ESTIMATION OF HANDLING QUALITY PARAMETERS OF A ROTORCRAFT USING OPEN-LOOP LINEARIZED AND NONLINEAR FLIGHT DYNAMIC MODELS

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Abstract

Flight dynamic analysis and estimation of handling quality parameters have become important aspects in the design and development of helicopters. This paper presents a detailed analysis of the procedure for estimating the handling quality parameters such as quickness parameter, bandwidth and phase delay. The flight dynamic model used in this study considers rigid flap model for blade structural dynamics, three states dynamic inflow for inflow calculation and modified ONERA dynamic stall model for sectional aerodynamic loads calculation. The applicability of open loop linearized (uncoupled, and coupled) and nonlinear flight dynamic models in estimating the handling quality parameters is studied. For linearized models, only pulse input is used, whereas in the nonlinear model, two different types of input, namely pulse and step inputs, are used to estimate the attitude quickness parameters. The bandwidth and phase delay are calculated from the frequency responses of helicopter attitude in pitch and roll axes, which are obtained from the time response of nonlinear flight dynamic model for the harmonic excitation of cyclic pitch input. The results show that the attitude quickness parameter depends on the duration of input pulse and the nonlinear open system provides attitude quickness parameter which is different from that of the linearised system. In addition, it is noted that linearized flight dynamic models (8x8) cannot be used for bandwidth-phase delay calculations, due to their lower order nature.

LIST OF SYMBOLS

ADS	***A**eronautical Design Standard*
I_{xx}, I_{yy}, I_{zz}	***M**ass moment of inertia of helicopter x,y and z directions* $(kg.m^2)$
I_{xz}	***P**roduct of Inertia* $(kg.m^2)$
g	***G**ravitational acceleration* (m/s^2)
L, M, N	***N**et moments at helicopter center of gravity along x,y and z directions* $(N.m)$
m_h	***M**ass of the helicopter*(kg)
p, q, r	***F**uselage angular velocity components along x, y and z directions* $(rad.s^{-1})$
X, Y, Z	***N**et forces at helicopter center of gravity* *along x, y and z directions* (N)
u, v, w	***F**uselage translational velocity components along x, y and z directions*(ms^{-1})
μ	***A**dvance ratio*
Φ	***R**oll attitude*
Θ	***P**itch attitude*
$\theta_0, \theta_{1c}, \theta_{1s}$	***M**ain rotor collective and cyclic pitch angles*
θ_{tr}	***T**ail rotor collective pitch angle*

1. INTRODUCTION

In the early days, due to limited usage, helicopters were required to perform gentle maneuvers like hover and forward flight. In recent times, the utility of helicopters has increased in both civil and military sectors; and the helicopters operational environment has also expanded. They have to perform various maneuvers and operate in extreme weather conditions. These demands require new design of helicopters or modifications in the existing design to provide better performance. To design a new helicopter or to modify the existing design, detailed analyses in various aspects have to be carried out. They are: (i) flight dynamic analysis, (ii) handling quality evaluation, and (iii) aeroelastic loads and response.

The flight dynamic analysis includes prediction of trim states, stability characteristics, and open loop control response characteristics to pilot input and external disturbances. Handling qualities are

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judged based on both qualitative and quantitative assessments. Qualitative assessment of handling qualities are based on flight test and pilots rating; and hence it is a subjective evaluation of individual pilots. Whereas, quantitative measures of handling qualities can be evaluated from simulation models and later can be verified with flight test data. The quantitative measures are used as design targets to achieve compliance with airworthiness requirements.

In the quantitative assessment, handling qualities or flying qualities are divided into two categories[1]. They are: (i) handling qualities which are related to the response characteristics of helicopter to pilot's input, and (ii) ride qualities which are associated with the response characteristics of the helicopter to external disturbances. Evaluation of response characteristics and subsequently the handling qualities require a reliable mathematical model representing the flight dynamics of the helicopter in general maneuvers.

Mathematical modelling of helicopter dynamics includes modelling of individual components and their integration to represent the dynamic equations of the helicopter. There are several flight dynamic models starting from simple to sophisticated comprehensive models. The comprehensive aeroelastic analysis[2,3] is used to predict the vibratory loads, blade response and also flight dynamics. However the comprehensive models are computationally intensive and time consuming. Hence they are not widely used for flight dynamic studies from the point view of stability, control response and handling quality evaluation. Therefore, relatively simple models are required for flight dynamics analysis and handling quality evaluation[4,5,6].

Nowadays, MIL-H-8501A[7], Cooper and Harper ratings[8], and ADS-33E[9] are used as standard specifications for helicopter handling qualities. MIL-H-8501A specifies the response characteristics of a rotorcraft to control stick movement, control power, force and moment gradients to control stick input. Flight regimes covered by MIL-H-8501A are limited to hover and forward flight. Based on a critical review of handling qualities specifications used from earlier days to MIL-H-8501A, it was noted that only the stability and control characteristics were considered as important from the point of safety of flight and pilots[10].

Later Cooper and Harper[8] brought forth the importance of other factors such as cockpit interface, aircraft environment and loads on various components that influence the handling qualities and introduced a method of evaluating the handling qualities by using pilot rating for fixed wing aircraft. Cooper and Harper defined three levels of handling qualities. Level-1 corresponds to numerical rating of 1 to 3 and the design is acceptable and satisfactory. Level-2 corresponds to the numerical rating of 4 to 6 and the design is acceptable but unsatisfactory. Level-3 corresponds to the numerical rating of 7 to 9 and the design is unacceptable.

MIL-H-8501A was reviewed based on Cooper and Harper rating with available flight test data. Clement et al.[11,12] and, Chalk and Radford[13] proposed a new structure of handling qualities specification by updating MIL-H-8501A from the point of view of mission task elements. The new structure incorporated the following aspects: (i) variety of rotorcraft types, (ii) mission flight phases, (iii) flight envelopes, (iv) flight environmental characteristics, (v) failures and reliability, and (vi) external vision aids. All these aspects were critically analysed and the new handling qualities specification ADS-33E[9] was proposed. ADS-33E provides mission oriented specifications. It defines operational missions and Mission-Task-Elements, response characteristics, agility parameters, operational environment, levels of handling qualities, flight envelopes, configurations, loadings, flight conditions and rotorcraft failures.

Most of the requirements specified by ADS-33E are assessed from flight data and pilots ratings. Some of the important specifications such as attitude quickness, agility parameter, bandwidth and phase delay can also be evaluated from simulation models. Mission task elements can be also simulated by using inverse simulation technique[14,15,16].

Handling qualities of the helicopter can be evaluated with and without augmented control system. The handling qualities of the base helicopter without control system (open loop) predict the safety of the baseline design. The handling qualities of the helicopter with control system (closed loop) predict the effectiveness of the control system. In the open literature, the study on the handling qualities evaluation of the helicopter without control augmentation system is very limited. The main objective of the current paper is to estimate the handling quality parameters of the helicopter without control augmentation system (i.e. base helicopter).

The objectives of this study are as follows

- Development of a relatively simple flight dynamic model of a helicopter which can be used to analyze trim, control response under various manoeuvring conditions and handling qualities.
- Develop an approach for estimating attitude quickness, bandwidth and phase delay using linearized and nonlinear flight dynamic models.

- Estimate attitude quickness parameter using linearized and nonlinear flight dynamic models.
- Estimate bandwidth and phase delay using nonlinear flight dynamic model.

2. FLIGHT DYNAMIC MODEL

Mathematical modelling of helicopter dynamics involves modelling of individual components such as, main rotor, tail rotor, fuselage and empennage; and integration of all the individual components to represent the dynamic equations of the helicopter. In the development of the mathematical model, the main rotor is given more importance while the other components are modelled in a relatively simple manner. In this study, the flight dynamic model is developed using individual blades so that nonlinear transient response of the vehicle as well as linearized system control response can be analysed using one general formulation. The following simplifications and assumptions are made in the modelling.

- Rotor blades are assumed to be rigid with an equivalent hinge offset having a root spring.
- Only blade flapping is considered. Lead-lag and torsion modes are ignored.
- Blades are rectangular with linear twist.
- Fuselage is modelled as a rigid body.
- Aerodynamic drag on fuselage is represented by an equivalent flat plate drag.
- Inertial loads from tail rotor are ignored and all the aerodynamic loads are considered.
- Empennage is modelled as a plate with equivalent aerodynamic coefficients.
- Wake interaction effects from main rotor on tail rotor, fuselage and empennage are neglected.

Modelling of the main rotor involves calculation of inertial and aerodynamic loads produced on the rotor. The formulation starts with derivation of velocity and acceleration at a given point on the blade. Acceleration components are used to calculate sectional inertial loads and the velocity components are used to calculate the inflow and sectional aerodynamic loads. In the present flight dynamic model, three state dynamic inflow model[17] is used for the calculation of induced velocities through the main rotor. The model consists of three first order differential equations which is integrated in time domain using fourth order Runge-Kutta method.

Modified ONERA dynamic stall model[18] is used for the calculation of sectional aerodynamic loads. It provides the time variation of lift, drag and pitching moment acting on an airfoil in arbitrary motion. This model assumes that the aerodynamic forces and moment are acting at quarter chord point of the airfoil.

The sectional (inertial and aerodynamic) forces and moments are integrated over the length of the rotor blade. Integrated loads from all the blades are added and transformed to hub. Hub forces are calculated at every azimuth location. Forces and moments from all the components are then transferred to helicopter center of gravity. Using the forces (X, Y and Z) and moments (L, M and N) at the helicopter center of gravity, flight dynamic equations for a general maneuver are written as follows.

Force Equations:

$$\dot{u} = -(wq - vr) + \frac{X}{m_h} - g\sin\Theta \tag{1a}$$

$$\dot{v} = -(ur - wp) + \frac{Y}{m_h} - g\cos\Theta\sin\Phi \tag{1b}$$

$$\dot{w} = -(vp - uq) + \frac{Z}{m_h} - g\cos\Theta\cos\Phi \tag{1c}$$

Moment Equations:

$$I_{xx}\dot{p} = (I_{xx} - I_{zz})\,rq + I_{xz}(\dot{r} + pq) + L \tag{2a}$$

$$I_{yy}\dot{q} = (I_{zz} - I_{xx})\,rp + I_{xz}\left(r^2 - p^2\right) + M \tag{2b}$$

$$I_{zz}\dot{r} = (I_{xx} - I_{yy})\,pq + I_{xz}(\dot{p} - rq) + N \tag{2c}$$

Kinematic relations:

$$\dot{\Theta} = q\cos\Phi - r\sin\Phi \tag{3a}$$

$$\dot{\Phi} = p + q\sin\Phi\tan\Theta + r\cos\Phi\tan\Theta \tag{3b}$$

3. SOLUTION PROCEDURE AND ANALYSIS

The flight dynamic model of the helicopter can be expressed in symbolic form as,

$$\dot{\mathbf{x}} = F(\mathbf{x},\mathbf{U}, t) \tag{4}$$

Where $\mathbf{x}$ is the state vector $(u, v, w, p, q, r, \Theta, \Phi)$ and $\mathbf{U}$ is the control vector $\theta_0, \theta_{1c}, \theta_{1s}, \theta_{tr}$. System of equations (4) is linearized using perturbation method to derive the linearized flight dynamic model. Using small perturbation theory, helicopter motion is described as a perturbation from the equilibrium state as given below:

$$\mathbf{x} = \mathbf{x}_e + \Delta\mathbf{x} \tag{5}$$

Where, $\mathbf{x_e}$ refers to equilibrium state and $\Delta\mathbf{x}$ is a small perturbation about the equilibrium state.It is assumed that right hand side of Eq. (4) can be represented as analytic functions of the vehicle motion variables and their derivatives. Using the Taylor series expansion theorem for analytic functions, the function can be written in the following approximate form:

$$(6) \quad F_i = F_{ie} + \Delta F_i$$

By substituting the above series approximation in the equations of motion, the equilibrium and perturbation equations are obtained separately. Equilibrium part of the equation is used to predict the trim states for a given flight condition. The current study adopts the solution procedure given in Ref.[19,20] for the prediction of trim states. The trim parameters are given to the stability module to calculate the stability and control derivatives by using forward difference scheme. The stability and control matrices are formed from these derivatives. The linearized system dynamics about the trim condition is given by the following equation

$$(7) \quad \dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

Where, **A** is system matrix and **B** is control matrix.

Nonlinear response of the vehicle for a prescribed control input is obtained by integrating the system of equations (4) in time domain. The procedure for control response calculation is shown in Fig. 1. The trim parameters (states, flap and inflow) are considered as initial conditions because the system is perturbed from the trim position. Using the states of the system, flap and inflow at i^{th} time step, the loads are calculated at i^{th} time step. Using the loads at i^{th} step, the states of the vehicle, the blade flap and rotor inflow are calculated at $(i+1)^{th}$ time step. The detailed procedure for evaluating control response is given in Ref[19].

3.1. HANDLING QUALITY: ATTITUDE QUICKNESS

In ADS-33E, the parameters defining the handling quality characteristics of a helicopter in roll and pitch motions are attitude quickness, and bandwidth and phase delay. The attitude quickness parameter is applicable for moderate to large change in attitude angle. Bandwidth and phase delay are defined for small amplitude and high frequency motions.

From ADS-33E, the attitude quickness is defined as follows,

$$(8) \quad \text{Quickness}_{Roll(Pitch)} = \frac{p_{peak}(\text{or}\, q_{peak})}{\Delta\Phi_{peak}(\text{or}\, \Delta\Theta_{peak})}$$

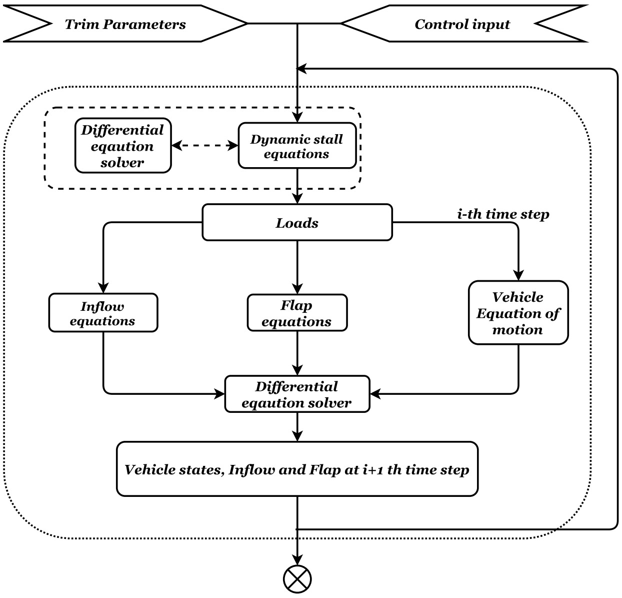


Figure 1: Flowchart for non-linear open loop control response

These quickness parameter values are plotted against minimum change in attitude angle $\Delta\Phi_{min}(\Delta\Theta_{min})$ to estimate the level of handling quality. In the present study, the quickness parameter is evaluated from the vehicle response for a given control input from both linearized (uncoupled and coupled) and nonlinear models.

The coupled linearized flight dynamic model is given by the Eq. (7). Uncoupled models are extracted from Eq. (7) by eliminating off-diagonal terms from stability matrix (**A**) and off-axis control derivatives from control matrix (**B**). Uncoupled equations for roll and pitch motions are given as follows

$$(9) \quad \text{Roll:} \quad \dot{p} = L_p p + L_{\theta_{1c}}\theta_{1c}$$

$$(10) \quad \text{Pitch:} \quad \dot{q} = M_q q + M_{\theta_{1s}}\theta_{1s}$$

Where, L_p is rolling moment derivative with respect to roll rate and $L_{\theta_{1c}}$ is rolling moment derivative with respect to lateral cyclic pitch input. M_q is pitching moment derivative with respect to pitch rate and $M_{\theta_{1s}}$ is pitching moment derivative with respect to longitudinal cyclic pitch input.

To obtain control response from coupled linearized model, Eq. (7) is integrated in time domain for a given control input. Since **x** and **u** in Eq. (7) refer to perturbation in state and control angles, the initial condition for the integration is taken as zero in all the states ($\mathbf{x} = \{0,0,0,0,0,0,0,0\}$). The input control vector is given as ($\mathbf{u} = \{0, m_1, 0, 0\}$) for roll axis response and ($\mathbf{u} = \{0, m_2, 0, 0\}$) for pitch axis response (m_1, m_2 - prescribed control inputs). In the case of uncoupled linearized model, Eqs. (9)

and (10) are integrated in time domain to obtain the time response of roll rate and pitch rate. These time responses of roll rate and pitch rate are integrated once again to obtain the corresponding attitude responses.

3.2. HANDLING QUALITY: BANDWIDTH AND PHASE DELAY

Bandwidth and phase delay are defined based on the frequency response behavior of attitude angles. The definition for the various parameters used to evaluate the bandwidth and phase delay can be found in ADS-33E. Among those parameters, crossover frequency (ω_{180}) is an important parameter. And it is defined as the frequency at which the phase angle is -180 degree.

Frequency response of the helicopter attitude angles can be obtained from both linearized and nonlinear models. In the case of linearized model, the transfer functions are used for evaluating the frequency responses. The transfer functions corresponding to uncoupled roll and pitch attitude angles can be obtained from Eqs. (9) and (10) by applying Laplace transformation. For example, the transfer function for the roll axis is can be written as,

$$G_{\Phi}(s) = \frac{\Phi(s)}{\theta_{1c}(s)} = \frac{L_{\theta_{1c}}}{s(s - L_p)} \tag{11}$$

By substituting $s = j\omega$, the transfer function of roll attitude in frequency domain can be written as;

$$G_{\Phi}(j\omega) = \frac{L_{\theta_{1c}}}{j\omega(j\omega - L_p)} = \frac{L_{\theta_{1c}}}{(-\omega^2 - j\omega L_p)} \tag{12}$$

The magnitude and phase angle from Eq. (12) can be obtained as

$$\text{Magnitude } (dB) = 20 \log_{10} \left(\frac{L_{\theta_{1c}}}{\omega\sqrt{\omega^2 + {L_p}^2}} \right) \tag{13}$$

$$\text{Phase angle } (deg) = \tan^{-1}\left(-\frac{L_p}{\omega} \right) \tag{14}$$

The magnitude (Eq. (13)) and phase angle (Eq. (14)) are evaluated for the range of frequency (0 to ∞) to obtain the frequency response.

In the case of nonlinear models, the nonlinear time responses are generated for harmonic cyclic input for wide range of frequencies. The phase difference between input and attitude response and the magnitude of attitude angle are noted down. The phase and magnitude are plotted against excitation frequency to obtain the frequency response of the system. Since the base system is unstable, the vehicle responses diverge as time evolves. Hence only initial few cycles in response are considered for frequency response calculation.

4. RESULTS AND DISCUSSION

All the results presented in this paper are pertaining to the helicopter data given in Tables. 1 and 2. The results are presented in two parts. In the first part, results corresponding to attitude quickness are presented. In the second part, the results of bandwidth and phase delay are presented.

Table 1: Main rotor and Tail rotor

	Main rotor	Tail rotor
Radius(m)	6.6	1.275
Angular speed(rad/s)	32.88	160
No. of blades	4	4
Lift curve slope	5.73	5.73
Chord(m)	0.5	0.19
Twist(deg)	-12	-12
Position of Hub from CG(m)	0.05, 0, -1.6	-7.9, 0, -2

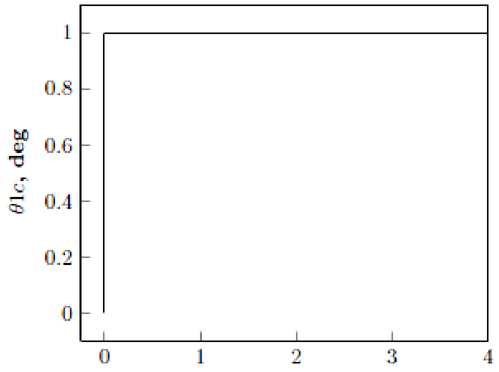
Table 2: Fuselage and Empennage

Mass(kg)	4500
$I_{xx}(kg.m^2)$	5000
$I_{yy}(kg.m^2)$	20000
$I_{zz}(kg.m^2)$	16700
$I_{xz}(kg.m^2)$	3700
Fuselage flat plate area(m^2)	1.8
Horizontal tail area(m^2)	1.326
Vertical fin area(m^2)	1.2036
Position of horizontal tail from CG(m)	-7.325, 0, -0.535
Position of vertical fin from CG(m)	-7.313, 0, -0.452

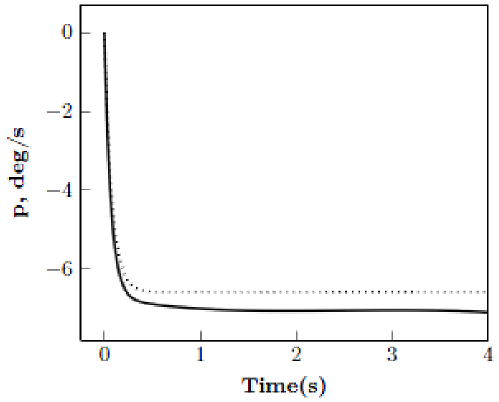
4.1. ATTITUDE QUICKNESS

The attitude quickness parameter is evaluated from both linearized (uncoupled and coupled) and nonlinear models. Pulse input is used for linearized models. For nonlinear model, two different types of input, namely, pulse input (1 deg magnitude for 1 second duration) and step input (1 deg magnitude), are used. Two different flight conditions, namely, hover and an advance ratio of 0.20, are used in this study.

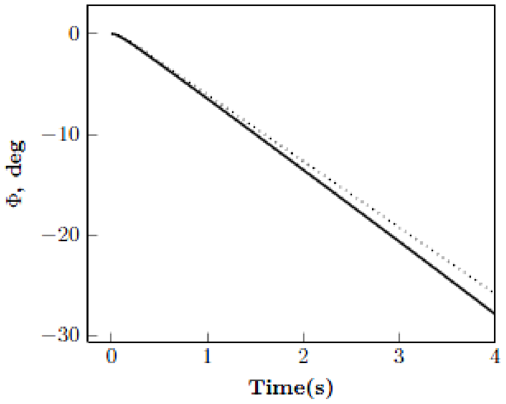
Figure 2 shows the roll axis responses from linearized (uncoupled and coupled) models for a lateral cyclic step input of 1 deg magnitude at hover. The responses corresponding to uncoupled linearized model are represented by continuous lines and the responses corresponding to coupled linearized model are represented by dashed lines. It can be noted that the roll rate attains the nonzero steady state value in 0.25 seconds. The roll attitude increases in the negative direction continuously. The maximum change in attitude angle ($\Delta\Phi_{peak}$) cannot be evaluated from these curves. Hence the step input cannot be used in linearized models in the estimation of quickness parameter and only pulse input with different time duration (in roll axis : 1 second, 2 seconds and 3 seconds; and in pitch axis: 1 second, 2 seconds and 5 seconds) is used for the estimation of quickness parameter using linearized models.



(a) Lateral cyclic pitch



(b) Roll rate



(c) Roll attitude

Figure 2: Roll axis responses from linearized models for step input

Figure 3 shows the lateral axis responses obtained from linearized models for lateral cyclic pulse input (1 deg magnitude for 1 second duration) at hover. It is noted that the responses obtained from linearized uncoupled and coupled models are very close. The roll attitude increases continuously till the end of the pulse. After the pulse input ceases to exist, the roll attitude attains the steady state for linearized models. Maximum and minimum change in attitude angles ($\Delta\Phi_{peak}$ and $\Delta\Phi_{min}$) cannot be obtained separately from these curves. Hence, the maximum change in attitude angle ($\Delta\Phi_{peak}$) is used instead of minimum change in attitude angle ($\Delta\Phi_{min}$).

Figure 4 shows the lateral axis responses obtained from the non-linear model for lateral cyclic input at hover. The responses corresponding to a pulse input are represented by continuous lines and the responses corresponding to a step input are represented by dashed lines. It is noted that the responses for both inputs are same till 1 second. In the case of pulse input, roll rate decreases suddenly once the input becomes zero. Whereas for step input, the roll rate decreases gradually after 1 second. Roll attitude increases and then decreases in the negative direction for both of the inputs.

The attitude quickness parameters evaluated from control response characteristics are marked against the change in maximum attitude in Fig. 5. The continuous lines represent the limit ratings as Level 1, Level 2 and Level 3, given in ADS-33E. Fully filled symbols represent the attitude quickness parameters corresponding to uncoupled linearized model. Open symbols represent the attitude quickness parameters corresponding to coupled linearized model. Partially filled symbols represent the attitude quickness parameters obtained from nonlinear model. From Figs 5(a) and 5(c), it is noted that in roll axis, the attitude quickness parameters obtained from nonlinear model for both step and pulse inputs correspond to Level-2 handling quality. The quickness parameter corresponding to step input from nonlinear model differs significantly from the quickness parameter corre-

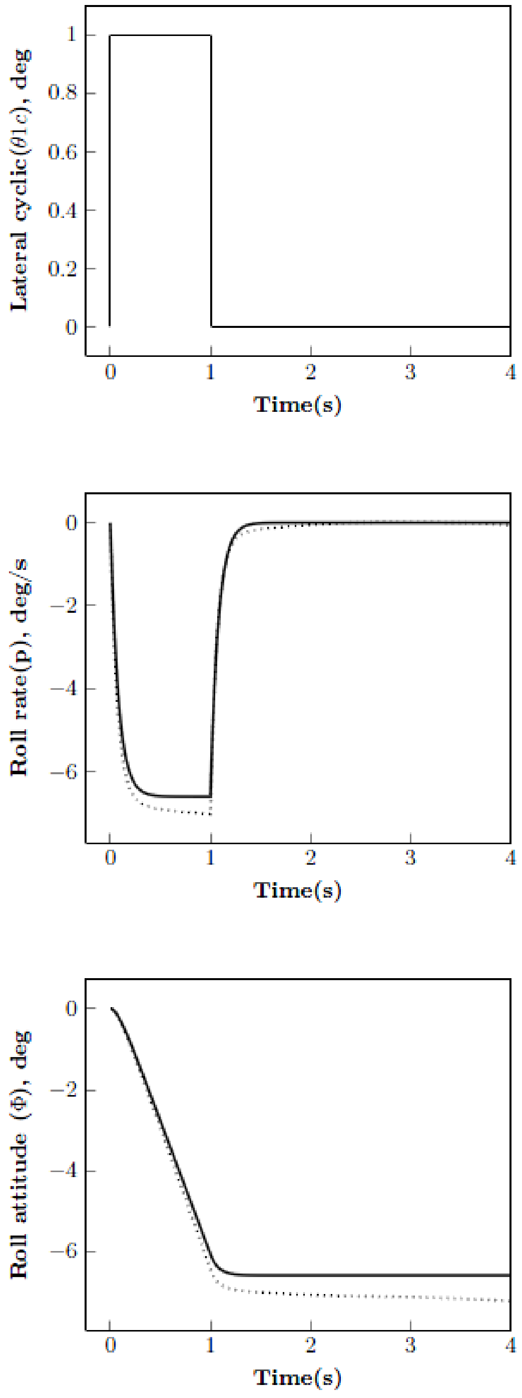
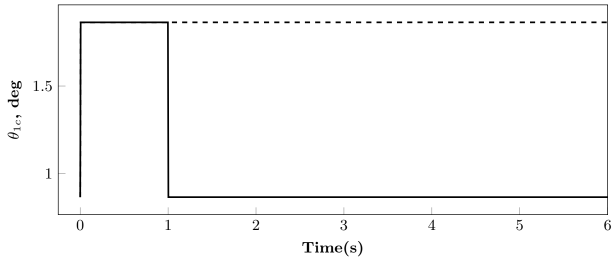
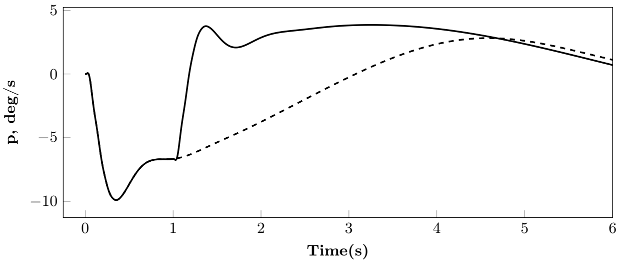


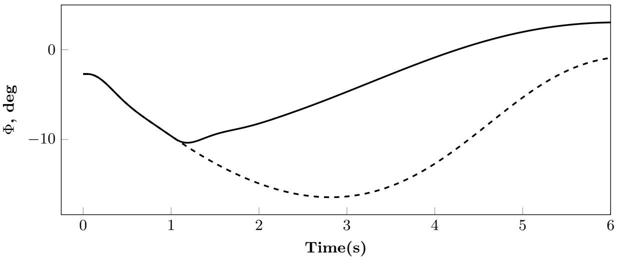
Figure 3: Roll axis responses from linearized models for pulse input at hover



(a) Lateral cyclic pitch



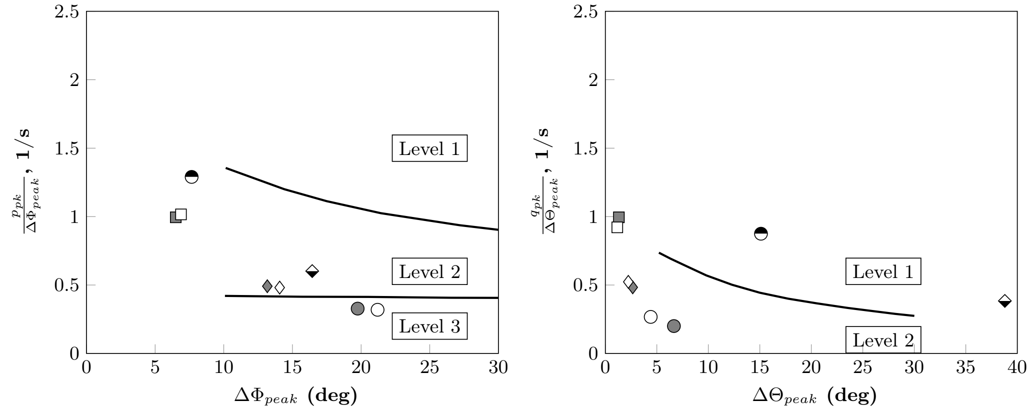
(b) Roll ratee



(c) Roll attitude

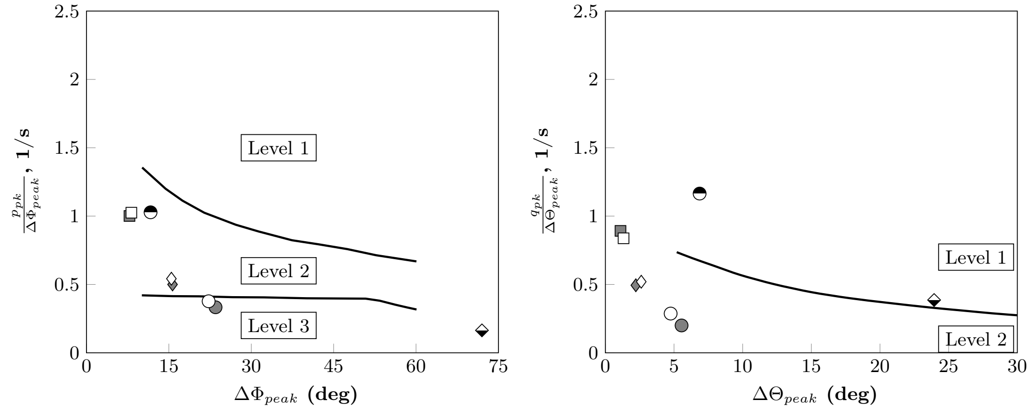
Figure 4: Roll axis responses from non-linear model for pulse and step inputs at hover

sponding to pulse input. The quickness parameters corresponding to pulse input of 1 second duration from linearized (coupled and uncoupled) and non-linear models are very close. For linearized models, the quickness parameters corresponding to 1 second and 2 second pulse inputs indicate Level-2 handling quality, whereas for 3 second pulse input, it degrades to Level-3 handling quality. From Figs 5(b) and 5(d), it is noted that in pitch axis, the nonlinear model predict almost Level-1 handling quality. The quickness parameters evaluated for pulse input of 1 second duration from linearized models are different from the quickness parameter obtained from nonlinear model. For the case of linearized models, as the duration of pulse input increases, the quickness parameter decreases and it shows Level-2 handling quality. The results indicate that for linearized models, a large time duration pulse is required to generate large change in pitch attitude which can be attributed to the large value of pitch inertia as compared to roll inertia.



(a) Hover (Roll)

(b) Hover (Pitch)



(c) $\mu = 0.20$ (Roll)

(d) $\mu = 0.20$ (Pitch)

■ Linearized-uncoupled-1 sec pulse
◆ Linearized-uncoupled-2 sec pulse
● Linearized-uncoupled-3 sec(roll) or 5 sec(pitch) pulse
□ Linearized-coupled-1 sec pulse
◇ Linearized-coupled-2 sec pulse
○ Linearized-coupled-3 sec(roll) or 5 sec(pitch) pulse
◓ Nonlinear - 1 sec pulse
⬙ Nonlinear -Step

Figure 5: Attitude quickness parameter comparison for different types of input

4.2. BANDWIDTH AND PHASE DELAY

Figure 6 shows the frequency response transfer function of roll attitude to lateral cyclic input in hover. Form Fig. 6(b), it can be seen that the phase angle varies from 90 deg. to 0 deg.. As mentioned earlier, bandwidth and phase delay, are calculated based on the cross over frequency (ω_{180}). From Fig. 6(b), it is clearly evident that the bandwidth and phase delay parameters cannot be evaluated from the linearized first order transfer function. However these parameters can be calculated for a system, if the order of transfer function is more than 1. Hence nonlinear model is used for the estimation of bandwidth and phase delay.

The time responses are generated for harmonic excitation of lateral and longitudinal cyclic pitch inputs with various frequencies in the range 0.05 Hz to 5 Hz (0.05, 0.08, 0.1, 0.25, 0.5, 1, 1.5, 2, 2.5, 3, 4 and 5). Frequency response of the attitude angles are evaluated from these responses. This study is carried out for two different amplitudes (1 deg and 2 deg) of input to bring out the effect of system nonlinearity. Two flight conditions, namely hover and advance ratio of 0.20, are considered in this analysis.

Figure 7 shows the frequency responses of roll attitude for harmonic excitation of 1 deg. and 2 deg. magnitudes of lateral cyclic pitch in hover. It is noted that the magnitude and phase deference are very close in both the cases. Whereas in the lower frequencies, the magnitude of the input influences the magnitude and phase of the attitude angle. It can be seen that the cross over frequency (ω_{180}) is very

Table 3: **Band width and phase delay parameters**

Flight condition	Axis	Magnitude of input	ω_{180} (rad/s)	$\Delta\phi_{2\omega_{180}}$ (deg)	$\omega_{BW_{phase}}$ (rad/s)	$\omega_{BW_{gain}}$ (rad/s)	τ_p (s)
Hover	Roll	1 deg	9.9425	75.45	6.035	4.804	0.06
		2 deg	10.36	75.19	6.081	5.8119	0.0633
	Pitch	1 deg	3.094	47.38	1.572	2.456	0.134
		2 deg	3.113	39	1.62	1.961	0.109
$\mu = 0.20$	Roll	1 deg	8.98	77	5.05	5.6146	0.0748
		2 deg	9.167	75.8	5.3624	5.6537	0.07215
	Pitch	1 deg	8.501	48.8	3.0512	5.0841	0.0501
		2 deg	6.0742	44.8	2.1993	3.9574	0.06435

close (9.945 rad/s for 1 deg. magnitude and 10.36 rad/s for 2 deg. input) in both the cases.

The bandwidth and phase delay are calculated from the frequency responses and are tabulated in Table 3. It is noted from Table 3 that the crossover frequency (ω_{180}) decreases in roll axis in forward flight compared to hover. From the frequency response corresponding to 1 deg. input in roll axis, the crossover frequency is 9.9425 rad/s at hover and 8.98 rad/s at $\mu = 0.20$. In roll axis, the gain bandwidth is less than the phase bandwidth at hover, but the gain bandwidth is more than the phase bandwidth at $\mu = 0.20$. In pitch axis, the crossover frequency (ω_{180}) increases in forward speed as compared to hover. From the frequency response corresponding to 1 deg. magnitude of input, the crossover frequency is 3.094 rad/s at hover and 8.501 rad/s at $\mu = 0.20$. In pitch axis, the phase bandwidth is less than the gain bandwidth at both at hover and forward speed $\mu = 0.20$. In Fig. 8, the phase delay τ_p is plotted against the bandwidth to predict the level of handling quality as defined in ADS-33E. It is noted that the results predict level-1 handling quality in pitch and roll axis for hover and $\mu = 0.20$.

5. CONCLUDING REMARKS

This study presents the estimation of handling quality parameters for a helicopter using open loop linearized and nonlinear flight dynamic models. The handling quality parameters corresponding to attitude quickness, bandwidth and phase delay in pitch and roll behavior are discussed. The important observations of this study are:

- Attitude quickness can be evaluated using either linearized (uncoupled and coupled) flight dynamic model or nonlinear flight dynamic model. For linearized models, only pulse input with different time duration is used, whereas in the nonlinear model, two different types of input, namely pulse input and step input, are considered.
- For linearized models, the duration of pulse input influences the level of handling quality in roll axis.
- The attitude quickness parameters evaluated from nonlinear rotor-fuselage coupled dynamic model are different for pulse input and step input.
- The bandwidth and phase delay cannot be evaluated from open loop linearized models. They can be calculated using open loop nonlinear coupled rotor-fuselage dynamic model.
- In roll axis, the bandwidth is gain limited at hover and is phase limited at $\mu = 0.20$, whereas in pitch axis, the bandwidth is phase limited both at hover and $\mu = 0.20$.

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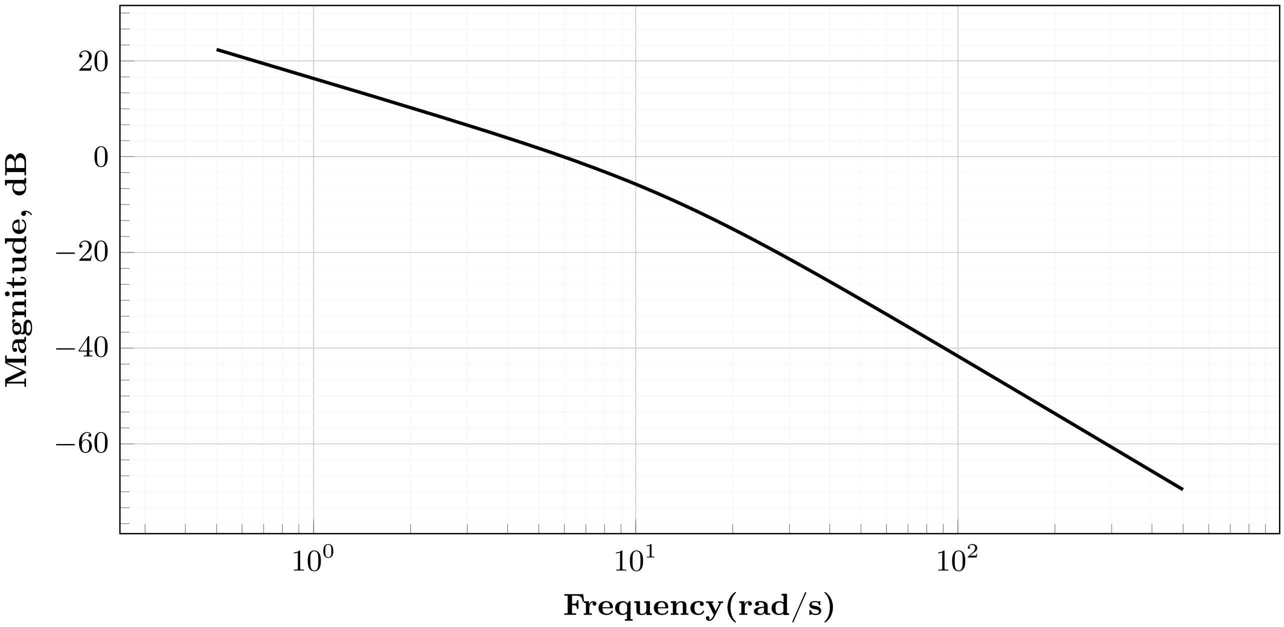
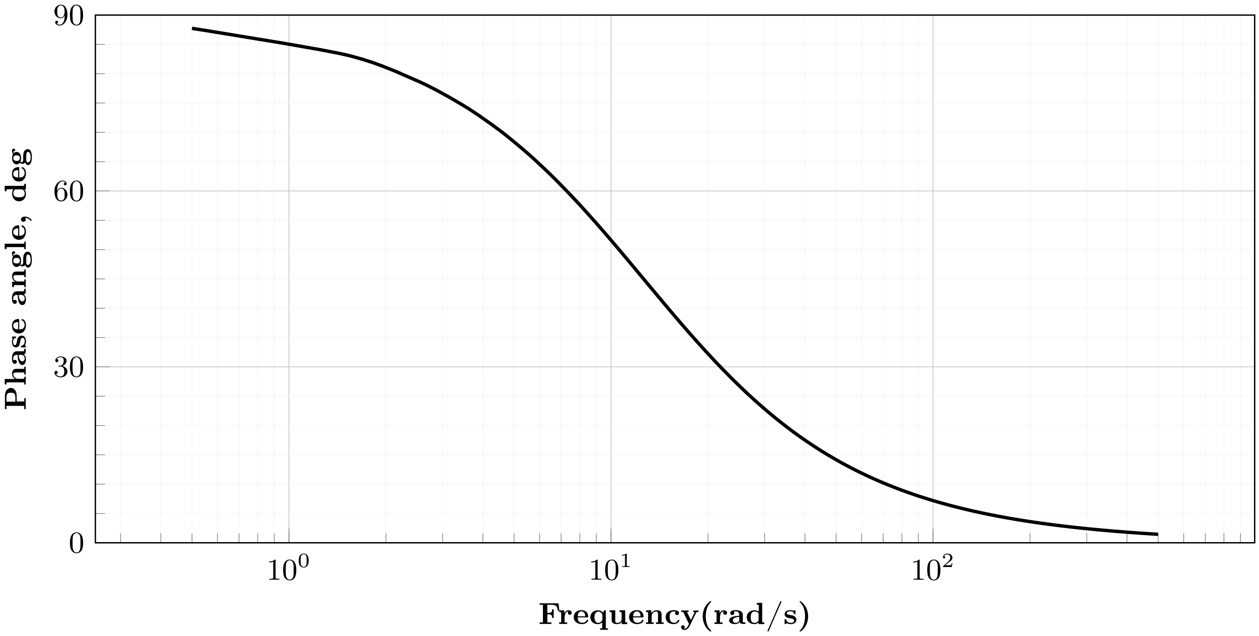


(a) Magnitude



(b) Phase angle

Figure 6: Frequency response of roll attitude obtained from linearized model transfer function at hover

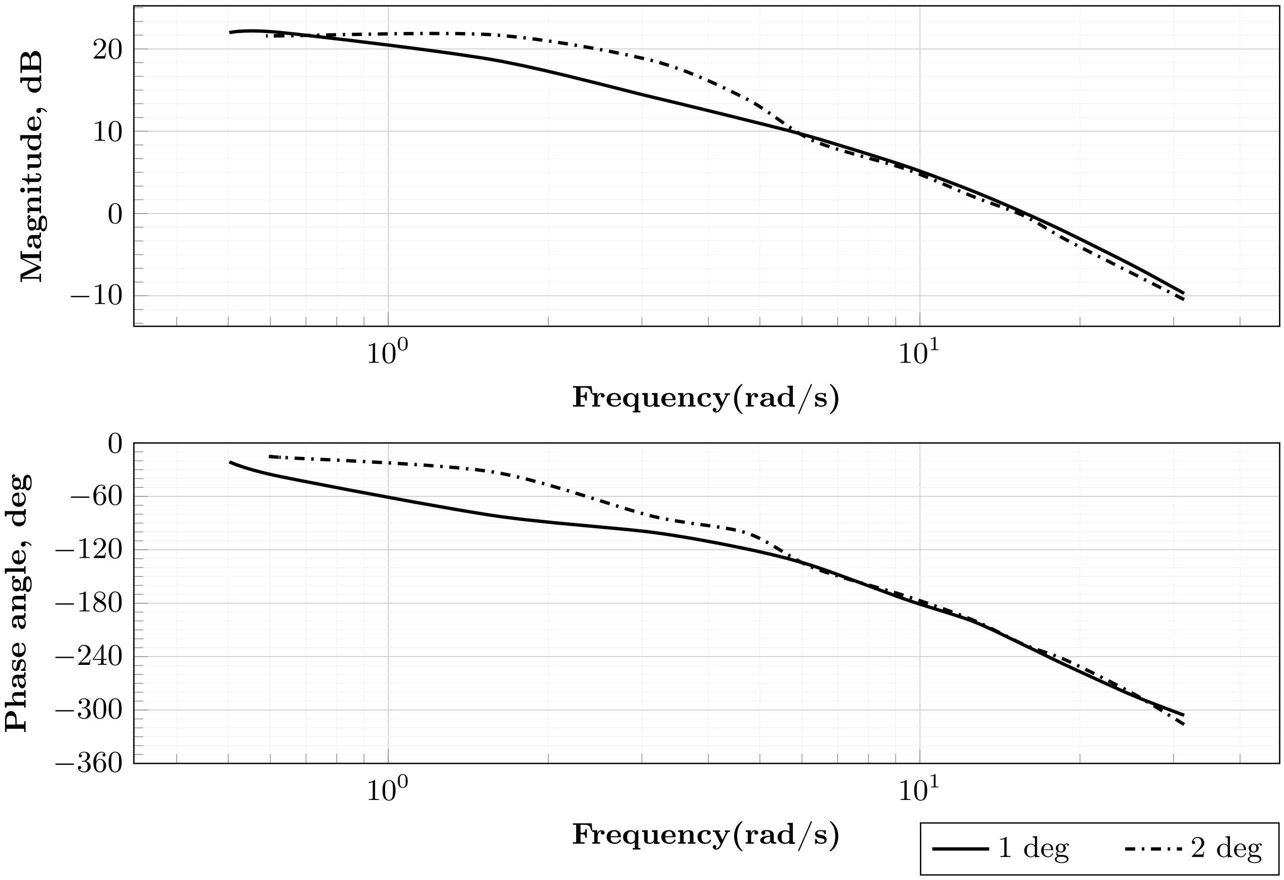
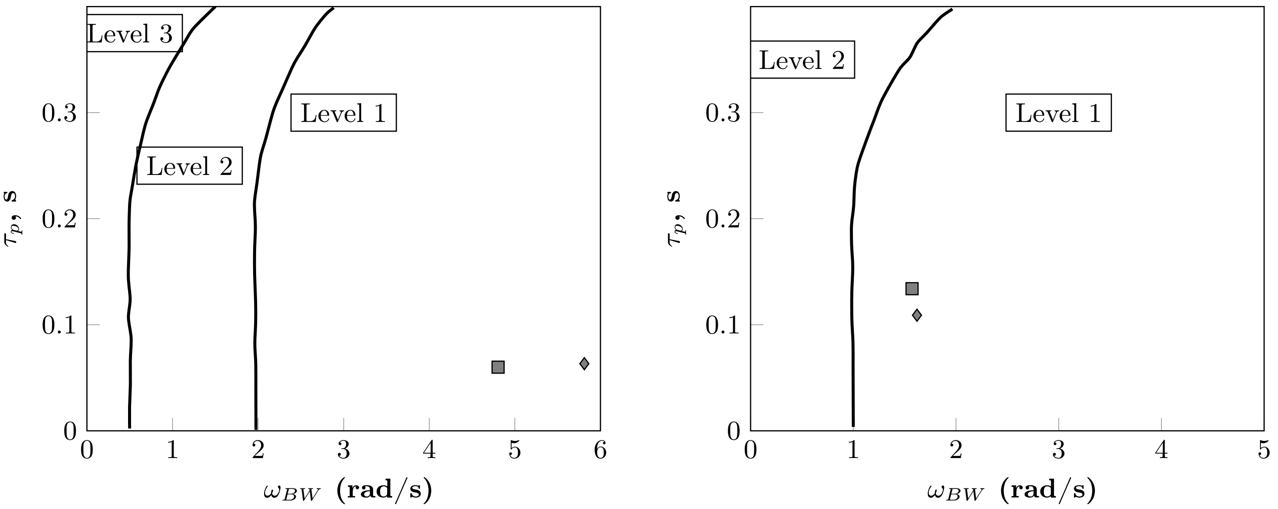
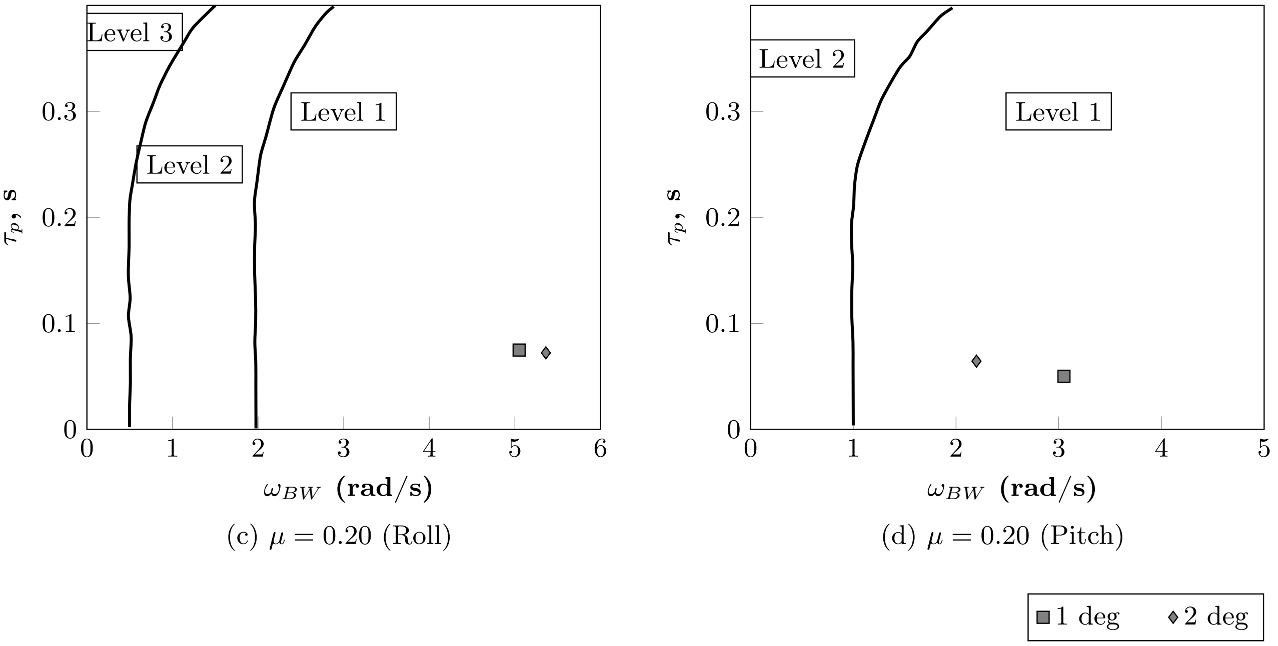


Figure 7: Frequency response of roll attitude at hover



(a) Hover (Roll) (b) Hover (Pitch)



(c) $\mu = 0.20$ (Roll) (d) $\mu = 0.20$ (Pitch)

Figure 8: Handling quality levels with respect to bandwidth and phase delay