GLOBAL SOLUTION OF OPTIMIZATION PROBLEMS IN SUPPORT OF CERTIFICATION AND FORMULATION OF OPERATIONAL PROCEDURES FOR ROTORCRAFT VEHICLES

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ABSTRACT

In this paper we present a procedure for the global solution of trajectory optimization and parameter estimation problems for rotorcraft vehicles using evolutionary algorithms. Our approach makes use of a repair heuristic to handle problem constraints, that is based on a sequential quadratic programming method. This way, no specialized evolutionary operators or ad hoc modifications of the objective function need to be considered. The performance of the proposed procedure is assessed with applications dealing with the flight testing for system identification of a rotorcraft unmanned aerial vehicle, and with the determination of the Category A take-off decision point for a medium-size helicopter.

1 INTRODUCTION

The software program STOP (System Identification and Trajectory Optimization Program) was developed at the Dipartimento di Ingegneria Aerospaziale of the Politecnico di Milano and conceived as a tool to be used in support of certification and formulation of operational procedures for rotorcraft vehicles. STOP has the ability to treat both trajectory optimization problems [13, 15], also referred to in the following as Maneuver Optimal Control Problems (MOCPs), and Parameter Estimation Problems (PEPs) [14] under a common framework. In fact, it can be shown that both can be formulated as two-point boundary value constrained optimization problems defined over a temporal domain of known or unknown duration; moreover, both can be discretized in time using the same techniques, thereby obtaining a constrained Non-Linear Programming (NLP) problem that is formally identical in the two cases [11].

In rotorcraft flight mechanics, problems such as, for instance, continued and rejected take-off procedures following an engine failure (Category A certification [1]), optimal auto-rotation, landing procedures after tail-rotor loss, and the analytical, i.e. non-piloted, simulation of ADS-33 mission task elements [2], can be profitably studied with the help of trajectory optimization. In all these cases, the analyst is interested in computing an extremal maneuver for a given vehicle model; one looks for a solution that minimizes a cost function while satisfying given constraints that translate various requirements including the boundaries of the performance envelope of the vehicle. Clearly, the quality of the results strongly depends on the fidelity of the model to the actual vehicle. If flight-test data are available, parameter estimation techniques can be used to tune the model parameters, thereby enhancing the ability of the model to represent the actual vehicle behavior. In this circumstance, the analyst is interested in finding the values of the parameters in the given mathematical model such that the model-computed response best matches (in a statistical sense) the experimentally observed one.

Some of the features of STOP have already been discussed in a series of previous publications by the authors [13, 14, 11]. In this work, we describe the interfacing of STOP with Evolutionary Algorithms (EAs) [5] for the global solution of MOCPs and PEPs.

It was found in [20, 16] that trajectory optimization problems might lead to NLP problems that are non-convex and hence with multiple solutions. In this case, gradient-based methods are likely to converge to local optimal solutions. Designed to solve non-convex problems, EAs borrow their working principle from the mechanisms that govern the process of natural evolution of biological organisms. By the application of genetic operators (selection, crossover, mutation), a population of possible solutions (individuals) is let evolve through successive generations so as to promote the individuals that better meet the design requirements. Since EAs are unconstrained optimization methods, their successful application to the solution of constrained problems requires the use of suitable constraint-handling techniques; a comprehensive survey of the different techniques that have been used to handle constraints in EAs is given in [17].

Published works that have investigated the use of EAs for the solution of optimal control problems, rely on the use of either penalty functions [25] or specialized genetic operators [24, 22]. In the former case, one transforms a constrained problem into an unconstrained one by penalizing with suitably high weights constraint violations in the objective function. Despite its simplicity, the major drawback of such an approach is that it requires fine tuning of the penalty parameters, which might not be a trivial task when dealing with highly-constrained search spaces. The latter approach, on the other hand, is more involved: by taking advantage of the knowledge of the constraint conditions imposed on the problem at hand, ad hoc modifications of the genetic operators are introduced in order to preserve the feasibility of the solutions at all generations. However, this leads to specialized computer codes, i.e. codes capable of solving just one specific problem type. Clearly, this is not desirable when one is interested in developing a tool that can be applied to an ample variety of problems.

The approach proposed in this work makes use of a split of the design variables: a first set (typically represented by model parameter, control policies and/or initial conditions) is handled by the global EA optimizer, while a second set (typically involving state variables but also possibly control inputs, see later on for details) is handled by a local optimizer using a Sequential Quadratic Programming (SQP) method. When working at the level of vehicle states, the SQP optimizer effectively implements a repair heuristic on these quantities, making them compatible with the problem constraints. Since this can be a time consuming process, the SQP method is typically run only for a limited number of iterations. When achieving a feasible solution within the specified maximum number of iterations proves to be difficult, the repair heuristic is given a limited authority on the control time histories; however, the repaired individual is never returned to the population (never replacing approach [17]). Using a repair technique reduces the search space of EA to feasible solutions only; hence, no special operators or modifications of the objective function need to be considered. Hence, the resulting code can be used to solve the different classes of problems that find applicability in the general area of rotorcraft flight mechanics.

The paper is organized as follows. Section 2 briefly describes the formulation and solution of optimization problems in rotorcraft flight mechanics, with particular reference to the STOP code. After having formulated MOCPs and PEPs as optimization problems using a single common notation in Sections 2.1 and 2.2, we briefly describe the architecture of STOP in Section 2.3. Next, in Section 3 we illustrate the proposed methodology for the use of EAs in the solution of maneuver optimal control problems. Finally, in Section 4 we present the application of the global optimization version of STOP to the solution of problems arising in the context of rotorcraft flight mechanics.

2 OPTIMIZATION PROBLEMS IN ROTORCRAFT FLIGHT MECHANICS

2.1 *The Maneuver Optimal Control Problem*

Consider a generic flight mechanics model $\mathcal{M}$ described in terms of the following set of non-linear differential equations

$$\boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{p}, \boldsymbol{w}, t) = \boldsymbol{0}, \tag{1a}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \tag{1b}$$

where $\boldsymbol{x}$ is the state vector, which groups together the structural dynamics (including states that describe rigid and possibly flexible rotor(s), fuselage, engine, etc.) and aerodynamics states (e.g. dynamic inflow variables), $\boldsymbol{u}$ is the control input vector, $\boldsymbol{p}$ is a set of model parameters, and $\boldsymbol{w}(t)$ models other exogenous inputs and disturbances acting on the system (e.g., gusts and air turbulence). Equations (1b) specify a set of outputs $\boldsymbol{y}$, which typically represent some vehicle states describing its gross motion, or other quantities useful for the analysis of the vehicle dynamics. Finally, the notation $\dot{(\cdot)} = \mathrm{d}(\cdot)/\mathrm{d}t$ indicates a derivative with respect to time t.

A general MOCP [9, 10] for model $\mathcal{M}$ can be formulated as:

$$\min_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{u},T} \quad J^{\text{MOCP}}(\boldsymbol{y}, \boldsymbol{u}, T, T_i), \tag{2a}$$

$$\text{s.t.:} \quad \boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{p}^*, \boldsymbol{w}^*, t) = \boldsymbol{0}, \tag{2b}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \tag{2c}$$

$$\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, t, T, T_i) \leq \boldsymbol{0}. \tag{2d}$$

The problem is defined over the interval $\Omega = [T_0, T]$, $t \in \Omega$, where the final time T is typically unknown and must be determined as part of the solution. Specific events might be associated with unknown time instants T_i, $T_0 < T_i < T$ (for example, the jettisoning of part of the cargo or other instantaneous conditions).

In Equation (2a), J^{MOCP} indicates the to-be-minimized cost, which, depending on the problem at hand, might account for maneuver duration, control activity, fuel consumption, etc., or some other given function of interest that typically expresses an index of performance of the vehicle.

The maneuver definition is completed by providing a set of problem-dependent equality and inequality constraints (Equations (2d)) which translate the operating envelope of the vehicle, the performance and procedural requirements as dictated by norms and regulations (for example, certification rules), and all other necessary maneuver-defining constraints. All such constraints are typically expressed in terms of the outputs $\boldsymbol{y}$. Finally, equations (2d) also include initial and final conditions on the vehicle states $\boldsymbol{x}$.

Notice that the problem is formulated for fixed values of the model parameters $\boldsymbol{p} = \boldsymbol{p}^*$, where the symbol $(\cdot)^*$

indicates a know assigned value. Similarly, if exogenous inputs are present, these are also known, so that $\boldsymbol{w}(t) = \boldsymbol{w}^*(t)$.

2.2 *The Parameter Estimation Problem*

A general PEP for the parametric model $\mathcal{M}(\boldsymbol{p})$ can be formulated as

$$\min_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{p}} \quad J^{\text{PEP}}(\boldsymbol{z} - \boldsymbol{y}), \tag{3a}$$

$$\text{s.t.: } \boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{u}^*, \boldsymbol{p}, \boldsymbol{w}, t) = \boldsymbol{0}, \tag{3b}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \tag{3c}$$

$$\boldsymbol{g}(\boldsymbol{p}) \leq \boldsymbol{0}, \tag{3d}$$

where $\boldsymbol{z}$ are measurements of the outputs gathered at N discrete sampling time instants t_k during the experimental test,

$$\boldsymbol{z}(t_k) = \boldsymbol{y}(t_k) + \boldsymbol{v}(t_k). \tag{4}$$

The available measures are affected by noise $\boldsymbol{v}$ with covariance $\boldsymbol{R}_k = E[\boldsymbol{v}_k \boldsymbol{v}_k^T]$, $E[\cdot]$ being the expected value operator. The presence of measurement noise, together with the possible presence of a process noise term $\boldsymbol{w}$ for modeling disturbances acting on the system (e.g., air turbulence), makes the problem of a stochastic nature. Hence, the to-be-minimized cost function J^{PEP} is typically a statistical measure of the match between quantities $\boldsymbol{z}$ and model outputs $\boldsymbol{y}$.

A maximum likelihood estimator is obtained by choosing

$$J^{\text{PEP}} = \det(\boldsymbol{R}), \tag{5}$$

where $\boldsymbol{R} = 1/N \sum_{k=1}^{N} \boldsymbol{\nu}(t_k)\boldsymbol{\nu}(t_k)^T$, with $\boldsymbol{\nu}(t_k) = \boldsymbol{z}(t_k) - \boldsymbol{y}(t_k)$. Alternatively, a weighted least squares estimator is obtained if

$$J^{\text{PEP}} = \frac{1}{2} \sum_{k=1}^{N} \boldsymbol{\nu}(t_k)^T \boldsymbol{W} \boldsymbol{\nu}(t_k), \tag{6}$$

where $\boldsymbol{W}$ is a weight matrix. This method can be seen as a particular case of the Maximum Likelihood method for known measurement noise covariance matrix, $\boldsymbol{W} = \boldsymbol{R}^{-1}$ [18]. In the Filter Error Method [18], the system states obtained by integrating model (3b) are corrected by a Kalman filter, whose role is to stabilize the integration around the measurements and to account for the presence of process noise; details are omitted for brevity, but the PEP can still be expressed in a form resembling (3).

Inequality (3d) enforces possible constraints on the model parameters. Such constraints ensure that the estimated parameters lie within acceptable bounds and do not take at convergence values which are non-physical.

Notice that in this case the model inputs are known and fixed to the values $\boldsymbol{u}(t_k) = \boldsymbol{u}^*(t_k)$ measured during the experimental test (values in between the sampling instants may be interpolated, if necessary). Similarly, the temporal domain $\Omega = [T_0, T^*]$ is also known.

We remark that rotorcraft vehicles are typically unstable, at least in certain flight conditions, and hence they are usually artificially stabilized by means of a control system. This fact has important consequences on the parameter estimation process and must be explicitly taken into account when formulating estimation methods for such vehicles; further details are given in [14].

2.3 *STOP Architecture*

The architecture of the `STOP` program is shown in Figure 1.

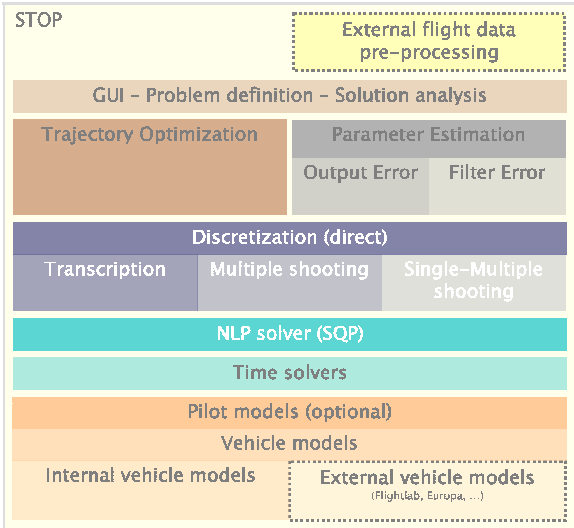

Figure 1: Architecture of the `STOP` code.

A graphical user interface supports the definition of MOCPs and PEPs. The common thread between the solution of the two classes of problems is the discretization in the temporal domain. `STOP` implements the so-called *direct* approach [7], which leads to a non-linear constrained optimization problem that writes

$$\min_{\boldsymbol{\pi}} \quad J^{\text{NLP}}(\boldsymbol{\pi}), \tag{7a}$$

$$\text{s.t.} \quad \boldsymbol{a}(\boldsymbol{\pi}) = \boldsymbol{0}, \tag{7b}$$

$$\boldsymbol{b}(\boldsymbol{\pi}) \leq \boldsymbol{0}, \tag{7c}$$

where $\boldsymbol{\pi}$ is a set of algebraic unknowns (design variables), and J^{NLP} is a scalar objective function which represents an approximation of the cost of Equation (2a) or (3a). The equality constraints (7b) are generated by the discretization of the equations of motion (2b) or (3b), whereas the inequality constraints (7c) by the discretization of Equations (2d) or (3d).

Three discretization techniques are available in `STOP`, namely the direct transcription and multiple shooting methods [13] and the recently developed hybrid single-multiple shooting [15]. In [11], the authors propose a classification of optimal areas of applicability of these methods and, for each one of them, derive the specific form of the vector of design variables.

Time marching can be based either on algorithms available in external vehicle models, or with built-in explicit or implicit time solvers.

The vehicle models include an optional layer that models the pilot, which is useful in certain MOCP applications for computing maneuvers considering pilot-in-the-loop effects [12]. The vehicle itself can be simulated using an internal model, or by external codes through a generic interface which supports all necessary operations.

3 GLOBAL SOLUTION OF MOCPs and PEPs

In this section we develop a formulation that makes use of EAs for the solution of problem (7) in the context of MOCPs and PEPs.

When using EAs, the computational cost is proportional to the population size. In general, if the population size is too small, then EA might not be able to thoroughly explore the solution space; on the other hand, increasing the population size generally enables EA to obtain better results. However, it is clear that, the larger the population size, the longer it takes EA to compute each generation. As a rule-of-thumb, population size is usually set to 5–10 times the number of design variables [5].

In light of these considerations, for the solution of optimal control problems using EA in rotorcraft flight mechanics, we favor the use of shooting methods over direct transcription ones, since the latter tend to generate potentially large NLP problems [11]. However, even in this case, we are faced with a problem of possibly overwhelming computational cost. In fact, when using shooting methods, the problem unknowns are defined as the discrete values of the states at the interfaces between shooting segments, the discrete values of the controls within each segment, and possibly the final time. Thus, one would need to consider extremely large populations in order to cover all possible feasible values of the design variables.

Another challenging task in the framework of EAs is the satisfaction of the gluing constraints on the states, which ensure the continuity of their time history across the boundaries of the shooting segments (see Figure 2), and of all other problem constraints, since EAs can tackle only unconstrained optimization problems.

To address these issues, we propose a procedure based on a split of the design variables: EA is used to compute an optimal solution in terms of the sole controls and/or initial conditions, whereas the feasibility of the computed solution is ensured through the use of a Repair Heuristic (RH) [17].

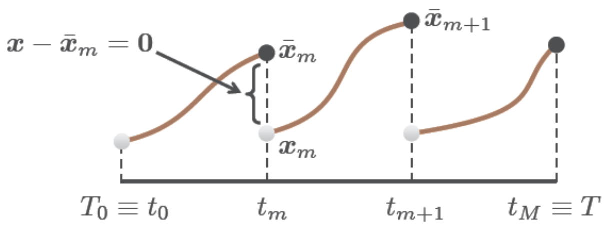

Figure 2: Basic principle of the multiple shooting method. The time domain Ω is partitioned as $T_0 \equiv t_0 < t_1 < \ldots < t_m < \ldots < t_M \equiv T$ with $\Omega^m = [t_m, t_{m+1}]$, $m = (0, M-1)$, where each Ω^m is a shooting segment. Next, the vehicle equations of motion are marched forward in time within each shooting segment Ω^m, starting from the initial conditions provided by the value of the states $\boldsymbol{x}_m$ at the left boundary of the segment. Segments are then glued together by imposing the equality constraints $\boldsymbol{x}_m - \bar{\boldsymbol{x}}_m = \mathbf{0}, m = (1, M)$.

The solution of problem (7) proceeds as follows. First, the temporal domain $\Omega = [T_0, T]$ is partitioned into N intervals $T^i = [t_i, t_{i+1}]$ of size h^i, $i = (0, N-1)$. A computational grid $\mathcal{T}_h^{\text{EA}}$ is associated with this partition. Since T is in general unknown, it is convenient to consider a fixed time domain by introducing the map $s\colon \Omega \to [0, 1]$, $s = (t - T_0)/(T - T_0)$, for all $t \in \Omega$. This yields the following expression for the step length $h^i = (T - T_0)(s_{i+1} - s_i)$, $i = (0, N-1)$. Typically we consider uniform grids and hence we simply have $h^i = h = (T - T_0)/N$.

Next, for every individual in the population, depending on the problem at hand, the set of design variables is chosen to include one or more of the following quantities: all values of the controls at the nodes of $\mathcal{T}_h^{EA}$, the problem initial conditions (if free), possibly the final time (if unknown). In order to limit the population size we consider coarse temporal grids, i.e. with a reduced number of nodes.

Remark 1 *Evolution Strategies (ESs) [8] use a representation (encoding) of the design variables as real numbers. We have observed that in the case of MOCPs, where very small variations in the controls or in the maneuver duration typically result in small (negligible) variations in the objective function, such encoding might lead to the premature stop of the algorithm due to an excessively low diversity of the individuals in the population. We have found that this problem is somehow alleviated by decreasing the resolution with which EA explores the solution space, which can be achieved by forcing the problem unknowns to take only values specified at a small number of equidistant points between their lower and upper bounds. For the generic variable* π_i *lying in the interval* $I_{\pi_i} = [\pi_i^{\min}, \pi_i^{\max}]$*, the set* Π_i *of admissible values is constructed as* $\Pi_i = \left\{\pi_i^{\min} + j\Delta\pi_i, j = 0, \ldots, L\right\}$*, with*

$L = (\pi_i^{\max} - \pi_i^{\min})/\Delta\pi_i$*, where* $\Delta\pi_i$ *is the resolution of the discretization of* I_{π_i}*. This turns the original problem into a discrete combinatorial optimization problem.*

Controls computed by EA are then projected onto a finer grid $\mathcal{T}_h^{\mathrm{RH}}$, associated with a partition of Ω into M shooting segments and a suitable discretization of the controls within each segment. This projection can be expressed with the notation

$$\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}} = \mathcal{P}(\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{EA}}}), \tag{8}$$

where $\mathcal{P}(\cdot)$ is an appropriate projection operator. The time history of vehicle states that are compatible with the given control policy and maneuver-defining constraints is found from the solution of the following NLP problem:

$$\min_{\boldsymbol{\theta}} \quad K(\boldsymbol{\theta}, \boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}}), \tag{9a}$$

$$\text{s.t.} \quad \boldsymbol{g}(\boldsymbol{\theta}, \boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}}) = \boldsymbol{0}, \tag{9b}$$

$$\boldsymbol{\theta} \in [\boldsymbol{\theta}^{\min}, \boldsymbol{\theta}^{\max}], \tag{9c}$$

where the unknowns $\boldsymbol{\theta}$ are the values of the states at the interfaces between shooting segments. Objective function K represents a measure (in a given norm) of the violation of constraints (7b) and (7c) expressed in terms of $\boldsymbol{\theta}$ and $\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}}$, whereas Equation (9b) represents the gluing constraints, which are evaluated by marching the vehicle equations of motion forward in time under the action of the controls $\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}}$. Problem (9) is solved using a SQP method with Jacobians computed through centered finite differencing by perturbation of the unknowns [6].

Finally, cost J^{NLP} of Equation (7a) is evaluated using the computed time histories of the states and corresponding outputs within each shooting segment.

Remark 2 *Solving problem (9) might be a time consuming process and hence the SQP method is typically run only for a limited number of iterations. When achieving a feasible solution within the specified maximum number of iterations is difficult, RH is allowed a limited authority on the inputs by modifying the given control time history as*

$$\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}} + \Delta\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}}, \tag{10}$$

where $\Delta\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}}$ *are bounded corrective terms, i.e.* $\Delta\boldsymbol{u}|_{\mathcal{T}_h^{\mathrm{RH}}} \in [\Delta\boldsymbol{u}^{\min}, \Delta\boldsymbol{u}^{\max}]$*, that are computed as part of the solution to problem (9). However, the repaired individuals are never returned to the population (never replacing approach [17]).*

The solution of PEPs can be developed along similar lines, although things are simpler in this case given the fact that control inputs are known. Therefore, the global optimizer operates at the level of the model parameters, while the local optimizer is used for the satisfaction of the gluing constraints at the interface between shooting arcs.

4 NUMERICAL APPLICATIONS

In this section we present the application of `STOP`, equipped with the proposed global optimization procedure, to the solution of problems arising in the context of rotorcraft flight mechanics applications.

Vehicle model equations are derived based on three-dimensional rigid body dynamics. Rotor forces and moments are computed analytically by combining actuator disk and blade element theory, considering a uniform inflow [23]. The rotor attitude is evaluated by means of quasi-steady flapping dynamics with a linear aerodynamic damping correction. Look-up tables are used for the quasi-steady aerodynamic coefficients of the vehicle lifting surfaces, and simple corrections for compressibility effects and for the downwash angle at the tail due to the main rotor are included in the model. Further details on the model structure are given in [10, 21].

4.1 *Design of Optimal Inputs for Parameter Estimation of a Small Autonomous Helicopter*

We consider the design of globally optimal input signals for parameter estimation flight trials. This example is chosen here because it combines the characteristics of a MOCP with those of a PEP. In fact, in this case, the idea is to formulate a MOCP that maximizes the identifiability of a given set of parameters in the vehicle model.

Multistep inputs are commonly used input signals during flight tests for parameter estimation. They consist of a sequence of alternating positive and negative amplitude pulses with different duration. In this case, the design problem consists in finding the optimal amplitude and width of the pulses so that the information content in the data from the experiment, as embodied in the Fisher information matrix [19], is maximized. Therefore, for this problem the EA optimization variables are represented by the control inputs, while the SQP optimizer is used for compatibilizing the vehicle states. Solutions are obtained with the self-adaptive EA implemented in the commercial software `OPTIMUS` [3].

We consider a small-size RUAV; the test condition is a forward level flight at a very low advance ratio, $\mu = 0.04$ ($V = 5\,\mathrm{m/s}$). Only the longitudinal dynamics of the vehicle is considered. We start from the controls set to the trim value and perturb the main rotor longitudinal cyclic, while holding the others fixed. Control perturbations are restricted to lie in the interval $I_A = [-1, 1]$ deg. For such an experiment, the model parameters of principal interest are the main rotor aerodynamic parameters, namely the rotor blade lift curve slope C_{L_α} and mean drag coefficient C_D.

The interval I_A is discretized with a resolution of $0.25\,\mathrm{deg}$ and the population size is set to 100. The computed optimal input is shown in Figure 3. Table 1 gives the results of the maximum likelihood estimation of the main rotor aerodynamic parameters for the optimal multistep input maneuver. Results are compared with those for the DLR 1-1-2-3 input[1], a widely known input form that has been shown to be very effective for flight vehicle parameter estimation [18]. The accuracy of the estimate of the blade lift curve slope is increased by 18% with respect to the 1-1-2-3 value. Figure 4 compares the Power Spectral Density (PSD) of the closed-loop swashplate deflection for the optimal multistep and 1-1-2-3 inputs. Notice how the optimal input better excites the low frequencies, leading to experimental data with a higher information content that yield more accurate parameter estimates.

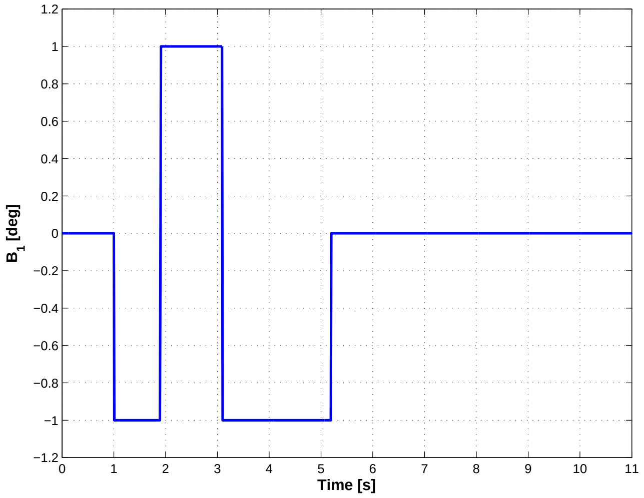

Figure 3: Optimal longitudinal cyclic input.

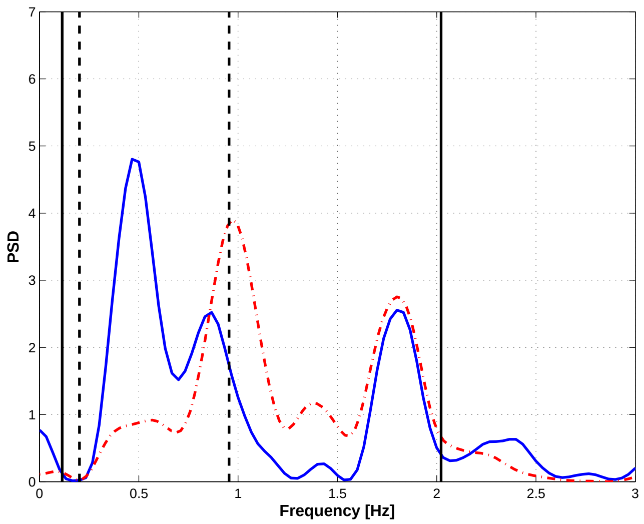

Figure 4: Power spectral density of closed-loop swashplate deflection. Solid line: optimal input. Dash-dotted line: 1-1-2-3 input.

[1]We consider a 1-1-2-3 input sequence with a time step length of 0.6 s and an amplitude of 1 deg. It was found in [21] that such an input allows for the adequate excitation of the vehicle natural frequencies.

4.2 *Category A Continued Take-Off Maneuver for a Medium-Size Helicopter*

In this section we consider the take-off maneuver for a ten ton twin-engine four-bladed helicopter model under Category A certification requirements [1]. In order to meet the certification requirements, the helicopter must be able to safely take-off after an engine failure occurring after the decision point has been reached. Such an emergency maneuver was previously studied in [9, 10].

The goal here is to investigate which is the best initial condition in terms of climb velocity W, horizontal velocity U and heading angle ψ that produces the minimum altitude take-off decision point. The presence of heading implies a non-planar solution and it is also associated with the pilot visibility. In this case, the EA optimization variables are represented by the initial conditions, and the SQP optimizer deals with control inputs, vehicle states and maneuver duration. Solutions are computed with the self-adaptive EA implemented in the commercial software NEXUS [4].

We performed two different global optimization runs. In the first case, the search region for the three EA variables is within the following ranges:

$$U \in [-2, 0] \text{ KTS}, \quad (11a)$$
$$W \in [-5, 0] \text{ KTS}, \quad (11b)$$
$$\psi \in [-45, 0] \text{ deg}, \quad (11c)$$

with a resolution of 1 KTS for the velocities and 5 deg for the heading angle.

The optimization cost function accounts for the control input rates and the initial vertical position H (take-off decision point altitude), i.e. it reads

$$J^{\text{MOCP}} = -H + \frac{1}{T}\int_T \dot{\boldsymbol{u}} \cdot \boldsymbol{W}\dot{\boldsymbol{u}}\, \mathrm{d}t. \quad (12)$$

The principal optimization constraints are: the minimum rotor speed should not fall below 90% of the nominal value, ground clearance should be at least 15 ft, minimum pitch angle should not exceed -10 deg. The exit conditions are: a climb velocity of 100 ft/min, rotor speed at 100% of the nominal value and null angular velocities. The starting guess is provided by a previously calculated STOP solution with a heading of -45 deg and null vertical and horizontal velocity components.

Results in terms of minimum altitude H vs. heading ψ, vertical velocity W and backup speed U are plotted in Figure 5, from top to bottom, respectively. Each point in the graphs represents an individual in the population generated by the EA solver. The optimal solution uses the maximum available climb velocity, a null horizontal speed (i.e., vertical climbing) and a heading of 35 deg.

This problem nicely illustrates the danger of remaining trapped in a local minimum when using local optimizers. In fact, the same problem was solved again

	1-1-2-3		Optimal Multistep	
Parameter	Est. Value	Std. Dev.	Est. Value	Std. Dev.
$C_{L_{\alpha\,\mathrm{MR}}}$ [rad^{-1}]	5.62012	0.00576	5.61134	0.00472
$C_{D_{\mathrm{MR}}}$	0.00974	0.00034	0.00973	0.00033

Table 1: Estimated value and standard deviation of main rotor aerodynamic parameters for 1-1-2-3 and optimal longitudinal cyclic inputs.

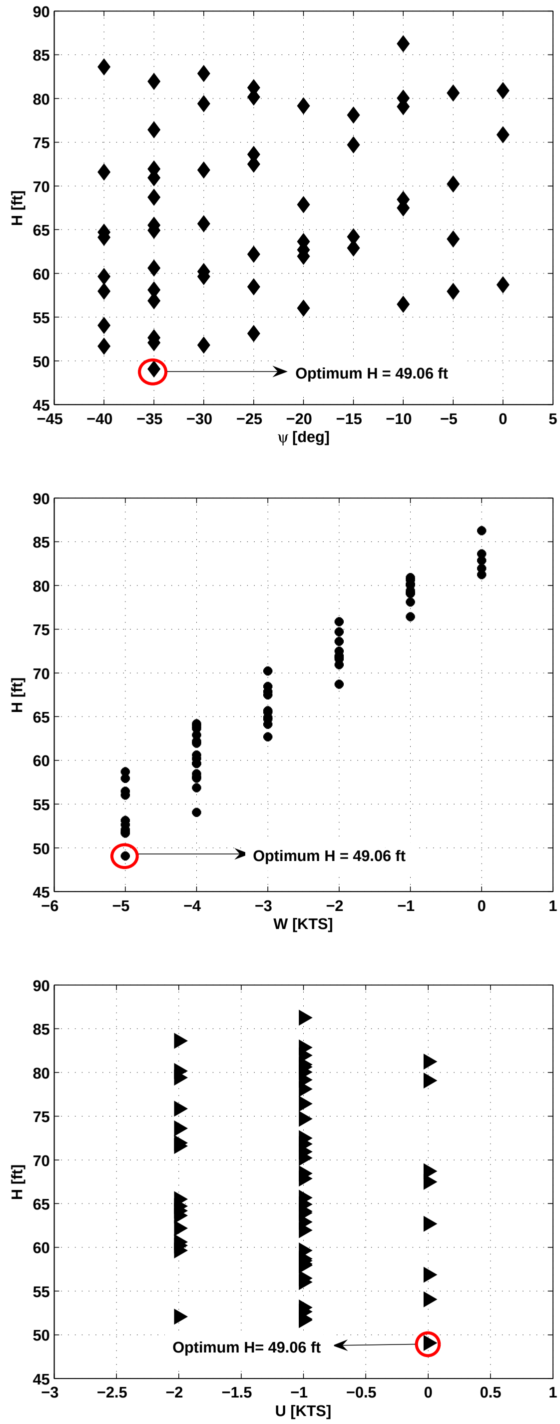

Figure 5: Category A continued take-off. Minimum altitude H vs. heading ψ (top), vertical velocity W (middle) and horizontal speed U (bottom).

using SQP, for $U = 0$ KTS, $W = -5$ KTS (the global optimum values) and with an initial heading constrained between 0 and -45 deg. The converged SQP solution has an initial heading of -45 deg, which however exhibits a higher associated take-off decision point. This means that the EA solution is a lower minimum than the one found by SQP.

To better illustrate the possible presence of local minima in rotorcraft flight mechanics optimal control problems, the Category A optimization was repeated again. The range of yaw angles was set to be between -45 deg and 45 deg, with a resolution of 1 deg, while the velocity components were held fixed at their global optimum values ($U = 0$ KTS, $W = -5$ KTS). Furthermore, the heading angle was added to the cost function so as to try to reduce it, since pilots typically prefer to work with small yaw angles (which improve visibility), resulting in the new cost

$$J^{\mathrm{MOCP}} = -H + \frac{1}{T}\int_T (w_\psi \psi^2 + \dot{\boldsymbol{u}} \cdot \boldsymbol{W} \dot{\boldsymbol{u}})\, \mathrm{d}t. \quad (13)$$

The results, illustrated in Figure 6, show that there is a global optimum at -43 deg and local optima at 23 and 31 deg; notice that such minima are present both in the cost function (top plot) and in the take-off decision point altitude (bottom plot). Some scatter of the points on the plots are due to generous tolerances in the solution of the SQP problems. We remark that any point in the plots is an optimal solution for the local optimizer, which again highlights the potential danger of being trapped in a local minimum when using non-global optimization algorithms.

5 CONCLUSIONS

In this paper we have presented a numerical procedure for the global solution using evolution algorithms of trajectory optimization problems in rotorcraft flight mechanics. Based on our experience, local minima are usually not a major issue for many flight mechanics optimization problems, in the sense that one is typically able to compute solutions of engineering interest by simply using gradient-based methods. However, as the applicability of such techniques to a variety of rotorcraft flight mechanics problems is progressively expanded, it becomes important to guarantee that one is not missing

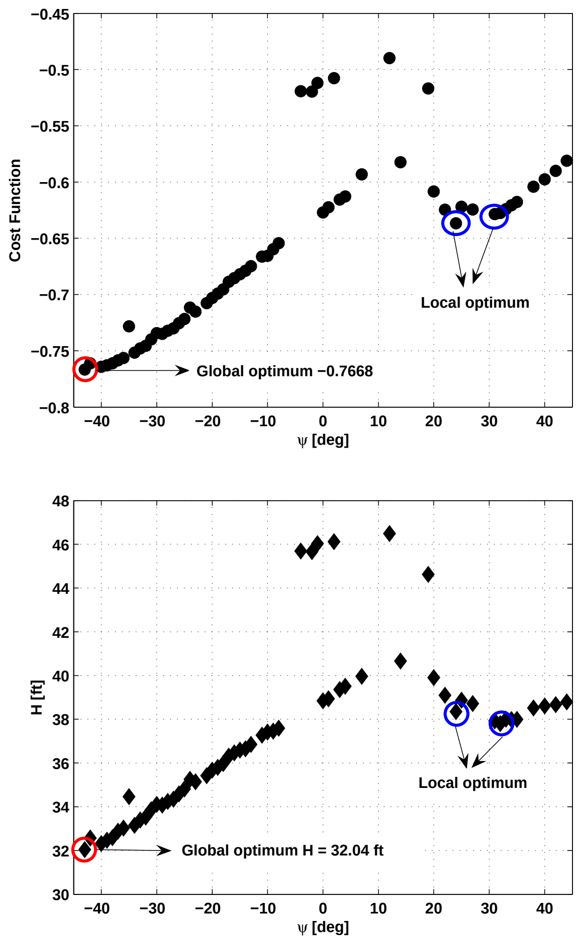

Figure 6: Category A continued take-off. Cost function (top) and take-off decition point altitude (bottom) vs. initial heading angle ψ.

relevant and better solutions. Furthermore, a well performing global optimizer reduces the need of generating good quality initial guesses, which is sometimes hard and is often a problem dependent issue.

The proposed approach makes use of a global EA coupled to a repair heuristic which ensures the feasibility of the computed solution. Using a repair heuristic reduces the EA search space to feasible solutions only; hence, no special evolutionary operators or modifications of the objective function need to be considered. This way, the resulting code can be used to solve different classes of optimization problems, including optimal control and parameter estimation ones.

Two application problems have been used for demonstrating the proposed methodology. In the first, we have designed a control input time sequence for identification trials of a small unmanned helicopter, that improves the commonly adopted 1-1-2-3 signal. In the second, we have shown that Category A take offs can present multiple local minima when the initial heading is considered.

The applications proposed in this work constitute preliminary results, that however allow one to draw some conclusions. In particular, they suggest that the coupling of a global optimizer like EA with a local optimizer based on SQP allows one to effectively explore the space of solutions. Typically, for this to work one has to avoid conflicts between the two optimizers, which therefore work on different sets of variables: a small set that includes control inputs, model parameters, initial conditions, etc., for the global optimizer, and the remaining set for the local optimizer, which is in charge of satisfying all nonlinear constraints.

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REFERENCES

[1] *Advisory Circular 29-2C, Certification of Transport Category Rotorcraft*, Federal Aviation Administration, Department of Transportation, USA, 1999.

[2] *Handling Qualities Requirements for Military Rotorcraft, Aeronautical Design Standard*, U.S. Army Aviation and Missile Command, Aviation Engineering Directorate, Rept. ADS-33E-PRF, Redstone Arsenal, AL, USA, 2000.

[3] `OPTIMUS`, Optimization Environment, Ver. 5.3. Noesis Solutions, Belgium, 2007. `http://www.noesissolutions.com`.

[4] `NEXUS`, iChrome Ltd. `http://ichrome.eu/nexus/overview`.

[5] T. Bäck, D. Fogel, Z. Michalewicz, editors. *Handbook of Evolutionary Computation*. Oxford University Press, New York, 1997.

[6] A. Barclay, P.E. Gill, and J.B. Rosen. SQP Methods and their Application to Numerical Optimal Control. Report NA 97-3, Department of Mathematics, University of California, San Diego, 1997.

[7] J.T. Betts. *Practical Methods for Optimal Control Using Non-Linear Programming*. SIAM, Philadelphia, 2006.

[8] H.-G. Beyer, H.-P. Schwefel. *Evolution Strategies: A Comprehensive Introduction*. Natural Computing, **1**, 3–52, 2002.

[9] C.L. Bottasso, A. Croce, D. Leonello, L. Riviello. Optimization of Critical Trajectories for Rotorcraft Vehicles. *Journal of the American Helicopter Society*, **50**, 165–177, 2005.

[10] C.L. Bottasso, A. Croce, D. Leonello, L. Riviello. Rotorcraft Trajectory Optimization with Realizability Considerations. *Journal of Aerospace Engineering*, **18**, 146–155, 2005.

[11] C.L. Bottasso, F. Luraghi, G. Maisano. A Unified Approach to Trajectory Optimization and Parameter Estimation in Vehicle Dynamics. Keynote lecture, CMND 2009, International Symposium on Coupled Methods in Numerical Dynamics, Split, Croatia, September 16-19, 2009.

[12] C.L. Bottasso, G. Maisano, F. Scorcelletti. Proceedings of AHS 65th Annual Forum and Technology Display, Grapevine, TX, USA, May 27–29, 2009.

[13] C.L. Bottasso, G. Maisano, F. Scorcelletti. Trajectory Optimization Procedures for Rotorcraft Vehicles, their Software Implementation, and Applicability to Models of Increasing Complexity. *Journal of the American Helicopter Society*, **55**, 2010, doi: 10.4050/JAHS.55.032010.

[14] C.L. Bottasso, F. Luraghi, A. Maffezzoli, G. Maisano. Parameter Estimation of Multibody Models of Unstable Systems from Experimental Data, with Application to Rotorcraft Vehicles. *Journal of Computational and Nonlinear Dynamics*, **5**, 2010, doi: 10.1115/1.4001390.

[15] C.L. Bottasso, F. Luraghi, G. Maisano. Efficient Rotorcraft Trajectory Optimization Using Comprehensive Vehicle Models by Improved Shooting Methods. *CEAS Aeronautical Journal*, to appear.

[16] R. Celi. Analytical Simulation of ADS-33 Mission Task Elements. Proceedings of AHS 63rd Annual Forum and Technology Display, Virginia Beach, VA, USA, May 1–3, 2007.

[17] C.A. Coello Coello. *Theoretical and Numerical Constraint-Handling Techniques used with Evolutionary Algorithms: A Survey of the State of the Art*. Computer Methods in Applied Mechanics and Engineering, **191**, 1245–1287, 2002.

[18] R.V. Jategaonkar. *Flight Vehicle System Identification. A Time Domain Approach*. AIAA Progress in Astronautics Aeronautics, Reston, VA, USA, 2006.

[19] S. Kullback. *Information Theory and Statistics*. John Wiley & Sons, New York, 1959.

[20] F. Luraghi. *Time-Domain Parameter Estimation Techniques for First-Principle Rotorcraft Models*. Ph.D. thesis, Politecnico di Milano, Dipartimento di Ingegneria Aerospaziale, Milano, Italy, 2009.

[21] A. Maffezzoli. *Procedures for the Estimation of Model Parameters for a Small Rotorcraft UAV*. M.Sc. thesis, Politecnico di Milano, Dipartimento di Ingegneria Aerospaziale, Milano, Italy, 2009.

[22] C. Onnen, R. Babuška, U. Kaymak, J.M. Sousa, H.B. Verbruggen, R. Isermann. Genetic Algorithms for Optimization in Predictive Control. *Control Engineering Practice*, **5**, 1363–1372, 1997.

[23] R.W. Prouty. *Helicopter Performance, Stability, and Control*. R.E. Krieger Publishing Co., Malabar, FL, 1990.

[24] C. Seren, F. Bommier, A. Bucharles, L. Verdier, D. Alazard. Flight Test Protocol Optimization Using Genetic Algorithms. Proceedings of 14th IFAC Symposium on System Identification, Newcastle, Australia, March 29–31, 2006.

[25] F.S. Wang, J.P. Chiou. Optimal Control and Optimal Time Location Problems of Differential-Algebraic Systems by Differential Evolution. *Industrial & Engineering Chemistry Research*, **36**, 5348–5357, 1997.