24^{th} EUROPEAN ROTORCRAFT FORUM
Marseilles, France - 15^{th}-17^{th} September 1998

"Acoustics" AC05

Transonic Noise Prediction of Rotating Blades by means of the Kirchhoff Formulation

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Over the last few years a large effort has been devoted at CIRA to the analysis of capabilities and limits of the FW-H formulations. Nevertheless following the trend of most research institutions and industries, a new code implementing the Kirchhoff formulation has been developed and tested. The possibility of using in parallel this new code with the one solving the Ffowcs Williams and Hawkings equation permits to increase the CIRA capabilities in the prediction of the acoustic disturbance. The main aim of this paper is to show the reliability of the new Kirchhoff solver in rotor and propeller noise predictions.

1 INTRODUCTION

Over the last decade rotorcraft and propeller-driven aircraft are playing an increasingly important role in the civil transportation market. The relevant features (wide manoeuvrability, limited take-off and landing spaces, narrow detectability) make the helicopter the most suitable tool for civil as well as military uses. The development of high-speed propellers able to produce the same jet thrust but with less fuel consumption and pollutant emission is making the propeller-driven aircraft a very competitive machine in the aviation market. To reduce, as required by the stringent certification rules, the enviromental impact of rotorcraft and propeller-driven aircraft, it is mandatory to improve the prediction methods used in the design phases and not be limited just to the study of the propulsion performances of these machines. The aeroacoustic analysis of helicopter blades and propellers represents nowadays one of the most active and useful research areas in the large field of the applied sciences. In order to mantain a high level of noise prediction capability and enhance the tools for the computational analysis of rotating blades, the need for an indipendent assessment of accuracy and efficiency of numerical codes for the prediction of rotor and propeller noise has been facing. This calls for the development of numerical tools based on alternative approaches.

Two are the techniques mostly used in the numerical prediction of noise generated by rotating blades. The first one is the Lighthill's analogy whose basic idea consists of the subdivision of the field in two domains: a near field describing the non linear generation of the noise and a far field where the linear propagation of the sound is computed. The solution in the far field can be obtained through the solution of an inhomogeneous wave equation with the right side term representing the aerodymanic disturbance. Exploiting the theory of generalized functions [1] Ffowcs Williams and Hawkings in 1969 extended the acoustic analogy to the noise generated by bodies in arbitrary motion and derived a governing differential equation for the acoustic pressure [2]. A lots of efforts were devoted to the analytical treatment of this equation in order to have some integral expressions suitable for a numerical manipulation, and several different solution forms, both in the time and frequency domain, have been proposed and implemented. In the FW-H equation the acoustic pressure is expressed as the sum of three contributions called *thickness*, *loading* and *quadrupole* source terms. The first two terms can be found through an integration on the body surface, while the last one requires a time-demanding volume integration, even though its own contribution is significant for only the high tip speed blades. Nevertheless the difficulties arising in the evaluation of the quadrupole term of the FW-H equation for the HSI noise prediction, pushed the research towards alternative methods.

A more recent formulation for computing the aeroacoustic field is the Kirchhoff approach, which takes advantage of the mathematical similarities between the aeroacoustic and electromagnetic equations. The Kirchhoff formula was first published in 1882 and primarily used in the theory of diffraction of light and in other electromagnetic phenomena. The use of this formula for predicting the noise from high speed propellers and helicopter rotors was proposed by Hawkings who suggested to surround the rotating blade with a closed surface moving at the same forward velocity as the machine. Inside this surface, non linear aerodynamic calculations are carried out giving the blade loads, the pressure and its spatial and temporal derivatives on the surface. Outside this surface a formula similar to the Kirchhoff one can be used to calculate the sound propagation in terms of the surface values. Then the Kirchhoff formulation has been extended by F. Farassat and M. Myers [3] to a deformable surface in arbitrary motion and has been successfully applied to both hover and forward flight conditions [4]-[10]. This method allows to compute the aeroacoustic field through the knowledge of the fluidodynamic quantities on a surface enclosing all the non linearities and noise sources, and which behaves as a source irradiating to the far field. The aerodynamic solution on the control surface can be obtained by a CFD method, while outside the acoustic pressure can be computed solving the wave equation.

2 THEORETICAL BACKGROUND

The theoretical basis for the analysis of sound generated by a body moving in a fluid is represented by the Ffowcs Williams-Hawkings and Kirchhoff equations. They can be derived from the basic conservation laws of mass and momentum taking into account the effect of the body by means of an appropriate surface which represents a discontinuity for the flow variables. In the FW-H approach the discontinuity surface is assumed to be coincident with the body where a condition of non penetration is imposed. In the Kirchhoff formulation some simplifying hypoteses are introduced to derive the solving formula, while no limitations are imposed on the location of the control surface.

In order to retrieve the Kirchhoff formula let us consider a body moving in a fluid and a closed surface S of arbitrary shape and motion described by the equation $f(\mathbf{x},t) = 0$ with $|\nabla f| = 1$ for $f = 0$. If the surface is far enough from the body, the fluid outside S can be considered to be inviscid, the disturbances small and the fluctuations of pressure and density connected by the relation $p' = p - p_0 = c^2(\rho - \rho_0)$. With these hypoteses, the classical wave equation is obtained, which for the pressure disturbance is written as:

$$\Box^2 p' \equiv \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0 \qquad (1)$$

The solution of the equation (1) can be found by means of the Green function for the wave equation in unbounded space [11].

The general Kirchhoff formula used in the numerical applications is the following [3] :

$$4\pi p(\mathbf{x},t) = \iint_S \left[\frac{E_1}{r(1-M_r)} + \frac{E_2 p}{r^2(1-M_r)}\right]_{\tau^*} \mathrm{d}S \quad (2)$$

where

$$\begin{aligned} E_1 = {} & (M_n^2 - 1)p_n + M_n \mathbf{M_t} \cdot \nabla_2 p - c^{-1} M_n \dot{p} \\ & + \frac{1}{c(1-M_r)}\left[(\dot{n}_r - \dot{M}_n - \dot{n}_M)p + (\cos\theta - M_n)\dot{p}\right] \\ & + \frac{1}{c(1-M_r)^2}\left[\dot{M}_r(\cos\theta - M_n)p\right] \end{aligned} \qquad (3)$$

and

$$E_2 = \frac{(1-M^2)}{(1-M_r)^2}(\cos\theta - M_n) \qquad (4)$$

If the control surface is assumed to be stationary, the (2) assumes the following form :

$$4\pi p(\mathbf{x},t) = \iint_S \left[\frac{p}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p}{\partial t}\right]_{\tau^*} \mathrm{d}S \quad (5)$$

Surrounding the body with a closed surface, it is possible, knowing on $f = 0$ the fluid pressure and its time history, to compute the acoustic disturbance in points located outside the control surface.
The position of the Kirchhoff surface must be treated carefully, since a correct estimation of the pressure signal requires that all the non linearities and noise sources of the flow are included in the region $f < 0$. If the control surface is located too near to the body, some noise sources can be neglected and the pressure signal can be underestimated; but, on the other hand, a too large surface could introduce a numerical dissipation due to the unaccuracy of the aerodynamic data far from the body.

Two approaches are used in the Kirchhoff method. In the first one the body is surrounded

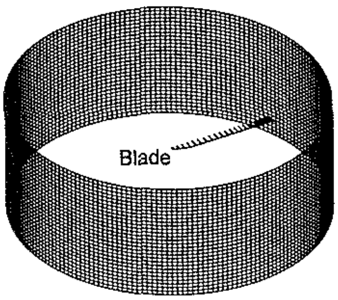

Figure 1: Kirchhoff stationary surface

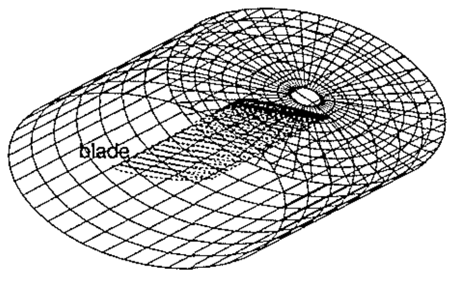

Figure 2: Kirchhoff rotating surface

with a stationary surface which must be chosen large enough to enclose the region of non linear behaviour since the linear wave equation is assumed to be valid outside the surface. As control surface is generally used a cylinder with the axis matching with the rotational axis of the body and the radius along the span direction (fig.1). Although the control surface, from a theoretical point of view should be closed, the contribution of the two base-surfaces can be neglected depending on the extension of the computing grid along the *normal* direction. A second approach uses a surface rotating with the same angular velocity of the body and moving at the same forward velocity. Usually a cylindrical surface with the axis along the span direction is assumed (fig.2). The same considerations, concerning the size and the location of the surface for the stationary Kirchhoff method are considered to be valid, but in this case only the contribution of the root base can be neglected because a significant contribution to the noise signal arises from the base end surface.

3 NUMERICAL RESULTS

A new code implementing both the rotating and fixed Kirchhoff surface method has been developed and tested. In order to validate the code, some particular set of aerodynamic data (kindly provided by DLR) and the corresponding available experimental data have been used [7]. Three test cases concerning the untwisted UH-1H hovering blade at M_{tip} of 0.85, 0.90 and 0.95 have been carried out. The Kirchhoff fixed surface approach has been adopted for all the test cases, while the rotating surface method has been only used at the lowest rotational velocity. At the two higher Mach numbers, the computations by means of the rotating Kirchhoff approach, have been limited inside the sonic circle. In all the test cases the observer is located in the rotor plane on the span axis at a distance from the rotor hub of 3.09 radii.

3.1 STATIONARY FORMULATION

The height of the cylinder, used as control surface in the stationaty formulation, has been chosen to be $2 * R_{tip}$ in order to not account for the contribution of the base surfaces. It is the same for all the three cases considered and 100 points in the vertical direction are used. The radius and the azimuthal discretization, on the contrary, change according to the tip Mach number of the blade. At the tip Mach number of 0.85, the cylinder has a radius of $1.2 * R_{tip}$ and an azimuthal step of one degree has been used, while at $M_{tip} = 0.90$ and $M_{tip} = 0.95$, a radius of $1.4 * R_{tip}$ and 980 points, strechted around the plane $y = 0$ corresponding to the initial position of the blade, have been employed.

In order to get the flow pressure on the Kirchhoff surface nodes, a geometric interpolation of the aerodynamic grid is required. This process is performed in a module which, for each Kirchhoff surface point, determines the cell including the point itself. Then, a trilinear interpolation between the eight points of the aerodynamic cell is carried out (a sketch of the Kirchhoff cylinder and a plane of the aerodynamic grid is shown in fig.3).

This procedure takes much CPU time introducing numerical errors and is the weak point of the stationary surface approach. On the other hand, being the surface points fixed, no particular difficulties arise increasing the body velocity, and this method can be successfully used for the evaluation of the acoustic pressure of flows even at critical delocalized conditions.

The results of the interpolation in the rotor plane

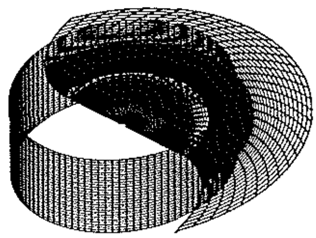

Figure 3: Z-constant plane of Euler grid and Kirchhoff cylinder

are presented in the figg.4 - 6. Increasing the Mach number the negative peak of the input aeroacoustic pressure becomes larger mainly turning from $M_{tip} = 0.90$ to $M_{tip} = 0.95$.

The aeroacustic pressure at $M_{tip} = 0.85$, obtained by means of the Kirchhoff method, is plotted togheter with the experimental results in the fig.7. The agreement is good for both the resulting waveform and the acoustic pressure peak values.
The variation of the acoustic signal with the integration surface radius is presented in fig.8. The surface radius has a little influence just on the negative peak of the signal. At a radius of $1.1 * R_{tip}$, the surface is located too near to the blade and some non linear terms are neglected resulting in a peak value lower than the experimental one. The best result is achieved at a radius of $1.2 * R_{tip}$, while the accounting for a larger radius of the Kirchhoff cylinder provides a progressive decrease of the pressure disturbance.

The aeroacoustic signal, computed at $M_{tip} = 0.90$ by means of the Kirchhooff stationary surface formulation, is shown in fig.9 together with the experimental data. The asymmetrical shape of the signature, due to the shock delocalization is well predicted, while the peak is underestimated. Nevertheless this behaviour is probably due to some unaccuracies in the aerodynamic data. In fact the same test case tested by different authors either through the Kirchhoff method [7, 10], and the Ffowcs Willimas-Hawkings [13] equation exhibits the same underprediction.

The variation in half a revolution period of the three integral (5) kernels is presented in fig.10. The term proportional to the pressure is negligible with respect to the other terms and to make it visible

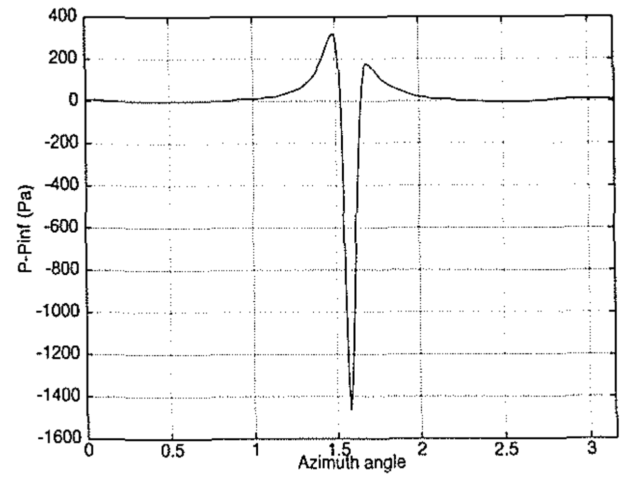

Figure 4: Input pressure distribution on the Kirchhoff cylinder in the rotor plane at $M_{tip} = 0.85$

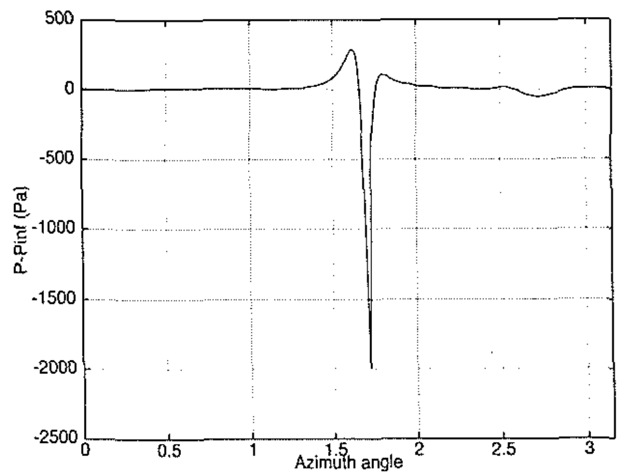

Figure 5: Input pressure distribution on the Kirchhoff cylinder in the rotor plane at $M_{tip} = 0.90$

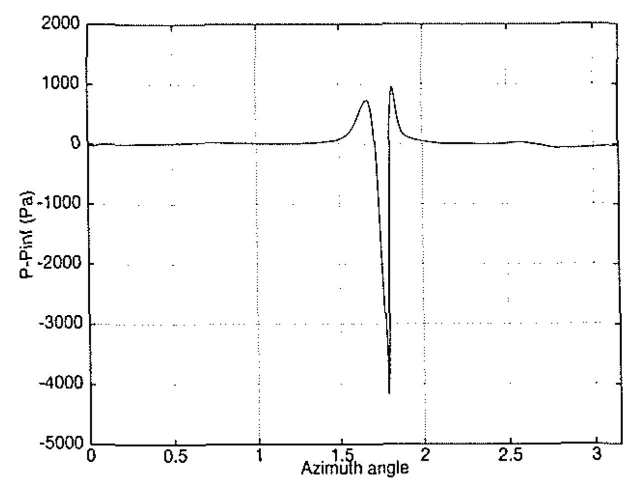

Figure 6: Input pressure distribution on the Kirchhoff cylinder in the rotor plane at $M_{tip} = 0.95$

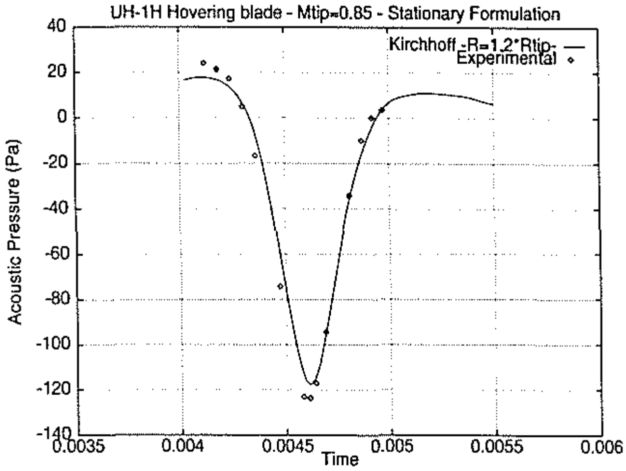

Figure 7: Aeroacoustic Pressure at $M_{tip} = 0.85$

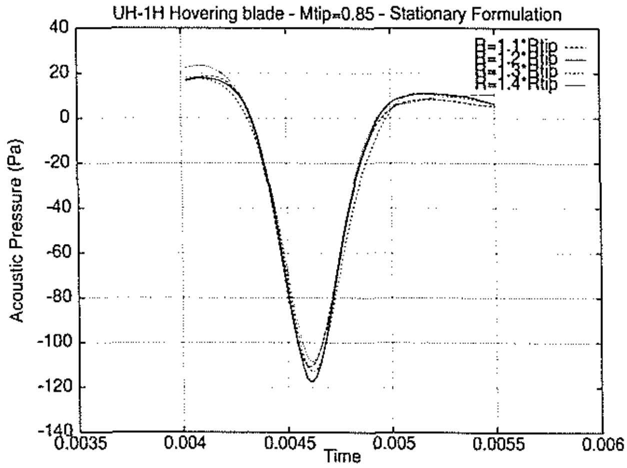

Figure 8: Pressure signal as a function of the radial position of the Kirchhoff surface at $M_{tip} = 0.85$

in the figure is multiplied by a factor 10. The two terms connected with the pressure derivatives sum, each contributing half of the acoustic disturbance in the vicinity of the signal, while they cancel away from the region of impulsive noise. This phenomenon, although present at $M_{tip} = 0.85$, is much more evident in this case than in the previous one due to the highly impulsive character of the resulting waveform.

The acoustic pressure signal as a function of the radial position of the Kirchhoff surface is shown in fig.11. Using a surface radius of $1.1 * R_{tip}$, the resulting signal has a symmetrical shape and exhibits a step in the recompression region. This particular behaviour is due to the absence of the supersonic non linear sources contribution. Looking at the same test case carried out through the FW-H approach [12, 13], it is possible to split

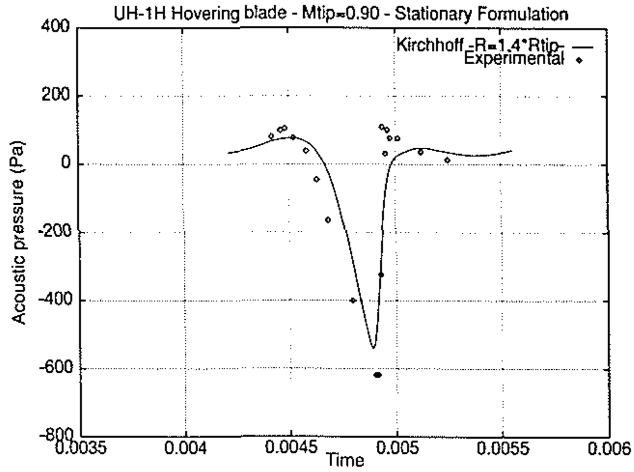

Figure 9: Aeroacoustic pressure at $M_{tip} = 0.90$

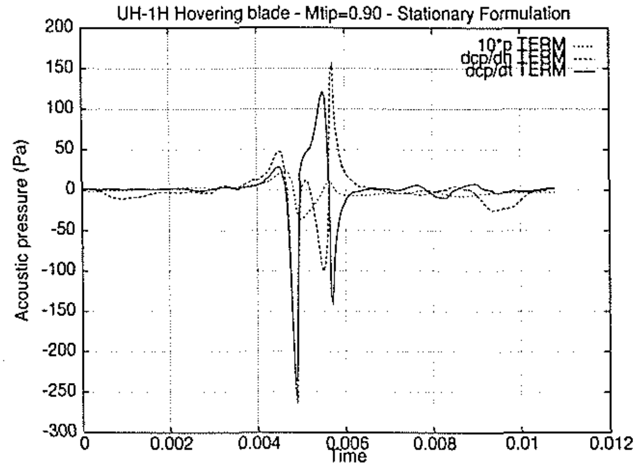

Figure 10: The three integral terms of the Kirchhoff fixed surface approach at $M_{tip} = 0.90$

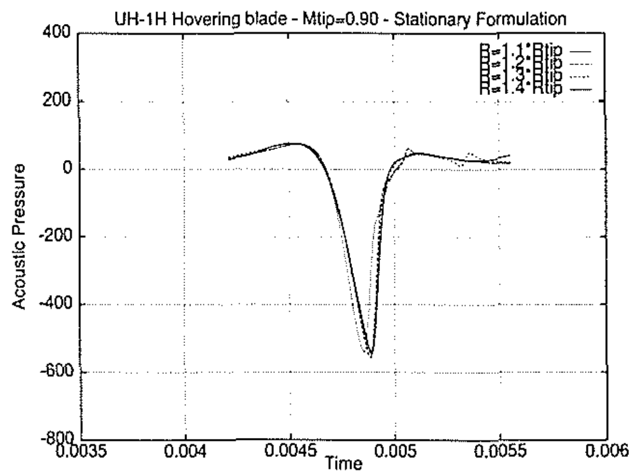

Figure 11: Pressure signal as a function of the radial position of the Kirchhoff surface at $M_{tip} = 0.90$

the total signal in its components (linear, non linear subsonic, and non linear supersonic). The supersonic non linear terms are out of phase and have a larger negative peak value with respect to the non linear subsonic noise component. The adding of the supersonic contribution provides a time shift and an increase of the negative peak of the resulting quadrupole source term. Thus a signal obtained using a surface located in the nearby of the sonic circle, built up only of the linear and quadrupole subsonic components, provides an underprediction of the negative peak value and a not correct evaluation of the resulting noise waveform.

The aeroacoustic pressure computed at $M_{tip} = 0.95$ together with the experimental results is presented in the fig.12 : the agreement is very good. This result has been obtained without any particular difficulty with respect to the case of $M_{tip} = 0.85$ where the delocalization phenomenon does not occur. A finer grid on the Kirchhoff cylinder has been required but the corresponding increase of CPU time is acceptable. No problems arise in the evaluation of high speed flow in delocalized conditions by means of the Kirchhoff stationary formulation, while the methods based on the FW-H equation require the computation of the non linear terms which introduces considerable drawbacks.

The same behaviour, concerning the integral terms of the equation (5), as in the case of $M_{tip} = 0.85$ and $M_{tip} = 0.90$, can be retrieved looking at fig.13 where the variation of the pressure and pressure derivatives terms of the Kirchhoff stationary formula in half a blade revolution period is reported.

A study of the influence of different radial positions of the Kirchhoff surface is presented in fig.14. Using a cylinder with a radius of $1.1 * R_{tip}$, a large overprediction of both negative and positive peaks values occurs. This should be due to the fact that, being the control surface located just after the sonic circle, not all the supersonic noise sources are taken into account. At this Mach number the effect of the supersonic non linear noise sources, is to provide a time shift and, unlike at $M_{tip} = 0.90$, only a slight increase in the negative peak value of the quadrupole term [12, 13]. Thus, the sum of the linear and subsonic non linear terms yields a signal with larger peak values with respect to the signal computed considering all the noise sources. The acoustic pressure time history computed moving the Kirchhoff surface further from the sonic circle, agrees much better with the experimental data,

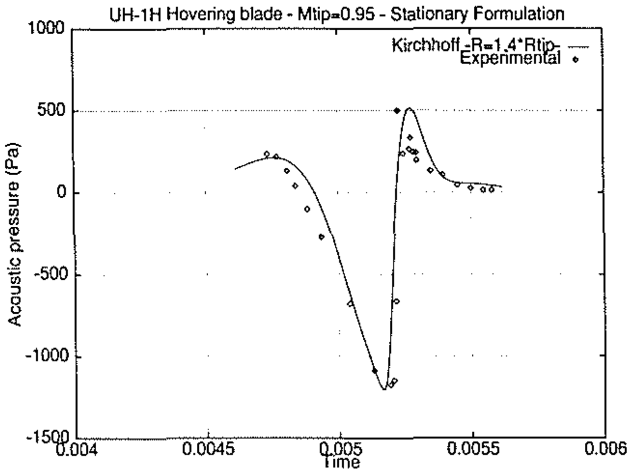

Figure 12: Aeroacoustic pressure at $M_{tip} = 0.95$

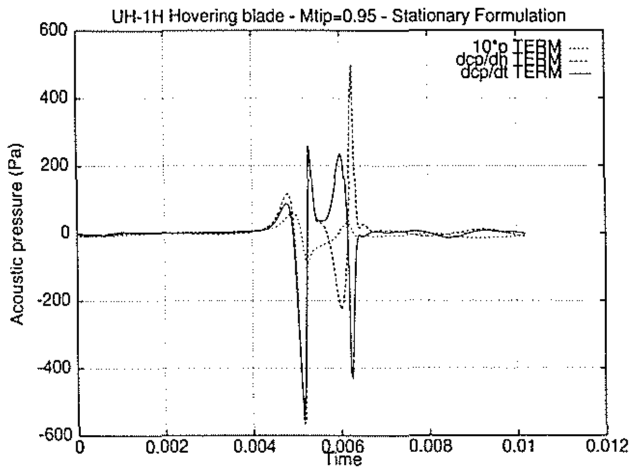

Figure 13: The three integral kernels of the Kirchhoff fixed surface approach at $M_{tip} = 0.95$

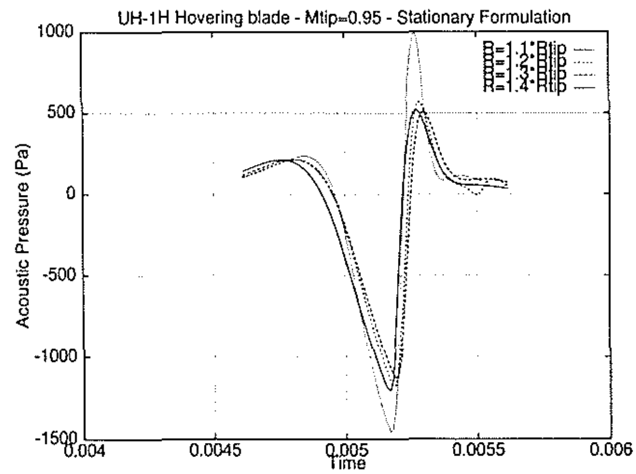

Figure 14: Pressure signal as a function of the radial position of the Kirchhoff surface at $M_{tip} = 0.95$

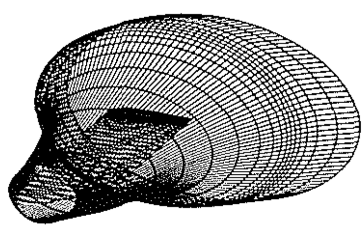

Figure 15: Kirchhoff surface directly extracted from the aerodynamic mesh with a constant vertical index

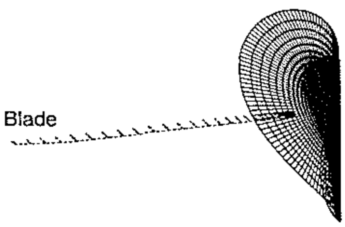

Figure 16: Kirchhoff surface directly extracted from the aerodynamic mesh with a constant span index

although some little variations in the predicted negative peak values arise by accounting for different cylindrical integration surfaces.

3.2 ROTATING FORMULATION

In the rotating formulation, the Kirchhoff surface can be directly extracted from the aerodynamic mesh. The blade is surrounded by a surface (shown in fig.15), which corresponds to a constant vertical index of the aerodynamic mesh and is closed through an end-base surface (fig.16).

In order to better evaluate the sensitivity of the Kirchhoff approach to the size and location of the control surface, and to better compute the pressure normal derivative, a cylindrical surface (fig.2) has been extracted from the aerodynamic mesh. The cylinder, with the axis perpendicular to the rotor hub, has an height of $1.15 * R_{tip}$ and has been discretized with a costant step of one degree in the azimuthal direction, and 50 equally spaced spanwise stations. The end base surface has the same number of azimuthal points, and is built up of 50 circles with radii decreasing.

The points of the control surface move at the same velocity as the blade. In this manner the method does not require any interpolation process of the aerodynamic data, but an important constraint is introduced. In fact, the equation (2) becomes singular when the velocity of the points of the Kirchhoff surface approaches the sound speed. This forces to locate the surface inside the sonic circle so that it has not been possible to perform a correct evaluation of the pressure disturbance at $M_{tip} = 0.90$, and $M_{tip} = 0.95$ because they would have required some computations in the supersonic region.

The aeroacoustic signals at $M_{tip} = 0.85$, computed using surfaces with constant vertical indices and 51 stations along the span, are shown in fig.17.

Moving far from the blade, the computed values become closer to the experimental results, but the oscillations of the signals at the highest k indices, indicate that some points of the closure end surface have velocities approaching or exceeding the sound speed. This forced to decrease the number of the spanwise stations.

Figure 18 shows the acoustic pressure calculated by considering the closer end surface having a j index of 49. By considering a surface further from the sonic circle allows to avoid partially the oscillations but an underprediction of the negative peak value occurs.

Carrying the computation out, with a cylindrical surface (fig.2), yields as result the acoustic disturbance signal presented in fig.19

The agreement with the experimental data is very good and also the pressure signal amplitude is well predicted likely because the cylindrical surface considered is more extended in the span direction than the surface directly extracted from the aerodynamic grid. A decreasing in the amplitude of the computed aeroacoustic signal, in fact, occurs considering a cylindrical surface with a height of $1.1 * R_{tip}$, as shown in fig.20.

The acoustic pressure computed by accounting for a different radius of the cylindrical surface, each proportional to the blade chord (constant), is shown in fig.21. The size of the control surface has influence either on the peak and on the shape of the pressure signal, as can also be noted from figures 17 and 18.

From a theoretical point of view the Kirchhoff surface has to be closed because it must divide the space surrounding the body in an inner non linear region and in an outer region where the fluid is assumed to be linear. The contribution of a root

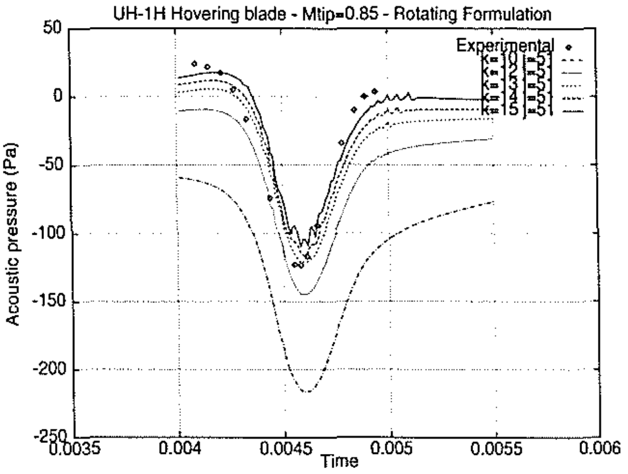

Figure 17: Comparison of aeroacoustic pressure computed at $M_{tip} = 0.85$ using k-constant layers of the aerodynamic mesh - 51 span stations -

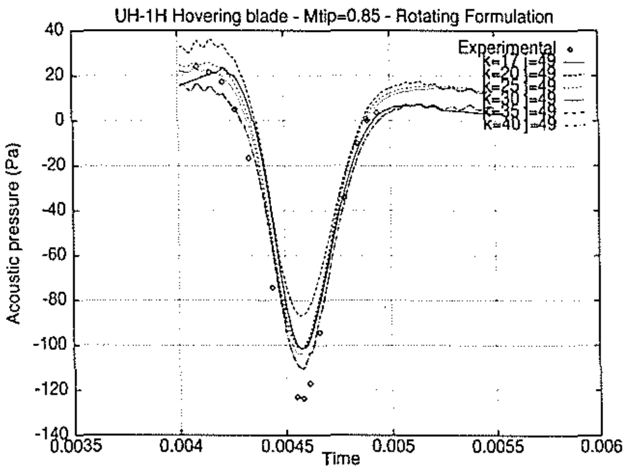

Figure 18: Comparison of aeroacoustic pressure computed at $M_{tip} = 0.85$ using k-constant layers of the aerodynamic mesh - 49 span stations -

base surface is negligible at all, while an end-base surface must be considered because it provides an important part of the signal.

Figg.22 and 23 present the acoustic pressure coming from the lateral and from the end-base surfaces respectively. As expected increasing the radius of the cylindrical surface, the contribution of the lateral part decreases because the surface moves away from the blade, while the contribution of the base becomes more important being more layers of the aerodynamic grid taken into account.

At a tip Mach numbers of 0.90 and 0.95, the computation has been performed considering a cylindrical surface, extracted through an interpolation from the aerodynamic mesh, with a height slightly

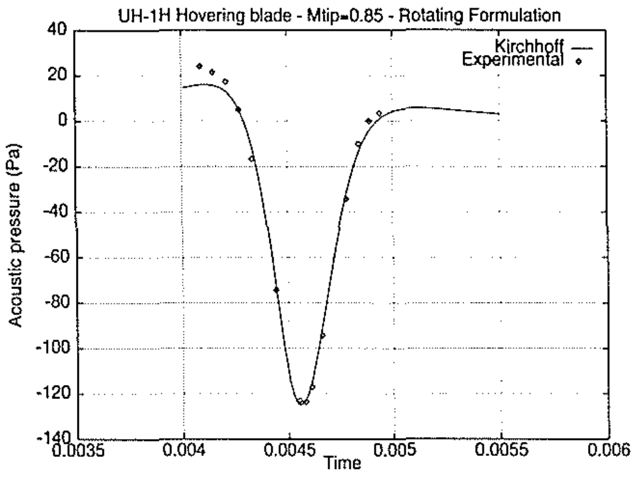

Figure 19: Aeroacoustic pressure at $M_{tip} = 0.85$

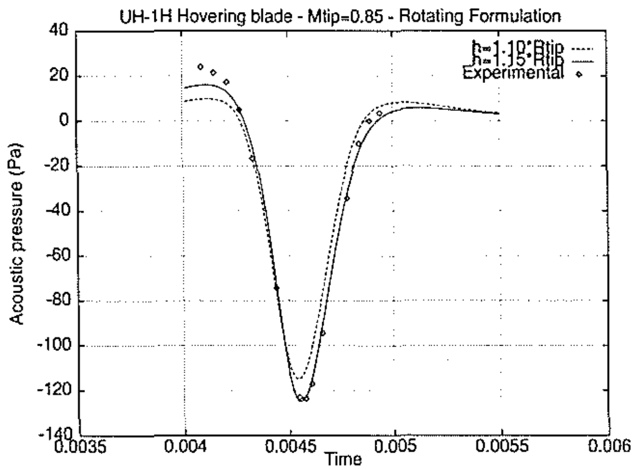

Figure 20: Aeroacoustic pressure at $M_{tip} = 0.85$ as a function of the height of the Kirchhoff surface

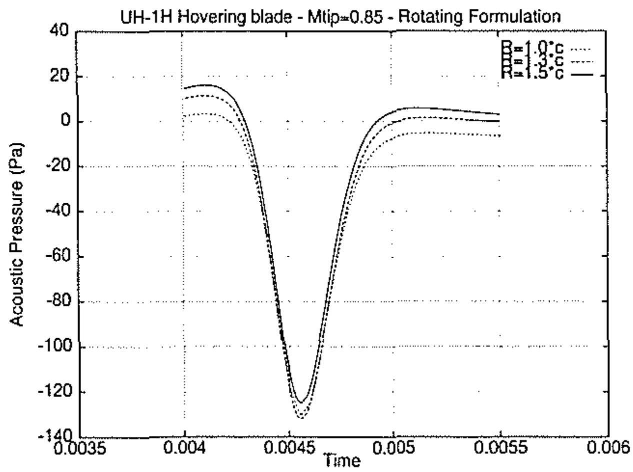

Figure 21: Aeroacoustic pressure at $M_{tip} = 0.85$ as a function of the radius of the Kirchhoff surface

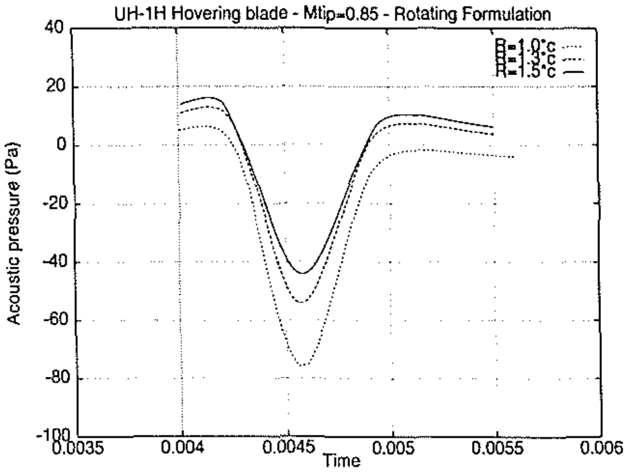

Figure 22: Aeroacoustic pressure at $M_{tip} = 0.85$ - Lateral surface contribution

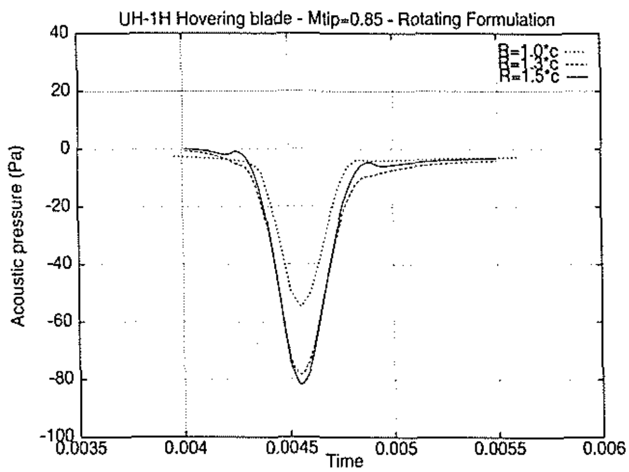

Figure 23: Aeroacoustic pressure at $M_{tip} = 0.85$ - Base end surface contribution

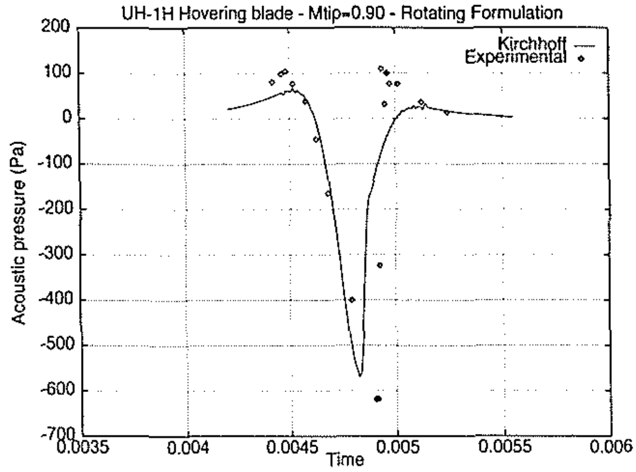

Figure 24: Aeroacoustic pressure at $M_{tip} = 0.90$ - limiting the integration inside the sonic circle

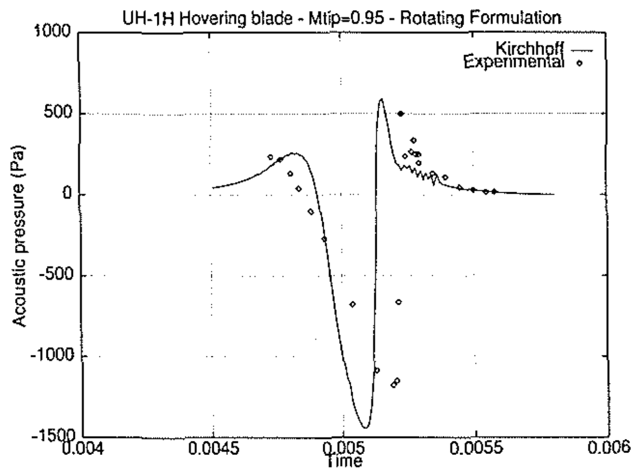

Figure 25: Aeroacoustic pressure at $M_{tip} = 0.95$ - limiting the integration inside the sonic circle

less than the sonic circle.

The result obtained at $M_{tip} = 0.90$ is shown, together with the experimental results in fig.24

The same underprediction of the signal negative peak has been found in the analysis performed through the stationary surface formulation; therefore the amplitude of the pressure disturbance is well predicted even limiting the computation inside the sonic circle. The limited spatial integration, however, accounts for the discrepancy in the signal shape and depends on having missed the supersonic non linear noise sources in the integration.

Fig.25 shows the pressure signal computed at $M_{tip} = 0.95$ by limiting the integration inside the sonic circle : there is a bad prediction of both the amplitude and the signal shape. The overprediction of the negative peak value means that the non linear supersonic noise sources oppose to the contribution of the subsonic component.

4 CONCLUSIONS

In order to increase the *CIRA* capabilities in the prediction of the acoustic pressure, and following the trend of most research institutions and industries, a code implementing the aeroacoustic Kirchhoff formulation has been developed and validated. Three test cases, concerning a hovering blade of the UH-1H rotor in non lifting conditions at tip Mach number of 0.85, 0.90, and 0.95, have been carried out, obtaining a good agreement with the corresponding available experimental data.

The code will be extended and tested in case of an unsteady aerodynamic input (helicopter rotor in forward flight) and the reliability of the new Kirchhoff code in the prediction of the propellers noise will be checked.

The problem of the Doppler singularity present in the equation (2) will be dealt with. The use of the supersonic Kirchhoff formula [15] seems to be a very hard task. This is due either to the behaviour of the acoustic surface in supersonic motion (problem faced at *CIRA* [14]), and to the difficult implementation of the integrand terms. The possibilty of using alternative approaches [10], suitable for an integration on a supersonic domain, is being investigated.

ACKNOWLEDGMENTS

The authors wish to thank Dr. M. Kuntz from DLR, for kindly providing the different sets of aerodynamic data used as input for the aeroacoustic calculations presented in this paper.

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