# DESIGN OF COMPOSITE HELICOPTER ROTOR BLADES TO MEET GIVEN CROSS SECTIONAL STIFFNESS PROPERTIES

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## Abstract

This paper examines the design of a composite helicopter rotor blade to meet given cross-sectional stiffness properties from a computational perspective. The problem is non-linear and non-convex, which would require the use of stochastic optimisation methods. Since the objective is evaluated by finite element analysis, the computational expense of stochastic methods is prohibitive.

It is shown that by appropriate simplifications, the problem becomes convex. This allows deterministic optimisation methods to be used, which is considerably more computationally efficient than stochastic methods.

The response of a single objective function to the design variables was highly non-linear. By choosing appropriate design variables, the response of each individual target variable was closely modelled by a linear approximation. The problem was therefore reformulated into a number of simultaneous linear equations that are easily solved by matrix methods, thus allowing an optimum to be located with a minimum number of computationally expensive finite element analyses.

## Background

A helicopter blade is designed to meet constraints on both inertia and stiffness properties. Whilst section mass and centre of gravity (*CG*) locations are important to ensure adequate blade stability, the blade stiffness properties are designed to meet target values. Geometrically, a helicopter blade is a long, slender structure and is routinely idealised as a 1-D beam. From a design perspective, it is important not only that the structural properties of the cross section can be accurately determined (usually from a finite element model), but that these properties can also be tailored to achieve desirable characteristics of the structure being designed.

In order to illustrate the concept of current work, this paper examines the design of a typical helicopter rotor blade with the objective that the cross section meets given values of axial ($EA$), bending ($EI_{xx}$ and $EI_{yy}$) and torsional ($GJ$) stiffness.

The use of composite materials allows the structural designer new degrees of freedom with which to tailor the structural properties of a design. This is a good feature in itself, but it also increases the dimensionality of the design space – a negative trait from an optimisation viewpoint. Design optimisation is further complicated by the fact that a number of design variables are discrete – typically due to manufacturing considerations. Examples include ply thickness (typically 1/8$^{th}$-mm increments), and ply orientation (typically 45-degree increments).

Many studies [1-13] have been directed towards the optimisation of composite aerospace structures, however most of these studies (e.g. Kameyama *et al* [1], Chattopadhyay *et al* [2]) use simplified geometrical models such as modelling the wings as flat plates, or helicopter blades as a rectangular torsion boxes. Although these give important physical insight and useful results for preliminary design, they are of little practical use to the aerospace designer seeking to produce a detailed design. Indeed, Chattopadhyay *et al*[2] conclude that "the results obtained must be viewed within the context of the modelling assumptions used in the analysis".

Finite element analysis is an established design tool in the aerospace industry, and the use of rigorous optimisation techniques is gradually becoming more widespread. An optimisation method is therefore required that is capable of interfacing with commercially available analysis tools, thereby allowing the design to be optimised at whatever level of detail is necessary.

Although this study examines the design of a simplified helicopter blade, the lessons learned from this problem and the methods developed to solve it are applicable to more complex structural design problems.

### Problem formulation

The problem formulated is to design a composite helicopter rotor blade (shown in Figure 1) to meet predetermined target values of the four cross sectional stiffnesses.

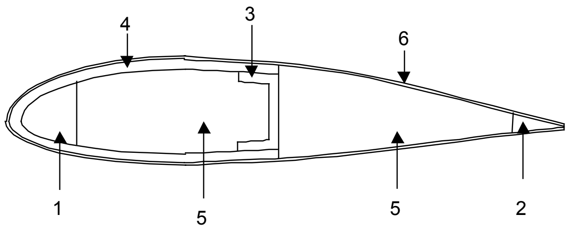



**Figure 1: Generic helicopter blade design**

The main features of this generic blade design are

1 = nose weight
2 = rear wedge
3 = 8-ply composite spar wall
4 = 8-ply composite blade wall
5 = foam filler
6 = glass fibre surface layer

The ***design variables*** are

*X[0]* = chordwise location of end of nose weight
*X[1]* = chordwise location of start of rear wedge
*X[2]* = thickness of each 8-ply composite

The ***target variables*** were chosen as

$EI_{XX} = 2.72 \times 10^8\ Nm^2$ *(1a)*
$EI_{YY} = 4.79 \times 10^9\ Nm^2$ *(1b)*
*mass* = 0.397 kg/m *(1c)*
*CG* = 23.0 % chord *(1d)*

The behaviour of the structure has been assessed using a scaled down model of Hill and Weaver's [14] and is essentially the approach of Bartholomew and Mercer [15]. This approach uses finite element analysis of a 3-dimensional slice of the cross-section of any prismatic beam with any number of materials to produce equivalent 1-D beam properties, i.e. a stiffness matrix. It achieves this by linking the two faces of the slice model with multi-point constraint equations, which allow relative motion of the faces according to linear bending, axial displacement and torsion through 'scalar freedoms'. The individual components of the stiffness matrix are found by taking the displacements of these scalar freedoms for the four load cases of axial tension, torsion and the two bending cases (which have the same result for the cylinder). The 3-D slice itself is free to deform (warp) in the plane of the section and also out-of-plane, if necessary.

Common practice is to formulate a single function that gives an objective measure of how good the design is. In this problem, the fitness of the design is determined by how closely the target values of cross sectional stiffness are met, so the following objective function was formulated, where subscripts *a* and *t* refer to actual and target values of the stiffness, respectively.

$$Obj = \left(\frac{EI_{XX\,a} - EI_{XX\,t}}{EI_{XX\,t}}\right)^2 + \left(\frac{EI_{YY\,a} - EI_{YY\,t}}{EI_{YY\,t}}\right)^2 + \left(\frac{mass_a - mass_t}{mass_a}\right)^2 + \left(\frac{CG_a - CG_t}{CG_t}\right)^2 \quad (2)$$

It should be obvious that the objective is always positive, but reduces to zero when all of the target values have been met exactly.

The cross sectional properties of the blade cannot be accurately determined from a simple analytical model. Using Hill and Weaver's method, it is relatively straightforward to obtain these results from a finite element analysis of the cross section.

However, in this problem (as in most real world problems), we have the additional complication of discretised ply thicknesses – we cannot choose the exact ply thicknesses that give the required ξ1 and ξ2 values, but must round the ply thicknesses to the nearest $1/8^{th}$ mm. An exhaustive search of the solutions for different designs will locate the discrete point that most closely matches the required solution.

**Investigation of feasible design space**

In order to gain a full understanding of the exact nature of the objective function, the design space was searched. Due to the simple (3-variable) nature of this problem, a sufficiently high-resolution search was obtained by discretising the entire design space into 512 design points. This design space required 2 days to exhaustively search.

Despite the limitations imposed by discretisation, it is possible to meet the target values to a mean error of less than 1% at the optimum point. The optimum design values are summarised in Figure 2, below.

| Design variable | X[0] | X[1] | X[2] |
|---|---|---|---|
| Value | 4.0 | 98 | 1.375 |

**Figure.2. Optimum discrete design**

The results of the search indicated that if stacking sequence effects are ignored, the design space becomes convex. Figure 3, below, illustrates this within the three dimensional design space, with the variation of objective function in grayscale.

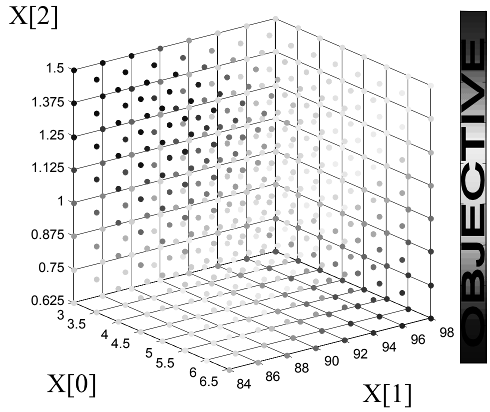



**Figure.3. Variation of objective function over 3-D design space**

Work with the composite cylindrical shells indicates that this design space is convex because the design variables in this is problem are wall locations and ply thicknesses. The inclusion of ply orientations, or stacking sequences would lead to a non-convex objective function.

If this were the case, the problem would either have to be solved using stochastic methods, or simplified by the use of lamination parameters.

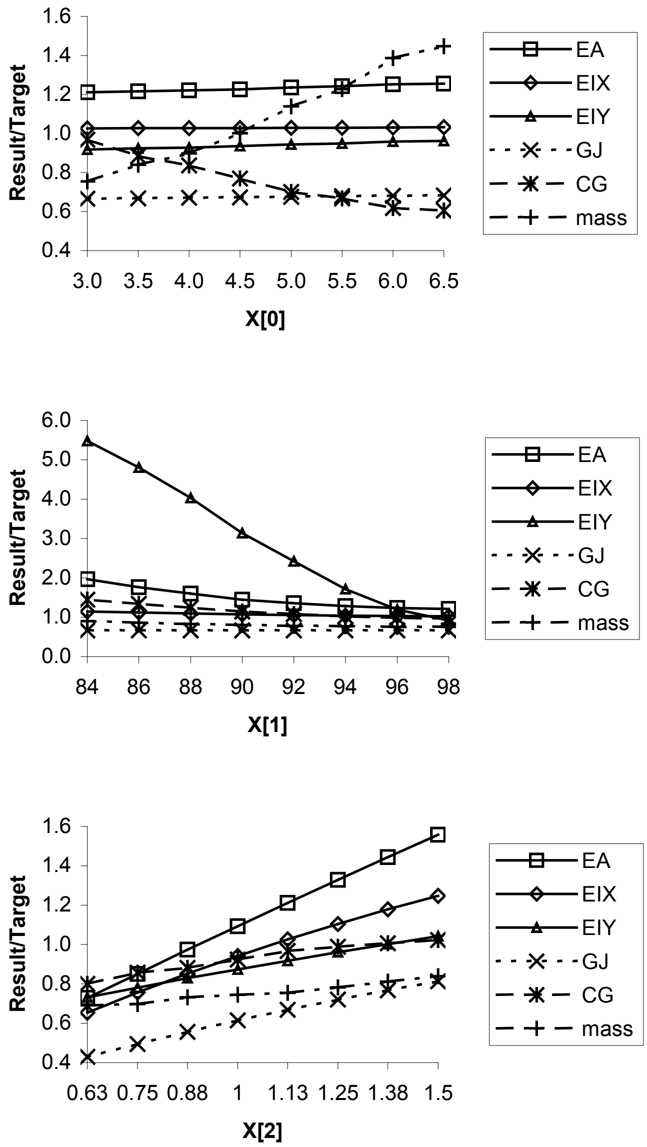
The visualisation of an objective function within a design space that has more than three dimensions is not intuitive for most people, and is certainly not easy to represent in a concise pictorial format. However, despite the obvious difficulties of representation, these results presented above have significant consequences for the optimisation of helicopter blades.

**An efficient solution method**

With the knowledge that the design space is convex, it is possible to apply sequential linear programming (SLP) techniques to solve the problem.

A typical approach may use the current objective function and apply a steepest descent method with appropriate move limits - a method that reliably finds the optimum of a convex problem. Since the objective function for this problem is highly non-linear, the analysis will require several iterations - each requiring the gradients of the objective function to be evaluated via a computationally expensive finite-difference method. It is therefore desirable to minimise the number of times that such gradients are calculated.

A better method linearises the variation of the target variables ($EI_X$, $EI_Y$ and $CG$) about an initial design point. The following charts (Figures 4a-4c), which plot the values of each cross-sectional property (normalised to the target values) against the variation of design, show that each property varies almost linearly with the design variable over almost the entire design space. Note that although only the target variables $EI_X$, $EI_Y$, $CG$ and *mass* are considered in this problem, although the graphs show that other variables (*EA* and *GJ*) vary linearly as well.




**Figures 4a-4c: Variation of results with design variables**

The problem can now be expressed as *N* simultaneous linear equations, where *N* is the number of target variables.

$$\begin{aligned} EI_{Xnew} &= EI_{Xold} + A_{11}\Delta x_1 + A_{12}\Delta x_2 + A_{13}\Delta x_3 \\ EI_{Ynew} &= EI_{Yold} + A_{21}\Delta x_1 + A_{22}\Delta x_2 + A_{23}\Delta x_3 \\ mass_{new} &= mass_{old} + A_{31}\Delta x_1 + A_{32}\Delta x_2 + A_{33}\Delta x_3 \\ CG_{new} &= CJ_{old} + A_{41}\Delta x_1 + A_{42}\Delta x_2 + A_{43}\Delta x_3 \end{aligned} \qquad (3a\text{-}3d)$$

Obviously the desired new values of the target variables are the target values, so it is straightforward to solve for $\Delta x_i$. Also, since the linearisation is valid across the entire design space, the change in each design variable $x_i$ (i.e. $\Delta x_i$) does not have to be small. These simultaneous linear equations can be conveniently represented in matrix form as

$$\mathbf{A}\,\Delta \underline{x} = \underline{b} \qquad (3d)$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \\ A_{41} & A_{42} & A_{43} \end{bmatrix} \begin{Bmatrix} \Delta x_0 \\ \Delta x_1 \\ \Delta x_2 \end{Bmatrix} = \begin{Bmatrix} EI_{Xnew} - EI_{Xold} \\ EI_{Ynew} - EI_{Yold} \\ mass_{new} - mass_{old} \\ CG_{new} - CG_{old} \end{Bmatrix} \qquad (3e)$$

There are three cases of problem that arise

**1. The number of design variables is less than the number of target variables**

This 3-design-variable problem requires that 4 target values ($EI_X$, $EI_Y$, *mass* and *CG*) are met. It is not necessarily possible to obtain an exact solution.

In general, it is not possible to obtain an exact solution where the number of target values exceeds the number of design variables. However, it is possible to obtain a least squared error solution by solving the following matrix equation

$$\mathbf{A}^{T}\mathbf{A}\,\Delta \underline{x} = \mathbf{A}^{T}\,\underline{b} \qquad (4a)$$

which gives $\Delta \underline{x}$ as

$$\Delta \underline{x} = (\mathbf{A}^{T}\mathbf{A})^{-1}\mathbf{A}^{T}\,\underline{b} \qquad (4b)$$

**2. The number of design variables is equal to the number of target variables**

If the problem considered here, had only required that $EI_X$, $EI_Y$ and *CG* values are matched, the number of target values is equal to the number of design variables. In this case there is one unique optimum solution, which may be found directly from

$$\Delta \underline{x} = \mathbf{A}^{-1}\,\underline{b} \qquad (4c)$$

**3. The number of design variables is greater than the number of target variables**

In this case, there is a range of solutions. This is analogous to redundancy in a structure. The appropriate number of design variables may therefore be fixed, until the remaining number of design variables is equal to the number of target variables. A unique solution may then be obtained from equation *4c* above.

**The effect of discretisation**

The above discussion does not consider the effect of discretisation. This problem is additionally constrained by the discretisation of the ply thickness (variable $x_2$) to 0.125-mm increments.

For this problem, the simplest solution is to evaluate the discrete designs either side of the continuous optimum and re-optimise any remaining continuous variables for each point. Each solution can be obtained analytically by solving the matrix equation above, using the same values of $A_{ij}$ and searching through all the appropriate discretised combinations of $\Delta x_i$.

In the general case with more than just one discretised variable, there will be $2^N$ discretised solutions, where $N$ is the number of discretised variables. The best of these discrete solutions will be the discretised optimum. Depending on the accuracy of the linearisation and the degree of discretisation, the calculated discrete optimum may then be verified using Finite Element Analysis.

**Conclusions**

Given existing computing power, it is not feasible to search the entire design space for a complex design of helicopter rotor blade. Even a simple 3 variable problem takes two days to run. Problems with more variables would require the design space be reduced in order to make the search feasible.

It is well known that while stochastic methods are useful for locating several near optimal points in a non-convex design space, they are poor at locating a single best point. Conversely, deterministic methods efficiently locate an optimum point, but do not guarantee global optimum for non-convex problems.

Since the design space of this problem is convex, the optimum point can be located using computationally efficient, deterministic methods. The original choice of a single objective function led to a non-linear response, but by linearising the target variables in terms of the design variables about a given design point, the problem is solved by matrix methods.

It is a straightforward procedure to code this into an appropriate language and interface this with finite element analysis packages to reliably and efficiently design complex composite structures to meet given structural properties.

In order to ensure that the design space is convex, lay-up and stacking sequence have been fixed. Work with cylindrical shells indicates that changing stacking sequence does not significantly affect cross sectional properties, provided that the structure is thin walled. Existing composite helicopter blades are not thin walled, so these design freedoms could affect the design envelope significantly. This work needs to be extended to allow variation of lay-up and stacking sequence to be used in finding the optimum design.

**Acknowledgement**

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