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NEW ASPECTS ON HELICOPTER ROTOR DYNAMICS

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ABSTRACT

Mathematical modeling of the elastic rotor blade in flight mechanical investigations is based on the known stubstitute model of rigid beam with phantom flapping - that is respective lagging hinge in the vicinity of blade clamping place. The elasticity of blade is respesented equivalently by installation of a spring on the hinge. The blade model serves sufficiently for statements on the first harmonic oscillations.

In case of dynamic investigations it is however necessary to represent higher harmonic oscillation forms of blade. The necessary local deviations for this on the blade supplies the solution of the partial differential equations of blade deflections.

These coupled differential equations for flapping, lagging and torsion are derived by J. C. Huboldt and G. W. Brooks.

For the solution of equations a variation formulation according to Ritz with Hermite-polynomials as formulation functions is drawn up. Based on this solution formulation, a calculation program is set up on which blade oscillation forms and bending procedures for various flight cases can be determined and discussed.

The study was carried out at the Institute for Flight Mechanics and Flight Control at the Technical University of Munich by order of the Federal Ministery for Research and Technology.

LIST OF SYMBOLS

$\underline{\underline{A}}$	system matrix
c_i	parameter for Ritz-formulation
c_A	lift coefficient
$\underline{c}$	vector of boundary deriviations
EI_1, EI_2	bending resistance according to main axis
EI	bending resistance in the simplified flap differential equation
F_A	lifting force
GI	torsional resistance of the blade
L	total blade length
m *	mass per unit length
q	line load resulting from lift distribution
t	time
$\bar{t}$	dimensionless time
t_{bl}	blade chord
T	centrifugal force on the rotor blade
$\underline{\underline{T}}_{GH}, \underline{\underline{T}}_{HR}$	matrices for transformation of coordinates
u_∞	freestream velocity
U	kinetic energy
$\underline{v}$	total velocity in rotor fixed coordinate system
$\underline{v}_g$	velocity vector, geodetical coordinate system
$\underline{v}_T$	velocity vector of translation
$\underline{v}_R$	velocity vector of rotation
$\underline{v}_H$	total velocity vector, helicopter fixed coordinate system
v_{res}	resulting flow velocity of the rotor blade
V	potential energy
W	exterior work
x_E, y_E, z_E	deflection of the blade
x	radial coordinate of the blade
$\bar{z}$	dimensionless deflection in flap plane
α_A	preset angle of incidence
α_i	induced angle of incidence
α_{eff}	effective angle of incidence
θ_u	twisting deformation of the blade
θ_E	torsion angle
ϕ, θ, ψ	angle of roll, pitch, yaw
ψ_{bl}	azimuth blade angle
ρ	air density
ω_{Ro}	rotational frequency
$\underline{\omega}$	rotational vector of the helicopter

1 INTRODUCTION

The derived and coupled differential equations in [1] are of the following form:

Torsion:

$$- \{(GI + Ti_A^2 + EB_1\theta_u'^2)\theta_E' - EB_2\theta_u' (y_E''\cos\theta_u + z_E''\sin\theta_u)\}'$$

$$- Te_A (z_E''\cos\theta_u - y_E''\sin\theta_u) + \omega_{Ro}^2 m^* ex (z_E'\cos\theta_u - y_E'\sin\theta_u)$$

$$+ \omega_{Ro}^2 m^* e \sin\theta_u \, y_E + \omega_{Ro}^2 m^* [(i_{m\zeta}^2 - i_{m\eta}^2)\cos 2\theta_u + ee_o\cos\theta_u]\,\theta_E$$

$$+ m^* i_m^2 \ddot{\theta}_E + m^* e (\ddot{z}_E\cos\theta_u - \ddot{y}_E\sin\theta_u) =$$

$$= M_L^* + (Ti_A^2\theta_u')' - \omega_{Ro}^2 m^* [(i_{m\zeta}^2 - i_{m\eta}^2)\sin\theta_u\cos\theta_u + ee_o\sin\theta_u]$$

Flapping:

$$\{(EI_1\cos^2\theta_u + EI_2\sin^2\theta_u)\, z_E'' + (EI_2 - EI_1)\sin\theta_u\cos\theta_u \, y_E'' \qquad (1)$$

$$- Te_A\cos\theta_u\theta_E - EB_2\theta_u'\sin\theta_u\theta_E'\}'' - (Tz_E')' - (\omega_{Ro}^2 m^* ex \cos\theta_u\theta_E)'$$

$$+ m^* (\ddot{z}_E + e\cos\theta_u\ddot{\theta}_E) = L_z^* + (Te_A\sin\theta_u)'' + (\omega_{Ro}^2 m^* ex \sin\theta_u)'$$

Lagging:

$$\{(EI_2 - EI_1)\sin\theta_u\cos\theta_u \, z_E'' + (EI_1\sin^2\theta_u + EI_2\cos^2\theta_u)\, y_E''$$

$$+ Te_A\sin\theta_u\theta_E - EB_2\theta_u'\cos\theta_u\,\theta_E'\}'' - (Ty_E')' + (\omega_{Ro}^2 m^* ex \sin\theta_u\theta_E)'$$

$$+ \omega_{Ro}^2 m^* e \sin\theta_u \,\theta_E + m^* (\ddot{y}_E - e\sin\theta_u\,\ddot{\theta}_E) - \omega_{Ro}^2 m^* y_E =$$

$$= - L_y^* + (Te_A\cos\theta_u)'' + (\omega_{Ro}^2 m^* ex \cos\theta_u)' + \omega_{Ro}^2 m^* (e_o + e\cos\theta_u)$$

The solution of this equation falls under the problems of elasticity theory.

Mechanical problems of this kind however, frequently do not permit any proper solution. This applies especially for the so called boundary value problems, that is problems as such where a differential equation or a system of differential equations with specified boundary conditions is to be solved. In such cases one frequently refers to approximation solutions adapted to the boundary conditions. The proper methods to find such approximation solutions, especially in boundary value problems in elasticity theory, are based on variation calculations where the Ritz - or the Galerkin procedure are the most significant.

2 SIMPLIFICATION OF DIFFERENTIAL EQUATIONS

The now following method of solution is based on the variation method by Ritz.

First of all to make the procedure clear the simplified uncoupled flap- equation is derived.

For this the following neglections are made:

a) Not only the stiffness of blade EI, but also the mass per unit m* are assumed to be constant along the blade.

b) Only displacements in the z-direction are considered.

$$\longrightarrow \qquad y_E = \theta_E = \theta_u = 0$$

c) The center of gravities-, tension - and elastic axes coincide

$$\longrightarrow \qquad e = e_A = 0$$

d) Time integration ensues in small time intervals, so that the process can be regarded as quasistationary.

$$\longrightarrow \qquad L_z^{\,*} = q(x)$$

The differential equation thus receives the form:

$$EIz_E'''' - (Tz_E')' + m^*\ddot{z}_E = q(x) \tag{2}$$

in which for centrifugal force holds:

$$T = m^*\omega_{Ro}^{\;2}\int x\,dx$$

$$T = \frac{1}{2}\,m^*\omega_{Ro}^{\;2}x^2 \tag{3}$$

Thus it follows

$$EIz_E'''' - m^*\omega_{Ro}^{\;2}xz_E' - \frac{1}{2}\,m^*\omega_{Ro}^{\;2}x^2z_E'' + m^*\ddot{z}_E = q(x) \tag{4}$$

3 THE RITZ-PROCEDURE

In the elasticity theory the elastic bending line

$$w = w\ (x)$$

is according to the Ritz-procedure generally approximated by

$$\hat{w}(x) = c_1 w_1(x) + c_2 w_2(x) + \dots + c_n w_n(x) = c_i w_i(x) \qquad (5)$$

The formulation functions $w_i(x)$ are arbitrary chosen functions which must be sufficient for the boundary conditions of system.

The parameters c_i must be determined, that is with the assistance of the already mentioned variation, which can be deduced from the energy formulation due to Hamilton [2]

$$\int_{\Delta t} [\delta(U - V) + \delta W]\, dt = 0 \qquad (6)$$

From the kinetic energy U, the potential energy V and the exteriour work W follows for the virtual derivations

$$\delta U = \int_0^L m^* \dot{z}\delta\dot{z}\, dx$$

$$\delta V = \int_0^L [EIz''\delta z'' - \frac{1}{2} m^* \omega_{Ro}{}^2 x^2 z'\delta z']\, dx \qquad (7)$$

$$\delta W = \int_0^L (\delta z q(x) + m^* \omega_{Ro}{}^2 x z\delta z' + m^* \omega_{Ro}{}^2 x\delta z z')\, dx$$

In this existing dynamic problem for the approximation function according to eqn. (5) holds

$$\hat{z}(x,t) = \sum_{i=1}^{n} c_i(t)\ z_i(x) \qquad (8)$$

From this formulation with the derivations $\hat{z}'$, $\hat{z}''$ ... follows the energy equation

$$m^* \int_0^L \Sigma \ddot{c}_i z_i\ \Sigma\delta c_i z_i dx + EI \int_0^L \Sigma c_i z_i''\ \Sigma\delta c_i z_i'' dx$$

$$+ \frac{1}{2} m^* \omega_{Ro}{}^2 \int_0^L x^2 \Sigma c_i z_i'\ \Sigma\delta c_i z_i' dx - 2m^* \omega_{Ro}{}^2 \int_0^L x \Sigma c_i z_i\ \Sigma\delta c_i z_i' dx \qquad (9)$$

$$- \int_0^L q(x)\ \Sigma\delta c_i z_i dx = 0$$

THE HERMITE- POLYNOMIALS AS FORMULATION FUNCTIONS

Now formulation functions must be found sufficing the boundary conditions. The Hermite- polynomials are deduced according to ref. [3] from a boundary value consideration; they fulfill thus this requisition. According to the degree of the based derivations of the boundary values, one distinguishes Hermite- polynomials of the 4th, 6th and 8th order.

In the following tables and figures the various polynomials are illustrated.

4^{th} order

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \bar{x} \\ \bar{x}^2 \\ \bar{x}^3 \end{bmatrix}$$

6^{th} order

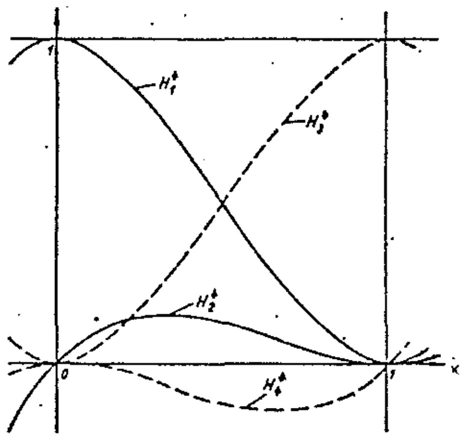
$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -10 & 15 & -6 \\ 0 & 1 & 0 & -6 & 8 & -3 \\ 0 & 0 & \frac{1}{2} & -\frac{3}{2} & \frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 10 & -15 & 6 \\ 0 & 0 & 0 & -4 & 7 & -3 \\ 0 & 0 & 0 & \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \bar{x} \\ \bar{x}^2 \\ \bar{x}^3 \\ \bar{x}^4 \\ \bar{x}^5 \end{bmatrix}$$

8^{th} order

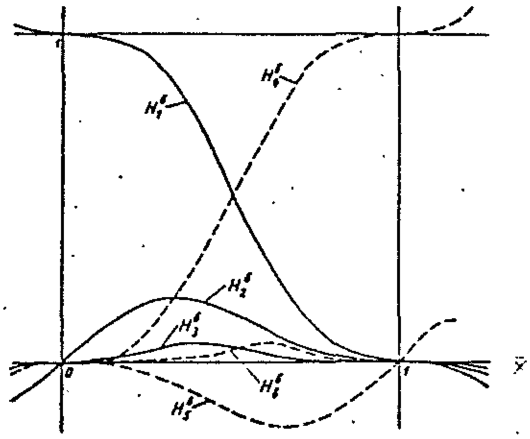
$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \\ H_7 \\ H_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -35 & 84 & -70 & 20 \\ 0 & 1 & 0 & 0 & -20 & 45 & -36 & 10 \\ 0 & 0 & \frac{1}{2} & 0 & -5 & 10 & -\frac{15}{2} & 2 \\ 0 & 0 & 0 & \frac{1}{6} & -\frac{4}{6} & 1 & -\frac{4}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 35 & -84 & 70 & -20 \\ 0 & 0 & 0 & 0 & -15 & 39 & -34 & 10 \\ 0 & 0 & 0 & 0 & \frac{5}{2} & -7 & \frac{13}{2} & -2 \\ 0 & 0 & 0 & 0 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \bar{x} \\ \bar{x}^2 \\ \bar{x}^3 \\ \bar{x}^4 \\ \bar{x}^5 \\ \bar{x}^6 \\ \bar{x}^7 \end{bmatrix}$$

Table 1: Hermite- polynomials

4th order



6th order



8th order

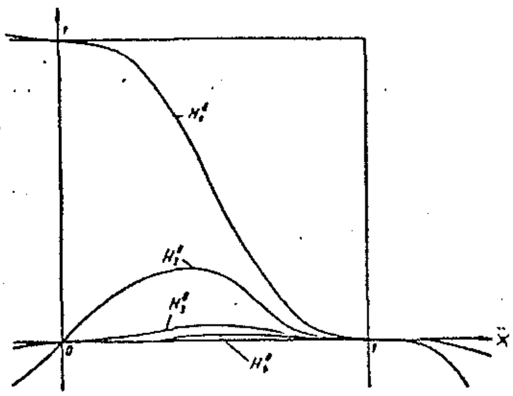


Fig.1: Graph of Hermite- polynomials

Variable $\bar{x}$ of the polynomial is hereby without any dimension. Thus the later integrations result across the blade radius in the interval [0,1] which will be of an advantage.

From the figures it can be seen that the approximation function $\hat{z}(x)$ from eqn. (8) originates from a superposition of several polynomials. Every polynomial H_i is hereby emphasized with the respective parameter c_i corresponding to the specific boundary derivation.

For first investigations we choose Hermite- polynomials of the 4th order. Since, as is well known, polynomials are dimensionless quantities, it is necessary to get energy equation (9) into a normalized form.

Normalizing with total blade radius L follows in vectorial representation.

$$\underline{c}^T \int_0^1 \underline{\bar{z}}''^2 d\bar{x} + \frac{1}{2EI} m^* \omega_{Ro}{}^2 L^4 \underline{c}^T \int_0^1 \bar{x}^2 \underline{\bar{z}}'^2 d\bar{x} - \frac{2}{EI} m^* \omega_{Ro}{}^2 L^4 \underline{c}^T \int_0^1 \bar{x} \underline{\bar{z}} \underline{\bar{z}}' d\bar{x}$$
$$+ \frac{1}{EI} m^* \omega_{Ro}{}^2 L^4 \underline{\ddot{c}}^T \int_0^1 \underline{\bar{z}}^2 d\bar{x} - \frac{L^3}{EI} \int_0^1 q(\bar{x}) \underline{\bar{z}} \, d\bar{x} = 0 \qquad (10)$$

in which for the vector $\underline{c}$ of boundary derivations it is valid that

$$c^T = [z_0, z_0', z_1, z_1']$$

Vectors $\underline{\bar{z}}$, $\underline{\bar{z}}'$, $\underline{\bar{z}}''$ are the Hermite- polynomials and their derivations.

Numerical integration of the with each other multiplied vectors $\underline{H}$ of the Hermite- polynomials, lead to constant matrices, the so called Hermite- integral matrices.

Hereby the following definitions apply:

$$\int_0^1 \underline{\bar{z}}''^2 d\bar{x} = \underline{\underline{H}}_{22} \qquad \int_0^1 \bar{x}^0 \underline{\bar{z}} \, d\bar{x} = \underline{h}_0$$
$$\int_0^1 \bar{x}^2 \underline{\bar{z}}'^2 d\bar{x} = \underline{\underline{H}}_{X11} \qquad \int_0^1 \bar{x}^1 \underline{\bar{z}} \, d\bar{x} = \underline{h}_1$$
$$\int_0^1 \underline{\bar{z}}^2 d\bar{x} = \underline{\underline{H}}_{00} \qquad \vdots \qquad (11)$$
$$\int_0^1 \bar{x} \underline{\bar{z}} \underline{\bar{z}} \, d\bar{x} = \underline{\underline{H}}_{X01} \qquad \int_0^1 \bar{x}^n \underline{\bar{z}} \, d\bar{x} = \underline{h}_n$$

The insertion of the from eqn. (11) derived integral matrices is equivalent with the local integration. Thus the time variant differential equation system of the 2nd order remains:

$$E^{*}\underline{\underline{H}}_{OO}\ddot{\underline{c}} + (\dot{\underline{\underline{H}}}_{22} + \frac{1}{2} E^{*} \underline{\underline{H}}_{X11} - 2E^{*}\underline{\underline{H}}_{XO1})\ \underline{c} - G^{*}\Sigma q_i \underline{h}_i = 0 \qquad (12)$$

One recognizes now the great advantage in the application of Hermite- polynomials integrated numerically only once according to eqn. (11). The matrices and vectors derived from this integration can be used for further relevant problems at once.

For the solution of differential equation system (12) it is significant to transform on a system 1st order of general form: [ref. 5]

$$\dot{\underline{y}} = \underline{\underline{A}}\ \underline{y} + \underline{b} \qquad (13)$$

With the introduction of a member $\dot{\underline{c}}$ the system (12) is extended in the following manner:

$$\begin{bmatrix} \dot{\underline{c}} \\ \hline \ddot{\underline{c}} \end{bmatrix} = \left[\begin{array}{c|c} \underline{\underline{O}} & \underline{\underline{E}} \\ \hline \underline{\underline{K}}_m & \underline{\underline{O}} \end{array}\right] \begin{bmatrix} \underline{c} \\ \hline \dot{\underline{c}} \end{bmatrix} + \frac{G^{*}}{E^{*}} \begin{bmatrix} \underline{O} \\ \hline \Sigma q_i \underline{k}_i \end{bmatrix} \qquad (14)$$

whereby it is valid that

$$\underline{\underline{K}}_m = \underline{\underline{H}}_{OO}^{-1}\ (2\underline{\underline{H}}_{XO1} - \frac{1}{2}\underline{\underline{H}}_{X11} - \frac{1}{E^{*}}\dot{\underline{\underline{H}}}_{22})$$

$$\underline{k}_i = \underline{\underline{H}}_{OO}^{-1}\ \underline{h}_i$$

Vector $\underline{c}$ contains, as is well known, the degrees of freedom $\bar{z}_o$, $\bar{z}'_o$, $\bar{z}_1$, $\bar{z}'_1$ of system. On behalf of both boundary conditions

$$\bar{z}_o = \bar{z}_o' = 0$$

the corresponding equations of system (12) drop out, which is equivalent to eliminate both first rows and columns of the integral matrices. We receive thus differential equation system in component notation

$$\begin{bmatrix} \dot{\bar{z}}_1 \\ \dot{\bar{z}}'_1 \\ \ddot{\bar{z}}_1 \\ \ddot{\bar{z}}'_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ K_{11} & K_{12} & 0 & 0 \\ K_{21} & K_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{z}_1 \\ \bar{z}'_1 \\ \dot{\bar{z}}_1 \\ \dot{\bar{z}}'_1 \end{bmatrix} + \frac{G^*}{E^*} \begin{bmatrix} 0 \\ 0 \\ \Sigma q_i k_{i1} \\ \Sigma q_i k_{i2} \end{bmatrix} \qquad (15)$$

5 DETERMINATION OF EXTERIOUR FORCE OF AIR DISTRIBUTION

The inhomogenous part q(x) in eqn. (2) is equivalent to the force of air distribution prevailing on blade. Because of the different oncoming flow along the blade radius this force of air distribution is a variable load distributed across distance. Generally for the uplift generated by a profile

$$F_A = \frac{1}{2}\rho c_A v_{res}^2 S$$

holds.

Since the force of air is variable on the blade radius, it must be calculated in segments.

$$dF_A = \frac{1}{2}\rho c_A v_{res}^2 t_{bl} dx \qquad (16)$$

Calculation of dF_A thus divides with ρ and t_{Bl} known in the determination of v_{res} and c_A. Both parameters are dependent on blade radius x; that is why a simple integration of dF_A is not possible. The velocity determining the uplift v_{res} will be calculated from the momentary state of flight, in which

$$v_{res} = v_{res}\,(u_\infty, \psi_{bl}, x)$$

For the calculation of v_{res} one needs the velocity components v_x, v_y, v_z in the co - revolving coordinate system with reference to rotor. For this first of all from the given velocity components v_{xg}, v_{yg}, v_{zg} in the geodetic coordinate system across the transformations matrix

$$\underline{\underline{T}}_{GH} = \begin{bmatrix} \cos\theta\cos\phi + \sin\phi\sin\theta\sin\psi & \sin\phi\sin\theta\cos\psi - \cos\theta\sin\psi & \sin\theta\cos\phi \\ \cos\phi\sin\psi & \cos\phi\cos\psi & -\sin\phi \\ \cos\theta\sin\phi\sin\psi - \sin\theta\cos\psi & \sin\theta\sin\psi + \cos\theta\sin\phi\cos\psi & \cos\theta\cos\phi \end{bmatrix}$$

the velocity components from the translational motion of coordinate system v_{xT}, v_{yT}, v_{zT} with reference to helicopter, are calculated to

$$\underline{v}_T = \begin{bmatrix} v_{xT} \\ v_{yT} \\ v_{zT} \end{bmatrix} = \underline{\underline{T}}_{GH}\,\underline{v}_g \quad . \tag{17}$$

In this system the rate of revolution of total helicopter (pitching, rolling, yawing) are superimposed on the translational velocity $\underline{v}_T$. If one describes this motion by an angular velocity vector $\underline{\omega}$, the thus resulting velocity $\underline{v}_R$ is calculated from

$$\underline{v}_R(r_p) = \underline{\omega} \times \underline{r}_p \tag{18}$$

where $\underline{r}_p$ is the vector from center of rotation to the investigated blade point.

For the velocity vector $\underline{v}_H$ resulting from the superposition of translational and rotational velocity it then holds that

$$\underline{v}_H(\underline{r}_p) = \underline{v}_T + \underline{v}_R(\underline{r}_p) \tag{19}$$

Velocity components v_x, v_y, v_z of coordinate system with reference to rotor under consideration of mast installing angle κ_m are calculated from the components of vector $\underline{v}_H$ with reference to helicopter with transformation matrix

$$
\underline{\underline{T}}_{HR} = \begin{bmatrix} -\cos\psi_{bl}\cos\kappa_m & \sin\psi_{bl} & -\cos\psi_{bl}\sin\kappa_m \\ -\sin\psi_{bl}\cos\kappa_m & -\cos\psi_{bl} & -\sin\psi_{bl}\sin\kappa_m \\ -\sin\kappa_m & 0 & \cos\kappa_m \end{bmatrix}
$$

to

$$
\underline{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \underline{\underline{T}}_{HR}\ \underline{v}_H \qquad (20)
$$

In the coordinate system with reference to blade the part

$$
v_{Ro} = \omega_{Ro} \cdot x \qquad (21)
$$

supplied by rotation of rotor is added scalarly to the y-component of $\underline{v}$.

Flapping velocity v_β supplies a contribution to the component v_z of $\underline{v}$. It reckons out stepwise from two temporal succeeding blade derivations. Since for every time interval for determining the local blade derivations an initial flapping velocity $v_{\beta o}$ is presupposed, the flapping velocity v_β must be determined iteratively.

Generally for the flapping velocity holds

$$
v_\beta = \frac{z_{i+1} - z_i}{\psi_{i+1} - \psi_i} \qquad (22)
$$

A share to the uplift give only both components v_y and v_z. For the resulting velocity v_{res} it thus follows that

$$
v_{res} = \sqrt{v_y^2 + v_z^2} \qquad (23)
$$

The uplift coefficient c_A depends on the effective angle of incidence α_{eff} which again depends on radius x and also on the Mach-number.

The effective angle of incidence is made up of a set angle of incidence α_A and a variably induced angle of incidence α_i For the induced angle of incidence α_i holds:

$$
\alpha_i = \arctan \frac{v_z}{v_y} \qquad (24)
$$

For the profile NACA 23012 exists a data sheet from "MBB" [6] which contains the coefficients c_A depending on angle of incidence α for several Mach-numbers.

If both the actual effective angle of incidence α_{eff} and the actual Mach-number are not comprehended in the data sheet, the c_A-coefficient is determined by a linear interpolation.

The values for v_{res} and c_A received thus result, according to eqn. (16), the uplift force dF_A/dx relevant to blade radius. The execution of this calculation on plurality of blade supporting points leads to the searched for line segment load $q(x)$ that is after normalizing to $q(\bar{x})$ respectively.

Now the integration of function $q(\bar{x})$ requires with the application of Hermite- polynomials according to eqn. (11) rational functions $f(\bar{x}^n)$. Because of this, according to eqn. (16) the line segment load

$$q(\bar{x}) = \frac{dF_A}{dx} = \frac{1}{2}\rho c_A v_{res}{}^2 t_{bl}$$

is approximated by a Newton-interpolation-polynomial of the 4^{th} order to

$$q(\bar{x}) = q_0 + q_1\bar{x} + q_2\bar{x}^2 + q_3\bar{x}^3 + q_4\bar{x}^4 \qquad (25)$$

The constants $q_0 \ldots q_4$ are hereby determined from the interpolation procedure. From integration acc rding to eqn. (11) the term $\sum q_i h_i$ in eqn. (12) results.

6 NUMERICAL SOLUTION OF SYSTEM

The setting up of the differential equation system (15) and its computational solution is carned out with the computer program "EBLAMO". The determination of the system matrix $\underline{A}$ is done with elementary matrix operations requiring short computation times. The time integration following these after is based on the approximation procedure due to "Runge-Kutta".
For this a library-routine "DVERK" exists [7].
After every small time step Δt, the solution vector

$$\underline{y}^T = [\bar{z}_l, \bar{z}_l', \dot{\bar{z}}_l, \dot{\bar{z}}_l']$$

of the equation system is determined from this.

By insertion of components $\bar{z}_1$, $\bar{z}_1'$ and the Hermite- polynominals in the Ritz formulation from eqn. (8), for every time interval with various blade support points the approximated blade bending line reckons to

$$\bar{z}(\bar{x}) = (3\bar{x}^2 - 2\bar{x}^3)\bar{z}_1 + (-\bar{x}^2 + \bar{x}^3)\bar{z}_1' \qquad (26)$$

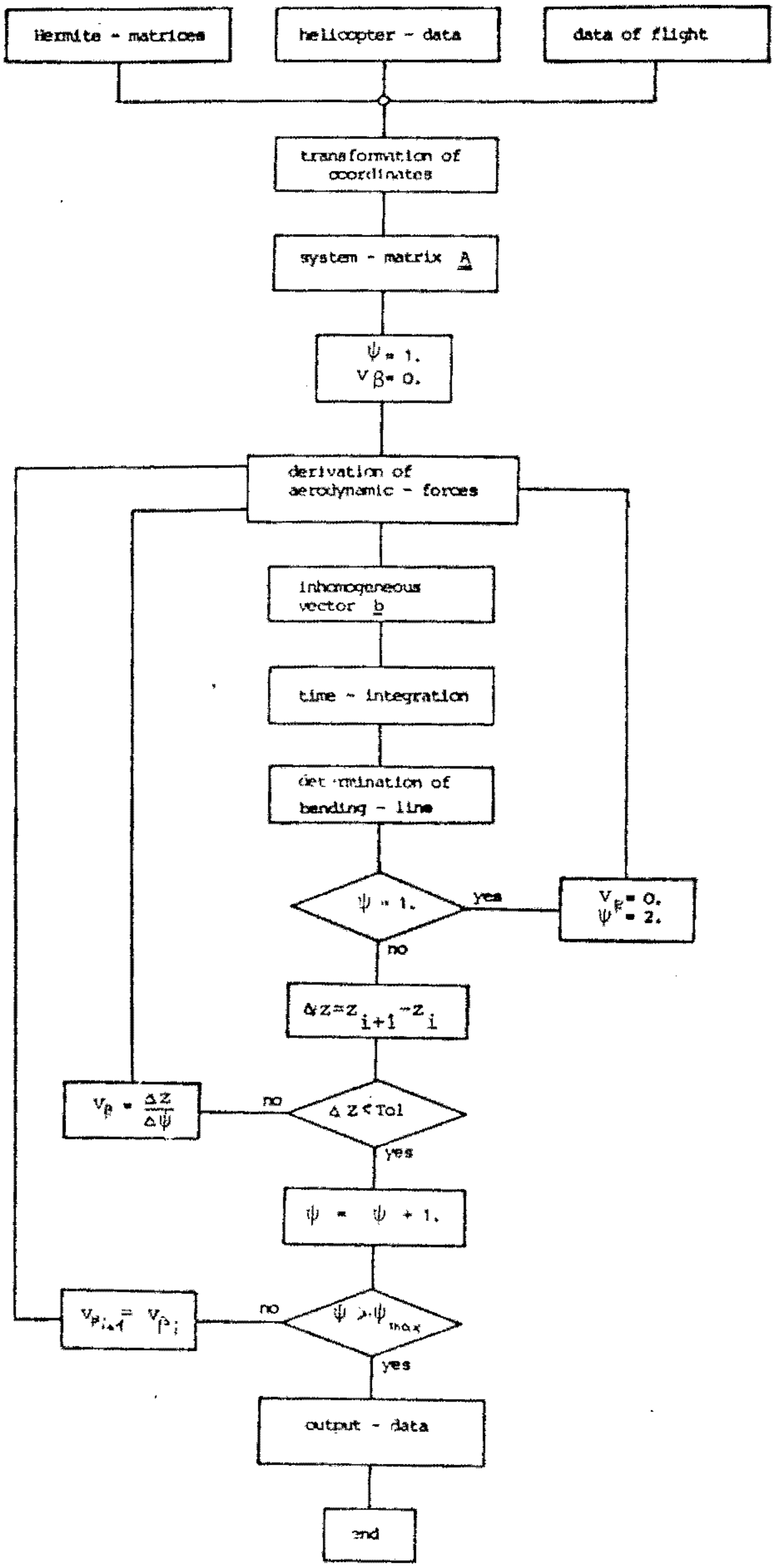
For small time intervals (f. e. $\Delta\psi_{bl} = 1^\circ$) oscillations can be represented for various blade support points (f. e. blade tip).

The determined program "EBLAMO" is now applied to th helicopter BO 105 from MBB, West Germany. The input data hereby are as follows:

$$EI = 6800\ Nm^2$$
$$m'' = 5.54\ kg/m$$
$$L = 4.912\ m$$
$$t_{bl} = 0{,}27\ m$$
$$\omega_{Ro} = 44{,}5\ m/s^2$$

The calculations are executed for both flight cases hove ing flight and horizontal forward flight by 200 km/h.

The working of the program "EBLAMO" is shown in the following flow chart.



Hermite - matrices
helicopter - data
data of flight
transformation of coordinates
system - matrix A
ψ = 1.
Vβ = 0.
derivation of aerodynamic - forces
inhomogeneous vector b
time - integration
determination of bending - line
ψ = 1.
yes
Vβ = 0.
ψ = 2.
no
Δz=z i+1 -z i
Vβ = Δz / Δψ
no
Δ z < Tol
yes
ψ = ψ + 1.
Vβi+1 = Vβi
no
ψ > ψmax
yes
output - data
end

7 RESULTS AND DISCUSSION

Fig. 2 presents the oscillation of the blade tip for the first six rotations, whereby the normalized deviation $\bar{z}$ of the blade tip is plotted versus azimuth angle ψ_{bl}. Figure 2a) shows that in hovering flight the system earlier gets into a stationary status than in forward flight. After the first rotation, the blade tip in both cases oscillates exactly with the frequency of excitation.

The flapping velocity v_β versus the normalized blade radius $\bar{x}$ is presented in fig. 3 for an interval of 45 degrees. A comparison with fig. 2 shows the correlation with the oscillation of the blade tip: Flapping velocity v_β is positive for increasing and negative for decreasing deviation.

In fig. 4 bending lines of the blade are presented, which show the elastic behaviour of the blade in radial direction - approximated by Hermite- polynomials.

The effective angle of incidence versus blade radius is shown in fig. 5. In case of the advancing blade radial variation of the angle of incidence is very small. Only at the retreating blade great variations are recognized near the clamping place of the blade (forward flight). The behaviour of uplift coefficient c_A, shown in fig. 6 is logiacally similar. In case of seperated flow the angle of incidence α and the uplift coefficient c_A are set to zero for plotting.

8 CONCLUDING REMARKS

The results in the previous chapter 7 make evident, that the approximation solution based on the Ritz-procedure with Hermite-polynomials as formulation functions is thoroughly applicable for such problems. The setting up of the system matrix $\underline{\underline{A}}$ only bases on elementary matrix operations and so it requires short calculation times. Only determination of the flapping velocity v_β necessitates more computation time. The outlined procedure is the beginning of a series of continous investigation possibilities. The application of this procedure for the other degrees of freedom of blade motion (lagging, torsion) is already in work.

9 REFERENCE

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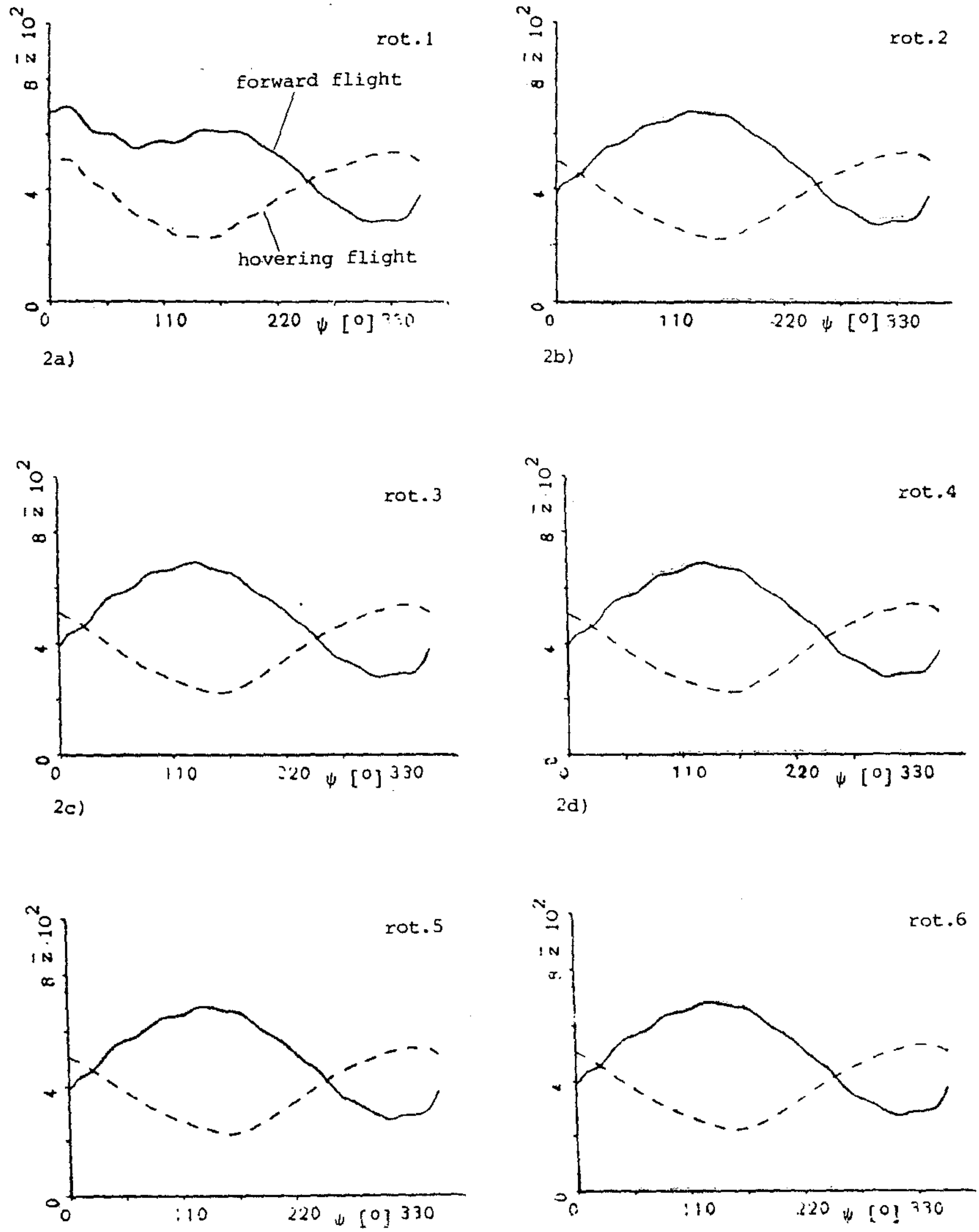

Fig.2: Deviations of blade tip $\bar{z}$ vs. azimuth angle ψ_{bl}

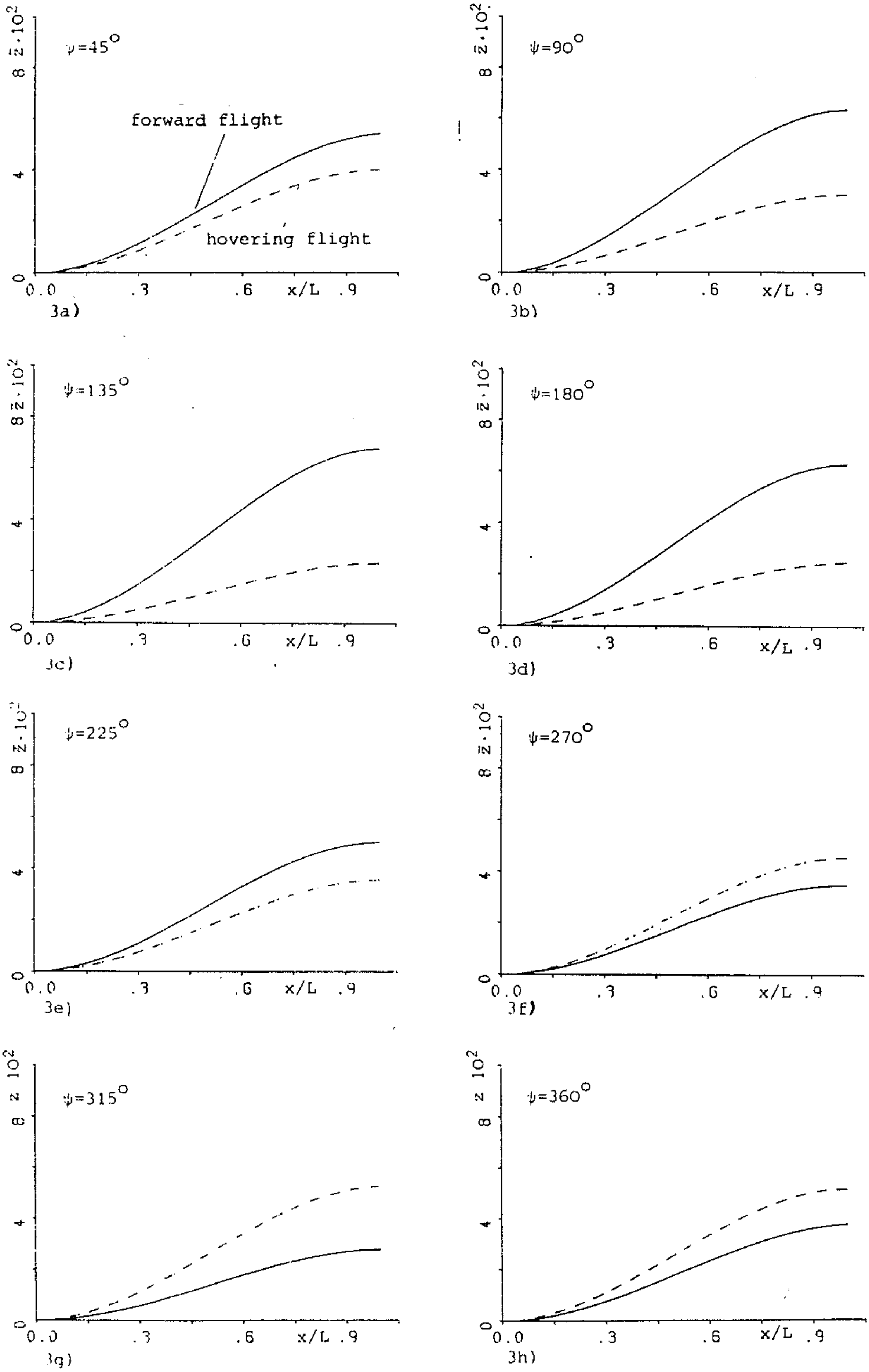

Fig.3: Bending line of rotor blade

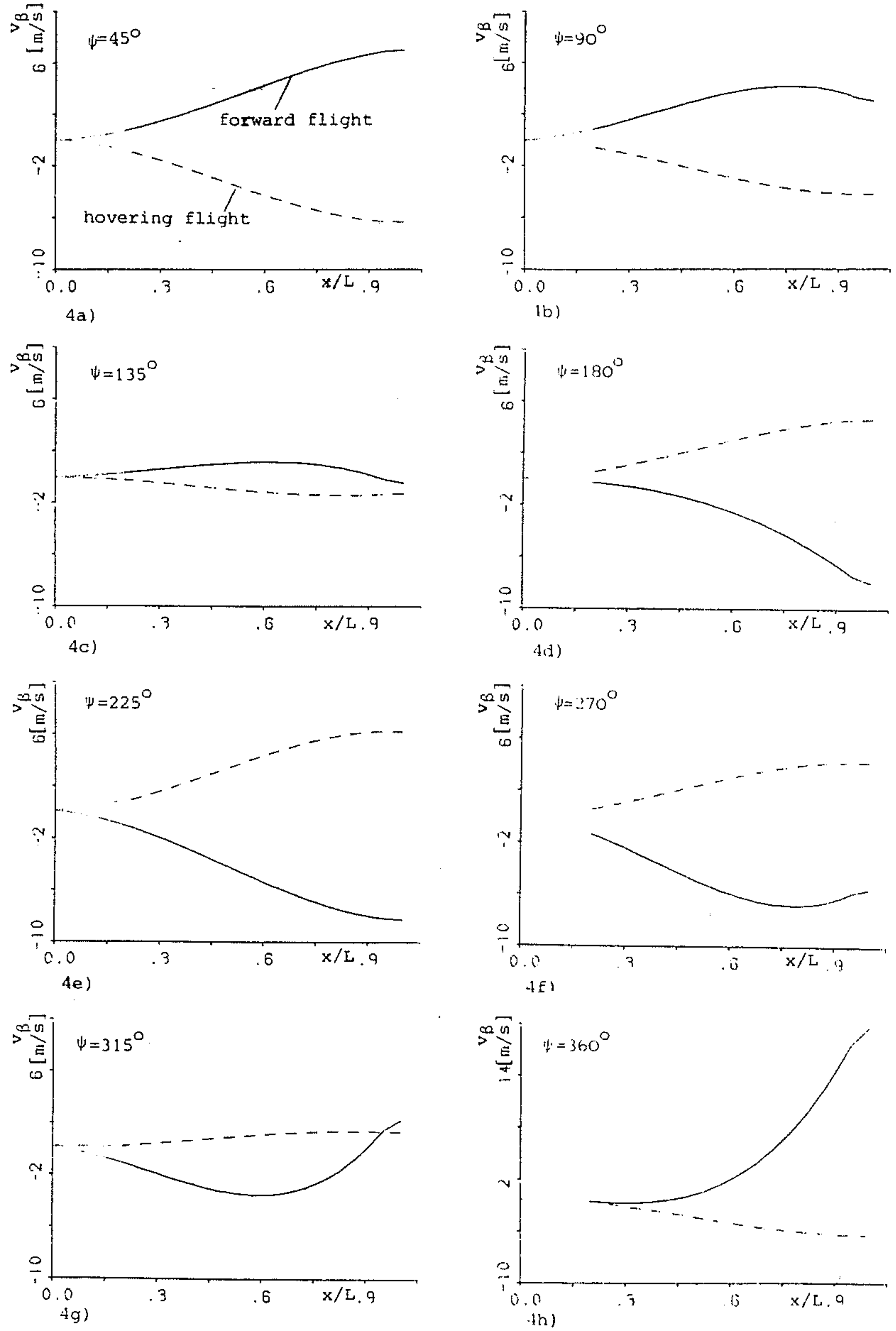

Fig.4: Flapping velocity v_β vs. blade radius x

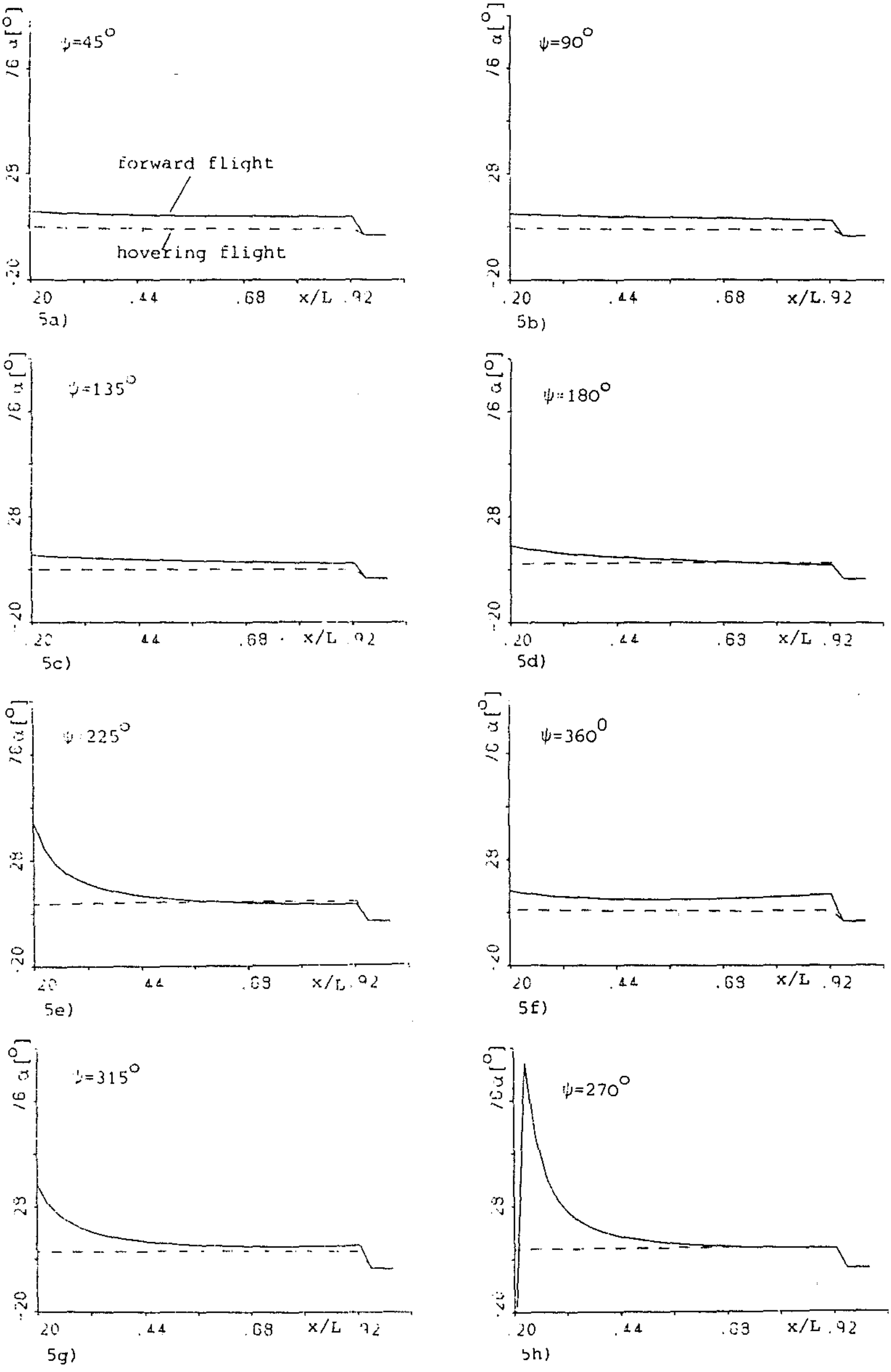

Fig.5: Effective angle of incidence α vs. blade radius x

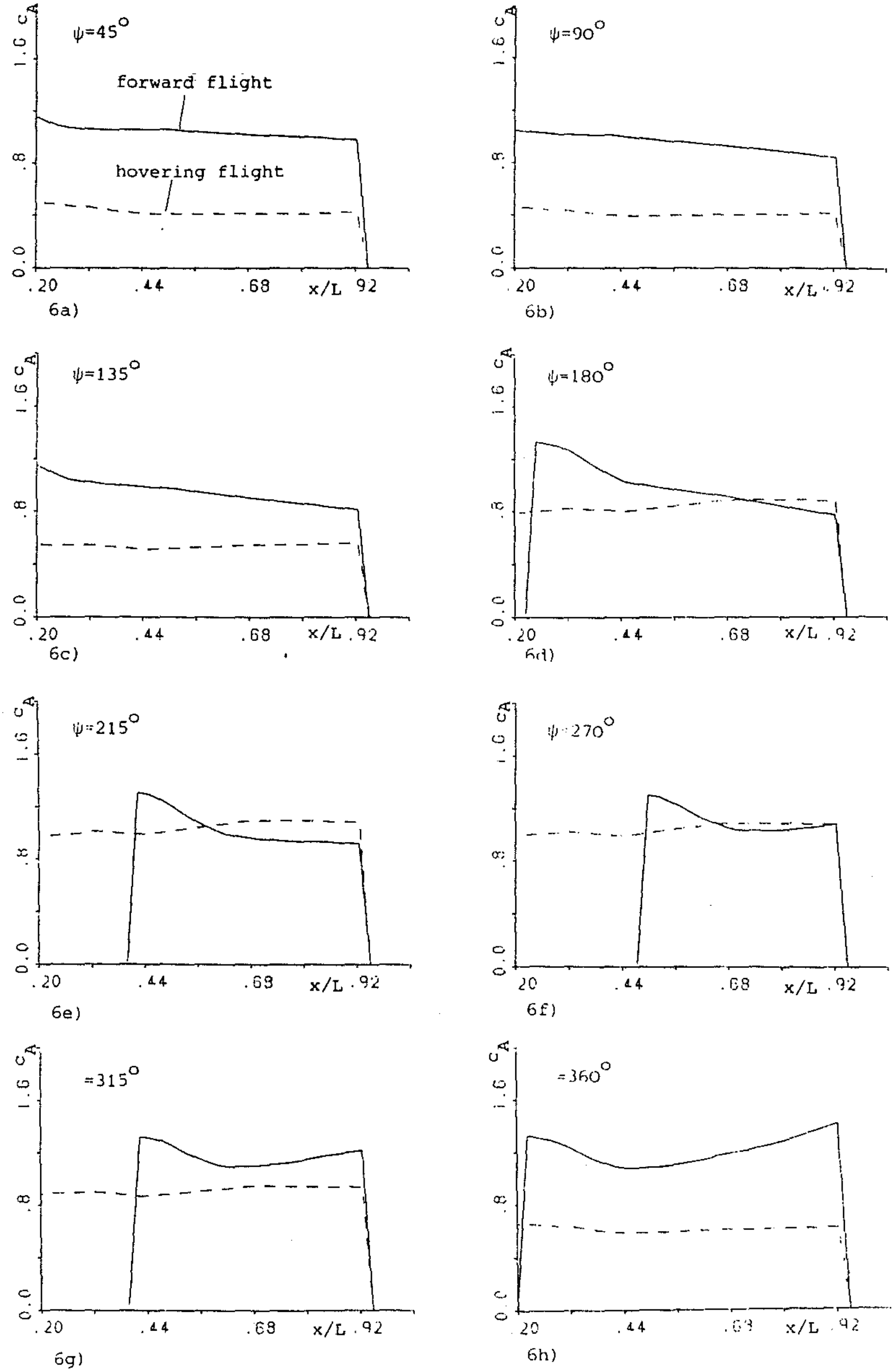

Fig.6: Uplift coefficient c_A vs. blade radius $\bar{x}$