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DESIGN OF AIRFOILS FOR A SPECIFIED MOMENT COEFFICIENT

S. De Ponte L. Manfriani

Politecnico di Milano
Dipartimento di Ingegneria Aerospaziale

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Sergio De Ponte - Leonardo Manfriani
Politecnico di Milano
Dipartimento di Ingegneria Aerospaziale
Via Golgi 40 - I-20133
MILANO (Italy)

ABSTRACT

A simplified representation of boundary layer properties allows to express them in simple closed form relationships in order to express lift and moment coefficients in form of boundary layer parameters. In this way it is possible to show the penalties in maximum lift do to a moment constraint and to an increase in airfoil thickness. Compressibility is shortly discussed and finally approximated data are compared to exact and **to** experimental results.

1) INTRODUCTION

The need of a limit for pitching moment coefficients in aeronautical application, mainly in rotary wing applications, suggested in most cases the use of old airfoils, with no camber or nose droop. Other attempt of airfoil design for such problems, based on thin airfoil theory, were not fully successful, either for drag considerations or for unsufficient maximum lift.

On the other hand, compressibility effects on rotary wings may be significant at lower Mach numbers and larger lift coefficients than on fixed-wing aircrafts. In this sense, not all airfoils designed for aircrafts have a desireble behaviour at Mach numbers of helicopter interest.

Starting from those condiderations, it is important to consider as starting point boundary layer properties, in order to consider maximum lift, range of lift coefficients in which the drag is limited, and good shock-wawe boundary layer interaction.

The problems which should be considered for this aim are:

a) the penalty introduced in maximum lift by a moment coefficient limit,
b) the penalties on maximum lift and moment coefficient due to a reasonable airfoil thickness,
c) the penalties introduced by an increase in width of the low-drag band.

The leading idea is to try a simplification of those problems, in order to allow an analytical approach. This is very significant in the first stages of design, giving a first step in optimization process limited in cost and time.

2) BOUNDARY LAYER ASSUMPTIONS

In order to evaluate the validity of some simplification, it is useful to recall some limit of airfoil design at its first stage. Due to the fact that the pressures are known only along the chord and not along the airfoil contour, the pressure gradients are known only with large approximation.At the first stage of design, any assumption of this order of approximation is acceptable. As second consideration, wall curvature effects are not known.

Th boundary layer assumptions are therefore the following ones:

a)Only equilibrium boundary layers are considered, both in the laminar and the turbulent case.In general this is valid in the decelerating part of the boundary layer, except for a short length after the transition ramp, but not on the laminar part, which usually starts from stagnation point flow and comes to flat-plate flow. Although the difference between equilibrium flow and airfoil flow are not completely negligible,the external velocity distribution is reasonably unaffected.

b) Transition is neglected.The decelerating transition ramp has a length which is decreasing with increasing Reynolds number and affects lift and moment coefficients only at low Reynolds numbers.

c) Laminar boundary layers are calculated by $Walt_z$'s approximation.

d) Turbulent boundary layers are considered as equilibrium layers when the shape parameter H is constant, any thickness is increasing linearly with arc length. and external velocity is related to arc length by a power law . This means that skin friction coefficient should be constant, which is relatively true only for decelerating boundary layers. As in airfoil design it is desirable to have laminar flow in accelerating streams, this latter condition is rather well verified.For the accuracy of the last assumptions the experiments of East and Sawyer may give a good error estimation.(1)

In this family of turbulent boundary layers also the well-known Stratford pressure distribution may be represented as approximation by a power law and the exponent is close to -1.

3) POTENTIAL-FLOW CONSIDERATIONS

Equilibrium boundary layer correspond to wedge flows and therefore do not describe airfoils by themselves. In order to get a closed, not crossing airfoil contour, it is necessary to satisfy a set of conditions, which are well-defined theoretically for the closure, not well defined for the non crossing.

The potential flow out of the boundary layer does not obey to classical airfoil conditions for two reasons. The first is that the contour generating the outer flow is open at trailing edge by a distance equal to the sum of displacement thicknesses on upper and lower surface,the second is that a trailing stagnation point cannot exist, due to the boundary layer, thus leading to cusped shapes only. It means that the trailing-edge velocity is not zero and has a finite value. Experience shows that this value may be approximatet by a certain fraction of free-stream velocity. Liebeck, for example, takes a pressure coefficient of 0.2 at trailing edge(2).

The former considerations will mean that the original design pressure distribution should be modified and has some constraint. This may be very important from the pratical aspect, because it affects the choice of the inverse potential method. The way in which the original pressure distribution is corrected in order to get an airfoil is very important when it affects the boundary layer properties in critical points. A short rew of the methods shows that:

- iterative singularity thecniques may take into account curvature effects and arc length in a general iteration procedure,
- best fit conformal mapping tend to smooth out peaks even where they are required by design,
- simple conformal mapping are very cost effective if properly used. In the present work the choice was the rather old Eppler method which seems to be the most pratical for interactive computer design and very cost-effective (3 and 4)

The choice of pressure distribution is related to the aim, which is first-stage design in incompressible conditions.For this, as first , the pressure distribution of fig. 1 was studied. It regards only the airfoil upper-surface and is built-up by a constant pressure , from the leading edge to a certain point x_1 along the chord, then an approximated Stratford distribution up to the trailing edge. This is not exactly the Liebeck airfoil (2) due to the larger simplification of the problem. Because the resulting airfoil would have a cusped leading edge and result in any case too thin, a second pressure distribution, with an accelerating boundary layer, corresponding to a family of thicker airfoils, was studied to see the penalties of airfoil thickness. Lastly, it is described the effect of the unloading of the rear part of the airfoil, which can help in reducing the moments and solving some tecnological problem, related to a too thin airfoil tail.

4) GENERAL VELOCITY RELATIONSHIPS

The considered velocity distributions, made by two or more parts, require some previous matching conditions. The first is on velocity derivatives as will now be discussed.

In any boundary layer in similarity conditions the non-dimensional velocity derivative must be a constant, being constant the other dimensionless quantities. Denoting V' the velocity gradient along the arc, and v' its dimensionless value, we have the following relationship:

$$v' = V'\Theta/V \qquad (4,1)$$

where V is the external velocity and Θ the momentum thickness. Application of this relationship to the decelerating turbulent boundary layer requires the matching of the momentum thickness at the initial point, where both V and Θ are those of the preceding boundary layer. In this case the constant of the velocity law

$$V = A\,x^{m} \qquad (4,2)$$

is defined , because the dimensionless velocity gradient v' is function of the exponent m.

The next step is the determination of the virtual origin of the turbulent boundary layer, x_i, i.e. the point where momentum thickness vanishes and external velocity tends to infinity.If the first part of the boundary layer is a Blasius solution, matching displacement thickness at x_1, we have:

$$\Theta_1 = 0.664\ x_1\sqrt{R_{e\circ}/(v_m\ x_1)}\quad C \qquad (4,3)$$

where C is the chord length.

Introducing this value and the velocity from 4,2 into 4,1 we get:

$$x_i/x_1 = 0.664/v'\sqrt{R_{e\circ}/(v_m\ x_1)}$$

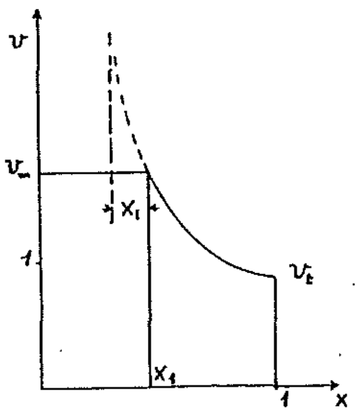


Fig. 1
Definition of aerodynamic and geometric parameters

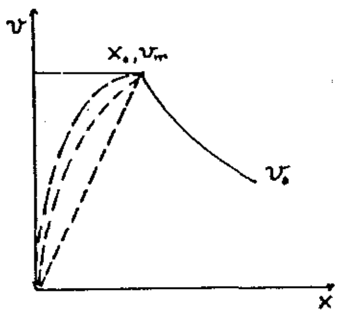


Fig. 2
Acceleration ramp

The momentum thickness at matching point x_1 is function of the length x_1 and of the Reynolds number, and therefore also of the velociti ratio

$$v_m = V_m/V_\circ$$

between the maximum velocity V_m at point x_1 and the free-stream velocity $V_\circ$.

The trailing edge velocity V_t, as seen before, is proportional to the free-stream velocity, and we may call this ratio

$$v_t = V_t / V_o$$

If we assume constant velocity between the leading edge and the matching point, as first approach to the problem, once we assign three of the four quantities:

$$R_{e^o},\ x_1,\ v_m,\ v_t$$

the fourth is known for any exponent m, in particular for Stratford pressure distribution, which enables us to shift as forward as possible the point x_1 , giving the minimum pitching moment. In this sense, in the following considerations, both the exponent m and the velocity gradient v' are not considered as variables.

Taking Blasius boundary layer solution up to x_1 and developing simple algebrical relationships we arrive to the formula:

$$x_1 = 1 - (v_m / v_t \ -1)\ 0.664 / v' \quad 1/\sqrt{R_{e^o}\ v_m} \qquad (4,4)$$

where the exponent m is taken -1 and the velocity gradient v' may be approximated as $2.7\ 10^{-3}$ although a more correct representation of this number should take into account that this is dependant upon the Reynolds number.

5) LIFT AND MOMENT RELATIONSHIPS

Once the point x_1 is known by equation 4,4 , it is possible to integrate the pressure on the upper surface of the airfoil, in order to obtain lift and pithching moment contribution.

The pressure coefficient:

$$c_p = 1 - v^2$$

Where v is the dimensionless velocity gives by integration from leading edge to x_1 the first contribution to the lift:

$$c_{11} = x_1 (1 - v_m^2)$$

and integrating from x_1 to 1 :

$$c_{12} = (1 - x_1) + v_m^2\ x_i^2 / (1 - x_1)$$

and adding the two contributions we obtain the upper-surface contribution to the lift:

$$c_{1u} = v_m^2\ ((x_i^2 / (1 - x_1)) - x_1) + 1 \quad (5,1)$$

in a similar way we may obtain moment coefficients referred to the leading edge. The first part of the airfoil gives:

$$c_{m1} = x_1^2(1-v_m^2)/2$$

and the second part gives:

$$c_{m2} = (1-x_1)^2/2 - v_m^2\,(x_1-x_i)^2\left(\ln\frac{1-x_1+x_i}{x_i} + (x_1-x_i)\cdot\left(\frac{1}{x_i} - \frac{1}{1-x_1+x_i}\right)\right)$$

Dividing 5,2 by 5,1 one can obtain the expression for the center of pressure x_p, and translating the moment pole to the aerodynamic center it is possible to obtain the relevant moment coefficient.

The total contribution of the upper surface to the moment coefficient is therefore:

$$c_{mu} = \frac{1}{2} - v_m^2\left(\frac{x_1^2}{2} + (x_1-x_i)^2\left(\ln\frac{1-x_1+x_i}{x_i} + (x_1-x_i)\cdot\left(\frac{1}{x_i} - \frac{1}{1-x_1+x_i}\right)\right)\right) \qquad (5,2)$$

In particular, the zero moment coefficient airfoil is obtained when:

$$c_{lu}/4 = c_{mu}$$

this giving for any Reynolds number a value for x_1 to which the airfoil corresponds.

6) THICKNESS PROBLEMS

Among the ways to obtain thick airfoils, two are commonly used because of their simplicity. The first is to assign constant velocity on the first part of the chord, at an incidence higher than the design one, and is suitable for conformal mapping design, like in the Eppler method(3 and 4), the other is to assign a specified acceleration in the same part of the chord. Both give similar results, while the second seems to be more suitable in singularity methods. The resultant velocity distribution are enough close to equilibrium boundary layer solutions as expressed by equation 4,2, with the exponent m comprised between 0 and 1, but generally rather small.

Of course, this change in pressure distribution will change the lift, which decreases,the moment coefficient,adding a nose-up component, but also the displacement thickness at point x_1.

Assuming as external velocity:

$$V = B\,x^{m}$$

and Waltz's approximation, we have:

$$\theta_1^2 = \nu \frac{0.47}{B\,x_1^{6m}} \int_0^{X_1} X^{5m}\,dX \qquad (6,1)$$

which is integrated in the form:

$$\theta_1^2 = \frac{0.47\,X_1^{1-m}}{(5m+1)\,B}\,\nu$$

At this point we can use different approaches to the problem. The first is to see the conditions to have the same momentum thickness at point x_1, the second is to keep the velocity ratio v_m constant and change x_1, the third is to rearrange all the quantities. But the simplest approach is to assume a ficticious Reynolds number R*, at which both the velocity ratio and the momentum thickness are constant at the same value of x_1. This does not affect the turbulent boundary layer, which does not depend upon Reynolds number.

The flat-plate flow of former analysis corresponds to m=o, thus it is possible to calculate the velocity ratio between the velocity of our boundary layer, V_m and the corresponding flat-plate velocity V_o which gives the same momentum thickness at the same point X_1 resulting:

$$V_m/V_o = \frac{1}{5m+1} \qquad (6,2)$$

this latter giving the Reynolds number ratio.

Equation 6,2 gives the penalty of increasing airfoil thickness, in the sense of the change in boundary layer parameters related to the change. The thickness behaves like a reduction in Reynolds number, i.e. a reduction in maximum lift and an increase in parasitic drag. The term is not very high, as m remains small compared to 1/5 and usually tends to this value for airfoils approaching 20% thickness ratio.

Beyond the change in ficticious Reynolds number, the decrease in lift and change in moment may be expressed as:

$$c_{1t} = v_m^2\,x_1\left(1 - \frac{x_1^m}{m+1}\right)$$

and

$$c_{mt} = \frac{1}{2} v_m^2\,x_1^2\left(\frac{1}{m+1} - 1\right)$$

and give an approximation to the penalty of increasing airfoil thickness.

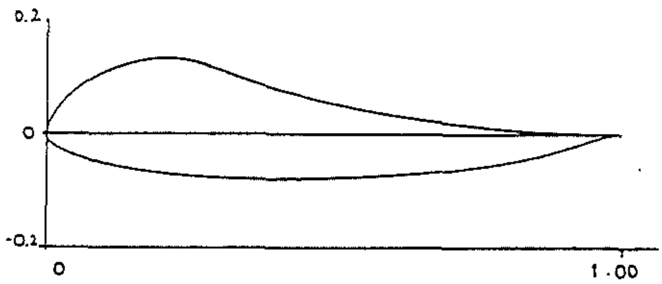
7) REAR UNLOADING

If the constraint of moment coefficient cannot be satisfied by simple shift in point of start of deceleration, it is possible to cross the velocity distribution at a certain point, in order to obtain negative lift in the rear of the airfoil. This has the obvious disadvantage to produce drag without lift benefit, thus reducing airfoil efficiency. It has an advantage, i.e. the increase in airfoil thickness near the trailing edge, which may be required for some tecnological problem.

In fixed-wing aircrafts, on the other hand, this thickness may be useful at the hinge station, in order to obtain efficient control surfaces, because a small radius of curvature of the moving part will anticipate flow separation.

An extreme case of this problem is shown in fig. 3 , where it is represented an airfoli designed for a tailless glider, where control surface problems are very important.

In this case the drag penalty is corresponding to the increase in displacement thickness in the part of unloading, with respect to the loaded part of the airfoil considered as single airfoil



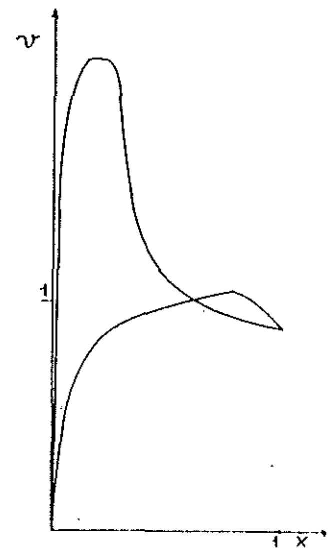


Fig. 3
Velocity distribution and airfoil shape for a rear unloaded autastable airfoil

8)COMPRESSIBILITY EFFECTS

As basis, the compressible airfoil design according to Sobieczky(5) is the most suitable to boundary layer considerations, as it starts from subsonic airfoil design.

Compressible boundary layer solutions can be referred to incompressible ones by slight changes when the Mach number is neither too high as in transonic flow,nor the thermal effects are too large.

The corresponding Blasius solution is the well-known Chapman-Rubesin flat plate solution(6) which is transformed from incompressible flow. The more general Howarth-Stewartson transformation allows to know similar solutions in compressible laminar boundary layer(7). In turbulent flows, approximations suitable for our aim may be obtained in many ways;one of the best is derived by Huo for the same problem on blade cascades (8) . The shape factor depends on Mach number in the form:

$$H = H_{Incompressible}(1 + 0.1145\ M^2) + 0.2728\ M^2$$

which is said to be valid up to M= 1.5 . This Mach number will not be exceded in transonic unseparated flows.The shape variations on a limiting deceleration as H = 3.5÷ 4 are not too large if we consider first approximation design.

The design method of Sobieczky does not change the subsonic part of the airfoil contour and pressure distribution, and when supersonic compression is limited, the boundary layer will not be affected in a large amount. This means that an attached boundary layer will be kept attached in the modification of the airfoil which produces shockless flow.

We may therefore conclude that the former considerations on pressure distributions may be extended to compressible flow, taking into account compressibility both on boundary layer and on potential flow, but the influence on the boundary layer is not very large.

9) COMPUTATIONAL PROBLEMS AND EXAMPLES

A first step design method requires a lot of judgement which could be hard to introduce in a computer in an economic form. An interactive program is therefore a good way to overcome some problem.

First step boundary layer may be calculated by very simple statements and has no computing problems.

Airfoil contour requires the largest part of computer time and requires a certain cost-effectiveness analysis, although personal taste would probably influence it very much. Simple thin airfoil approximations seems to be suitable, but they have the large disadvantage that the obtained pressure distributions are not acceptable for further airfoil analysis. In this sense conformal mappings may show some advantage, i.e. the following:

a) can give corrected and reliable pressure distribution after contour design, to compare with design distributions and to calculate boundary layers, drag and corrected lift.
b) closure modifications may be introduced in more versatile ways,
c) flow near stagnation does not introduce mathematical troubles as in some linearized approximation,
d) parts of the pressure distribution may be introduced at different angles of attack.

The last point is very important, as it allows to design an airfoil for a certain range of lift coefficients, in which flow is attached and therefore the drag is small. This is very important in helicopters, where lift is periodic.

To fulfill those requirements, the old Eppler method seems to be up to now one of the most suitable, as it allows to obtain airfoil closure by changing the velocities by an exponent, and to assign to different incidences parts of the pressure distribution.

An example of application of this method to an autostable airfoil is given in the last figures,where final velocity distribution is represented compared to first approximation one, as function fo airfoil contour. It can be sees that the method works rather well. Computed and measured pressure distributions show that a simple conformal mapping with boundary layer corrections gives results as accurate as careful pressure measurements and very simple airfoil analysis is acceptable as experimental results.

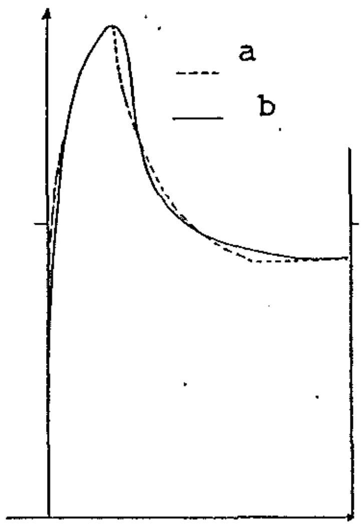


Fig. 4
First approximation (a) and final airfoil velocities

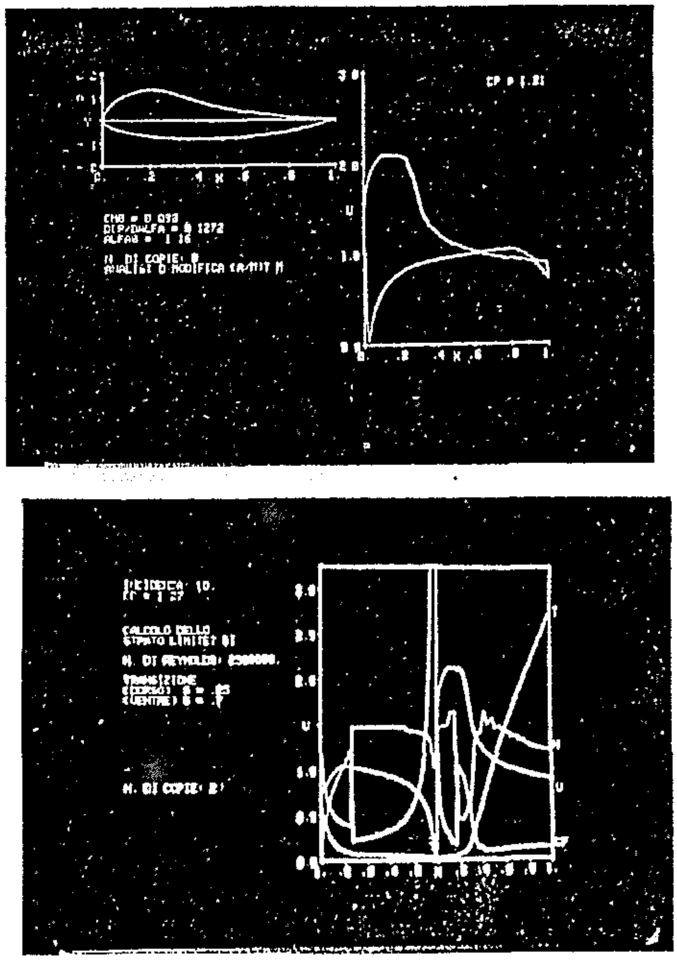


Fig. 5
Monitor photographs during design evaluation by interactive program: airfoil shape and velocities (High) and boundary layer parameters (Low)

10) CONCLUSIONS

As seen before, at least in an example completed up to accurate two-dimensional experiments, the simple design procedure of present work is suitable for airfoil design.

The analytical form of expressions for the lift and moment coefficient give a rapid estimation of penalties related to a specific design, in order to allow integrated structural and aerodynamic design, specially when composite materials allow more sophisticated blade concepts, firts of them non constant blade section.

Compressibility may be taken into account in a rather simple form, this also taking into account that tha blade section may be adapted to local Mach number range.

The thickness problem may be approached by changing aerodynamic and not geometrical parameters, in order to obtain an optimized airfoil for each thickness and not a family of affine airfoils. In this way , thickness penalties may be minimized al least in incompressible flow.

Interactive computer work may be performed very well even in very small computers, as in present work, where a PdP 11 was choosen.

The coupling of inverse (design) and direct (analysis) programs may give excellent final accuracy and good cost-effectiveness if properly used in an interactive way.

The final remark is that a similar procedure should be adapted to dynamic design when better understanding of some unsteady phenomena would be available.

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Symbols

A	Constant	m	exponent
B	Constant	R_e	Reynolds Number
C	Chord length	V	velocity
C_l	lift coefficient	v	dimensionless vel.
C_m	moment coefficient	X	chordwise length
c_l	increment in lift coefficient	x	X/C
c_m	increment in moment coefficient	x_p	pressure center
c_p	pressure coefficient	ν	kinematic viscosity
H	shape factor	ϑ	momentum thickness
M	Mach number		

subscripts

°	free-stream	m	maximum
o	flat-plate	t	trailing edge-thickness
1	first part of airfoil	u	upper-surface
2	second part of airfoil		
i	initial		