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Active control of vibrations in helicopters: from HHC to OBC

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Abstract

Starting from a critical analysis of *HHC* (an intrinsically sampling and hold type technique), the problem of counterbalancing vibrations in helicopters is embedded in a continuous time setting. In this way, one can assess the performance of the *HHC* compensator and understand its limitations. To improve performance, in this paper a continuous time controller based on observer concepts is proposed. The basic idea is to resort to a dynamic model to describe the effect of the swash plate commands on the hub force, and to enlarge the state of such a model by including a state-space model of the disturbance. Then, if one solves a state estimation problem by means of a suitable observer, one also obtains an estimate of the disturbance. Such an estimate can be used to compensate for vibrations. Compared to the low flexibility of the *HHC* regulator, the new Observer Based Controller (*OBC*) looks better in terms of robustness, time responsiveness and frequency constraints.

1. Introduction

The problem of reducing the helicopter fuselage vibrations by means of active control techniques has received considerable attention in the last decade, see e.g. [1]-[4]. The most common approach to reduce helicopter fuselage vibrations is the so called *Higher Harmonic Control* (*HHC*). The basic rationale of *HHC* is to superimpose to the pilot's inputs (swash plate commands) a small signal consisting of a sinusoid whose amplitude and phase are designed so as to counteract the first and most important harmonic of the vibration. As is well known, [5], due to harmonic filtering effects, such a harmonic is the *N/rev* one, where *N* is the number of blades.

HHC controllers are intrinsically sampling and hold type controllers where the control action is decided on the basis of a sinusoidal regime approach; as a consequence, notwithstanding the huge literature on *HHC*, it is still difficult to say on which controller parameters one could act to improve some basic dynamic properties of the control system, such as:

1. *Stability and robustness*
   Is there a guaranteed stability margin against mismodellings or modifications in the system dynamics?
2. *Disturbance rejection time*
   How quickly can the control system counterbalance the effects of amplitude and/or phase variations in the vibration harmonic?
3. *Non-interaction*
   To what extent does the active control system interfere with the pilot's action?

To gain insight into the stability issue, in [6] the *HHC* approach is embedded in a continuous time domain dynamic context. In this framework the performance of *HHC* can be assessed via classical control theory methods. It is shown that, in most cases, *HHC* based control systems are asymptotically stable; however, due to the particular structure of the *HHC* compensator, there is an intrinsic lack of flexibility in the control scheme, so that it seems difficult to find a way of improving the dynamic performances of the overall system within the *HHC* context.

The mentioned limitations may be overcome if a continuous time approach is adopted from the very beginning, by resorting to suitable design techniques for the rejection of periodic disturbances. This has the advantage of giving transparency to the control design methodology.

Along this route, a dynamic model of the influence of the swash plate commands on the vibrations is preliminary needed. Such a model can be obtained starting from the basic dynamic equations of the helicopters world, see e.g. [7], or by (black-box) system identification techniques [8].

As for the control strategy, a possibility [9], [10], is to resort to optimal control methods with frequency shaping: by emphasizing the disturbance frequency in the performance index, the designed controller can provide disturbance rejection. However, this approach requires the availability of all the state variables of the rotor model and, if an accurate rotor model is used, there are so many state variables that the requirement to measure all of them looks demanding.

In this paper, we consider the problem of *reducing the mast total vertical N/rev vibration* by acting on the collective command. The basic control design strategy we propose is to resort to an observer to estimate the hidden vibration from the mast force measurement, and then to compensate for vibrations by adding a suitable countervibration term to the blade pitch command. We will call our controller *Observer Based Control* (*OBC*) system. Interestingly enough, the *OBC* controller can be given a form which presents structural analogies with the *HHC* scheme. This observation turns out to be useful to reinterpret the *HHC* philosophy, and to put it in contrast with the observer based rationale. The main conclusions which can be drawn are summarized as follows:

- In the *HHC* context only a single frequency representation of the system is considered. On the opposite, the complete system dynamics is taken into consideration in the *OBC* approach. This enables one to probe the control system characteristics and evaluate its stability and robustness degrees.
- By tuning the loop gain of the *OBC* system, one can control the disturbance rejection time.
- The *OBC* action is confined to a narrow band around the N/rev frequency, so that the remaining frequencies are subject to minor modifications only. As a consequence, the pilot's commands (acting at low frequency only) are not perturbed by the active control signals.

2. *HHC*

2.1 The HHC approach

The classical *HHC* algorithm is based on a "quasi steady" model of the rotor: it is assumed that the helicopter can be represented by a constant matrix T. This matrix relates the *N/rev sin* and *cos* components of the *N/rev* swash plate inputs to the *sin* and *cos* components of the *N/rev* response of the helicopter.

As is well known, the harmonic filtering effect implies that the vibratory loads transmitted from the rotor to the hub are characterized by a discrete-frequency spectrum. In particular, vertical *forces contain the N/rev frequency and its multiple* integers. Nevertheless, in many practical cases the higher harmonics give neglectable contributions to the total vibration, so that the analysis can be focused on the *N/rev* frequency only.

In the sequel, we will denote by Ω_{rot} the rotor angular velocity and by $\Omega=N\Omega_{rot}$ the fundamental vibration frequency. The *HHC* approach is based on the following model:

$$y = Tu + \tilde{d} \qquad (1)$$

where $y=[y_c \;\; y_s]'$ and $u=[u_c \;\; u_s]'$ are vectors (phasors) containing the *sin* and *cos* components at frequency Ω of the vertical hub force $y(t)$ and the collective command $u(t)$ respectively, while $\tilde{d} = [\tilde{d}_c \; \tilde{d}_s]'$ the *baseline vibration* and represents the vibration to be rejected with the *HHC* compensator. The (2x2) matrix T relating u and y is known as *gain transfer matrix*.

The relationship between u and y is schematically as follows: the $\Omega=N\Omega_{rot}$ component of the swash plate collective command produces harmonics in the pitch angle variations at frequencies $(N-1)\Omega_{rot}$, $N\Omega_{rot}$ and $(N+1)\Omega_{rot}$. In turn, due to harmonic filtering effects, these variations result in a $\Omega=N\Omega_{rot}$ countervibration at the rotor hub.

For a practical use of model (1), the time axis is to be partitioned into discrete intervals of length τ (typically $\tau=2\Pi/\Omega$) during which data $y(t)$ are collected to be Fourier transformed. Denoting by k the sampled time index, model (1) is then given the discrete time form:

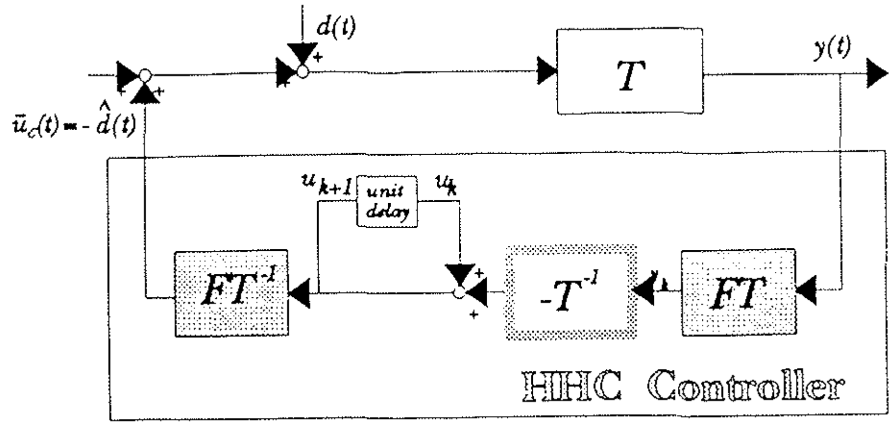

Fig. 1

$$y_k = T\,u_k + \tilde{d}\,, \tag{2}$$

where

$$y_k = \frac{1}{\tau}\int_{(k-1)\tau}^{k\tau}\begin{bmatrix}\cos\Omega t\\ \sin\Omega t\end{bmatrix} y(t)\,dt \tag{3}$$

is the vector containing the Fourier components at frequency Ω of the measured vibration, estimated over the last interval $[(k-1)\tau, k\tau)$. Analogously, u_k is the vector of the *sin* and *cos* components of the actual swash plate movement at the same frequency.

In order to simplify the subsequent analysis it is convenient to rewrite Eq. (2) in an equivalent form:

$$y_k = T\,(u_k + d) \tag{4}$$

where an $d = T^{-1}\tilde{d}$.

Eq. (4) is at the root of the *HHC* approach: if T is known or suitably estimated, [11], and y_k is evaluated over the last *rev* period from $y(\cdot)$ by Fourier transform techniques, the term $(-u_k + T^{-1}y_k)$ can be seen as an estimate $\hat{d}_k$ of the disturbance phasor over the last period. To achieve the objective of zeroing the vibration at the hub over the subsequent *rev* period, the natural cure is to take:

$$u_{k+1} = -\hat{d}_k = [u_k - T^{-1}\,y_k] \tag{5}$$

By this way, should disturbance d keep constant when passing from (k) to $(k+1)$, y_{k+1} would be zeroed. Notice that (5) is equivalent to updating the control action as follows:

$$\bar{u}_c(t) = [\cos\Omega t \;\; \sin\Omega t]\cdot u_{k+1}$$

$$\text{for } t \in [k\tau, (k+1)\tau) \tag{6}$$

This sinusoid is hold until the time instant $(k+1)\tau$, when all the procedure is restarted to compute the new sinusoidal segment over $[(k+1)\tau, (k+2)\tau)$, etc.

In conclusion, *HHC* is just a *Generalized Sample and Hold Function* (*GSHF*) strategy [12]: in place of the classical *Zero Order Hold* (*ZOH*) strategy (consisting in keeping u constant over the sampling interval), in *GSHF* u is generalized to be any periodic function.

The overall *HHC* control scheme is graphically depicted in Fig. 1, where blocks *FT* and FT^{-1} represent Eqns. (3) and (6) respectively.

2.2 HHC revisited

In [6] the *HHC* approach is embedded in a continuous time domain context as briefly outlined in the following.

In view of Eqns. (3), (5) and (6), it is possible to write:

$$u_k = u_{k-1} - \frac{T^{-1}}{\tau}\int_{(k-1)\tau}^{k\tau}\begin{bmatrix}\cos\Omega t\\ \sin\Omega t\end{bmatrix} y(t)\,dt$$

$$= u_{k-2} - \frac{T^{-1}}{\tau}\int_{(k-2)\tau}^{k\tau}\begin{bmatrix}\cos\Omega t\\ \sin\Omega t\end{bmatrix} y(t)\,dt$$

$$= -\frac{T^{-1}}{\tau}\int_{-\infty}^{k\tau}\begin{bmatrix}\cos\Omega t\\ \sin\Omega t\end{bmatrix} y(t)\,dt$$

and then

$$\bar{u}_c(t) = [\cos\Omega t \quad \sin\Omega t] \cdot \left\{ -\frac{T^{-1}}{\tau} \int_{-\infty}^{k\tau} \begin{bmatrix} \cos\Omega t \\ \sin\Omega t \end{bmatrix} y(t)dt \right\}$$

$$t \in [k\tau, (k+1)\tau] \qquad (7)$$

By simply eliminating the "sample and hold" structure implicit in Eq. (7) one gets

$$u_c(t) = [\cos\Omega t \quad \sin\Omega t] \cdot \left\{ -\frac{T^{-1}}{\tau} \int_{-\infty}^{t} \begin{bmatrix} \cos\Omega t \\ \sin\Omega t \end{bmatrix} y(t)dt \right\} \qquad \forall t$$

In [6] it has been shown that the so-computed control $u_c(t)$ can be seen as the output of a continuous time-invariant regulator fed by $y(t)$ and characterized by a transfer function

$$\bar{R}(s) = \frac{2}{\tau} \frac{(\alpha s + \beta\Omega)}{s^2 + \Omega^2} \qquad (8)$$

where, if

$$T = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

so that

$$T^{-1} = \frac{1}{a^2+b^2} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

(α, β) are exactly the elements of the first row of matrix T^{-1}.

The overall control scheme can then be represented as in Fig. 2, where

- $G(s)$ indicates the transfer function of the rotor dynamic model;
- $u_p(t)$ is a signal which takes into account the pilot's action.

2.3 A critical view on HHC

The interpretation of *HHC* given in Fig. 2 is enlightening from the control theory viewpoint. From it, the following main points are in order:

- Disturbance rejection

  The *HHC* regulator provides asymptotic rejection of the Ω component of the vibration. This is due to the presence of two poles in $\pm j\Omega$ in the regulator, which result in two zeros at the same frequency in the closed loop transfer function relating the disturbance to the output.

- Disturbance rejection time

  The *HHC* scheme does not provide any tuning knob to control the length of transients in disturbance rejection.

- Non interaction

  Apparently, there is no guarantee that the *HHC* regulator does not interfere with the pilot's action. Indeed, the guidance commands involve the low frequency helicopter dynamics, so that a non-interacting regulator should not affect the behavior of the machine at these frequencies. On the contrary, the insertion of regulator (8) in the control loop (Fig. 2) causes a modification of even the gain of

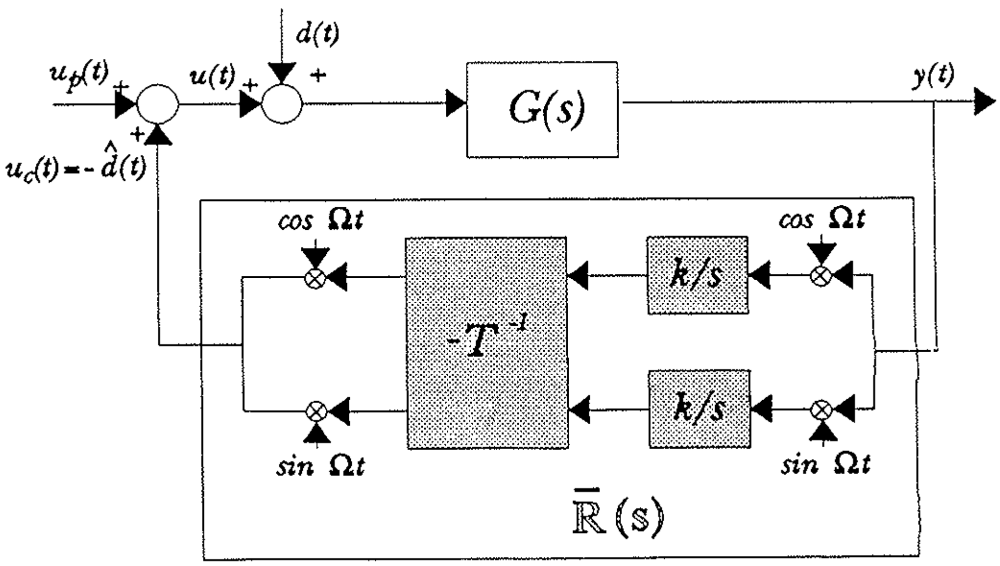

Fig. 2

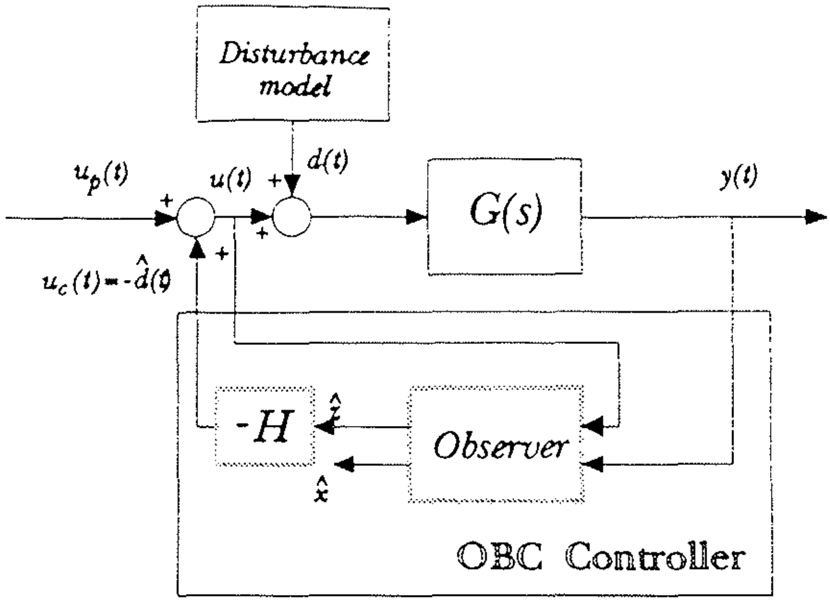

Fig. 3

the transfer function between the pilot's command and the total vertical force at the hub: such a gain changes from $g_{open}=G(0)$ to $g_{closed}=\dfrac{G(0)}{1+G(0)\bar{R}(0)}$ (observe that, if, as it often happens, $G(0)$ assumes large values then g_{closed} becomes approximately equal to $1/\bar{R}(0)$ which is almost independent of $G(0)$!).

3. OBC

3.1 Active control of vibrations by OBC

Consider now a dynamic model of the rotor in a state-space form:

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) + D\,u(t) + \tilde{d}(t) \end{cases} \qquad (9)$$

where (A,B,C,D) are matrices of dimension (nxn), $(nx1)$, $(1xn)$ and $(1x1)$ respectively. The pair (A,C) is supposed detectable, (A,B) stabilizable and matrices B and C with full rank. Moreover, we assume that (A,B,C,D) has neither zeros nor poles in $\pm j\Omega$. Disturbance $\tilde{d}(t)$ is modeled as a generic sinusoid at frequency Ω with unknown amplitude and phase.

Notice that, coherently with the *HHC* model (2), the disturbance is supposed to act on the system output. However, here too it is convenient to consider a model similar to (4) where a sinusoidal disturbance acts on the system input:

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,(u(t) + d(t)) \\ y(t) = C\,x(t) + D\,(u(t) + d(t)) \end{cases} \qquad (10)$$

The first step in the design of the controller consists in providing a *dynamic model* of the disturbance $d(t)$. One possibility is to describe it as the output of a second order autonomous system with two imaginary poles at $\pm j\Omega$:

$$\begin{cases} \dot{z}(t) = J\,z(t) \\ d(t) = H\,z(t) \end{cases} \qquad (11)$$

where

$$J = \begin{bmatrix} 0 & W^2 \\ -1 & 0 \end{bmatrix}, \quad H = [\,1 \;\; 0\,] \qquad (12)$$

Eqns. (10) and (11) give rise to the augmented system with state $[x(t)\; z(t)]$. Such a state can be estimated by a suitable observer. Denoting by $[\,\hat{x}(t)\; \hat{z}(t)\,]$ the estimate of the augmented state, the typical structure of an observer is:

$$\begin{cases} \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & BH \\ 0 & J \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} L \\ \Lambda \end{bmatrix} \varepsilon(t) \\ \varepsilon(t) = [y(t) - C\,\hat{x}(t) - D\,u(t) - D\hat{d}(t)] \\ \hat{d}(t) = H\,\hat{z}(t) \end{cases} \qquad (13)$$

L and Λ are the observer gain vectors (of dimension (nx1) and (2x1) respectively) and play the role of design parameters in the controller, as will be shown later.

Once an estimate $\hat{d}(t)$ of the disturbance $d(t)$ is available, it can be subtracted from the pilot's action to cancel the disturbance effect:

$$u(t) = u_p(t) - \hat{d}(t) \qquad (14)$$

The block diagram of the resulting control system is depicted in Fig. 3. Eqns. (13) and (14) together are a state-space description of the resulting control law. Laplace-transforming all terms in the above equations, (with zero initial conditions), one obtains:

$$U(s) = \{I+H(sI-J)^{-1}\Lambda(I-C(sI-A+LC)^{-1}L)D+ C(sI-A+LC)^{-1}B]\}U_p(s)+ -H(sI-J)^{-1}\Lambda[I-C(sI-A+LC)^{-1}(s)L]Y(s) \quad (15)$$

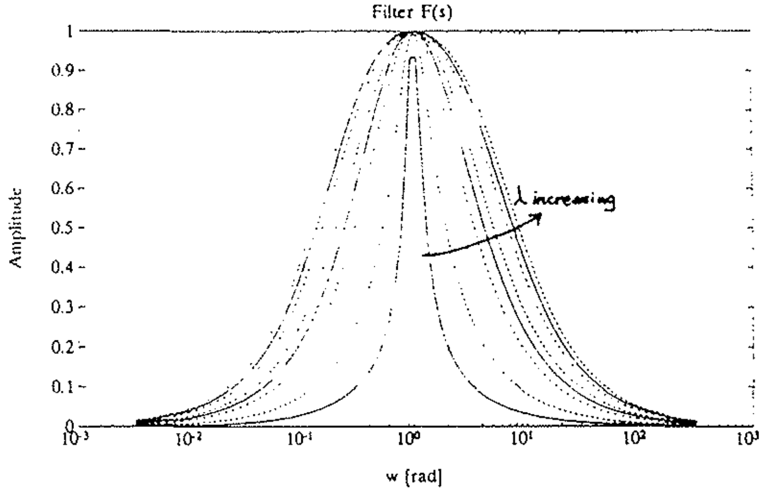

Fig. 4

Now, let

$$G(s) \equiv \frac{N(s)}{D(s)} = C\,(sI-A)^{-1}\,B$$

be the rotor transfer function, and select the observer gains as:

$$\Lambda = [0\ \ \lambda]' \qquad \lambda \in R$$

$$L = [l_1\ l_2 \dots l_n]$$

$$l_i \in R,\ i=1..n \qquad (16)$$

The structural zero introduced in the expression for Λ is necessary in order to achieve non interaction with the pilot's commands (Sect. 3.5).

Then, after some computations one can show that Eq. (15) can be given the following form:

$$U(s) = U_p(s) + \frac{s\lambda}{s^2+\Omega^2+s\lambda\frac{N(s)}{\tilde{N}(s)}}\left[\frac{N(s)}{\tilde{N}(s)}U(s) - \frac{D(s)}{\tilde{N}(s)}Y(s)\right] \qquad (17)$$

where

$$\tilde{N}(s) \equiv l_1 s^n + l_2 s^{n-1} + \dots + l_n$$

If one makes the assumption that the system is *improper* (order of $N(s)$ equal to order of $D(s)$) and *minimum phase* (all the zeros of $N(s)$ have negative real part), then it can be proven that it is possible to select parameters $\{l_i\}$ so that

$$\tilde{N}(s) \equiv N(s).$$

Then, Eq. (17) can be rewritten as

$$U(s) = U_p(s) + F(s)\left[U(s) - \frac{1}{G(s)}Y(s)\right] \qquad (18)$$

where

$$F(s) \equiv \frac{s\lambda}{s^2 + s\lambda + \Omega^2} \qquad (19)$$

3.2 OBC versus HHC

OBC and *HHC* present strong analogies. Indeed, compare expression (18) of the *OBC* with Eq. (5) of the *HHC*. In both cases, the expression in brackets is made up of two terms: the first one is obtained by filtering the output y through *the inverse of the rotor model* (either T^{-1} or $G(s)^{-1}$). The second one is simply the control itself (which, in the *HHC* context, is computed by Fourier transforms over the preceding *rev* period). The main structural difference between the two equations (apart from the guidance signal $U_p(s)$ which has been introduced in Eq. (18)) is the term $F(s)$, given by Eq. (19). This is the transfer function of a *narrow band filter* centered on the frequency Ω (Fig. 4). Since $F(j\Omega)=1$, a sinusoid at frequency Ω is transformed, by $F(s)$, into a sinusoid with the same frequency, amplitude and phase. If the filter is given a non-purely sinusoidal input, the regime output will contain the input component at frequency Ω, while all other harmonics will be strongly reduced.

Parameter λ plays a double role in determining the characteristics of the filter $F(s)$. On one hand, λ is directly connected with the responsiveness of the filter: suppose, e.g., to select

$$i(t)=\begin{cases}0 & t<0\\ A\ sin(\Omega t) & t\geq 0\end{cases}$$

as input of $F(s)$. The corresponding output will be

$$o(t)=\begin{cases}0 & t<0\\ A\ sin(\Omega t)+k\,e^{-\lambda t/2}\,[sin\,(\Omega' t+\psi)] & t\geq 0\end{cases}$$

$$\text{where }\ \Omega'=\sqrt{\Omega^2-\lambda^2/4} \qquad (20)$$

with k and ψ suitable constants. Therefore the response transients are characterized by an exponential decay with time-constant $2/\lambda$. Large values for λ result in rapid filtering transients. On the other hand, Fig. 4 reveals that the band of the filter enlarges as λ increases, so that the attenuation effect of frequencies different from Ω is reduced.

HHC scheme, through the cascade of the Fourier Transform block (*FT*) and the Inverse Fourier Transform block (FT^{-1}). As a matter of fact, also the block ($FT+FT^{-1}$) of Fig. 1, extracts from an input signal the component at Ω, while cancelling all other harmonics.

In view of the preceding observation, one can go further in comparing the continuous time *HHC* compensator structure (Eq. (8)) with the *OBC* one. The apparent asymmetry between Eqns. (5) and (18) is due to the fact that the classical continuous time *HHC* regulator leaves out the necessary link between the discrete time and the continuous time world. Taking into account such an interface, ($FT+FT^{-1}$), it surprisingly comes out that the two algorithms, though stemming from very different frameworks, are characterized by the same functional blocks. This can be graphically observed in Figs. 1 and 5.

3.3 Stability

With reference to the continuous time interpretation of the *HHC* algorithm, given in [6] and here outlined in Sect. 2.2, the dynamic behavior of the feedback compensator is examined by looking at the root locus of the closed-loop system. In [6], the conclusion is drawn that *HHC* provides a high gain margin (≅ 90°). However, this algorithm is based on the assumption that input and output of the helicopter rotor are linked by a purely algebraic relationship (the matrix *T*). If just part of the dominant dynamics of the rotor is taken into account, there might well be poles driven unstable by the feedback regulator, as

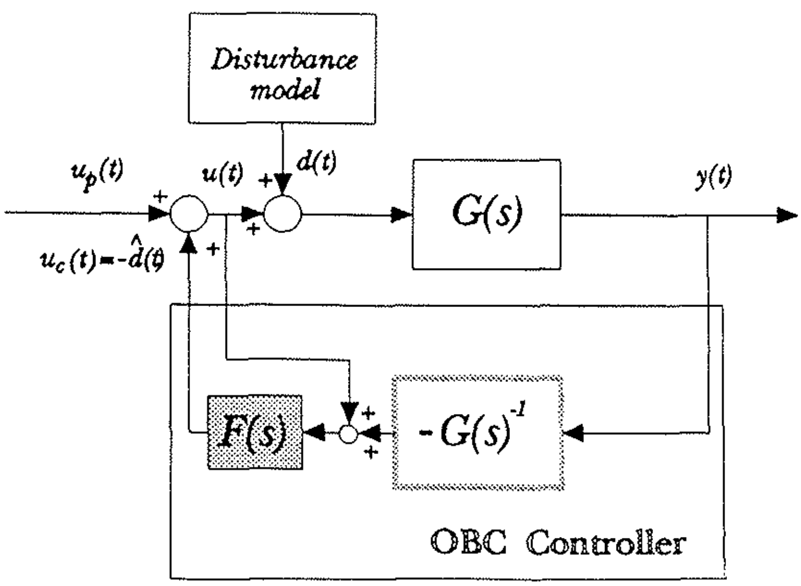

Fig. 5

schematically represented in Fig. 6.

Let's now analyze the stability of the control system based on the *OBC* controller. From Eq. (18), one can derive the relationship linking $U(s)$ to $U_p(s)$ and $Y(s)$:

$$U(s)=P(s)U_p(s) - R(s)Y(s) \quad (21)$$

where

$$P(s) = \frac{s^2+ \lambda s +\Omega^2}{s^2 + \Omega^2}$$

and

$$R(s) = \frac{\lambda s}{s^2 + \Omega^2} \frac{1}{G(s)} \quad (22)$$

The block $P(s)$ is just a filter modifying only the Ω frequency content of the pilot's action: considering that such a frequency is very high with respect to the low frequency dynamics of the pilot's commands, this should not be seen as a significant modification of the $u_p(t)$ signal.

Let us now focus on the effect of $Y(s)$ on $U(s)$. From Eq. (21) it follows that $R(s)$ is the regulator producing the feedback term added to the pilot's command in order to achieve the rotor control action. In view of (22), the open loop transfer function

$$L(s)=G(s)R(s)$$

simply becomes:

$$L(s) = \frac{\lambda s}{s^2 +\Omega^2}$$

The root-locus type analysis carried out in [6] can be applied to this case too, to conclude that, as λ increases, the poles of the feedback system move from the open loop ones (located at $\pm j\Omega$) into the left half plane with an angle of about 180°. Even more so, one can see that the closed loop poles are stable for any gain value. This fact depends primarily on the cancellation between the model and its inverse transfer function.

The Nyquist plot of $L(s)$ will present a phase jump of 180° (introduced by the two imaginary poles) approximately at 90°; this has the direct consequence of a good phase margin, so leading to a more than acceptable robustness degree in terms of phase margin.

In conclusion, the phase margin properties claimed for the *HHC* scheme on the basis of considerations relative to a "*sinusoidal regime*" model of the rotor, are in fact valid for the *OBC* feedback system.

As for the gain margin, it is consequent to the root locus behavior that, whatever large value is assumed by parameter λ, the closed loop poles both move in the left half plane so giving a limiting infinite gain.

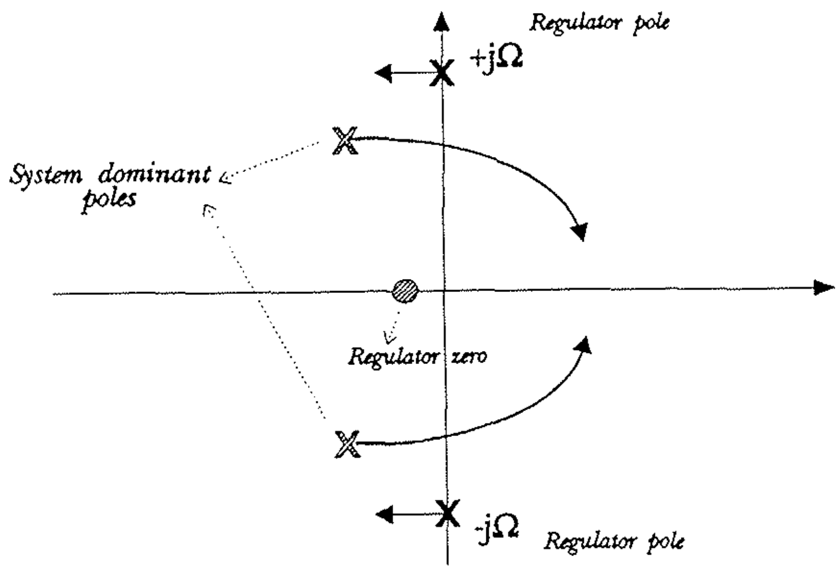

Fig. 6

3.4 Disturbance rejection time

In Sect. 3.2 it has been outlined how parameter λ affects the rapidity of the filter $F(s)$ in recovering an input sinusoid. Not surprisingly, this nice property reflects on the behavior of the closed loop system in rejecting the harmonic disturbance.

As a matter of fact, suppose that $u_p(t)=0$ and a sinusoidal disturbance suddenly enters the system "on its output", i.e.:

$$\tilde{d}(t) = \begin{cases} 0 & t < \bar{t} \\ \tilde{A}\, sin(\Omega t+\tilde{\psi}) & t \geq \bar{t} \end{cases}$$

then, recalling that the closed loop transfer function between $\tilde{d}(t)$ and $y(t)$ is

$$G_{\tilde{d}y}(s) = \frac{s^2 + \Omega^2}{s^2 + \lambda s + \Omega^2}$$

the regulator output results

$$y(t) = \bar{k}\, e^{-\lambda t/2}\, (cos\Omega' t + \bar{\psi})$$

where $\bar{k}$ and $\bar{\psi}$ are suitable constants and Ω' is given by Eq. (20) . Therefore, in the closed loop case too, the disturbance rejection transients are characterized by exponential decays with time constants $2\backslash\lambda$: the higher the value of λ the faster the disturbance rejection action.

3.5 Non Interaction

As outlined before, an important feature of active controllers is that their action must not interfere with the pilot's commands, which involve the low frequency dynamics of the system.

In this connection, recall that the control action $u_c(t)= - \hat{d}(t)$ is added to the pilot's action $u_p(t)$ to produce the actual swash plate command $u(t)$ (Fig. 3). Therefore, the *"non - interaction"* requirement is equivalent to imposing that $u_c(t)$ is such that signals $u(t)$ and $u_p(t)$ have the same low frequency behavior. This property is indeed enjoyed by the *OBC* regulator of Eq. (22). As a matter of fact, the zero in the origin, which automatically appears in $R(s)$ thanks to the zeroing of the first element of Λ (see Eq. (16)), makes the regulator act as a high pass filter, thus guaranteeing that the control action has poor content at low frequencies.

In particular, differently from what happens to regulator (8), the gain of the transfer function between the pilot's command $u_p(t)$ and the mast force $y(t)$ does not change when switching from an open loop to a closed loop configuration.

3.6 Generalizations

Among the assumptions underlying the analysis above, we underly the following two:

i) Disturbance with a single harmonic ;

ii) Function $G(s)$ improper and minimum phase.

The case of multi harmonic disturbances can be simply faced by considering more complex models of the form given in Eqns. (11) and (12), where $d(t)$ could be a sum of sinusoids at different frequencies. This would lead to regulators characterized by more than one couple of imaginary poles.

If assumption ii) is not met with, then the cancellation philosophy behind the simple *OBC* design technique discussed above cannot be applied any more. The observer parameters can then be tuned by resorting to optimal control techniques, as discussed in [13].

Last, but not least, a few words on the *SISO* character of our analysis. This is due to the fact that only the collective command was taken as control variable, and only the force at the rotor hub was considered as controlled variable.

A natural generalization concerns the input variable: better performances are obviously expected if the cyclic command is also used. Moreover, measurement points distributed along the fuselage could be considered in place of a unique measurement at the mast only. This would lead to a Multi Input Multi Output (*MIMO*) model. In this connection, it is worthwhile noting that, notwithstanding the obvious increase in computational complexity, the *OBC* approach, which originates from a state-space analysis. does not suffer of the passage from the *SISO* to the *MIMO* case.

In this case, as well as in the other ones mentioned above, the design of the observer can be dealt with by means of optimal filtering theory (Kalman Filter). It is worthwhile mentioning that the above presented *OBC* approach leads to a non standard *Linear Quadratic Gaussian* (*LQG*) control problem. In [13], the theoretical approach to the solution of such a problem is addressed in some existing techniques to counterbalance periodic disturbances are revisited and enhanced in order to concisely present a complete method for incorporating disturbance rejection capabilities in optimal controllers.

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5. References

1. J. Shaw and N. Albion, *Active Control of the Helicopter Rotor for Vibration Reduction*, Journal of the American Helicopter Society, vol. 26, n. 3, 1981.

2. M. Achache and M. Gauvrit, *Control Active des Vibrations sur Helicopters par Commandes Multicycliques Autoadaptatifs*, Proceedings of the 12th European Rotorcraft Forum, 1986.

3. M.W. Davis, *Development and Evaluation of Generic Active Helicopter Vibration Controller*, Proceedings of the 10th European Rotorcraft Forum, 1984.

4. N. Ham, *Helicopter individual blade control research at MIT / 1977 - 1985*, Vertica, vol. 11, n. 1/2, 1987.

5. A. Gessow and G. C. Myers, *Aerodynamics of the Helicopter*, Frederik Ungar Publishing Co., N.Y.

6. S. Hall and N. M. Werely, *Linear Control Issues in the Higher Harmonic Control of Helicopter Vibrations*, Proceedings of the 45th Annual Forum of the American Helicopter Society, Boston, MA, 1989

7. R. Du Val, C. Z. Gregory and N. Gupta, *Design and Evaluation of a State Feedback Vibration Controller*, Proceedings of the American Helicopter Society National Specialists' Meeting on Helicopter Vibration, Hartford, Connecticut, 1981.

8. S. Bittanti, P. Bolzern, P. Colaneri, P. Delrio, G. De Nicolao, F. Lorito, A. Russo and S. Strada, *Identification of a Helicopter Dynamic Model for Active Control of Vibrations*, Proceedings of the 9th IFAC/IFORS Symposium on Identification and System Parameter Estimation, Budapest, 1991.

9. N. Gupta, *Frequency-Shaped Cost Functionals: Extension of Linear-Quadratic-Gaussian Design Methods*, Journal of Guidance and Control, 1980, Vol. 3, No. 6, pp. 529-535.

10. N. Gupta and R.W. Du Val, *A New Approach for Active Control of Rotorcraft Vibration*, Journal of Guidance, 1982, Vol. 5, No. 2, pp. 143-150.

11. R. Kube, *Evaluation of a Constant Feedback Gain for Closed Loop Higher Harmonic Control*, Proceedings of the 18th European Rotorcraft Forum, Glasgow, 1990.

12. P. T. Kabamba, *Control of Linear Systems Using Generalized Sampled-Data Hold Functions*, IEEE Transactions on Automatic Control, 1987, Vol. AC-32, No. 9, pp. 772-783.

13. G. De Nicolao, F. Lorito and S. Strada, *Dual LQG Based Methods for Rejection of Narrowband Disturbances*, Proceedings of the 2nd IFAC Workshop on System Structure and Control, Prague, 1992.