FOURTH EUROPEAN ROTORCRAFT AND POWERED LIFT AIRCRAFT FORUM

Paper No. 9

GREEN'S FUNCTION METHOD FOR COMPRESSIBLE UNSTEADY POTENTIAL AERODYNAMIC ANALYSIS OF ROTOR - FUSELAGE INTERACTION

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September 13 - 15, 1978

STRESA - ITALY

Associazione Italiana di Aeronautica ed Astronautica

Associazione Industrie Aerospaziali

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1. Introduction

This paper deals with a very general theory of unsteady compressible potential aerodynamics of helicopters in hover and forward flight. Numerical results for an isolated rotor in steady and unsteady incompressible flows are also presented.

The formulation is very general (the only restriction is the assumption of potential aerodynamics) and is based upon the integral equation developed by Dr. Luigi Morino (Refs. 1 to 3; see also Section 3) for the exact nonlinear compressible three-dimensional unsteady velocity-potential equation for lifting bodies having arbitrary shapes and motions. For incompressible flows the formulation has been applied to rotor-fuselage helicopter configurations (Refs. 4 to 6), and to windmills (time-domain analysis of unsteady flows, Refs. 7 to 12). Unsteady compressible (subsonic and supersonic) flows have been considered for fixed-wing flexible aircrafts having arbitrary shapes and motions (Refs. 13 to 20); a very general purpose computer program, SOUSSA-P (Steady, Oscillatory and Unsteady, Subsonic and Supersonic Aerodynamics; Production version), has been completed (Refs. 21 and 22); time-domain nonlinear analysis is considered in Ref. 23. The integral equation of Ref. 1 is directly applicable to helicopters and is exact: nonlinear terms and moving shock waves are included in the formulation. The only approximations are due to the numerical implementation and are such that the error goes to zero as the number of unknowns goes to infinity. It is important to note that, despite the complete generality of the method, the resulting formulation is computationally very efficient as demonstrated by the results presented in Refs. 1 to 23.

An efficient and general method for unsteady, compressible potential aerodynamic analysis of helicopters in hover or forward flight is presented here. The availability of such a method (and corresponding computer program) would enhance considerably the present computational capability for an accurate evaluation of pressure and flow fields. Such evaluation is required for instance for the problem of drag-reduction (badly needed because of the energy problem) as a prerequisite for the boundary-layer analysis. Accurate pressure evaluation for compressible flow is also badly needed for flutter analysis (see Ref. 24). Therefore the effects of compressibility is analyzed rigorously for a helicopter in forward flight. The formulation is presented here in Sections 3 and 4 after a brief review of the state of the art. Numerical results are presented in Section 5.

2. Review of the state of the Art

An excellent review on aerodynamic technology for advanced rotor craft was presented by Landgrebe, Moffitt and Clark in Refs. 25 and 26. Additional reviews are presented in Refs. 24, 27-30. Compressibility effects in particular are reviewed in Ref. 30. Therefore only works which are not reviewed in Refs. 24 to 30 and which are relevant to the objective and the motivation of the present work are included in this brief review, which is not to be considered, by any means, complete.

Three items which are relevant to this paper and which need a discussion deeper than the ones presented in Refs. 24 to 30 are advanced computational methods (lifting-line, lifting-surface and panel methods), compressibility and wake roll-up. These items are briefly examined in the following.

Consider first the advanced computational methods. Two important ifting-line methods were published recently. The first one (Ref. 27) "points out several errors in the usual lifting-line methods of rotor analysis", and presents a higher-order lifting-line theory which takes into account all the unsteady, yawed effects encountered by helicopter blades. Unfortunately the theory does not include compressibility. Compressibility is considered in the second work (Ref. 28) which includes a lifting-line as well as a disk method. More advanced methods (lifting-surface theories) are presented in Refs. 29 and 30. Both methods are of interest here. The first (Summa, Ref. 29) introduces a time-domain analysis, but is limited to incompressible flow. The second one (Rao and Schatzle, Ref. 30) introduces in a simplified form (local Prandtl-Glauert, chordwise transformation) the effect of compressibility for rotors in unsteady flow (in addition, the importance of the correct geometry of the wake are clearly demonstrated in Ref. 30). A lifting-surface theory was presented in Refs. 8 to 12 (along with a more complex panel method based upon Green's function method). The lifting-surface method (program WILSA*) is very similar to the ones of Ref. 29 and 30. Results obtained with WILSA are very close to the results obtained by Rao and Schatzle[30]. Finally panel method designates a new methodology recently introduced in aircraft aerodynamics. This methodology consists in the finite-element solution of integral equations (over the actual surface of the body**) for potential aerodynamics. Typically, the surface of the aircraft and its wake is covered with source-panels (doublet-, vortex- and pressure-panels are also used). The intensity of the source distribution is obtained by imposing that the flow does not penetrate the surface of the body (a brief outline of panel aerodynamics is presented in Section 3). The most general formulation for panel aerodynamics is the Green's Function Method (Refs. 1 to 23; see also Sections 1 and 3). Other methods are briefly presented here. An early work on the flow field around three-dimensional bodies by Hess and Smith (Ref. 31) uses constant strength source-elements to solve the problem of steady subsonic flow around nonlifting bodies. This method has been extended to lifting bodies (Refs. 32-34) by including doublet and vortex panels. Woodward's method for steady subsonic and supersonic flow (Refs. 35 and 36) is a different approach that uses lifting surface elements for the representation of the body. Little work has been done in unsteady flow around complex configurations besides the work of Refs. 1 to 23: for oscillatory subsonic aerodynamics, extensions of the doublet-lattice method (Ref. 37) are obtained by either placing unsteady lifting surface elements on the surface of the body or by using the method of images combined with slender body theory (Ref. 38). In the supersonic range, complex configurations are analyzed in Refs. 35, 39 and 40. The program WBAERO (Ref. 41) is a modification of the program by Rubbert and Saaris (see Ref. 32), which in turn is based on the original program by Hess and Smith (Ref. 31). Applications of panel methods to helicopter aerodynamics include the work by Dvorak, Maskew and Woodward (Ref. 42) which present a method for calculating the complete pressure distribution on a helicopter fuselage with separated flow (the method uses WBAERO for the potential flow solution; the boundary layer is calculated up to the separation line; separated flow is modeled by streamwise panels of uniform vorticity). Helicopter applications of panel methods are also given in the already-mentioned work by Soohoo, Morino, Noll and Ham[4,5]: Ref. 4 presents an extension of the Green's Function Method to actuator disks with application

*Windmill Incompressible Lifting Surface Aerodynamics

**Note the difference with respect to the lifting surface formulations, in which the integral equation is over the mean-surface of the configuration. Panel method here indicates only those methods in which the actual surface is used.

to tilting proprotor aircraft, whereas Ref. 5 presents an extension of the method to study the rotor-wake effects on hub/pylon flow separation. Both works indicate that the Green's Fucntion Method is very powerful for applications to helicopter potential aerodynamics, with rotor/fuselage interference. The above remarks indicate that panel-aerodynamics methods are becoming available for the analysis of the complete configuration. The availability of such methods (and corresponding computer programs) enhances considerably the present computational capability for an accurate evaluation of pressure and flow fields. This evaluation is becoming more and more important because of a recent change in pattern in the field of helicopter aerodynamics. Wind tunnel experiments are very costly whereas computers are becoming less and less expensive*. Therefore the use of computers is becoming more attractive for the aerodynamic analysis of helicopter configuraitons. For instance, items such as higher performance (lower drag, higher speed, higher lift, higher reliability) requires more theoretical analysis: in particular as mentioned above, reduction of drag (badly needed because of the too well known energy problem) requires a very accurate evaluation of the flow field including separation of the potential field as a prerequisite for the boundary-layer analysis (Ref. 42). Therefore panel-aerodynamics methods deserve further attention: in particular the effects of compressibility and wake roll-up (examined in this paper) should be included in the panel method. (It should be noted that panel methods require an amount of computer time of the same order of magnitude as the lifting surface methods. This was clearly illustrated by the comparison between the lifting-surface program and the panel-method program, which is presented in Ref. 8.)

Next consider the effect of the compressibility of the air. The importance of compressibility was clearly demonstrated by Friedman and Yuan[24] for the problem of aeroelastic stability (i.e. flutter and divergence) of rotor blades. The work is based on simple aerodynamic strip-theories (Refs. 45 to 51). However the same effect is expected from more sophisticated unsteady three-dimensional compressible theories. Compressibility effects are included in the lifting-line theory by Johansson[28] and in the lifting-surface method by Rao and Schatzle[30]. However the theory of Ref. 28 is limited to steady flow. The work of Ref. 30 is more interesting and is based upon a theory developed by Rao and Jones[52]: However the work is limited to rotors in hover and it is apparent, from the more general theory developed by Dr. Morino[1-3], that the method used by Rao and Schatzle[30] is not applicable to rotors in forward flight, since in this case the time delays, θ , (from the dummy point of integration to the control point, see Section 3) are a complicated function of the motion of the rotor and cannot be obtained through the simple chordwise Prandtl-Glauert transformation used in Ref. 30. A possible alternative approach is the numerical solution of the differential equation using for instance finite differences (Caradonna et al., Refs. 53-55): this method however is extremely time consuming. Therefore the use of the correct integral equation proposed here (Section 3) appears to be considerably better (either for higher accuracy or for less computer time) than all the other existing methods.

Finally consider briefly the third item, the wake roll-up. Analytical methods for predicting the geometry of the rotor wake were developed by Landgrebe[56,57] and by Crews, Hohenemser and Ormiston[58]. Landgrebe model was used by Rao and Schatzle[30], in their lifting surface theory, to show that

*Chapman, Mark and Pirtle[43,44] of NASA Ames Research Center estimate that wind-tunnel simulation will be replaced by computer simulation in about ten years, if the present trend in computers continues.

a considerable improvement in the comparison with the experimental results of Ref. 59 may be obtained simply by using the correct wake geometry. Automatic generation of the wake is considered for instance by Scully[60].

3. Integral Equation

The perturbation velocity potential φ for a flow having free-stream velocity U_∞ in the direction of the positive x-axis is given by

$$\nabla^2\varphi - a_\infty^{-2}\left(\partial/\partial t + U_\infty\,\partial/\partial x\right)^2\varphi = F \tag{1}$$

where F is the contribution of the nonlinear terms. Let the surface of the body be described in the general form

$$S(x,y,z,t) = 0 \tag{2}$$

Then the boundary conditions on the body are given by DS/Dt=0 or

$$\psi = \partial\varphi/\partial n = \nabla\varphi\cdot\nabla S/|\nabla S| = -\left(\partial S/\partial t + U_\infty\,\partial S/\partial x\right)/|\nabla S| \tag{3}$$

In order to solve this problem it is convenient to transform it into an integral equation. Consider the Green function of the linearized equation of the subsonic potential

$$G = -\delta(t - t_* + \theta)/4\pi r_\beta \tag{4}$$

where δ is the Dirac delta function and

$$r_\beta = \left\{(x-x_*)^2 + \beta^2\left[(y-y_*)^2 + (z-z_*)^2\right]\right\}^{1/2} \tag{5}$$

$$\theta = \left[M(x-x_*) + r_\beta\right]/a_\infty\beta^2 \tag{6}$$

By using the classical Green theorem method one obtains (Refs. 1 to 3)

$$4\pi E\varphi(P_*,t_*) = -\oiint_{\sigma^\theta}\left[\left(\nabla S\cdot\nabla\varphi - \frac{1}{a_\infty^2}\frac{dS}{dt}\frac{d\varphi}{dt}\right)\frac{1}{r_\beta} - \left(\nabla S\cdot\nabla\frac{1}{r_\beta} - \frac{1}{a_\infty^2}\frac{dS}{dt}\frac{d}{dt}\frac{1}{r_\beta}\right)\varphi\right]^\theta\frac{d\sigma^\theta}{|\nabla S|}$$
$$-\frac{\partial}{\partial t_*}\oiint_{\sigma^\theta}\left[\nabla S\cdot\nabla\theta - \frac{1}{a_\infty^2}\frac{dS}{dt}\left(1 + U_\infty\frac{\partial\theta}{\partial x}\right)\right]^\theta\left[\varphi\right]^\theta\frac{1}{r_\beta}\frac{d\sigma^\theta}{|\nabla S|} - \iiint_{-\infty}^{\infty}\left[EF\right]^\theta\frac{1}{r_\beta}\,dV \tag{7}$$

where $d/dt = \partial/\partial t + U_\infty\,\partial/\partial x$ and

$$\begin{aligned} &E=0 \quad \text{inside } \sigma \\ &E=\tfrac{1}{2} \quad \text{on } \sigma \\ &E=1 \quad \text{outside } \sigma \end{aligned} \tag{8}$$

and $[\;]^\theta$ indicates evaluation at $t=t_*-\theta$. Similarly, σ^θ is the surface given by

$$S^\theta = S(x,y,z,t_*-\theta) = 0 \tag{9}$$

where (x,y,z) is the dummy point of integration on σ^θ. If initial conditions are prescribed Eq. (7) should be modified as indicated in Appendix F of Ref. 3.

For simplicity only the linear problem (F=0) is considered here. (The method of solution for the nonlinear problem is similar to the one of Ref. 23.) In this case, if the point is on the surface S=0, Eq. (7) yields an integro-differential equation which may be used for evaluating the value of φ on the surface. It may be noted that Eq. (7) is very general since it is valid even if the surface of the body is moving in time with respect to the frame of reference (helicopter blades, spinning missiles, etc.).

4. Numerical Formulation

Equation (7) is the basic equation for the analysis of unsteady compressible potential aerodynamics of helicopters. The contribution of the wake is discussed in Refs. 1 to 3 and is not repeated here. The contribution of the shock wave is similar and yields a source distribution over the surface of the shock wave. The approximate solution of Eq. (7) for arbitrary shapes may be obtained numerically. For simplicity only the numerical formulation for incompressible flow (used to obtain the results presented in Section 5) is presented here; the numerical formulation for compressible flow is similar to the one of Ref. 23, and is a simple extension of the one presented here.

Isolating the contribution of the wake and assuming P_* to be on σ, Eq.(7) yields (for incompressible flow)

$$2\pi\varphi(P_*,t) = -\oint\!\!\oint_{\Sigma_B}\left[\psi\,\frac{1}{r} - \varphi\,\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right]d\Sigma_B - \iint_{\Sigma_W}\Delta\varphi\,\frac{\partial}{\partial n}\left(\frac{1}{r}\right)d\Sigma_W \tag{10}$$

where Σ_B is the (closed) surface of the body Σ_W is the (open) surface of the vortex-layer wake emanating from the trailing edge. In addition $\Delta\varphi$ is the potential discontinuity on the wake and satisfies the equation

$$\frac{\partial}{\partial t}(\Delta\varphi) + \frac{1}{2}(\nabla\varphi_1 + \nabla\varphi_2)\cdot\nabla(\Delta\varphi) = 0 \tag{11}$$

where φ_1 and φ_2 are the values of the potential on side 1 and 2 of the wake.

Finally the normal wash $\psi = \partial\varphi/\partial n$ is known from the boundary conditions. Therefore, Eq. (10) is an integral equation relating the potential, φ, to the normal wash, ψ. This equation is the basis for the Green's function method used in Refs. 4 to 12.

In order to obtain an efficient computational procedure, the integral equation is approximated by a system of delay equations in time. This is obtained by using a finite-element representation, i.e., by letting

$$\psi(P,t) = \sum_{j=1}^{J}\psi_j(t)\,N_j(P) \quad ; \quad \varphi(P,t) = \sum_{j=1}^{J}\varphi_j(t)\,N_j(P) \tag{12}$$

where J is the total number of nodes on the body and $\psi_j(t)$ and $\varphi_j(t)$ are time-dependent values of ψ and φ at the node P_j at the time t; furthermore, the $N_j(P)$ are prescribed global shape functions, obtained by standard assembly of the element shape function. Similarly, the potential discontinuity on the wake is expressed as

$$\Delta\varphi(P,t) = \sum_{n=1}^{N}\Delta\varphi_n(t)\,L_n(P) \tag{13}$$

where N is the number of nodes on the wake, $\Delta\varphi_n(t)$ is the value of $\Delta\varphi$ at the nth node $P_n(W)$ on the wake at time t, and $L_n(P)$ is the global shape function relative to the nth node of the wake. Note that according to Eq. (11)

$$\Delta\varphi_n(t) = \Delta\varphi^{(TE)}_{m(n)}(t-\tau_n) \tag{14}$$

where $m(n)$ identifies the trailing-edge point, $P_m^{(TE)}$, from which the vortex-point, located at $P^{(W)}$ at time t, emanated at time $t-\tau_n$ (where τ_n is the time necessary for the vortex-point to be convected from the trailing-edge point $P_m^{(TE)}$ to the wake-point $P_n^{(W)}$). Note that $\Delta\varphi_m^{(TE)} = \varphi_{h_u} - \varphi_{h_\ell}$, where h_u and h_ℓ identify the upper and lower trailing-edge nodes on the body corresponding to the nth node on the trailing-edge. Hence it is possible to write

$$\Delta\varphi^{(TE)}_{m(n)} = \sum_{j=1}^{J} S_{nj}\,\varphi_j \tag{15}$$

where $S_{nj}=1$ ($S_{nj}=-1$), if j identifies the upper (lower) trailing-edge point P_j on the body corresponding to the point $P_n^{(W)}$ on the wake, and $S_{nj}=0$ otherwise.

Combining Eqs. (10) through (15), one obtains a system of delay equations relating φ_h to ψ_j and φ_j:

$$\varphi_h(t) = \sum_{j=1}^{J} B_{hj}(t)\,\psi_j(t) + \sum_{j=1}^{J} C_{hj}(t)\,\varphi_j(t) + \sum_{n=1}^{N}\sum_{j=1}^{J} F_{hn}(t)\,S_{nj}\,\varphi_j(t-\tau_n) \tag{16}$$

where

$$B_{hj} = \frac{-1}{2\pi}\oint\!\!\oint_{\Sigma_B} N_j\,\frac{1}{r}\,d\Sigma_B\Big|_{P_*=P_h} \; ; \; C_{hj} = \frac{1}{2\pi}\oint\!\!\oint_{\Sigma_B} N_j\,\frac{\partial}{\partial n}\left(\frac{1}{r}\right)d\Sigma_B\Big|_{P_*=P_h} \; ; \; F_{hn} = \frac{1}{2\pi}\iint_{\Sigma_W} L_n\,\frac{\partial}{\partial n}\left(\frac{1}{r}\right)d\Sigma_W\Big|_{P_*=P_h} \tag{17}$$

These coefficients are evaluated analytically for a general quadrilateral twisted element (hyperboloidal elements) with zeroth order shape functions (Ref. 14).

Note that if the surface is moving these coefficients are in general time-dependent.

Equation (23) gives the value of φ_n at time t in terms of the values of $\dot{\varphi}_j$, and φ_j at preceding times. Therefore Eq. (23) may be easily solved by a step-by-step procedure (for the details see Ref. 23).

In the case of helicopter rotors in compressible flow, the system resulting from the finite-element solution of Eq. (7), with F=0, would be similar to the one of Ref. 23 (which is a simple extension of Eq. (16)). In the case of helicopters however the coefficients are time-dependent. Finally note that the wake roll-up can be evaluated step-by-step by using the method used by Suciu and Morino[16].

5. Results

Extensive results have been obtained with the programs WILSA (Windmill Incompressible Lifting Surface Aerodynamics, Refs. 8-12), WICCA (Windmill Incompressible Complex Configuration Aerodynamics, Refs. 8-12) and SHAPES (Subsonic Helicopter Aerodynamic Program with Effects of Separation, Refs. 5 and 6). The programs WICCA and SHAPES are similar and are based upon the theory presented in Section 4. The program WILSA is similar to WICCA and is based upon a lifting-surface formulation (limiting case of the formulation of Section 4 as the thickness goes to zero; see Ref. 8). A few basic results are presented first, followed by a brief outline of the results obtained thus far.

Figures 1 and 2 present a comparison of the results obtained with WILSA and WICCA with the numerical results of Rao and Schatzle[30] and the experimental data obtained by Bartsch[59], for an isolated rotor in hover. The geometry of the rotor is defined by rotor radius R=17.5', chord c=1.083', root radius r=2.33', collective pitch angle at blade roots, α=10.61°, and blade twist, Θ =-5° (angle of attack decreasing along the span). The tip Mach number is M=.58. Classical wake (used in WILSA and WICCA) indicates a classical helicoidal wake with the induced velocity u obtained from the thrust coefficients $c_T = 2\,u^2/\Omega^2 R^2$ with c_T=.00186 (Ref. 30). Modified wake indicates a wake geometry derived by Landgrebe[57] from experimental data. One blade is used for the results of Figure 1 note the excellent agreement among the three classical-wake results (WILSA, WICCA and Rao and Schatzle[30]). Note also the increase in lift coefficient obtained by Rao and Schatzle by using the Landgrebe wake. This is important in analyzing Figure 2 (for a four blade rotor): WILSA is in excellent agreement with the classical-wake results of Rao and Schatzle[30], whereas their modified wake results are in excellent agreement with the experiments: it seems apparent that the concentration of vorticity into tip-vortices (taken into account by the Landgrebe's wake) is responsible for the spike of the section lift distribution near the tip. Therefore the method presented here is considered to be in excellent agreement with existing results with the understanding that the use of the correct wake geometry would enhance considerably the agreement with the experimental results.

Additional results which have been obtained using WICCA are presented in Refs. 8-12. In particular fully unsteady results in the time domain are presented in Refs. 9 and 12 for an isolated rotor. The results include transient response to a sudden change in collective pitch angle as well as time domain analysis for oscillatory flow (due for instance to windmill rotor in shear wind); the time-domain results are in excellent agreement with the frequency-domain results.

Numerical results which have been obtained using SHAPES are presented in Ref. 5. This includes flow around a rotor-fuselage configuration including the effects of separation: the separation is modeled with an infinitesimally-thin vortex layer emanating from the (empirically determined) separation line. Vorticity in the separation region is concentrated into a single isolated vortex: the intensity of the vortex is determined by imposing the condition that the tangential velocity changes sign at the separation line. The results are in excellent agreement with the experimental results of Refs. 61 and 62.

6. Comments

First a general assessment of the method is presented, followed by a detailed analysis of the effect of compressibility, and concluding remarks.

The Green's function method proposed here was assessed by Carrick[63] in his 3th Von Karman lecture. Therefore the following comments are only in reference to the specific application considered here. First, there should be no question about the generality of the method. Second it is obviously much more efficient than the finite-difference time-domain solution of the complete potential flow field. In addition it should be noted that, despite the generality, the method is also very efficient, when compared to lifting-surface theories; this was clearly illustrated by the comparison of the programs WICCA (Windmill Incompressible Complex Configuration Aerodynamics, based upon Green's function method) and WILSA (Windmill Incompressible Lifting Surface Aerodynamics based upon a lifting-surface theory similar to the one of Refs. 30 and 52): the more complex program WICCA yields similar results and takes only slightly more computer time than WILSA (for incompressible steady flow ; for unsteady compressible flow, the present method yields a much simpler integral equation than the lifting surface theory). Finally, the method uses the actual geometry of the surface and therefore takes into account the thickness effect and can be easily extended to complete rotor-fuselage configuration. Therefore the present method satisfies simultaneously both requirements of generality and efficiency.

Next consider the question of compressibility. Note that Eq. (7) is the exact integral equation for the non-linear compressible unsteady potential aerodynamic equation, Eq. (1). No simplifying assumption is used in deriving Eq. (7). In particular the compressibility effects are considered in an exact way. It may be worth examining the way in which the compressibility effects appear in Eq. (7). In order to simplify the discussion consider the linear case for an aircraft having infinitesimal oscillations around a steady-state configuration. In this case F=0; the surface σ^{θ} may be replaced by the time independent steady-state surface and the term $\partial S/\partial t$ is retained only in the boundary conditions. Then introducing the generalized Prandtl-Glauert transformation $X = x/\beta L$, $Y = y/L$, $Z = z/L$, $T = U_\infty t/L$ (where L is a characteristic length of the body) Eq. (7) reduces to

$$4\pi E\,\varphi(P_*,T) = -\oiint_{\Sigma}\left\{\left[\frac{\partial\varphi}{\partial N}\right]^{\Theta}\frac{1}{R} - [\varphi]^{\Theta}\frac{\partial}{\partial N}\left(\frac{1}{R}\right) - \left[\frac{\partial\varphi}{\partial T}\right]^{\Theta}\frac{1}{R}\frac{\partial\hat{\Theta}}{\partial N}\right\}d\Sigma \tag{18}$$

where $\vec{N}$ is the normal to the surface Σ of the space X,Y,Z, $R=|P-P_*|$ whereas $\Theta = U_\infty\theta/L$ and $\hat{\Theta} = [M(X_*-X)+R]M/\beta$. By comparing Eq. (18) to Eq. (10) it is apparent that, first, the Prandtl-Glauert transformation is used in the direction of forward-flight with $M=U_\infty/a_\infty$: this term takes into account forward-flight effect (it does not exist for hover conditions, $U_\infty=0$). Second, a third integral containing $\dot{\varphi}$ appears in the equation: this term exists in the integral equation for the wave equation and takes into account the presence of the $\partial^2\varphi/\partial t^2$ term in the differential equation. Third the functions are evaluated at delayed times $T-\Theta$: this is also a consequence of the $\partial^2\varphi/\partial t^2$ term in Eq. (1), like item 2. Next consider the effects of the motion of the rotor: these are obtained by comparing Eq. (7) to Eq. (18) and may be summarized by saying that: (1) $\partial S/\partial t$ appears in the equation; (2) the integrals are over σ^{θ} instead of σ and (3) $d\hat{\sigma} = d\sigma^{\theta}|\nabla S|^{\theta}/|\nabla S^{\theta}|$ is used in place of $d\sigma$. These three effects are important and have never been introduced in any integral equation used for the analysis of helicopter rotors. In particular the second item is very interesting and also peculiar of the present formulation: the surface σ^{θ} can be interpreted from a physical point of view by stating that the dummy point of integration, P, has to be considered in the position it had at time $t=t_*-\theta$ (θ is the time necessary for the disturbance to travel from the point P to the control point P_*). It should now be apparent that the local chordwise Prandtl-Glauert transformation used in Ref. 30 is reasonably acceptable for a rotor in hover (on a strip-theory intuitive approach), but not in forward flight: in this case the compressibility manifests itself in a much more compicated way.

In conclusion, a general method for the unsteady compressible potential aerodynamic analysis of helicopter rotor-fuselage configurations, in hover and forward flight, has been presented. The method has been validated for incompressible flows: steady-state results are included here, unsteady results (computer program WICCA) are presented in Refs. 9 and 12 and rotor-fuselage configurations with the effects of separation (computer program SHAPES) are considered in Ref. 5. The effects of compressibility have been discussed in detail; their inclusion in a general-purpose computer program are now underway.

7. References

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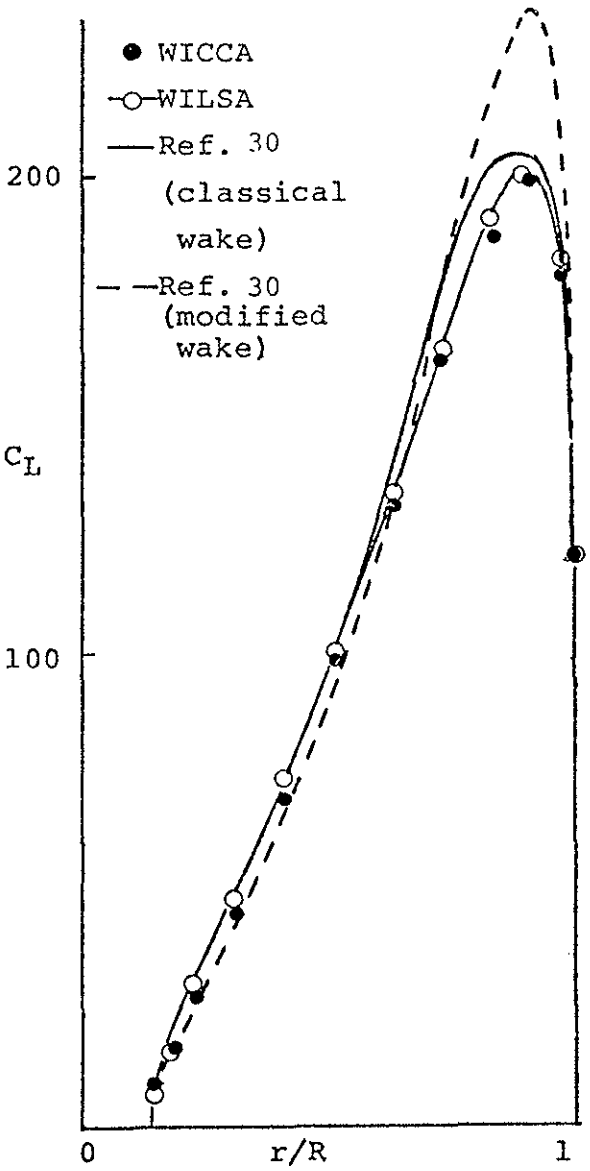

Figure 1. Section lift coefficient, C_L (lbs/ft), vs. non-dimensional blade radius for a 1-blade helicopter rotor in hover.

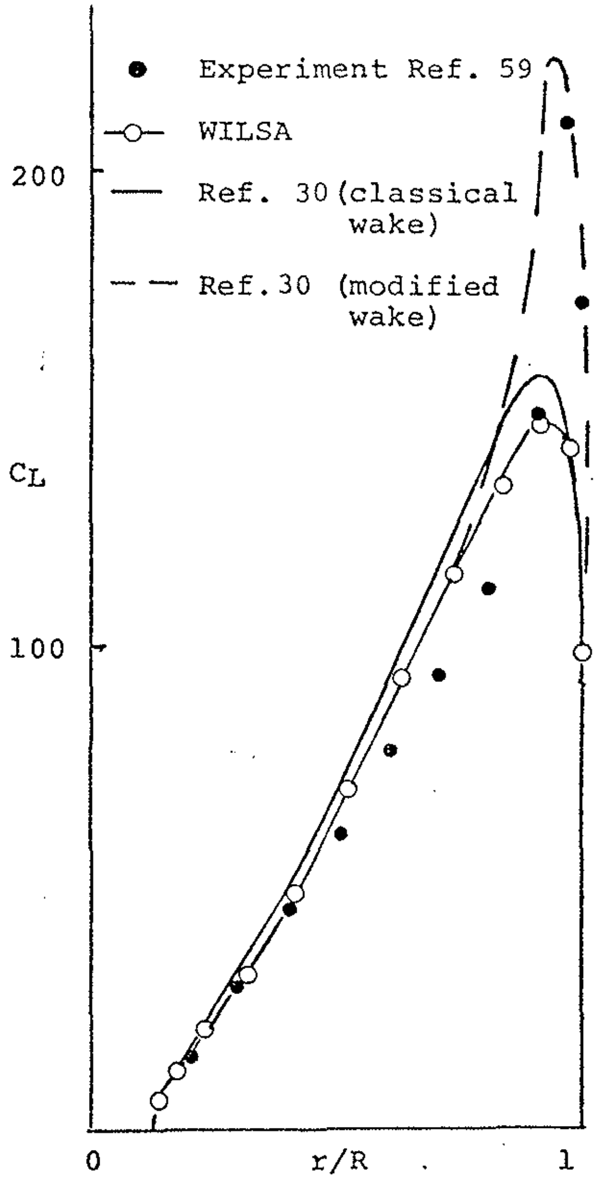

Figure 2. Section lift coefficient, C_L (lbs/ft), vs. non-dimensional blade radius for a 4-blade helicopter rotor in hover.