AN EMPIRICAL PREDICTION METHOD FOR
HELICOPTER PERFORMANCE IN LOW
SPEED LEVEL FLIGHT AND IN
VERTICAL AND FORWARD FLIGHT CLIMBS

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ABSTRACT

This paper presents methods for the prediction of helicopter power requirements in low speed forward flight and in vertical and forward flight climbs. The methodology is based on an extensive study of experimental data. Available analytical prediction methods are summarized and compared with the data. The data are reduced to non-dimensional forms that cause the data to collapse into a single trend for all of the aircraft considered. The low speed forward flight data yield a single non-dimensional curve that relates the power required at any given speed to that required at hover and at the minimum power speed. The vertical climb data also yield a single non-dimensional plot that relates the climb power requirement to the climb rate. For forward climb, a single climb constant was found that is valid for all climb rates.
LIST OF SYMBOLS

\( A \) Rotor swept disc area, \( \text{ft}^2, \frac{\pi R^2}{2} \)

\( \bar{C}_p \) Non-dimensional climb power coefficient,

\[
\frac{C_p - C_{p_m}}{C_{p_h} - C_{p_m}}
\]

\( C_p \) Power coefficient

\[
\frac{550 \text{ESHP}}{\rho A (AR)^3}
\]

\( C_{pG} \) Generalized power coefficient

\( C_{p_h} \) Hover power coefficient

\( C_{p_l} \) Level flight power coefficient

\( C_{p_m} \) Power coefficient at the minimum power speed

\( C_T \) Thrust coefficient, \( \frac{T}{\rho A (AR)^2} \)

\( \text{ESHP}_c \) Engine shaft power required to climb

\( GW \) Gross weight, lb.

\( K_c \) Climb constant, \( GW \frac{\nu_c}{\text{ESHP}_c} 550 \)

\( R \) Main rotor radius, ft.

\( \nu \) Airspeed, \( \text{ft/sec} \) (1.688 airspeed in kt.)

\( \bar{\nu}_c \) Non-dimensional climb rate, \( \frac{\nu_c}{\nu_i} \)

\( \nu_c \) Climb rate, \( \text{ft/sec} \).

\( \bar{\nu}_h \) Non-dimensional horizontal velocity, \( \frac{\nu_h}{\nu_i} \)

\( \nu_h \) Horizontal velocity, \( \text{ft/sec} \).

\[
\frac{\sqrt{\nu^2 - \nu_c^2}}{\nu_i}
\]

\( \nu_m \) Airspeed for minimum power, \( \text{ft/sec} \).

\( \nu_i \) Ideal hover downwash velocity, \( \sqrt{CT/2} \)

\( \eta_m \) Helicopter mechanical efficiency

\( \rho \) Atmospheric density, slugs/\( \text{ft}^3 \)

\( AR \) Main rotor tip speed, \( \text{ft/sec} \).

\( \Omega \) Main rotor rotational speed, \( \text{rad/sec} \).
1. INTRODUCTION

Helicopter performance has traditionally been measured in hover and in forward flight at speeds from just below the speed for minimum power out to the maximum speed attainable. Climb performance is sometimes measured at the speed for best rate of climb (about the same as the speed for minimum power), and less frequently, at zero airspeed, known as vertical climb. Generally no data are taken for the transition speed range between hover and the speed for minimum power. Most helicopters operate in this regime only briefly during takeoff and landing, and if the helicopter has sufficient power to hover, even in ground effect, it will usually have enough power to pass safely through transition. However, there are situations in which the low speed performance of a helicoper is very important. One is nap-of-the-earth flight in combat. Here, there will normally be ample power to remain airborne, but the amount of excess power available determines the amount of power available for turning, acceleration, or a vertical climb "pop up". Another critical area is the single engine landing capability of a twin engine helicopter. This is of increasing interest with the trend toward twin engine helicopters in all but the smallest sizes. Low speed capability is also important in mine countermeasures operations. Hence, there is a definite need for methodology to determine the level flight performance of helicopters at speeds between hover and the speed for minimum power.

Climb performance is also of increasing interest. As helicopter ranges increase, there is more interest in cruising at high altitudes to improve the cruising efficiency. This fuel savings must be balanced against the fuel used to climb, which can be a significant part of the mission fuel. Vertical and low speed climb capability is also important as a measure of power margin for takeoff or waveoff conditions and for maneuvering in nap-of-the-earth combat.

This paper presents test data for a variety of single and tandem rotor helicopters covering a wide range of sizes and speed capability. The data are analyzed and used to develop methods for quickly calculating the power required at speeds between hover and the speed for minimum power and for vertical and forward flight climbs.

2. Low Speed Flight

Helicopter performance has been well defined at hover and at speeds from the minimum power speed on up to the maximum speeds presently attainable. All the major manufacturers have analytical prediction techniques for both hover and cruise flight, which are reasonably accurate except at conditions involving significant amounts of blade stall. There is a very large amount of test data available in both regimes for a wide variety of helicoper sizes and configurations.

The low speed regime is much less well defined. It is difficult to analyze. Neither the simple momentum analysis used at hover, nor the fixed wing analogy commonly used for forward flight at speeds at or above the minimum power speed, is valid (reference 1). The equations for the rotor induced power in forward flight (reference 1) can be simplified by assuming a zero rotor angle of attack, which is nearly true, but these equations are based on the analogy between a rotor and a fixed wing with an elliptical
lift distribution. This analysis assumes that the induced downwash is uniform over the rotor diameter and small compared to the flight speed. As the flight speed decreases below the minimum power speed, the inflow becomes a significant fraction of the flight speed, and increasingly non-uniform. Hence, the classical forward flight assumptions become very questionable, although as shown in reference 2, they do give plausible results.

Computer analysis of low speed flight is possible. Some uniform inflow type programs will run in the low speed regime and give reasonable results, but the programs are also based on the same fixed wing analogy, so the results are of questionable accuracy. Wake modeling type programs do account for the non-uniform inflow and should be capable of giving good results, but these are expensive and many require considerable operator skill.

It is much more difficult to obtain good low speed test data than for hover or forward flight. The biggest problem is probably accurately holding and measuring airspeed. Conventional airspeed sensors typically become inaccurate below 40 to 50 kt. Low range air speed systems are available, and are a major improvement, but these are not installed on most aircraft. They must be carefully calibrated to avoid large errors caused by velocities induced by the helicopter itself. Alternate methods of measuring airspeed are to use a pace car or a theadelite system. Both can give good data, but they require calm weather conditions, since they measure the aircraft's speed relative to the ground.

An extensive literature search revealed a large amount of data for airspeeds down to about one-half of the minimum power speed, usually obtained with a conventional airspeed indicator. In addition, an extensive set of data exists covering the entire low speed range for the Bell UH-1 aircraft (reference 3). This was obtained at Edwards Air Force Base using a pace car. A total of 12 data sets were located and included several models of the Bell UH-1, the Bell AH-1J, the Boeing CH-47B, the Hughes OH-6A, five models of the Sikorsky H-3, and the Sikorsky RH-53D (references 3-14).

Two methods of reducing the data were considered. The first was to plot a non-dimensional power coefficient against a non-dimensional forward flight velocity as done by Boirun in reference 15. In this method, the non-dimensional coefficients $C_{pG}$ and $V_h$ are defined, where

$$C_{pG} = (Cp_{1} - C_{pH})/0.707 C_{p1.5}$$

and

$$V_h = V_h/W_1.\$$

With this analysis, the weight coefficient drops out and a single plot of $C_{pG}$ vs $V_h$ defines the helicopter's power requirements for all gross weights provided there are no compressibility or blade stall effects. The resulting curve is valid only for a given aircraft.

The second method considered is to define the non-dimensional power coefficient $C_p$, where

$$C_p = (Cp - C_{pH})/(C_{pH} - C_{pM})\$$

and to plot this against $V/V_m$. With this method $C_p$ always decreases from 1.0 to 0.0 as $V/V_m$ increases from 0.0 to 1.0. Note that the range shown in Figure 1 is only the difference between minimum and hover power, and is about half of the hover power.

Both methods offer advantages. The method of Boirun is more rigorous analytically and may be more accurate if a good power required curve is
available for one weight coefficient and one is desired for another weight coefficient. The second method offers the potential advantage that if the data collapsed to a single curve, the resulting curve could be used to determine the low speed performance of any helicopter if its hover and minimum power performance were known, as they frequently are.

All of the available data were plotted in the second format (see Figure 1). This figure shows that within a band of about $\pm 10\%$ of full scale (about $\pm 5\%$ of the hover power), the data do form a single curve. This curve includes 13 data sets for thirteen different aircraft, and the scatter of many of the data sets are nearly as large as the total scatter in Figure 1. The data were reviewed for any trend with blade or disc loading or configuration, but none was found. Therefore, a curve was fit through all the points in Figure 1, and this line is considered valid to predict the low speed performance of any conventional single or tandem rotor helicopter. The curve may not be valid for a helicopter carrying an external load if the load has a large amount of drag.

Use of the method is simple. The points at hover and the minimum power speed are first plotted. Then, a given fraction of the minimum power speed is chosen and the corresponding fraction of the power increment above the minimum power is obtained from Figure 1. This fraction is then multiplied by the difference between hover and minimum power to give the power at the chosen point. The process is repeated across the speed range from hover to the minimum power speed to define the curve.

3. Vertical Climb

Vertical climb has received a substantial amount of interest as a result of its being included in the specification requirements of several
recent military helicopter procurements. Unlike the low speed transition regime, vertical climb performance can be readily analyzed. The classical momentum analysis, used for hover can be extended to include vertical climb, as is done in page 103 of reference 1. In the classical hover momentum analysis, the ideal momentum power is usually multiplied by a factor of about 1.12 to account for the actual non-uniform inflow, and a profile power term is added to obtain the main rotor power. For a typical single rotor helicopter, the effective rotor thrust must be reduced by about 5% to account for the downwash on the fuselage and interference between the main and tail rotors. The total power required is then obtained by dividing the main rotor power by a mechanical efficiency term of about .85, which accounts for the transmission losses, accessory drives, and tail rotor power.

For vertical climb, assumptions must be made about the various losses discussed above, especially the losses caused by non-uniform inflow. This is extensively discussed by Moffitt and Sheehy in reference 16. The first possible assumption is that the losses caused by non-uniform inflow are a constant fraction of the momentum power, i.e., that they increase with climb rate. The second possibility is to assume that the losses remain constant, while the third is to assure that they decrease as the climb rate increases because of the more uniform inflow and more rapid downward displacement of the tip vortices. Reference 16 shows that the third method is most accurate, but that it is still slightly pessimistic. The fuselage download increases with climb rate because of the increased inflow, and must be accounted for. This is usually done by the classical strip analysis with the new inflow velocity at the rate of climb in question. Alternately, the download often is assumed to be increased by the factor $1 + \frac{V_c}{W_1}^2$, which implies that the shape of the inflow distribution does not change as the climb rate increases. Actually, one would expect it to become more uniform, since a uniform climb velocity is being superimposed on the non-uniform inflow. Moffitt and Sheehy discuss this in reference 16, and show that the strip method is somewhat more accurate, but that using the much more convenient $1 + \frac{V_c}{W_1}$ factor is slightly conservative and causes only a small error.

Computer analysis is also possible with either momentum-based or wake-modeling type programs. Both can give satisfactory results. The author has obtained acceptable correlations with several momentum type programs, while Moffitt and Sheehy show good correlation with their Vortex Wake analysis in reference 16.

The analysis presented in this paper is a statistical method based on the correlation of a large amount of flight test data. Some of the aircraft considered were the Bell Model AH-1J, Hughes OH-58A, Kaman UH-2C and HU2K-1, and Sikorsky HH-3A and CH-53E (references 17-24). These data were non-dimensionalized by defining the climb constant $K_c$ and the non-dimensional vertical velocity $V_c$, where $K_c = \frac{GW}{(V_c)}$ and $V_c = V_c$. $K_c$ is the ratio of the climb power that would be needed if the climb had no effect on the rotor performance to the actual climb power required. $V_c$ is the ratio of climb velocity to the ideal hover downwash velocity obtained with no tip losses and uniform inflow. In the idealized momentum analysis in reference 1, $K_c$ is 2.0 at zero rate of climb and decreases to 1.0 as the climb velocity becomes infinite.
All of the data were plotted in the non-dimensional format in Figure 2. They show a clear trend, although with a very large amount of scatter at the low climb rates. This scatter is to be expected, as measuring vertical climb performance accurately is very difficult, especially at low rates of climb. The pilot must hold a zero airspeed and steady climb rate, both of which are difficult to do, and even low wind velocities seriously affect the data. Another major difficulty is that the climb power is the difference between two much larger total power readings, and the data scatter can easily be as large as the increment being sought. As an example, a 100 fpm rate of climb requires only about a 1.3% increase in total power. For comparison, the curve resulting from the idealized momentum analysis in reference 1 is also shown.

The data were examined for any differences between aircraft, but none were apparent, with all of them showing a similar trend. It should be noted that non-dimensionalizing the climb rate over the induced velocity accounts for disc loading effects, while basing the $K_c$ on main rotor power removes any effects of different power extractions or transmission losses. The tail rotor power is also removed, but when the data are dimensionalized, this will normally have to be added in using a plot of main rotor vs. tail rotor power obtained at hover. If this is done, any effects of the climb rate on the tail rotor performance are neglected. This is probably not significant on most helicopters, except possibly at very high climb rates. The tail rotor effect should be greater for a helicopter with a canted tail rotor, since its tail rotor inflow would be directly increased by a climb rate. However, even a 10% savings in tail rotor power represents only about a 1% savings in total power, and test data for the CH-53E, which has a canted tail rotor, correlate well with the other data. The effect of the changes in fuselage download due to climb is included in the test data, and therefore accounted for.
Because of the lack of any trends between different aircraft, a single curve was fitted through all of the data. This curve lies slightly below the ideal momentum curve discussed above. This indicates that the increased fuselage download is being offset by the savings in induced losses resulting from the more uniform inflow caused by the climb rate. No data were found for tandem rotor aircraft, but their rotors should experience the same flow changes in a vertical climb as a single rotor aircraft, so there is no apparent reason why Figure 2 should not also be valid for them. Hence, the curve fitted through the data is considered valid for all single and tandem rotor helicopters. The curve should remain valid for helicopters carrying an external load, since this has no effect on the rotor aerodynamics, and the download on the load should increase with climb rate in the same manner as the fuselage download.

Vertical climb performance is easily calculated from Figure 2. For a given case, select a climb velocity, weight, and atmospheric density. This determines $V_{c}/w_{i};$ Figure 2 is then used to obtain $K_{c}.$ From $K_{c}$, the main power increment is obtained, and this is then converted to total power by adding in the power extraction, mechanical losses and tail rotor power. If this data is not available, it is suggested that the main rotor power increment be divided by .85 for single rotor helicopters and .95 for tandem helicopters.

The level of accuracy of this method of climb analysis is difficult to assess in view of the large amount of data scatter, but should be within ten percent of the climb power except at low climb rates, where the scatter becomes very severe. It should be noted that at low climb rates, the climb power itself is small, so that the error in total power remains low. Nevertheless, the large scatter leads to controversy about the vertical climb power, and the author recommends that in future procurements, if a given climb rate is desired, the excess power required for this could be calculated from Figure 2 and then specified as a required power margin at hover, i.e., the aircraft might be required to hover OGE at 95% power in lieu of a 400 fpm vertical rate of climb requirement.

4. Forward Climb Performance

Helicopter climb performance is well documented for climbs at or near the best climb speed. For speeds at or above the best climb speed, the fixed-wing analogy described in the discussion of low speed flight is valid. A climb has little effect on the mass flow through the rotor so the additional power required to climb is close to what is required to simply raise the helicopter's weight at its climb velocity. Hence, the climb performance for speeds at or above minimum power is normally calculated by multiplying the excess power available above that required in level flight by an experimentally determined constant and by the mechanical efficiency term which accounts for power extraction for accessories, transmission losses, and tail rotor power.

Forward flight climb performance is easier to measure than vertical climb performance for several reasons. The airspeed can be measured with a conventional calibrated airspeed indicator, and the effect of small amounts of wind is less. The pilot still needs to hold a steady climb, but this is usually easier, since the helicopter is more stable than at zero airspeed. The problem at low speed rates of subtracting two large power readings to obtain a small increment still exists, but it is not as
severe as in vertical climbs, since the power increment is about twice as large for a given climb rate.

A large amount of flight test data were located for climbs at the optimum climb speed. Data were obtained from references 6, 12, and 24-36 for the Bell UH-1B, Model 206, Model 209, AH-1G, H-13 and AH-1J, Boeing CH-46D, CH-47A, and CH-47C, Fairchild-Hiller FH-1100, Hughes OH-58A, and the Sikorsky SH-3A, CH-3C, CH-3E, and RH-53D. No data were found for a helicopter with an external load. All the data were non-dimensionalized using $K_c$ and $V_c$ as was done with the vertical climb data. The data for all the single rotor aircraft are shown in Figure 3. It should be noted that each point on this figure represents a fit through a number of data points. As was expected, there was less scatter than in the vertical climb data, and the data do not show a trend with climb rate parameter. The data for the Boeing tandem rotor aircraft are shown separately in Figure 4 and are similar.

The data were statistically analyzed and showed a mean of .867 and a standard deviation of .07 for the single rotor aircraft. For the tandem aircraft, the mean was .884 with a standard deviation of .089. The small differences between the single rotor and tandem aircraft are not considered significant, and a climb constant of .875 is recommended for single or twin rotor helicopters. This is nearly identical to the value of .86 obtained by Boirun in reference 15. This value should remain valid for a helicopter with an external load, unless the load changes attitude during the climb in such a way as to significantly affect lift or drag.
Calculation of forward flight climb performance is simple. The level flight power required is subtracted from the power available at the speed in question, to give the excess power available for climbing. The climb rate is then given by the equation $V_c = (0.875) \frac{(550) \cdot \text{ESHP} \cdot \eta_m}{GW}$.

The reader should note that $V_c$ is in ft/sec with the constants shown. The mechanical efficiency, $\eta_m$, is normally available from the level flight test data, but if not, typical values are .90 for single rotor helicopters and .95 for tandem rotor aircraft.

5. Low Speed Forward Climb

Low speed forward climb is probably the most difficult and least well defined area of helicopter performance. As was discussed in the section on low speed power, there is no simple momentum analysis that is rigorously valid, and uniform inflow blade-flapping type computer programs either do not run or involve assumptions of questionable accuracy. This area has been almost totally ignored in test programs, and only two sets of data are known to exist. These data were used by Boirun in reference 15 to develop climb constants. He shows that $K_c$ ($K_p$ in his notation) decreases rapidly with airspeed. Unfortunately, there are data for only one airspeed between hover and best climb speed. Because the author could not locate any other data other than that used by Boirun, no analysis was attempted, and the reader is referred to Boirun's paper. As a conservative alternative, the climb constant can be assumed equal to the .875 value obtained for climb at the speed for minimum power. The U.S. Navy LAMPS MK III aircraft has a specification requirement for climb performance at 30 kt, so its flight testing will include carefully measured low speed climbs. Hopefully this will yield further information on low speed climb efficiency.
6. Conclusions

This paper has presented a large body of experimental data for helicopters operating in the speed range between hover and the minimum power speed. A large amount of climb performance data for both vertical climbs and climbs at the best climb speed has also been presented. The low speed performance data has been non-dimensionalized and used to develop a method for rapidly predicting the low speed power requirements of any helicopter provided that its performance at hover and at its minimum power speed has been defined. The climb performance data were also non-dimensionalized and used to derive methods for predicting the power required for either a vertical climb or a climb at the best climb speed. A satisfactory method could not be developed for predicting the power required to climb at speeds between hover and the best climb speed because insufficient data were available.

The methodology presented is considered valid for any conventional single or tandem rotor helicopter. The climb analysis is considered valid for helicopters carrying external loads, but the low speed power analysis may not be valid if the drag of the load is large.
REFERENCES


