FREQUENCY DOMAIN TESTING OF HELICOPTER DYNAMICS USING AUTOMATED INPUT SIGNALS

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Abstract

This paper provides a description of a collaborative research project between Westland Helicopters and York University. The main object of the investigation is to provide a characterisation of all significant dynamic phenomena arising from flight test on a helicopter flown by Westland. Logarithmic swept sinewave test signals have been applied to the series actuators of an agile prototype helicopter during flight testing. Gain and phase data derived from auto- and cross-spectral estimates are examined to validate mathematical models used in flight simulation and to provide information about significant non-linearities not accounted for in the simulation model. York University has investigated complex curve-fitting techniques for obtaining estimates of linear system parameters. Magnitude-squared coherency function plots and linear system parameter estimates together enable a powerful comparison of the use of different forms of test signal (such as swept sinewave, m-sequence and Schröeder-phase). It is concluded that: the Schröeder phased-signal shows great promise for future flight application; the logarithmic swept sinewave is adequate for flight application and frequency domain testing is, in general, a powerful method for identifying the physical phenomena of the system under test.

1 Introduction

In recent years, control system development and evaluation work at Westland Helicopters has been concerned with the determination of helicopter frequency responses in order to validate the Flight Dynamics Simulator. Currently, a frequency response testing approach is being used by Westland as part of their flight development programmes. This has been linked with research at York University in frequency domain identification.

Frequency responses derived from flight traces allow identification of the various aircraft transfer functions of interest without recourse to a postulated mathematical model. This focuses attention on, and facilitates an understanding of, the dominant modes of the aircraft behaviour. The occurrence of non-linearities (hysteresis, backlash, saturation etc.) can be isolated for further investigation. These flight test results are compared with simulations from the helicopter and flight control system mathematical model. The comparison establishes confidence in the simulation model and identifies detailed differences for future development.

Studies of this kind, form a very useful preliminary stage to parameter identification, but only when meaningful model structure validation has been confirmed.

An automated test signal technique has been developed to provide experimental repeatability and an adequate excitation of helicopters in flight. Measures have been taken to ensure that the automated signal generation is safe, well controlled and repeatable. The safety issues dictate that the pilot should be free to observe, monitor and remove the test signal input.

A number of other investigators have reported useful results after conducting frequency response testing and identification on helicopters and other aircraft [1-4]. Some studies have been concerned with identification of the open-loop dynamics of a *stability augmented aircraft system*. Helicopters are unstable open-loop, hence frequency domain testing and identification experiments can only be worthwhile if performed under carefully controlled conditions, making use of an automated signal generation approach.

When performed correctly, frequency response testing can give clear engineering insight into system dynamic structure and the results can be readily interpreted in graphical form. Dominant modes and features such as resonances can often be observed immediately and related to known physical features. In designing a frequency domain experiment, use can be made of certain system-dependent information such as bandwidth and frequency resolution - if these are available. This feature, together with the need to tailor a test signal spectrum to the required application means that some *a priori* information about the system under test is required.

One of the difficulties often encountered with the frequency domain approach, is that of inadequate persistency of test input signal excitation. It is important to excite the aircraft actuation system over a over a wide enough bandwidth to obtain reliable estimates at both low and high frequencies.

Another important requirement of a test signal is that it should have a power spectral density whose shape can be tailored to provide the best excitation for the application under consideration. With some types of test signal this feature is not possible.

It is also necessary that the test signal should have a maximum excitation amplitude which is physically reasonable, given the normal operating characteristics of the actuation system to which it is to be applied. An essential feature is the need to control the amplitude of the signal as frequency domain testing is fatigue-damaging especially for hingeless rotor systems.

By careful design of the harmonic content of a multi-harmonic signal the signal power normally available from other higher amplitude test signals can be achieved. This can be optimized by minimizing a measure known as the signal *relative peak factor*.

The paper is organized as follows: First a brief description of the frequency domain identification method is given, followed by a description of the spectral analysis technique used.

The paper then gives a description of the approach used in evaluating the suitability of a number of binary and non-binary test signals. The magnitude-squared coherence function will be used as a basis for the comparison of these signals. This will be followed by a discussion of their relative advantages and disadvantages on the basis of suitability for the helicopter identification application.

2 Frequency Domain Methods

2.1 Description of Test Signals

In any frequency domain identification scheme the choice of signal used to excite the system so that measurements can be made, is crucial. No sensible attempt at identifying dynamic system parameters can be made unless the test signal excites all the significant modes of the system under test. A number of different types of test signals can be effective in providing suitable excitation, these range from swept sinewave types, binary m-sequence signals and multi-level extensions of the 2-level case, so-called 'optimal' multi-frequency binary signals to complex non-binary signals based on special choices of phasing between the harmonic components. This work has considered the comparison and use of the m-sequence, and non-binary cases.

The 3-2-1-1 (7 bit binary m-sequence) and linear swept sinewave are particularly simple and hence convenient forms of test input signals. Both of these signals have been used extensively for multi-frequency testing in aircraft systems [5, 2]. They do, however, suffer from the problem of providing an inadequate excitation to the system under test, over the frequency band of interest. Some improvement over the 7-bit m-sequence can be achieved by increasing the sequence length to, say 31 bits, however this has the disadvantage of lengthening the test signal required to achieve a given spectrum.

It is important that test signals should have a flat spectrum across a sufficiently wide bandwidth, with a sharp cut-off at very low and at high frequencies, however, this is not easy to arrange. The problem arises mainly as a consequence of destructive interference between the harmonics, a phenomenon which gives rise to a poor spectrum for identification purposes. To attempt to overcome this problem, two more complex test signals have been introduced into the work and compared. These are the *logarithmic swept sine wave* and a relatively little known test signal - the *Schröeder-phased harmonic signal* [6, 7]. The former has been applied in simulation and to helicopter flight testing, whereas the Schröeder-phased signal has only been investigated in simulation, in the current study [8, 9].

The swept sinewave signal is quite simple in concept. It is a sinusoid whose frequency increases as a function of time. The linear sweep has a rate profile with a fixed increment for a given time step, whereas the logarithmic swept sinewave has a sweep rate for which the frequency is multiplied by a fixed scale factor and increases by this factor at each time increment. These two sweep rates result in the test signal having different power spectral density functions. The linear sweep is deficient in spectral content at lower frequencies but has constant power spectral density at high frequencies. This occurs because a number of decades in the frequency response is considered and what may be a relatively small frequency increment at the high end of the frequency range is a very large one at low frequencies. Also, the number of cycles or the proportion of a single cycle applied at a particular low frequency is much less than that for a high frequency. The result is that relatively low frequency excitations are only applied for a very short time.

In contrast the logarithmic sweep concentrates the power in the low frequencies at the expense of the higher ones because the frequency increment itself increases with frequency. Such a signal will excite the low frequency or dominant dynamics of the system more, but does not have a flat spectrum across any portion of the frequency band of interest.

Appendix 1 gives the definition of the logarithmic swept sinewave signal together with a typical example used for helicopter multi-frequency testing.

The Schröeder-phased signal is a multi-frequency wave, which is especially suitable for providing a flat spectrum. It is, however, a complex multi-harmonic signal with the phasing of the harmonics especially selected to provide a *low peak factor.* Each harmonic is specified by a phase shift chosen so that, when all such harmonics are added together the resulting wave fits a given power spectral density. This type of signal was first described by Schröeder [6] who wanted to minimise the peak-to-peak amplitude, or peak-factor, of a multi-frequency signal for possible application in radar, sonar and other areas of signal processing. For identification of parameters of a dynamic system a low peak-factor test signal will avoid large perturbations being introduced into the system under test, whilst giving adequate excitation over the whole time for which the test is applied. These are clearly desirable characteristic of a test signal.

The design of the Schröeder-phased signal is quite straight forward as the specification of the phase angles of the harmonics depends on the relative magnitudes of the different frequency components. The objective is to select the phase angles so that the peak-factor or maximum envelope excursion is small. The standard derivation (see for example Flower et al [7]) does not solve the *minimum* peak-factor problem, it simply yields *low* peak-factors. However, the standard approach can be generalised with an optimal peak-factor to provide a competition for the so-called optimal binary test signals. Details of how to select the phases to design a Schröeder-phased wave are given in Appendix 2. For a spectrum with N_s consecutive harmonics of equal power $p_i = 1/N_s$, $i = 1$ to N_s, the phases should be

$$\Phi_n = (\pi/N_s)(n^2 + n) \qquad (1)$$

The typical form of a Schröeder-phased signal, with period T, as used in this study is shown in Fig. 1.

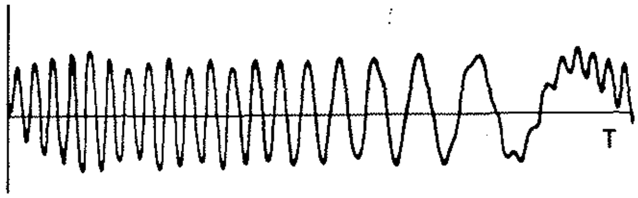

Figure 1 The Schröeder-phased signal

Fig. 2 shows a comparison of the power spectral densities of three signals used in a simulation study comprising a helicopter mathematical model. The amplitude of the sine wave and the mean amplitude of the Schröeder-phased wave are equal. The low peak-factor of the latter means that the peak amplitude is of similar magnitude, and the two types of signal will disturb the system under test to the same extent.

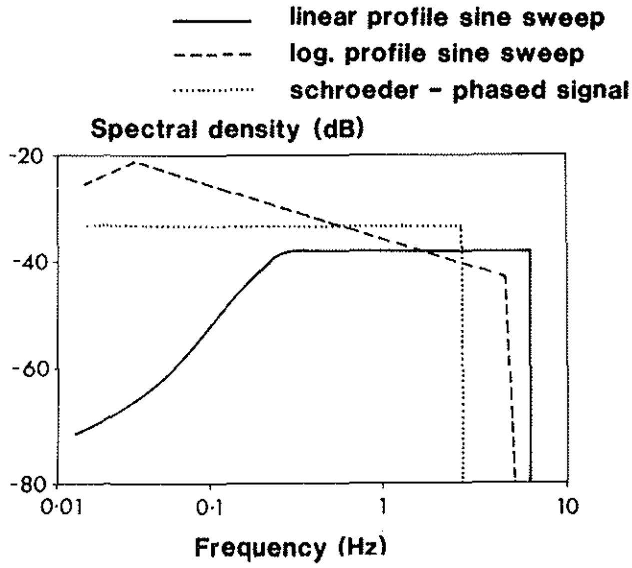

Figure 2 Power spectra of the test signals

The Schröeder-phased signal used has a period of T = 65.54 s and is repeated 16 times. It comprises 160 harmonics in the frequency range 0.015 to 2.5 Hz. Each frequency sweep range is one cycle long with a range of 0.015 to 4.0 Hz (linear) or to 2.5 Hz (logarithmic), but the total time for which the signal is applied is the same as for the harmonic signal.

2.2 Spectral Analysis

Spectral techniques are applied to input/output data sequences to obtain auto- and cross-power spectral density functions, frequency response estimates, and the magnitude-squared coherence function. Direct spectral estimation using overlapped fast Fourier transform (FFT) is used. Several steps are required for the estimation. The data are divided into K segments, each of M points. Each corresponding time segment pair (input and output) is selected and processed, in turn, to obtain a spectral density estimate. Then the data for a particular segment are multiplied by a cosine weighting function to reduce side-lobe leakage. As some data are lost when weighting is applied, a segment overlap of 62.5% in the FFT processing is used to compensate for this effect. Carter et al. [8] show that this value maximizes the improvement in the spectral estimate possible from over-lapping. The identification is not performed in real time and computation time is not critical; therefore the full 62.5% overlap is used. The discrete Fourier transform of the weighted data segments is taken using a radix-2 FFT algorithm. Averaging the squared-modulus of the Fourier coefficients obtained (at a given frequency) over all the segments gives an *estimate* of the spectral density function. This amounts to averaging a number of spectral estimates (each found from a data segment) over the whole data series. The auto- and cross-power spectral estimates are determined as

$$G_{xx}(k) = \frac{1}{K f_s M} \sum_{n=1}^{K} |X_n(k)|^2 \qquad (2)$$

$$G_{yy}(k) = \frac{1}{K f_s M} \sum_{n=1}^{K} |Y_n(k)|^2 \qquad (3)$$

$$G_{xy}(k) = \frac{1}{K f_s M} \sum_{n=1}^{K} |X_n^*(k)\ Y_n(k)|^2 \qquad (4)$$

where f_s is the sampling frequency, k is the discrete frequency variable, G_{xx} and G_{yy} are the input and output auto-spectral densities, and G_{xy} is the cross-spectral density.

Assuming un-biased spectral estimation, the transfer function can then be defined as

$$H(k) = [G_{xy}(k)] / [G_{xx}(k)] \qquad (5)$$

This relation is strictly true only when the measurements are uncorrelated with the actuation inputs - a situation which can break down in some closed-loop instances when feedback is applied between the measurements and the actuation signals. In the ideal case, the frequency response is found using Eq (5). The variance of spectral estimates is reduced by the averaging of estimates from successive data frames. An expression can be derived for the variance of frequency response estimates in terms of the coherence function [11, 9]. This variance is inversely proportional to the number of data frames K. When choosing suitable frame lengths for a fixed total data length, a balance must be found between this variance and the bias introduced by shorter frame lengths.

The precision and resolution of the spectral estimate depends on the choice of N, the number of data samples, and M, the segment size, respectively. The analysis of Priestley [12] can be applied here to evaluate suitable minimum values for M and N and the ratio between them. The minimum value of M depends on the required degree of resolution, that is, the ability to distinguish fine detail in the spectral density functions. It also depends on the data window used, in this case the Tukey-Hanning window. For a given value of M, there is a certain minimum value of N, which depends on the precision required in the spectral estimates. The frequency range of interest for the signals compared in Fig. 1 is, 0.015 - 4 Hz. Hence, for this case, the theoretical minimum sample rate (f_s) is 8 Hz. An acceptable sample rate needs much greater than this and f_s was chosen to be 31.25 Hz. A minimum resolvable frequency of 0.015 Hz was desired and, using Priestley's equations we obtain M = 1307. For a proportional error not exceeding 25% at the 90% confidence level, this would give N = 61103. However, with 62.5% segment overlap, fewer points are required as each is used more than once. In fact, the number of points needed is approximately N' = 3N/8 giving N' = 22914, for this example. To make use of the radix-2 FFT algorithm M was selected as 2048, and the corresponding value of N' was 32,768. A greater value of M would give better resolution as the window bandwidth is decreased and, in general, a larger value of N would allow M to be increased without loss of precision. About 17 min. of flight test data would be required to satisfy these criteria in practice. This seems a long time but can be justified as multiple runs can be concatenated.

2.3 Least-Squares Curve Fitting

The frequency-domain testing method can be used to obtain linear system parameter estimates. A transfer function is obtained by fitting the frequency response estimate algebraically as a ratio of two frequency-dependent polynomials:

$$\frac{P(s)}{Q(s)} = \frac{p_0 + p_1 s + p_2 s^2 + \ldots + \ldots p_m s^m}{q_0 + q_1 s + q_2 s^2 + \ldots + \ldots q_n s^n} \qquad (6)$$

where s= jω, the complex frequency variable. Least-squares techniques are employed to minimize a given error criterion and to derive the set of transfer function coefficients that gives the best algebraic fit to the complex curve. The following least-squares criterion is an obvious choice, but as it is non-linear in the parameters q_0, q_1, ...,q_n the minimization is quite complex

$$E = \sum_{k=1}^{N} \left| \frac{P(j\omega_k)}{Q(j\omega_k)} - H(j\omega_k) \right|^2 \qquad (7)$$

$H(j\omega_k)$ are the actual frequency response data resulting from an application of Eq.(5) to the windowed spectral estimates given by Eq. (4) and N is the number of frequency response data samples.

A number of linear approaches have been developed to simplify the approach to the solution, and a comparison of these has been made by Whitfield [13]. The method of Sanathanan and Koerner [14] has been used with some success by several authors [15]. However, it has been shown that, when convergent, the algorithm produces biased estimates [16]. In particular, if the frequency response data are noisy, the accuracy of the transfer function parameter estimates deteriorate with increasing noise level. This problem of biased-estimation is typical of linear least-squares methods.

The Levenberg-Marquardt form of the Gauss-Newton, non-linear, least-squares method [17] has been applied in this work to improve the identification of the transfer function parameters in the case of noisy systems. This is an iterative search procedure that interpolates between a Gauss-Newton method, which converges rapidly when the initial parameter estimates are good and the steepest descent algorithm which converges with poor starting estimates but can take rather a long time. By changing gradually between the methods (depending on how close the parameters are to the optimum set at each iteration), the dual advantage of both methods can be gained. A description of all these nonlinear least-squares methods is given given by Bevington [18]. To obtain suitable starting values, a linear least-squares optimization is carried out first using the method of Sanathanan and Koerner, and the evaluated transfer function coefficients are then used as the initial parameters in the non-linear optimization. Reasonably good starting values help to minimize computation time and reduce the likelihood of the optimization converging to a local minimum solution rather than the global one. In the authors' experience, this method always converges, finding the global minimum solution, although convergence is not rigourously proven.

This form of identification is considered an important approach to the structural and parameter estimation of a helicopter subjected to multi-frequency testing. The main object of the current work, however, is to identify the frequency response correspondence between estimates derived from spectral testing and frequency response data resulting from the flight simulator. The parameter estimation aspects will not therefore be discussed further in this paper as more appropriate details are available elsewhere [4, 9].

3 Comparison of Test Signals Applied to Helicopter Simulations

Fig.3 shows a comparison of the linear and logarithmic sweep, m-sequence and Schröeder-phased test signals applied to a linear digital simulation of a helicopter. The well-known *magnitude-squared coherence function* serves as an indicator of how good the first harmonic is as a model of the input-output dynamics, i.e. it is a measure of the linear-dependence of the output on the input defined in spectral terms. The magnitude-squared coherence function Γ^2_{xy} is given by:

$$\Gamma^2_{xy}(k) = \frac{|G_{xy}(k)|^2}{|G_{xx}(k)| \; |G_{yy}(k)|} \leq 1 \qquad (8)$$

G_{yy} is the output auto-spectrum, G_{xx} is the input auto-spectrum and G_{xy} is the cross-spectrum between the output and the input. By definition, the coherence function lies between 0 and 1. A totally noise-free linear system would yield $\Gamma^2_{xy}(k) = 1$. Three effects can cause the coherence function to take a value less than unity: non-linearities in the system under test, input and output noise and secondary inputs such as external disturbances. When a system is noisy or non-linear, the coherence function indicates the accuracy of a linear identification as a function of frequency. The closer it is to *unity*, the more reliance can be placed on an accompanying frequency response estimate, at a given frequency. For a real application, which will be non-linear and affected to some extent by noise, a plot of coherence against frequency will indicate the changing way in which one variable can be described as a linear function of the other. However, for a noise-free linear system, the coherence function gives an excellent comparison of the efficiency of a number of test signals in exciting the dynamic modes of a typical system.

These results correspond to an application of the test signals to a linear helicopter simulation as a part of a study to evaluate the performance of frequency domain identification methods [4, 9]. The coherence was computed from the estimated spectral response of yaw (r) rate from test signals applied in the tail- rotor collective pitch (Θ_t) actuator. Fig. 3(a) shows the comparison of the coherence plot of the linear swept sinewave, logarithmic swept sinewave and Schröeder-phased test signals. Fig. 3(b) extends this comparison to the m-sequence signals with the 3-2-1-1 and more complex 31 bit case considered. It is clear from this comparison that suitable test signals are obtained through using a higher complexity signal structure affording flexibility to achieve a flat signal spectrum.

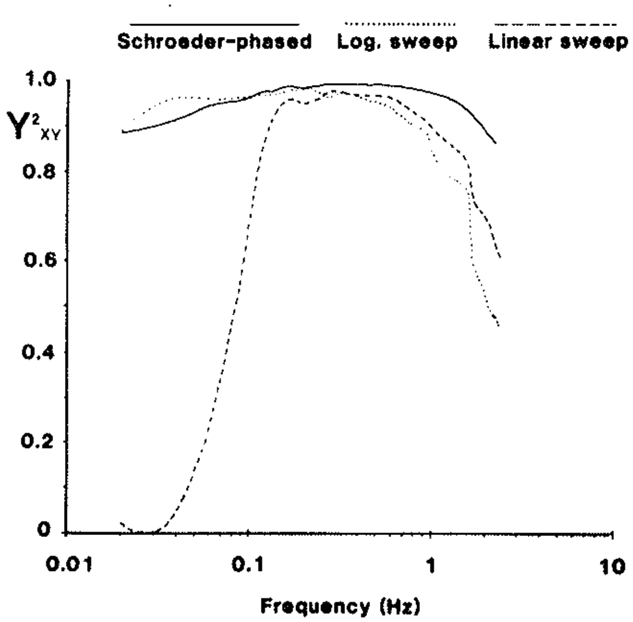

Figure 3(a) Γ^2 plots for non-binary tests

These results show that the coherence function is an important indicator of the effectiveness of the multi-frequency wave as a test signal.

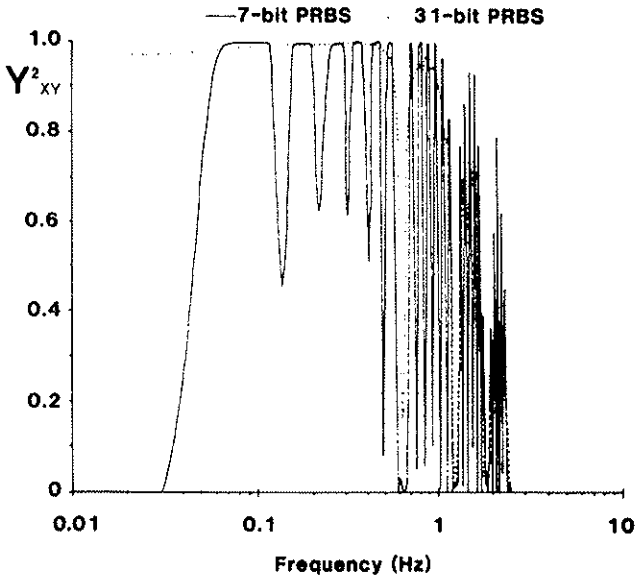

Figure 3(b) Γ^2 plots for binary tests

To further illustrate the relative properties of the *non-binary* test signals, the gain and phase estimates derived from Eq. 5 have been compared with the true frequency response (gain & phase), based on the simulation. Fig. 4(a) contrasts the application of the linear and logarithmic sine wave sweep signals. The comparison illustrates quite well the ability that the logarithmic sweep signal has in correctly exciting the low frequencies. Although, it can be said that neither the linear or log. sweep signals produce good high frequency fits, particularly in phase shift.

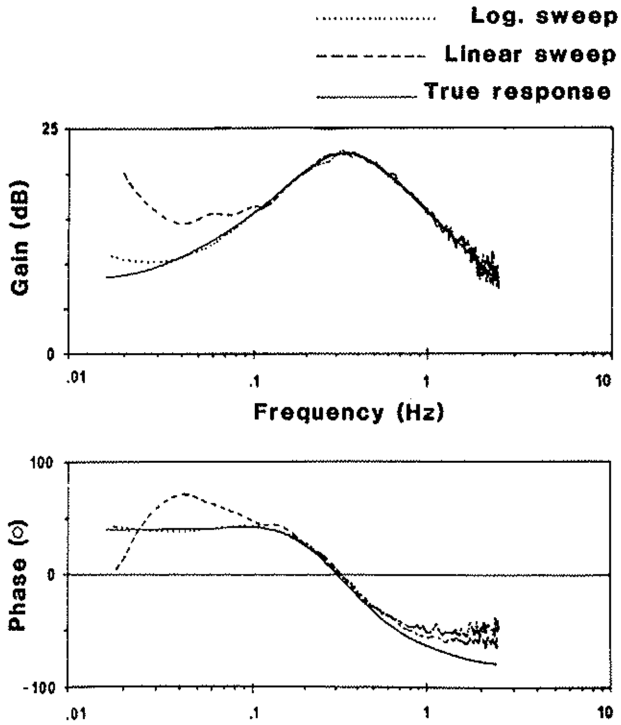

Figure 4(a) Sine/log sweep responses

This result is compared well with plots in Fig. 4(b) for the estimates resulting from using the Schröeder-phased test signal under the same simulation conditions (corresponding to yaw rate (r) response from tail rotor collective pitch Θ_t - in hover). It is clear that the Schröeder-phased test using the power spectrum shown in Fig. 2, generally gives rise to better spectral estimates at both low and high frequencies. This signal shows significant promise, particularly as it can easily be designed to suit the system's spectral characteristics. The advantages of gaining an improved estimation and the use of a relatively low amplitude signal (with low relative peak factor) indicate that this signal should be incorporated into a future flight programme.

The logarithmic swept sinewave provides quite an adequate excitation of the system and has been applied in flight by Westland. This form of testing is the subject of the next sections.

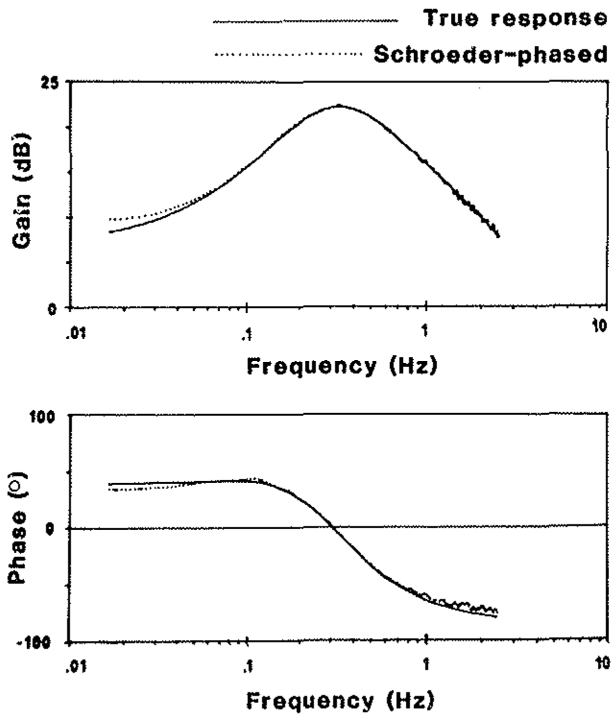

Figure 4(b) Schröeder-phased responses

4 Automated Test Signal Application

A conflict arises if these test signals are to be applied manually by the pilot; a simple signal sequence is all that is feasible in this situation and this is insufficient to ensure adequacy of spectral content and repeatability. An obvious way forward is to automate the generation of the signal which is to excite the aircraft. This enables the experiment to be sufficiently controlled, to be repeatable and also guarantees the desired spectral content. There are other important advantages: limiting of the inevitable fatigue damage that this type of testing incurs and leaving the pilot free to observe and monitor the experiment and to intervene if necessary. Automation allows us, therefore, to use more complex test signals to meet the above requirements.

The initial considerations of the design of the test technique were:

(a) The frequency range for excitation: This must avoid any lightly damped structural or aeroelastic mode frequencies; the start frequency is determined by how much discomfort the crew will tolerate.

(b) The sweep time length, T: The workload associated with keeping the aircraft on condition especially in hover and at low speeds; crew discomfort.

(c) The rigid body modes of interest should be excited sufficiently to give reasonable attitude and rate excursions; This enables the aircraft to be kept on condition without too much difficulty; the aircraft response should be typical; corrective control should be kept to a minimum.

A single 'frequency sweep' at one condition provides information at all frequencies within the range of interest. This results in a much lower number of flight hours compared with the classical steady-state testing.

Experience from piloted simulations had indicated that the quality of the Gain and Phase values are improved at low frequencies by:

(a) a low rate of change of frequency at low frequencies, achieved by logarithmically increasing the frequency,

(b) two cycles at the start frequency which begins to move the system and allows for transient effects.

The input signal, aircraft response and other outputs of interest are recorded on a system known as MODAS (Multiplexed On-board Data Acquisition System). The type of Automatic Stabilisation Equipment (ASE) fitted to Westland Helicopters are such that the series actuators facilitate the implementation of automatic test signal inputs. The signal is summed with the ASE signal at the series actuator input (single lane). This implementation conveniently avoids "switching" of the control laws which would occur if the test input were applied through the cyclic control stick.

In general, the series actuator amplitude (single lane) cannot exceed 5% of the blade pitch range. The test inputs are introduced into one lane only, for safety reasons. The swept sinewave is pre-recorded on tape and played back in flight. A change in any of the parameters previously discussed is easily accommodated by re-recording the signal.

The test signal applied to the helicopter is of constant amplitude, beginning at 25m Hz, ending at 2 Hz, and lasts for 180 seconds.

The amplitude is selectable in flight, 25%, 50% and 75% approximately of actuator travel, allowing shakedown and safe progression to attaining a reasonable attitude response. For overall confidence and improvement in the quality of the Gain and Phase information at low frequencies, a signal at a selected constant frequency is introduced, typical values being 0.1 Hz (low), 0.25 Hz, 1.0 Hz (high).

Prior to flight, the hardware was bench-tested and the techniques assessed in the Flight Dynamics Simulator. Results were analysed for possible fatigue damage.

The flight test procedure involves trimming the aircraft to the required condition with the ASE engaged, i.e. the aircraft is in equilibrium. The lane of the channel of interest, which does not include the test signal, is disengaged (this ensures adequate response by preventing the stabilisation from opposing the test excitation); the "autotrim" (where fitted) is switched to "manual" (this disengages the parallel actuator which would otherwise oppose the low frequency excitation); the amplitude of the test signal is selected. The tape of the test signal is played. The pilot monitors the aircraft response and trim condition, applying manual control as necessary (to date, this has only been necessary in hover). Critical loads are monitored on telemetry. When the tape has finished, the aircraft trim is checked before re-engaging the ASE and autotrim.

5 Flight Test Results

Typical flight results from frequency domain testing obtained by the method of Section 2 and meeting the requirements outlined in previous Sections are shown in Figs. 5 & 6. Fig. 5 illustrates typical time histories of

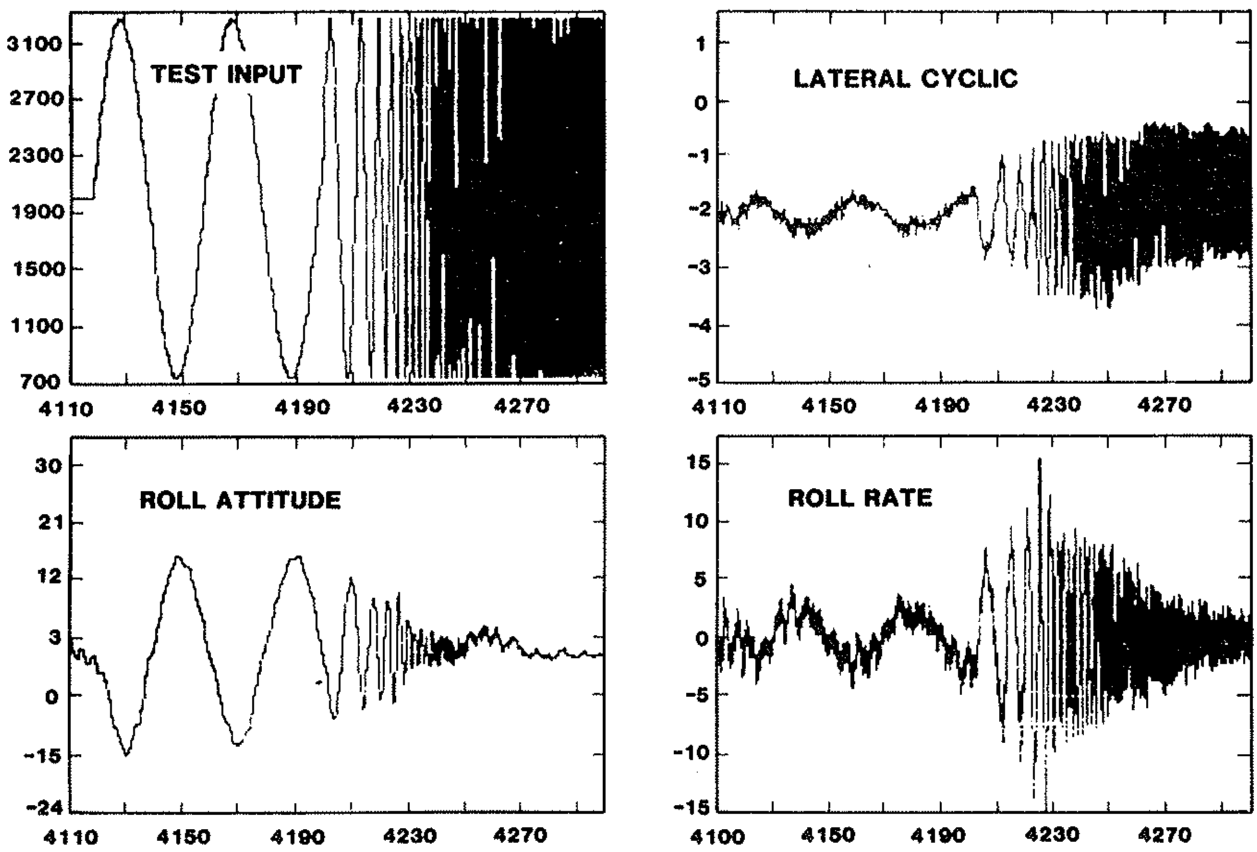

Figure 5 Selected time histories in roll at 80 Knots

variables of interest: input test signal; lateral cyclic blade pitch angle (combined test signal and stabilisation signal); aircraft body roll attitude; and rate. Particularly noticeable are the increased attenuation and phase lag of the estimated aircraft response at high frequencies.

Figs. 6(a) & 6(b) illustrate a conventional Bode diagram of Gain and Phase for aircraft roll attitude. The sign convention adopted is a *positive lateral cyclic stick input* (to the right) produces a positive lateral cyclic pitch change at the rotor and causes the aircraft to roll to the right (positive). The coherence plot (Fig. 6(c)) shows that the information is good for frequencies up to 1 Hz, i.e. a bandwidth containing the dominant rigid modes.

Figs. 7(a), 7(b) & 7(c) show corresponding Bode and coherence plots for the aircraft body pitch attitude to longitudinal cyclic blade angle. The sign convention adopted is a positive longitudinal cyclic stick input (forwards) which produces a positive longitudinal cyclic pitch change at the rotor and causes a nose-down pitch (negative) and produces a positive longitudinal cyclic pitch change at the rotor. Again the coherence (Fig. 7(c)) shows that the information is good for frequencies up to 1 Hz.

Figs. 8(a) and 8(b) illustrate the equivalent Bode plots to Figs. 7(a) and 7(b) produced from a non-linear simulation model. Overlaying Figs. 7(a) and 8(a) illustrates excellent agreement for the Gain over the frequency range of interest. Overlaying Figs. 7(b) and 8(b) shows a general good agreement of Phase. At the higher frequency there appears to be a tendency for the aircraft phase to roll off more rapidly than the simulation. Extending the frequency range would confirm this. The cause is: either missing additional degrees of freedom or too large a time constant in the modelled degrees of freedom; and non-linearities present in mechanical control runs and power servos.

The results demonstrate the adequacy of the log. swept sinewave test input for flight applications.

In the above cases, the output, body attitude, is correlated with the total signal at the power servo outputs resolved into the blade lateral or longitudinal cyclic pitch applied to the rotor. This allows direct comparison of the stabilised mathematical model to the stabilised aircraft but without the presence of the ASE and mechanical control runs. In principle, Bode diagrams between any two instrumented stations can be produced. A picture of the characteristics of the components within the closed-loop (ASE + aircraft) can be constructed. The comparison of these Bode diagrams with those derived from classical linear analysis identifies non-linearities which can then be addressed by "describing functions", giving a good physical understanding of the system characteristics.

The ability to demonstrate physical phenomena, in this way, demonstrates the power of the frequency domain approach. This is an important prelude to parameter identification; the estimation of system parameters should only be attempted once the dynamic system structure is well understood. This work shows that frequency domain testing facilitate this process, if the experiments are carried out with due care.

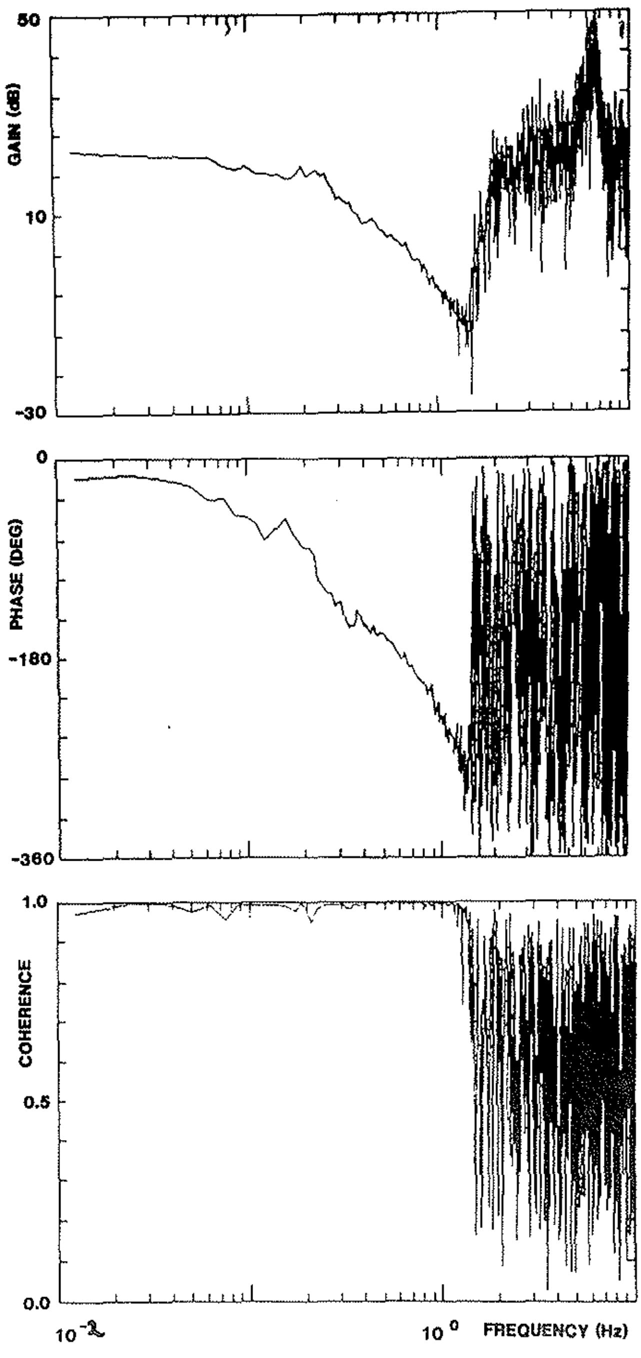

Figure 6: Aircraft Roll Attitude Gain, Phase & Coherence at 80 KTS

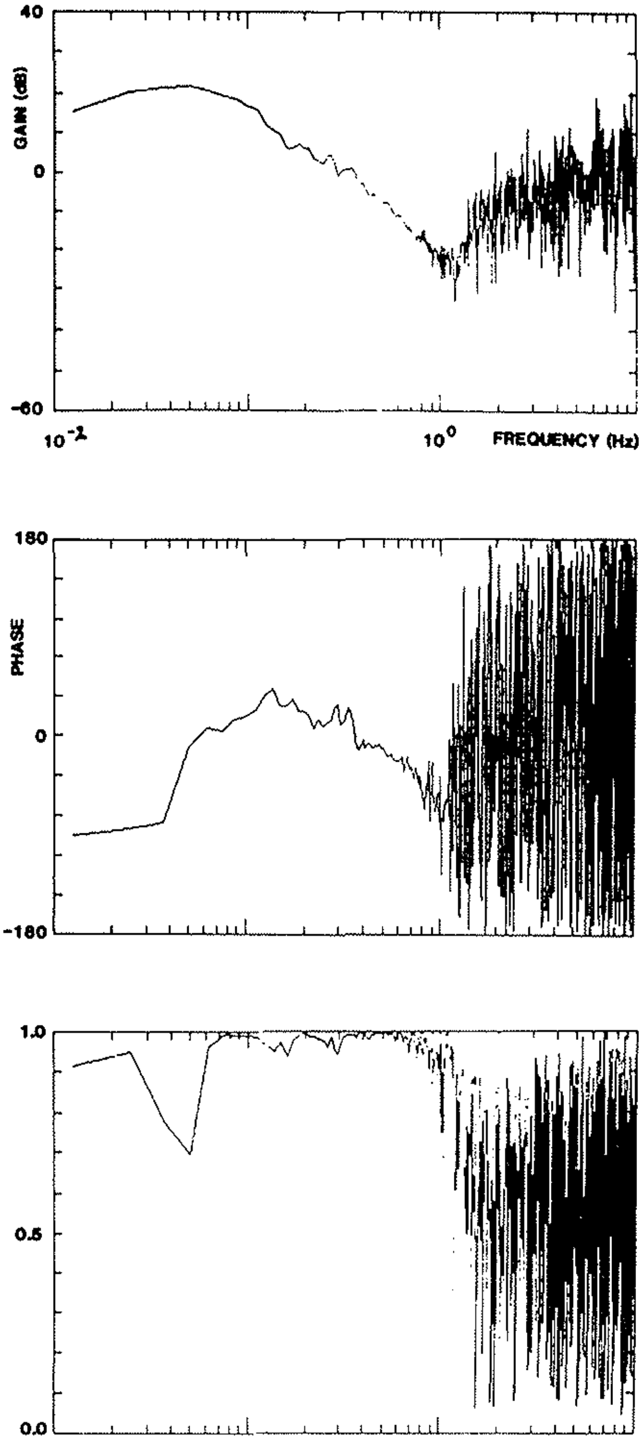

Figure 7: Aircraft Pitch Attitude Gain, Phase & Coherence at 80 KTS

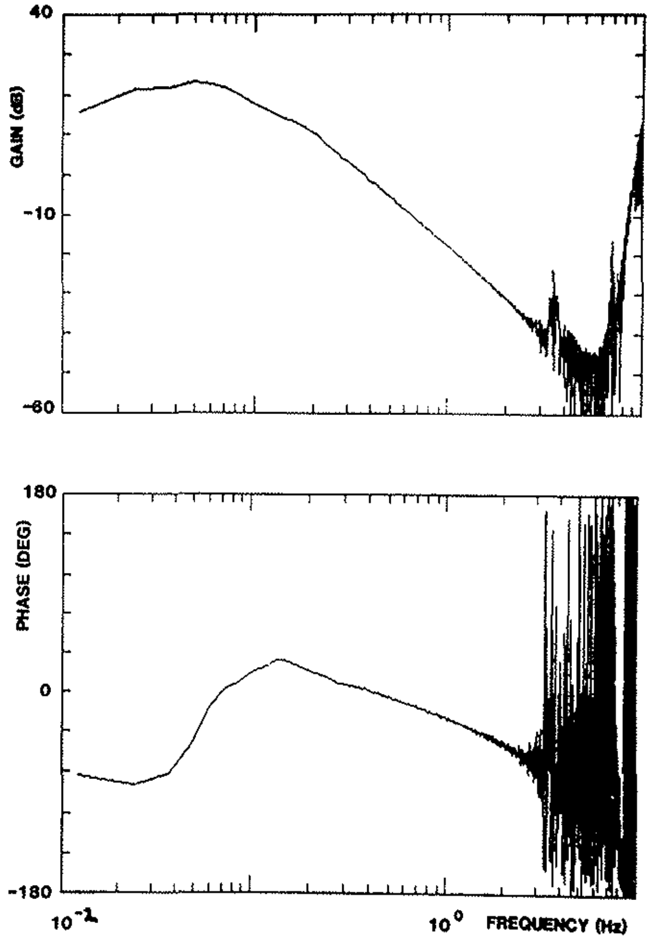

Figure 8: Simulated Pitch Attitude Gain, Phase

6 Later Developments

The mathematical models currently being developed and thus requiring validation include the rotor aeroelastic modes. The test technique, especially the flight safety implications, has been reassessed and the frequency range extended, in a particular instance, to 5 Hz at low amplitude. It was impossible to correlate the attitude response with any confidence but it was possible to correlate the rate response (see Figs. 9(a), 9(b) & 9(c)). Figs. 9(a) & 9(b) show the Bode Diagram for roll rate to lateral cyclic pitch angle in hover.The Gain and Phase both illustrate mode or modes around the 3.5 Hz frequency. This is again confirmed by Figs. 10(a) & 10(b) which show the equivalent simulation Bode Diagram. The coherence illustrated in Fig.10(c) is not so good as in forward flight where the aircraft is generally more stable. Pilot corrective action is evident at lower frequencies. Overlaying Figs. 9(a), 10(a) and 9(b), 10(b) shows excellent agreement for both Gain and Phase over a frequency range up to 4 Hz. (excitation 0.1 to 5 Hz). With the evident amplification causing large output amplitudes, the difference here is certainly due to amplitude-conscious non-linearities.

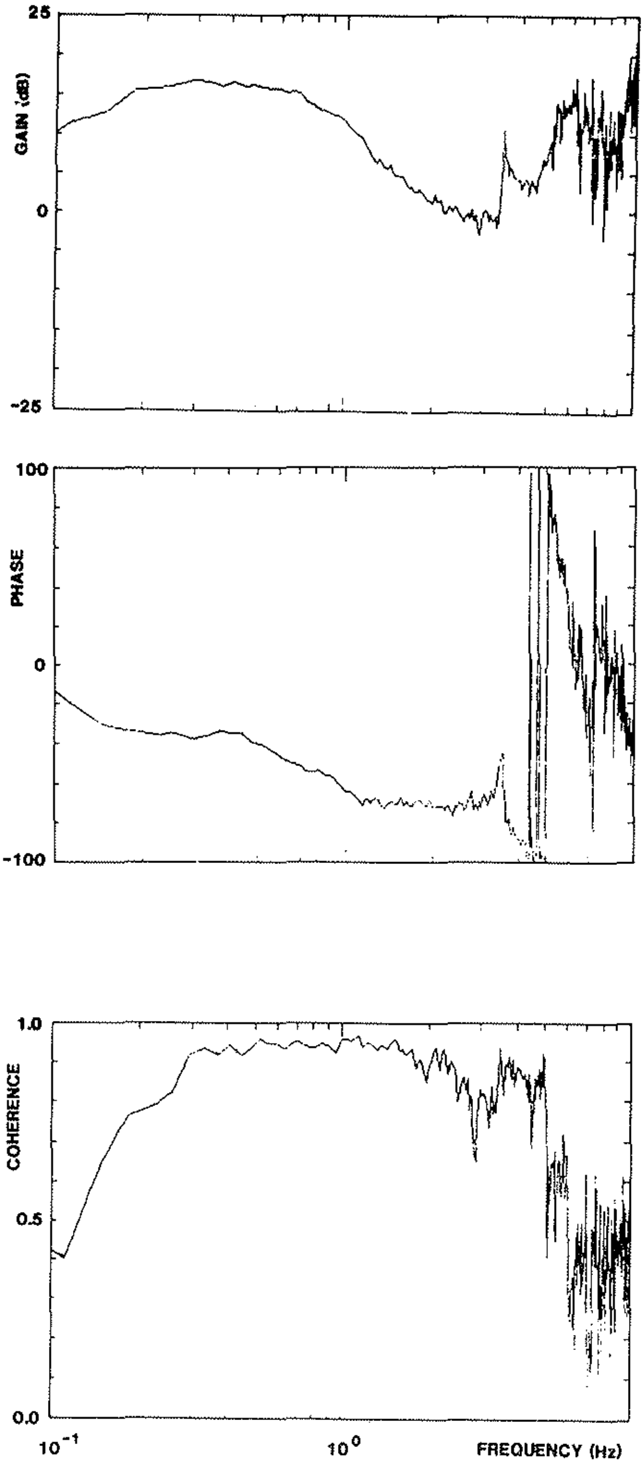

Figure 9: Aircraft Roll Rate Gain,Phase & Coherence in Hover

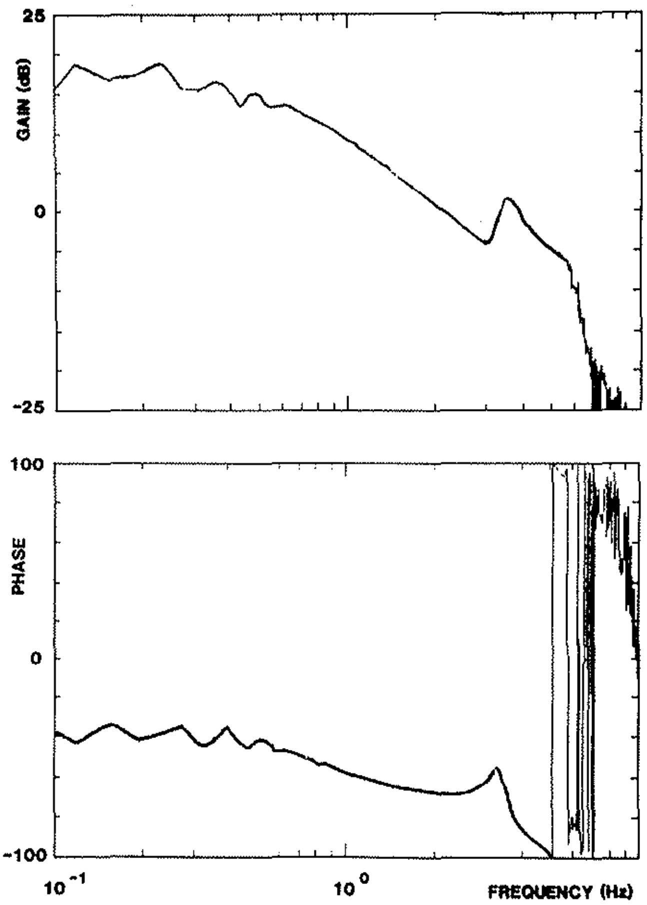

Figure 10: Simulated Roll Rate Gain,Phase

7 Conclusion:

The Schröeder phased test signal shows significant promise, and has now been recommended for incorporation into a future flight programme. The logarithmic swept sinewave is a suitable test input for flight applications. The frequency domain approach gives a powerful way of identifying physical phenomena and this has been well demonstrated in this study. The results have shown a very powerful validation of the flight simulator model structure; phase and gain discrepancies can be accounted for (only when necessary) by considering specific non-linear phenomena or time-delays. *It has not been considered appropriate to adopt the practice of automatic inclusion of pseudo time-delay parameters to account for phase mismatches at high frequency, as other authors do.* It is considered that the approach taken is realistic and a thorough way of validating dynamic structure for helicopters, based on flight-testing and simulation data. It is considered an important prelude to detailed and meaningful identification of system parameters.

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Appendix 1 Design of Logarithmic Sweep Signal

The logarithmic sweep signal is generated as follows:

The Rate of change of frequency δF (Hz/s) is

$$\delta F = 10^{**}\ \frac{\log(f_e) - \log(f_o)}{T} \qquad \text{(A1)}$$

where f_o and f_e are the start and end frequencies of the sweep, respectively and T is the test period.

The kth sweep frequency is then given by

$$F[kt] = \delta F\, F[(k-1)t] \qquad \text{(A2)}$$

For example, for the helicopter application, typical frequency sweep parameters are f_o = 0.015 Hz, f_e = 2.5 Hz and T = 65.536 s. The rate of change of frequency is then given by:

$$\delta F = 1.0812 \text{ Hz/s} \qquad \text{(A3)}$$

Appendix 2 Design of Schröeder-Phased Test Signals

The relative peak factor of a periodic signal $V(t)$ can be defined as:

$$\text{Relative peak-factor} = \frac{|V_{max} - V_{min}|}{2\sqrt{2}V_{rms}} \qquad \text{(A4)}$$

In identification, a test signal with a low peak factor will avoid large perturbations in the system which is clearly desirable. It is possible to select the phase angles of a number of harmonics so that, when they are summed together they produce a low peak-factor wave that fits a given power spectral density. Such a wave is a Schröeder-phased wave and it is derived as follows.

The Fourier series of a wide-band signal of period T is

$$V(t) = \sum_{k=1}^{N_s} \sqrt{2p_k}\cos(2\pi kt + \Theta_k) \qquad \text{(A5)}$$

Θ_k is the phase angle, p_k is the relative power of the kth harmonic, and N_s is the number of the harmonics.

By definition

$$\sum_{k=1}^{N_s} p_k = 1 \qquad \text{(A6)}$$

To design the wave from a particular set of values of p_k, the values of Θ_k are chosen to minimize $V_{max} - V_{min}$. Consider a phase-modulated signal:

$$s(t) = \cos[\Phi(t)] \qquad \text{(A7)}$$

$$\Phi(t) = \int_0^t \dot{\Phi}(t)\, dt \qquad \text{(A8)}$$

with $\dot{\Phi}(t) = 2\pi k/T$ in an interval between t_{k-1} and t_k and with

$$t_k = T \sum_{j=0}^{k} p_j = v \qquad \text{(A9)}$$

(i.e., $t_0 = 0$)

During a time interval from t_0 to t_1, the instantaneous phase of s(t) is

$$\Phi(t) = \Phi_0 + 2\pi t/T \qquad \text{(A10)}$$

Φ_0, the phase at $t = 0$ can be set to zero without loss of generality. Therefore

$$\Phi(t_1) = 2\pi t_1/T = 2\pi p_1 = \Phi_1 \quad (A11)$$

During the time interval from t_1 to t_2

$$\Phi(t) = \Phi_1 + \frac{2\pi}{T}\int_{t1}^{t2} 2\,dt = 4\pi(t-t_1)/T \quad (A12)$$

$$\Phi(t_2) = 2\pi(p_1 + 2p_2) = \Phi_2 \quad (A13)$$

In general

$$\Phi_n = 2\pi\sum ip_i, \quad i=1,....,Ns \quad (A14)$$

Equation (A14) is used to adjust Θ_k in equation (A5).

With N_s harmonics of equal power, the alternative expression is

$$\Phi_n = \frac{2\pi}{N_s}\sum_{1=1}^{n} i = \frac{\pi}{N_s}(n^2 + n) \quad (A15)$$

The term $\pi n/N_s$ is a linear-phase delay. A Schröeder-phased signal with a flat spectrum can be designed, using Eqs. (A8) and (A15), specifying the frequency range and the number of harmonics. In the more general case, Equations (A8) and (A14) can be used to design a low peak-factor wave with any power spectral density function.

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