Paper No 22

COMPRESSIBLE FLOW ABOUT HELICOPTER ROTORS

by

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1. Lifting-line theory for a rotor in vertical climb

1.1 Introduction

The problem of a helicopter rotor in vertical climb, hovering or slow descent (or a lightly loaded propeller in forward flight or operating statically) is treated with the method of matched asymptotic expansions. The fluid is assumed to be compressible, and the maximum velocities occurring are assumed to be subsonic. Two perturbation parameters are used. The first one, denoted by ϵ, is the ratio of a typical blade chord to the disk diameter. The matching in ϵ relates an inner limit region in the vicinity of an arbitrary blade section to an outer limit region far from that blade section. To solve the outer limit problem, there is introduced a second perturbation parameter, denoted by δ and defined as the ratio between the climb velocity of the rotor (or flight velocity of a propeller), V_0 (to which the mean induced velocity perpendicular to the disk, W, can be added, to obtain

$$V = V_0 + W), \tag{1.1}$$

and the tip velocity. The matching in δ relates (in the disk plane or the "Trefftz plane" in the far wake) two outer limits, the "blade limit" within the disk and the "outer flow limit" outside the disk, to two inner limits, the "root limit" for the root region and the "tip limit" for the tip region.

For the quantities of interest (local blade forces and moments, thrust, torque, power, propulsive efficiency et cetera) series solutions in ϵ and δ are obtained in the form of a two-term expansion in ϵ, where the first term is exact in δ, while the second term is a two-term expansion in δ. The tip correction obtained in the first δ approximation is identical to the one given by Prandtl. It is based upon the values at the tip of relative wind velocity, chord length and angle of attack. In the second δ approximation this tip correction is refined by taking into account also the radial variation of these quantities within a tip region of radial extension $O(\delta)$.

The major advantages of the present theory are that compressibility is included, that Prandtl's tip approximation has been improved and that the load distribution is allowed to vary arbitrarily: it is not restricted to be the optimum one, as in Prandtl's and Goldstein's theories. Another advantage is that it is very easy to evaluate numerically.

1.2 The computational model

We consider a rotor (or propeller) of B equal blades, rotating with the angular velocity Ω and situated in a rotor-fixed cylindrical coordinate system $Or\varphi z$ (see Fig. 1.1). The disk radius is taken as unit of length. The blades have the local two-dimensional lift-curve slope 2π and the chord length $2\epsilon h(r)$, where $h(r)$ is of order unity. The local zero lift direction of the blade foil forms the geometrical angle of attack $\alpha(r)$ with the relative wind velocity, $(-V\hat{z} - \Omega r\hat{\varphi})$. (The notation ^ indicates a unit vector in the corresponding coordinate direction.)

Writing the velocity $\vec{v}$ in component form,

$$\vec{v} = u\hat{r} + (-\Omega r + v)\,\hat{\varphi} + (-V_o + w)\,\hat{z}\,, \tag{1.2}$$

where u, v and w are the induced perturbation velocities, the linearized equations of motion are found to be:

$$\begin{cases} \rho_\infty\left[\frac{u}{r} + \frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \varphi} + \frac{\partial w}{\partial z}\right] - D\rho = 0 & (1.3) \\ Du = \frac{1}{\rho_\infty}\frac{\partial p}{\partial r} & (1.4) \\ Dv = \frac{1}{\rho_\infty r}\frac{\partial p}{\partial \varphi} & (1.5) \\ Dw = \frac{1}{\rho_\infty}\frac{\partial p}{\partial z} & (1.6) \\ Dp - a^2 D\rho = 0 & (1.7) \end{cases}$$

Here p is the pressure, ρ the density, ρ_∞ the undisturbed density, a the undisturbed speed of sound and D the differential operator

$$D = \Omega\frac{\partial}{\partial\varphi} + V\frac{\partial}{\partial z} \tag{1.8}$$

Combination of Eqs. (1.3) to (1.8) gives the differential equation for p:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \left(\frac{1}{r^2} - \frac{\Omega^2}{a^2}\right)\frac{\partial^2 p}{\partial \varphi^2} + \left(1 - \frac{V^2}{a^2}\right)\frac{\partial^2 p}{\partial z^2} - \frac{2\Omega V}{a^2}\frac{\partial^2 p}{\partial\varphi\partial z} = 0 \tag{1.9}$$

The boundary conditions for our problem are undisturbed flow far upstream, tangency at the blade surfaces, the Kutta condition at the trailing edge and p being continuous and the normal velocity zero through the wake vortex sheets.

1.3 First inner ϵ approximation

We introduce an inner ϵ limit, valid near the blade located at $\varphi = 0$, $z = 0$, by introducing the magnified inner variables

$$\Phi = \frac{\varphi}{\epsilon}\;; \qquad Z = \frac{z}{\epsilon}\;. \tag{1.10}$$

Inserting (1.10) into (1.9), the differential equation for the inner limit is obtained:

$$\left(\frac{1}{r^2} - \frac{\Omega^2}{a^2}\right)\frac{\partial^2 p}{\partial \Phi^2} + \left(1 - \frac{V^2}{a^2}\right)\frac{\partial^2 p}{\partial Z^2} - \frac{2\Omega V}{a^2}\frac{\partial^2 p}{\partial\Phi\partial Z} + \epsilon^2\left[\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right] = 0 \tag{1.11}$$

When higher-order ϵ terms are neglected, r appears only as a parameter in the differential equation and the boundary conditions. For every section r = constant the problem turns out to be that of two-dimensional flow around a wing section. The matching to the outer limit will be performed in terms of the lift distribution, $\ell''(r)$, obtained in the inner ϵ limit:

$$\ell''(r) = \epsilon 2\pi\rho_\infty\,\alpha(r)\,h(r)\,\frac{\Omega^2 r^2 + V^2}{\sqrt{1 - (\Omega^2 r^2 + V^2)/a^2}} \tag{1.12}$$

1.4 First outer ϵ approximation

In the outer ϵ approximation, the blades shrink to lifting lines, when ϵ tends to zero. Introducing an oblique coordinate system

$$\begin{cases} r = r \\ \eta = -\varphi + \frac{\Omega}{V} z \\ \zeta = -z , \end{cases} \tag{1.13}$$

and the notations

$$\begin{cases} v_1 = -v \\ w_1 = -\frac{\Omega r}{V} v - w , \end{cases} \tag{1.14}$$

and inserting Eqs. (1.13) and (1.14) into the equations of motion (1.3) to (1.7), it will be possible to introduce a perturbation velocity potential Φ, defined by

$$\begin{cases} \frac{\partial \Phi}{\partial r} = \frac{u}{V} \\ \frac{1}{r}\frac{\partial \Phi}{\partial \eta} = \frac{v_1}{V} \\ \frac{\partial \Phi}{\partial \zeta} = \frac{w_1}{V} = -\frac{p - p_\infty}{\rho_\infty V^2} , \end{cases} \tag{1.15}$$

(p_∞ being the undisturbed pressure) and satisfying

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \left(\frac{1}{r^2} + \frac{\Omega^2}{V^2}\right)\frac{\partial^2 \Phi}{\partial \eta^2} + \left(1 - \frac{V^2}{a^2}\right)\frac{\partial^2 \Phi}{\partial \zeta^2} - 2\frac{\Omega}{V}\frac{\partial^2 \Phi}{\partial \eta \partial \zeta} = 0 . \tag{1.16}$$

The problem is transferred to the "Trefftz plane" by introducing

$$\Phi_1(r,\eta) = \Phi(r,\eta,\zeta) + \Phi(r,\eta,-\zeta) , \tag{1.17}$$

satisfying

$$\frac{\partial^2 \Phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi_1}{\partial r} + \left(\frac{1}{r^2} + \frac{\Omega^2}{V^2}\right)\frac{\partial^2 \Phi_1}{\partial \eta^2} = 0 . \tag{1.18}$$

The second perturbation parameter,

$$\delta = \frac{V}{\Omega} , \quad \delta \ll 1 , \tag{1.19}$$

must also satisfy the condition

$$\frac{B\epsilon}{\pi\delta} \ll 1 , \tag{1.20}$$

in order not to invalidate the inner ϵ approximation. Since Φ_1 is found to be $O(\epsilon/\delta)$, it is convenient to introduce

$$\varphi_1 = \frac{\delta}{\epsilon\pi} \Phi_1 . \tag{1.21}$$

φ_1 is periodical in η with the period $2\pi/B$. The differential equation for φ_1 and the boundary conditions in the interval $0 \le \eta \le 2\pi/B$ are:

$$\frac{\partial^2 \varphi_1}{\partial \eta^2} + \delta^2 \left[\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi_1}{\partial \eta^2}\right] = 0 \tag{1.22}$$

$$\begin{cases} \frac{\partial \varphi_1}{\partial \eta}(r,0) = \frac{\partial \varphi_1}{\partial \eta}(r,\frac{2\pi}{B}) & (1.23) \\ \left.\begin{aligned} \varphi_1(r,0) &= f_1(r) \\ \varphi_1(r,\tfrac{2\pi}{B}) &= -f_1(r) \end{aligned}\right\} , & (1.24) \end{cases}$$

where

$$f_1(r,\delta) = \begin{cases} \sqrt{r^2+\delta^2}\,\dfrac{h(r)\,\alpha(r)}{\sqrt{1-\frac{\Omega^2}{a^2}(r^2+\delta^2)}} , & 0 \le r \le 1 \\ 0 , & r > 1 \end{cases} \qquad (1.25)$$

The requirement that the velocity field is continuous gives Eq. (1.23), while (1.24) is obtained by matching to the inner ε limit in terms of the lift distribution ($\ell''(r)$). Eq. (1.24) is not valid near the blade tips (for $r \le 1$). A refined boundary condition for this region will be obtained later.

The induced upwash velocity at a blade, normal to the blade chord, is found to be:

$$w' = \varepsilon\pi \frac{\Omega\sqrt{r^2+\delta^2}}{2r\delta}\left[\frac{\partial\varphi_1}{\partial\eta}\right]_{\eta=0} \qquad (1.26)$$

1.5 The δ approximations

To solve the problem defined by Eqs. (1.22) to (1.25), we introduce the blade, outer flow, root and tip limits, defined in the introduction, and denote them by B, O, R and T, respectively. In the root and tip limits magnified inner variables, also denoted by R and T, are introduced:

$$R = \frac{r}{\delta} \qquad (1.27) \qquad\qquad T = \frac{r-1}{\delta} \qquad (1.28)$$

In every limit, φ_1 is expanded as a power series in δ:

$$\varphi_{1N} = \varphi_{10N} + \delta\varphi_{11N} + \delta^2\varphi_{12N} + \delta^3\varphi_{13N} + \dots \qquad (1.29)$$

Here N takes the "values" B, O, R or T, denoting the limit in consideration. In the first δ approximation the following solutions are obtained:

$$\varphi_{10B} = f_1(r,\delta)\left[1 - \frac{B}{\pi}\eta\right] \qquad (1.30)$$

$$\varphi_{100} = 0 \qquad (1.31)$$

$$\varphi_{10R} = f_1(0,\delta)\left[1 - \frac{B}{\pi}\eta\right] \qquad (1.32)$$

$$\varphi_{10T} = \frac{2}{\pi} f_1(1,\delta)\left[\mathrm{Re}\ \mathrm{arc\,cos}\ e^{\frac{B}{2}(T+i\eta)} - \frac{B}{2}\eta\right] \qquad (1.33)$$

(We have chosen not to expand $f_1(r,\delta)$ as a power series in δ. All φ_{1nB} and φ_{1nO} can be calculated exactly. Every φ_{1nO} and every odd φ_{1nB} are equal to zero.)

From (1.33) a tip correction to the strip approximation lift distribution, $\ell''(r)$, is derived, to obtain the improved lift distribution $\ell'(r)$, which is then used to correct the boundary condition (1.24):

$$\ell'(r) = \ell''(r) \cdot \frac{2}{\pi} \text{ arc cos } e^{-\frac{B}{2}\frac{1-r}{\delta}} \qquad (1.34)$$

In the second δ approximation, the following solutions are found:

$$\varphi_{11B} = \varphi_{110} = \varphi_{11R} = 0 \qquad (1.35)$$

$$\varphi_{11T} = \frac{f_1(1,\delta)}{2} \left\{ \frac{1}{\pi} \left[- 2T \text{ Re arc cos } e^{\frac{B}{2}(T+i\eta)} + BT\eta \right] + \frac{3a^2 - \Omega^2}{a^2 - \Omega^2} P \right\}, \qquad (1.36)$$

where

$$P(T,\eta) = T \left[1 - H(T)\right]\left[1 - \frac{B}{\pi}\eta\right] + \frac{1}{\pi}\int_0^\infty \frac{1}{g^2}\left[\frac{B}{\pi}\eta - 1 - \frac{\sinh g(\eta - \frac{\pi}{B})}{\sinh \frac{g\pi}{B}}\right] \cos gT \, dg +$$

$$+ \frac{4}{B\pi^2}\int_0^\infty \frac{\sinh \frac{k}{2}(B\eta - \pi)}{\sinh \frac{k\pi}{2}} \left\{ k \left[\frac{2}{(1+k^2)^2} + \frac{1}{(9+k^2)^2}\right] \sin \frac{kBT}{2} - \right.$$

$$\left. - \left[\frac{1-k^2}{(1+k^2)^2} + \frac{9-k^2}{6(9+k^2)^2}\right] \cos \frac{kBT}{2} \right\} dk \qquad (1.37)$$

Here $H(T)$ is the Heaviside unit step function. The corresponding $[\partial\varphi_1/\partial\eta]_{\eta=0}$ value for $T \le 0$ is:

$$\left[\frac{\partial \varphi_{11T}}{\partial \eta}\right]_{\eta=0} = \frac{f_1(1,\delta)}{2\pi} \left[- \left(\frac{2a^2}{a^2 - \Omega^2}\right) BT + \left(\frac{3a^2 - \Omega^2}{a^2 - \Omega^2}\right) J(BT) \right] , \qquad (1.38)$$

where

$$J(BT) = \int_0^\infty \frac{1}{\varkappa^2}\left[1 - \varkappa \coth \varkappa\right] \cos \left(\frac{\varkappa BT}{\pi}\right) d\varkappa +$$

$$+ \frac{2}{\pi}\int_0^\infty k \coth\left(\frac{k\pi}{2}\right) \cdot \left\{ k \left[\frac{2}{(1+k^2)^2} + \frac{1}{(9+k^2)^2}\right] \sin\left(\frac{kBT}{2}\right) - \right.$$

$$\left. - \left[\frac{1-k^2}{(1+k^2)^2} + \frac{9-k^2}{6(9+k^2)^2}\right] \cos\left(\frac{kBT}{2}\right) \right\} dk \, . \qquad (1.39)$$

The function $J(BT)$ is given in Fig. 1.2.

1.6 Second inner ε approximation. Final formulas

In the second inner ε approximation, the lift distribution ℓ' is corrected for the influence of the induced upwash velocity, w', obtained in the first outer ε approximation. Introducing also $\Delta\alpha(r)$, the correction angle of $\alpha(r)$, which is related to w' by

$$\Delta\alpha = \frac{w' + W\cos\beta}{\sqrt{V^2 + \Omega^2 r^2}} = \frac{w' + W\cos\beta}{\Omega\sqrt{r^2 + \delta^2}} \;, \quad \beta = \text{arc tg } \frac{V}{\Omega r} \qquad (1.40)$$

the final lift distribution $\ell(r)$ is given by

$$\ell(r) = \ell'(r) \left(1 + \frac{\Delta\alpha(r)}{\alpha(r)}\right) \;, \qquad (1.41)$$

where

$$\Delta\alpha = -\frac{\varepsilon}{\delta}\cdot\frac{1}{2r}\left[B\ f_1(r,\delta) - \delta\,\frac{f_1(1,\delta)}{2}\left(\frac{3a^2-\Omega^2}{a^2-\Omega^2}\right) J\left(-B\,\frac{1-r}{\delta}\right) + O(\delta^2)\right] +$$

$$+\frac{W\cdot r}{\Omega(r^2+\delta^2)} \qquad (1.42)$$

The distribution of induced drag is

$$d_i(r) = -\Delta\alpha(r)\cdot \ell(r)\ . \qquad (1.43)$$

2. Disk approximation for a helicopter rotor in forward flight

2.1 Introduction

A theory is developed for calculation of the induced velocity field of a helicopter rotor in forward flight. The rotor is approximated by a disk of continuous thrust and in-plane force distributions, which are assumed to be known. Its wake is represented by a semi-infinite cylinder of distributed vorticity. The velocity field is obtained using a form of the Biot-Savart formula modified to take into account the compressibility effect of a parallel flow. Thus, the compressibility effects due to the flight speed are accounted for. The first-order compressibility effect due to the rotation speed (the Prandtl-Glauert factor) can be taken into account when setting up the formula for the initial load distribution. Second-order compressibility effects due to the rotation speed are lost when making the disk approximation, however.

2.2 The computational model

From the total aerodynamic force of the rotor a mean value of the induced velocity at the disk area can be calculated, using the momentum equation. By vector addition of this mean value to the flight velocity of the rotor we obtain the relative wind velocity, $\vec{V}$, forming the angle β with the rotor axis. A Cartesian coordinate system Oxyz is introduced, with the z axis coincident to the rotor axis and directed downwards and the x and y axes in the disk plane. The y axis is perpendicular to the $\vec{V}$ velocity vector and directed to the port side of the helicopter, while the x axis is directed aft. See Fig. 2.1. We also introduce a corresponding cylindrical coordinate system Orθz and an oblique system Opys by the following transformations:

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = -\ \mathrm{arc\ tg}\,\frac{y}{x} \\ z = z \end{cases} \qquad (2.1) \qquad\qquad \begin{cases} p = x - z\ \mathrm{tg}\,\beta \\ y = y \\ s = z \end{cases} \qquad (2.2)$$

Continuous distributions of thrust per unit area, $t(r,\theta)$, and aerodynamic force per unit area parallel to the disk plane, $\vec{q}(r,\theta)$, are obtained by time-averaging the real load for every point in the disk plane. The rotor thus is represented by a circular sheet of radial bound vortices of the strength per unit area $\gamma(r,\theta)$ or $\gamma(x,y)$ (γ being determined from t and $\vec{q}$). The axis of the wake cylinder is chosen to be parallel to the velocity $\vec{V}$, i.e., coinciding with the s axis. The distribution of vorticity in the wake is taken to be the same in every plane parallel to the disk, i.e., independent of s. Thus we neglect the influence of the vorticity on the location of the infinitesimal wake vortices.

When calculating the time mean value of the induced velocity, $\vec{v}$, we divide the task into two problems: That of calculating the velocity $\vec{v}_1$, induced by the bound vortices of the rotor disk, and that of calculating $\vec{v}_2$, induced by the wake vorticity distribution. Hence

$$\vec{v} = \vec{v}_1 + \vec{v}_2 \quad . \tag{2.3}$$

2.3 Calculation of the velocity induced by the bound disk vortices

Using the Prandtl-Glauert transformation, the Biot-Savart formula can be modified to take into account the compressibility effects of a subsonic parallel flow with the Mach number M in the ξ direction, ξ being a coordinate in a Cartesian frame $O\xi\eta\zeta$. It is found that

$$\overrightarrow{dv} = \mu^2 \frac{\Gamma}{4\pi} \cdot \frac{\overrightarrow{dl} \times \vec{r}_1}{r_{1i}^3} \quad , \tag{2.4}$$

where

$$\mu = \sqrt{1 - M^2} \tag{2.5}$$

and

$$r_{1i} = \sqrt{(\xi - \xi')^2 + \mu^2(\eta - \eta')^2 + \mu^2(\zeta - \zeta')^2} \quad , \tag{2.6}$$

(ξ, η, ζ) being the point where $\overrightarrow{dv}$ is calculated and the primmed quantities being the coordinates of the vortex element.

Using Eq. (2.4), $\vec{v}_1$ is found to be:

$$\vec{v}_1(x,y,z) = \sum_{i=x,y,z} \frac{1 - M^2}{4\pi} \left[\iint_{\text{The disk}} \frac{\gamma(x',y') \cdot E_i}{r_{1io}^3 [x'^2 + y'^2]^{\frac{1}{2}}} \, dx' \, dy' \right] \hat{i} \quad , \tag{2.7}$$

where

$$r_{1io} = \sqrt{(x' - x)^2 + (y'-y)^2 + z^2 - M^2 \left\{ [(x'-x) \cos\beta + z \sin\beta]^2 + (y'-y)^2 \right\}} , \tag{2.8}$$

$$\begin{cases} E_x = y'z \\ E_y = - x'z \\ E_z = x'y - y'x \ , \end{cases} \tag{2.9}$$

and where

$$M = \frac{V}{a} \quad , \tag{2.10}$$

V being the magnitude of $\vec{V}$ and a being the undisturbed speed of sound.

2.4 Calculation of the velocity induced by the wake

We denote the vorticity of the flow field by $\vec{\omega}(p,y,s)$ and the vorticity jump through the rotor disk by $[\vec{\omega}](p,y)$. According to Ref. [5],

$$[\vec{\omega}](p,y) = - \frac{1}{m} \hat{z} \times \text{grad } t - \text{curl } \frac{\vec{q}}{m} \quad . \tag{2.11}$$

Approximating m, the mass flow per unit area, by its mean value,

$$m = \rho V \cos \beta \tag{2.12}$$

and using that $\vec{\omega}$ is zero above the disk and independent of s in the wake cylinder, i.e.

$$[\vec{\omega}](p,y) = \vec{\omega}(p,y,0+) = \vec{\omega}(p,y,s), \qquad s > 0, \tag{2.13}$$

we obtain

$$\vec{\omega}(p,y,s) = \frac{1}{\rho V \cos\beta} \left\{ \frac{\partial t}{\partial y}(p,y)\,\hat{x} - \frac{\partial t}{\partial x}(p,y)\,\hat{y} + \left[\frac{\partial q_x}{\partial y}(p,y) - \frac{\partial q_y}{\partial x}(p,y)\right]\hat{z} \right\} =$$

$$= \omega_x\,\hat{x} + \omega_y\,\hat{y} + \omega_z\,\hat{z}\;, \quad s > 0\;, \tag{2.14}$$

ω_x, ω_y and ω_z being defined by Eq. (2.14).

Since

$$\Gamma\,\vec{dl} = \vec{\omega}\,dV\;, \tag{2.15}$$

dV being a volume element, Eq. (2.4) can be rewritten:

$$\vec{dv} = \mu^2 \frac{\vec{\omega} \times \vec{r}_1}{4\pi r_{1i}^3}\,dV \tag{2.16}$$

Integrating this equation over the wake cylinder, where the integration with respect to the variable s' can be performed analytically, $\vec{v}_2$ is obtained:

$$\vec{v}_2\,(x,y,z) = \sum_{i=x,y,z} (1-M^2)\,\frac{\cos^3\beta}{4\pi}\left[\iint\limits_{\text{The disk}} \frac{C_i + D_i \cdot A}{A^2 + B \cdot A}\,dx'dy'\right]\hat{i}\;, \tag{2.17}$$

where

$$\begin{cases} A = \sqrt{(x'-x)^2 + (y'-y)^2 + z^2 - M^2\left\{[(x'-x)\cos\beta + z\sin\beta]^2 + (y'-y)^2\right\}}\;\cos\beta \\ B = (x'-x)\sin\beta\cos\beta - z\cos^2\beta \\ C_x = \omega_y z + \omega_z (y'-y) \\ D_x = -\,\omega_y \\ C_y = -\,\omega_x z - \omega_z (x'-x) \\ D_y = \omega_x - \omega_z \operatorname{tg}\beta \\ C_z = -\,\omega_x (y'-y) + \omega_y (x'-x) \\ D_z = \omega_y \operatorname{tg}\beta \end{cases}$$

$$\omega_i = \omega_i(x',y')\,, \qquad i = x,y,z\;. \tag{2.18}$$

References:

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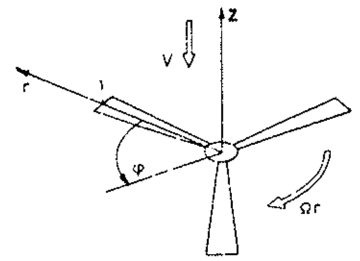

Fig. 1.1. The rotor in the rotor-fixed frame Orφz.

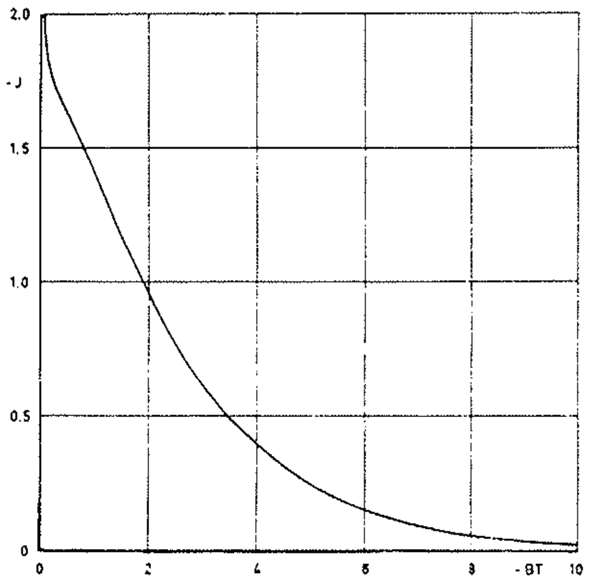

Fig. 1.2. The function J(BT).

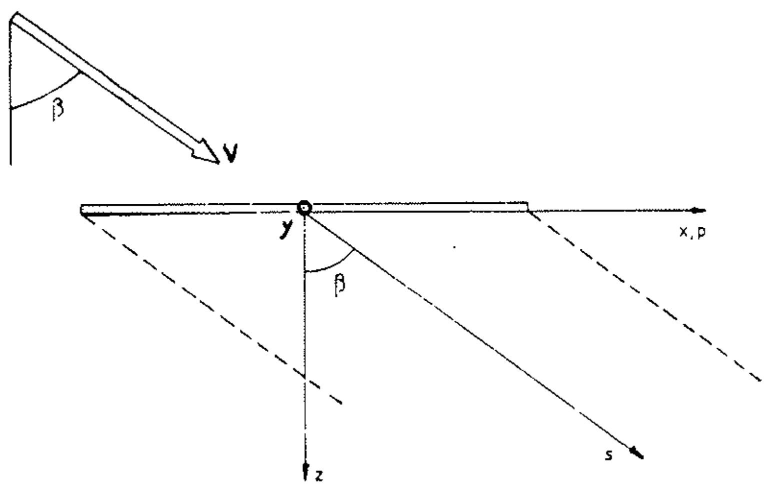

Fig. 2.1. The Oxyz and Opys coordinate systems.