TWENTY FIRST EUROPEAN ROTORCRAFT FORUM

Paper No II.15

APPLICATION OF A LEARNING CONTROL ALGORITHM TO A HELOCOPTER ROTOR BLADE WITH A TRAILING EDGE TAB

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August 30 - September 1, 1995

SAINT - PETERSBURG, RUSSIA

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Application of a learning control algorithm to a helicopter rotor blade with a trailing edge tab

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Abstract

A new control algorithm is applied to control deflection of a tab mounted at the trailing edge of a helicopter rotor blade. The goal of control is to obtain prescribed blade motion and it is shown by numerical simulation that the algorithm is effective in controlling blade torsion.

Notation

$\mathbf{A}(t)$ - state matrix of linearized system,
$\mathbf{A}(t)=[A_{mn}(t)]$, m, n=1,..,N
$\mathbf{B}(t)$ - control matrix of linearized system,
$\mathbf{B}(t)=[B_{m1}(t)]$, m=1,..,N
c - aerofoil chord
C_L - aerofoil lift coefficient
C_M - aerofoil moment coefficient
$\mathbf{C}(k)$ - state matrix of discrete system,
$\mathbf{C}(k)=[C_{mn}]$, m, n=1,..,N
$\mathbf{d}(k)$ - vector of discrete external disturbance,
$\mathbf{d}=[d_n]$, n=1,..,N
$\mathbf{D}(k)$ - control matrix of discrete system,
$\mathbf{D}=[D_{n1}]$, n=1,..,N
$e_i(k)$ - difference between required and actual values of states
$\mathbf{f}(t,x)$ - right hand sides of nonlinear system,
$\mathbf{f}(t,x)=[f_n]$, n=1,..,N
$\mathbf{G}(k)$ - matrix of control gain,
$\mathbf{G}=[G_{nm}]$, n=1,..,2N, m=1,..,N
i - number of period
$\mathbf{I}$ - identity matrix
k - time step number, 0<k<M
M - number of time steps
Ma - Mach number of undisturbed flow
N - number of states
$\mathbf{R}(t,x)$ - vector of higher order terms,
$\mathbf{R}(t,x)=[R_n(t,x)]$, n=1,..,N
t - time
$u_d(k)$ - required control
$u_i(k)$ - actual control in i-th period
$\mathbf{x}(t)$ - vector of state variables, $\mathbf{x}(t)=[x_n(t)]$, n=1,..,N
$\mathbf{x}_d(t)$ - vector of required values of states $\mathbf{x}(t)$
$\alpha(t)$ - aerofoil angle of incidence
$\delta(t)$ - angle of tab deflection
ε_δ- bound value of disturbance $\mathbf{d}(k)$
ε_0- tolerance of system motion
λ - control constant

Subscripts and indices
$(\hat{\ })$ - estimated value
$(^+)$ - generalised matrix inversion $\mathbf{A}^+=(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$
$(\dot{\ })$ - differentiation with respect to time

Introduction.

The suppression of vibration is of continuing interest in rotorcraft technology. Due to periodic excitation inherent especially in forward flight, a rotorcraft is subject to varying dynamic and aerodynamic loads. These variable loads act on main rotor, so attempts to alleviate these lead to diminishing the vibration level of the whole rotorcraft.

This provides the motivation for many different studies concerning various rotor design concepts, passive antivibration devices [1], active control of rotor pitch (HHC, IBC concepts) [2] and application of actively controlled additional devices [3].

Recently the application of blade mounted trailing edge tabs has stimulated the interest of many researchers.

The use of blade trailing edge tabs for primary control of rotorcraft has been successfully implemented by Kaman Company in their products, most recently on the K-Max helicopter. The use of tabs for primary control was analytically investigated in [4].

Several analytical and experimental studies have been carried out to obtain insight into different aspects of the application of a trailing edge tab for additional control. The use of tabs for vibration suppression was investigated in [5], for reduction of the effects of blade vortex interaction in [6] and for rotor performance optimisation in [7].

This interest has been caused by prospects of providing the driving mechanism for tabs through smart structure technology [8]. Tabs driven by piezoelectric benders were tested experimentally in [9] on a rotor model in hover.

Physical phenomena involved in applications of the "smart tab" are aeroelastic including both dynamic i.e. (inertia and elastic loads), and aerodynamic phenomena.

To achieve the required goal a proper control strategy should be applied to the system. Up till now the open loop systems have been considered [10] or control algorithms of mainly LQC or LGC type in the frequency domain [11] have been utilised. Also some heuristic approaches [7] in the time domain have been tested.

The objective of this study is to investigate the possibility of the application of a time domain „learning algorithm" for controlling a tab mounted at the trailing edge of a blade to diminish the rotor vibration level.

Properties of the chosen control algorithm are evaluated by computer simulation using an individual blade model adapted to the needs of this study by adding a trailing edge tab. The aerodynamic loads at the tab are calculated using static aerodynamic coefficients obtained from experimental data as functions of aerofoil angle of attack and tab deflection.

The vibration reduction considered here is expressed as the requirement for the blade to perform assumed motion. In the computational examples the particular goal of controlling deflection of the trailing edge tab is to remove one or several harmonics from the blade steady motion.

The algorithm demonstrates its efficiency in this aeroservoelastic case, allowing that tab size is adequate to influence the blade motion.

Background of the control method.

In rotorcraft aeroservoelastic problems, the plant to be controlled is periodic with respect to time. There have been attempts to develop control algorithms for such types of plant in rotorcraft research, and similar activity has been performed in the robotics area, although the plant considered in this field seem to be more easily handled.

The control algorithm applied during this study is a modification of that developed in [12,13]. The background of the method is presented here for completeness.

The discrete, linear, system periodic with respect to time with scalar control u(k) is considered:

$$x(k+1) = C(k)x(k) + D(k)u(k) + d(k) \tag{1}$$

where k=1,2,..,M.

Matrices C(k) and D(k) are periodic with respect to time, i.e. for all k

$$C_{i+1}(k) = C_i(k), \quad D_{i+1}(k) = D_i(k) \tag{2}$$

In the above, subscript i describes the number of the period.

The periodic and bounded disturbance d(k) for all k and i fulfils the condition

$$\|d_{i+1}(k) = d_i(k)\| \le \varepsilon_d \tag{3}$$

where ε_d is a prescribed constant.

The learning problem is stated as the requirement, that the state vector $x_d(k)$ is a realisable, periodic trajectory. The sequence of control applied $u_i(k)$,i=1,2,... should provide that, starting from some period of time, the system trajectory $x_i(k)$ will satisfy the condition

$$\|x(k) - x_d(k)\| \le \varepsilon_0 \tag{4}$$

where ε_0 is assumed tolerance bound.

It was proved in [14], that the control defined as

$$\begin{aligned} u_{i+1}(k) &= u_i(k) + \\ &\lambda\,[\hat{D}_i^T(k) : -\hat{D}_i^T(k)\hat{C}_i(k)] \times [e_i(k+1)^T : e_i^T(k)]^T \\ e_i(k) &= x_d(k) - x_i(k) \end{aligned} \tag{5}$$

fulfils the learning condition if, for initial error $e_i(0) = 0$, the estimate of matrix D(k) satisfies the condition

$$\left|1 - \lambda\,\hat{D}_i^T(k)D(k)\right| < 1 \tag{6}$$

If the external disturbance is periodic, then

$$\|e_i(k)\| \to 0, \quad \text{for } i \to \infty \tag{7}$$

These expressions form the basis for application of this algorithm to a nonlinear, continuous system.

Application to a nonlinear, continuous system.

The mathematical model of a helicopter rotor blade can be expressed as a nonlinear system of ordinary differential equations periodic with respect to time, with scalar control u(t) corresponding to the angle of deflection of the trailing edge tab

$$\dot{x} = f(t, x, u) \tag{8}$$

For assumed nominal tab control $u_d(t)$ the desired periodic solution for this equation is $x_d(t)$.

The system (8) is linearized about $x_d(t)$

$$\dot{x} = A(t)x + B(t)u(t) + R(x, x_d, u(t), u_d(t), t) \tag{9}$$

The matrices A(t), B(t) in (9) are defined as

$$A = [A_{ij}] = \left[\frac{\partial f_i}{\partial x_j}\right]_{x_d, u_d}, \quad B = [B_i] = \left[\frac{\partial f_i}{\partial u}\right]_{x_d, u_d} \tag{10}$$

and the quantity $R(x, x_d, u(t), u_d(t), t)$ contains the higher order terms.

Approximating the time derivative by the forward finite difference

$$\dot{x} = \frac{x(t+\Delta t) - x(t)}{\Delta t} \tag{11}$$

and inserting it into the linearized equation (9) transfers the linearized equation to the discrete time domain

$$\begin{aligned} x(t+\Delta t) &= [I + A(t)\Delta t]x + B(t)\Delta t u(t) + \\ &+ R(x, x_d, u(t), u_d(t), t)\Delta t \end{aligned} \tag{12}$$

and by substitution

$$\begin{aligned} &t = k\Delta t, \quad C(k) = [I + A(k\Delta t)\Delta t], \quad D(k) = B(k\Delta t)\Delta t, \\ &d(k) = R(x(k\Delta t), x_d(k\Delta t), u(k\Delta t), u_d(k\Delta t), k\Delta t, \Delta t) \end{aligned} \tag{13}$$

equation (12) can be reformulated to the form (1).

The proposed application of the algorithm described in the previous section to the nonlinear case consists of:

I. Dividing the time period into M steps by prescribing the points of time $t_k = k\Delta t$, k=1,2,...,M and calculating at these points:

1. The desired solution $x_d(k)$,
2. Matrices A(k), B(k) of the linearized, continuous system (10),
3. Matrices C(k), D(k) according to (13),
4. The gain matrix G(k according to the formula

$$G(k) = [\hat{D}_i^*(k) : -\hat{D}_i^*(k)\hat{C}_i(k)]^T \quad (14)$$

II. Assuming the value of λ, as the theory gives no indication for selecting its value.

III. Starting from $e_0(k)=0$ the motion of the system is controlled in each period of time according to the formulae

$$u_{i+1}(k) = u_i(k) + \lambda\, G(k) \times [e_i(k+1)^T : e_i^T(k)]^T$$
$$e_i(k) = x_d(k) - x_i(k) \quad (15)$$

In this approach the higher order terms R(t,x,u) rejected in linearization are treated as a disturbance vector d(k) and matrices C(k) and D(k) as estimates of the matrices of the discrete periodic system.

Application of the algorithm to blade motion.

In this study the algorithm is applied to obtain the required motion of a helicopter rotor blade with an attempt to suppress the prescribed harmonics of steady motion.

The sequence of calculation during numerical simulation is:

1. Solve the nonlinear system (8) of blade equations of motion for the prescribed number of rotor revolutions. The blade motion during the last rotation is blade steady motion x(k).
2. Perform a Fourier analysis of the steady motion x(k).
3. Reconstruct the blade required motion $x_d(t)$ using the selected Fourier series coefficients from step 2.
4. Calculate the control gain matrix according to (14) by linearizing the blade equations about the reconstructed steady motion and assumed initial control.
5. Apply the learning algorithm to simulate controlled blade motion for the assumed number of blade rotations.

For the numerical simulation of helicopter rotor blade motion, a well tried and tested computer model of an individual blade is utilised, which is described fully in [14].

Blade model.

The motion of a single rotor blade of a helicopter in a steady flight is studied. The angular velocity of the rotor shaft is constant.

The model applied for the blade modelling allows the selection of different arrangements of the rotor hub and the deflection modes of the blade.

In the general case the blade has a straight elastic axis and is pretwisted about it. The blade stiffness loads are obtained from a Houbolt-Brooks model. It can bend lag-wise, flat-wise and twist about the elastic axis. The blade cross sections have symmetry of elastic properties about a chord and there is no section warping.. Viscous structural damping of blade deformations is included.

The blade deflections are discretized by free vibration modes.

The aerodynamic loads are calculated from a two-dimensional, quasi-steady, nonlinear model based on a table look-up procedure described in Appendix 1. The induced velocity is calculated from the Glauert formula.

The vector of generalised coordinates of the blade motion contains elastic degrees of freedom resulting from discretization of blade deformations by normal modes.

For numerical integration of equations of motion, Gear's algorithm is used, which allows for solution of "stiff equations".

The scope of the numerical study concerns the following aspects associated with the proposed control algorithm:

1. Application of linear methodology to the nonlinear case.
2. Simple form of continuous system discretization to the time domain.
3. No rational indications for choosing parameter λ.
4. Rotor blade aeroelastic behaviour which influences controllability of the system.

Insight into these aspects of the control can be gained only by numerical simulation

Sample results of numerical calculations.

The effectiveness of the control strategy depends both on the plant properties and the control algorithm. As one of the assumptions of the method comprises controllability of the system, the tab mounted at the blade should produce aerodynamic loads sufficient to influence blade motion. The magnitude of these loads depends on the blade dynamic properties and the velocity of the helicopter flight.

A tab would most likely influence the blade twisting moment, so this was the reason for selecting this blade degree of freedom for investigation here.

From the available blade models, the hingeless blade stiff in bending and elastic in torsion was selected to test the control algorithm.

The base blade configuration selected for the study of the properties of the control algorithm comprises the blade deformable in twist attached to the shaft via a stiff element. It can be controlled in pitch about a feathering bearing.

Numerical results are obtained using as the base data that corresponding approximated the Westland Lynx blade [15]. The main values of blade parameters are given in Table I.

Table I Blade data

rotor angular velocity Ω	rad/s	34.0
air density	kg/m^3	1.226
blade chord (aerofoil+tab)	m	0.395
rotor radius	m	6.40
blade mass	kg	57.2
blade length	m	5.61
natural frequency of twist	$1/\Omega$	6.29
damping of aerofoil twist	% crit	0.01
linear twist of the blade	deg	from 4.3 to -2.2

The flight conditions concern an untrimmed rotor having collective pitch of 10^0 and no cyclic control. The flow velocity expressed as rotor advance ratio varies from 0 to 0.35 in 0.05 intervals.

As the first result of numerical simulation it was found that due to high fundamental torsional frequency of the blade, a tab of chord 0.1c, which can influence blade twisting deflection should elongate from 23.3% to 95% of the blade span.

The next important factor for control efficiency, the control constant λ can be adjusted by trial and errors. It can be neither to large, which would make control too aggressive nor too small which slows the learning process.

In the case considered, the smallest value of λ which was found to be effective was 0.05.

For the chosen tab chord and control parameter, the sample results of blade control are given in the figures for helicopter advance ratios of 0.15 and 0.35. These show the motion of the nonlinear system after 10 rotations (which is regarded as the blade steady motion) the required motion for the case considered and the controlled motion after 10 rotations of the algorithm being applied.

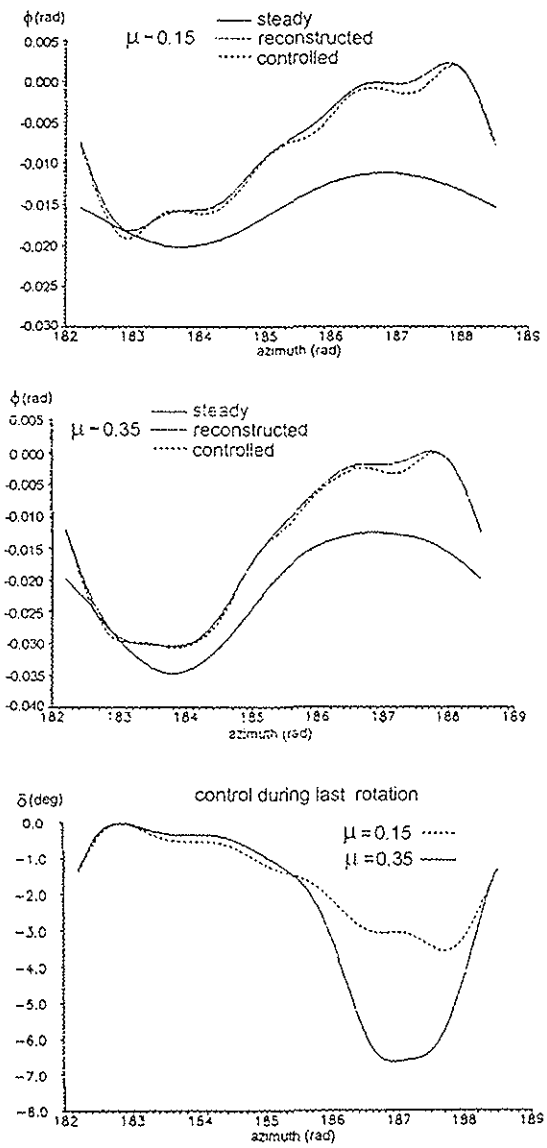

Fig.1. Required motion composed of selected harmonics

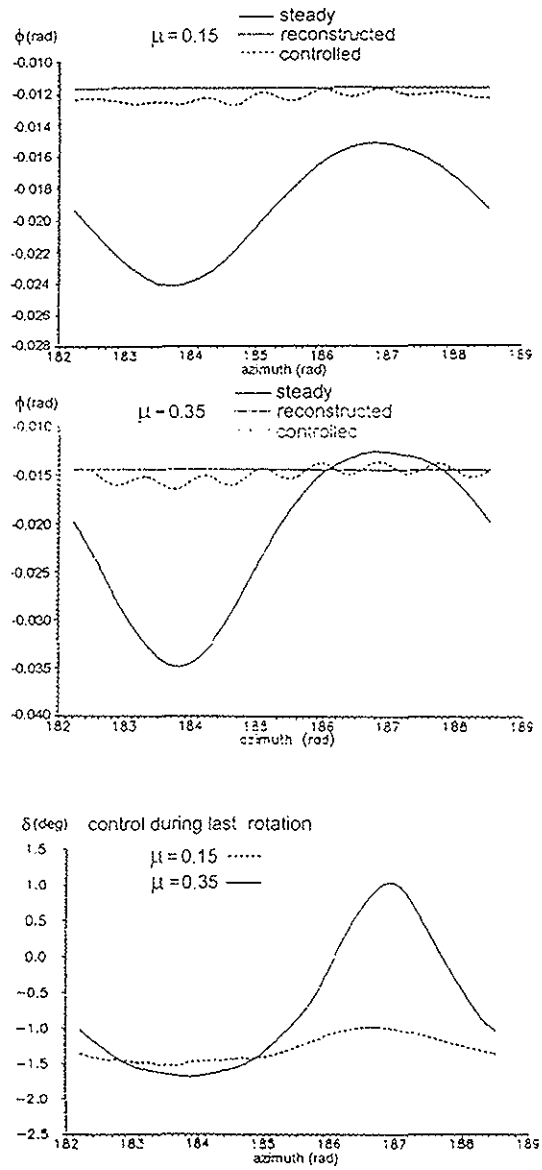

Fig.2. Required motion of constant values.

Two cases of blade required motion are considered.

In the first case shown in Fig.1, the required motion is reconstructed from the Fourier coefficients of steady motion up to the seventh order but without the third and fourth harmonics.

In the second case, Fig.2. the constant component of steady motion was forced by the control algorithm.

Both of these cases are completely artificial and are aimed at demonstrating the effectiveness of the control algorithm; it is not intended that they represent real flight situations.

In both cases the control algorithm proved to be effective, driving the blade twist to the vicinity of required motion. The required tab deflections are within the acceptable limits, although the time dependence varies with the type of the motion required.

Conclusions

A learning control algorithm taken from the field of robotics has been modified and applied to the nonlinear periodic model of a helicopter rotor blade. The numerical simulations have shown that with an appropriate tab length and selected control constant, the blade twist angle can be influenced in such a way that the blade follows the required motion. This demonstrates the efficiency of the control strategy being applied to a periodic aeroelastic system.

The somewhat unrealistic tab length needed to fulfil this task suggests that the mass and stiffness tailoring of the blade should be a necessary follow-up exercise in order to obtain effective control within acceptable design parameters.

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Appendix 1. Aerofoil static characteristics

The Prandtl-Glauert correction factor is applied to accounting for influence of Mach number on aerofoil characteristics

$$C_L = \frac{C_{LN}}{\sqrt{1 - Ma^2}}$$

The static characteristics of an aerofoil with a tab are based on the characteristics of an aerofoil without a tab, which are modified to account for tab deflection. A table look-up procedure is utilised here for obtaining the static characteristics of an aerofoil.

To account for tab deflection, the lift coefficient is calculated as

$$C_{LS} = C_{LaS} + \left| \frac{\partial C_{LaS}}{\partial \alpha} \right| \Delta_{\alpha LS}$$

According to [16] the correction $\Delta_{\alpha LS}$ to the angle of attack has the form

$$\Delta_{\alpha LS} = \tau \delta \eta$$

The flap effectiveness factor τ is calculated as

$$\tau = 1 - \frac{\theta - \sin\theta}{\pi}, \theta_T = \cos^{-1}\left(\frac{2c_t}{b} - 1\right)$$

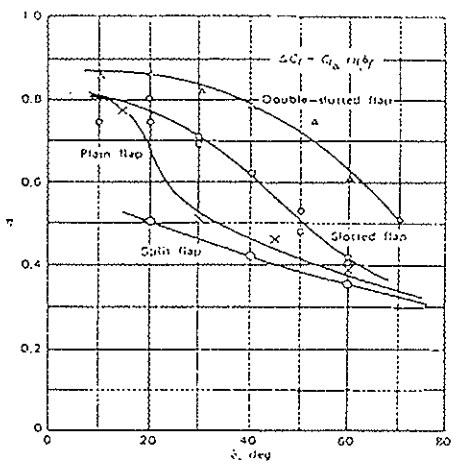


Fig. A

The correction factor η is approximated for the plain tab and deflection angle less than 20^0 from Fig.A [17] by the formula

$$\eta = -0.00045\delta^2 + 0.0005\delta + 0.85$$

The moment coefficient for an aerofoil with a tab is calculated as

$$C_{MaS} = C_{MS} + \Delta C_{MSS}$$

The correction for the tab is calculated from the formula

$$\Delta C_{MSS} = f\left(\frac{c_f}{c}\right) \Delta C_{LS}$$

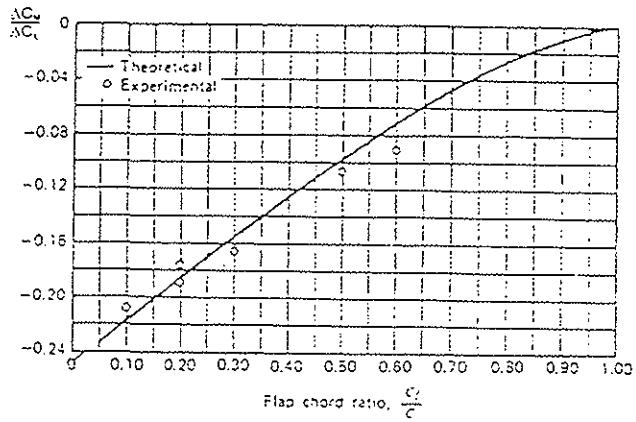
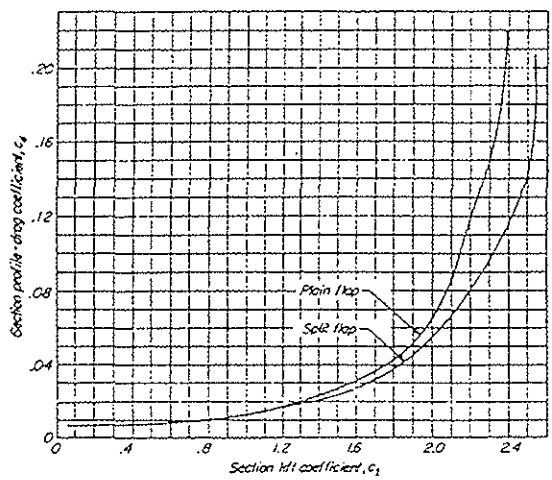


Fig.B

The function $\Delta f(c_f/c)$ is approximated from Fig.B for a flap ratio less than 20% by the formula

$$f = 0.3\frac{c_f}{c} - 0.245$$

The drag coefficient is obtained from a table look-up procedure of the data taken from Fig.C [18] as a function of lift coefficients.