_x^{\eta} = \dfrac{\Delta M_x^{\eta}}{M_x^{\eta}} \approx \dfrac{\Delta\dot{\omega}_{x\,\max}}{\dot{\omega}_{x\,H\,\max}}$.

It is clearly seen that the negative effect on the lateral control effectiveness is proportional to the load weight and does not depend any significantly on the speed of transportation (contrary to the longitudinal control effect). This is accounted for by the fact that $N = G/\cos\varphi_x$ in the steady straight-line level flight and hence $\Delta M_x = G\cdot l\cdot \sin\varphi_z$, i.e. is influenced solely by lateral components of the aerodynamic forces acting on the helicopter and the external-sling load.

The impact of the external-sling load on helicopter lateral stability manifests itself in the following way: assume that the «helicopter-external-sling load» system suffers from a right-hand wind gust $\beta w>0$ (see figure 8), which results in the left banking of the cross-stable helicopter to make up for the side slipping; the external-sling load appears left from the helicopter plane of symmetry $\varphi_z>0$ and creates a lateral moment $\Delta Mx>0$ to prevent banking of the helicopter and decreasing of its sideslip. The external-sling load thus worsens helicopter lateral stability in sideslip.

To analyze the external load – lateral sideslip stability effect we employed a technique similar to the above method of the angle of attack stability analysis. Figure 10 gives results obtained from calculations. Here

$$\Delta\overline{M}_z^{\beta} \equiv \frac{\Delta M_z^{\beta}}{M_z^{\beta}} \approx \frac{\Delta\dot{\omega}_{x\,W}}{\dot{\omega}_{xH\,W}}\cdot 100\%,$$

$\Delta\dot{\omega}_{x\,W}$ is the difference between the maximum angular accelerations induced by the wind gust equivalent to $\Delta\beta w=5^{\circ}$ for the helicopter with the external-sling load and free helicopter, $\dot{\omega}_{xH\,W}$ is the maximum wind gust-induced angular acceleration of the free helicopter.

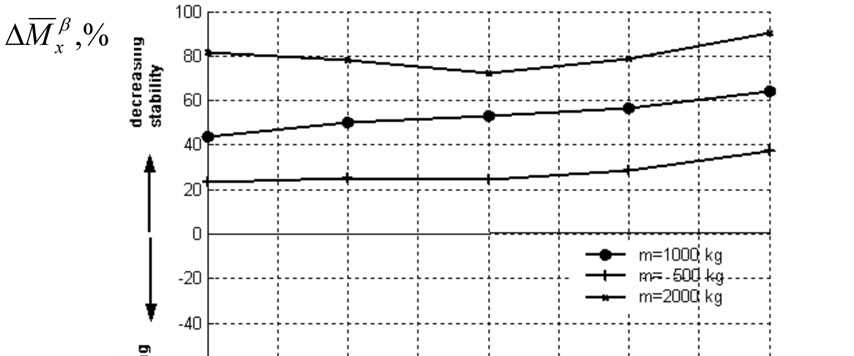

Figure 10. The external-sling load effect on helicopter lateral stability in sideslip

As would be expected, the external-sling load worsens the helicopter lateral stability in sideslip with this effect becoming more pronounced under heavier loads.

As a result of the analysis we can make a conclusion that an external-sling load effects static stability and controllability of the helicopter to a great extent:

- Control effectiveness worsens both in longitudinal and lateral control channels;
- Although helicopter stability in angle of attack somewhat improves, yet its stability in speed and lateral stability in sideslip worsens.

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