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ANALYTICAL STUDY ON AERODYNAMIC CHARACTERISTICS OF THE HELICOPTER SHROUDED TAIL ROTOR

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Abstract

One of the advantages for the shrouded tail rotor of a helicopter is its better aerodynamic characteristics over the conventional (isolated) tail rotor. In this paper, in order to make a deeper understanding for the shrouded tail rotor, a thrust division factor q, which represents the ratio of the shroud thrust to total thrust of the shrouded tail rotor, is introduced and with the help of q, the slipstream theory for the static and axial flow states of the shrouded tail rotor are fully derived, so that variations of the thrust, power and disk area against q for different cases are analysed.

Next, an estimated method, based on combined blade element theory and slipstream theory, is developed to evaluate the aerodynamic characteristics of a shrouded tail rotor against it's pitch. As an example, the thrust and required power of Helicopter Z — 9 shrouded tail rotor are calculated and compared with those of the isolated tail rotor.

Finally, several conclusions are presented from above analysis and calculations.

Nomenclature

A : disk area
P, C_P : ideal induced power and its coefficient
q : thrust division factor (see Eq. (2))
T, C_T : total thrust and its coefficient
T_R, C_{T_R} : rotor thrust in shroud and its coefficient
T_S, C_{T_S} : shroud thrust and its coefficient
V_0 : flow velocity far upstream
V_1, v_1 : resultant velocity and induced velocity at the disk
V_2, v_2 : resultant velocity and induced velocity far downstream
ρ : air density
φ_0, φ_7: pitch at $\bar{r}=0$ and $\bar{r}=0.7$
$\Delta\varphi$: linear blade twist

Superscript
$(\ )'$: shrouded rotor system
$(\bar{\ })$: nondimensional

Introduction

The development of a shrouded tail rotor (Fenestron) may be traced back to the idea of earilier shrouded propeller or ducted fan in the 1930s'. The shrouded propeller was not seriously studied until the 1950s' — 1960s' when it was used as the propulsion system on ground effect machines and the lift system on V/STOL aircrafts (Ref. 1). Compared to a conventional propeller, a shrouded one provides a greater thrust for equal power with same diameter, or produces same thrust with smaller diameter under the condition of equal power. It can be explained as follows:

a. the presence of a shroud substantially changes the slipstream downwards and converts more kinetic energy below the disk into pressure energy.

b. the tip losses are reduced by weakening the three-dimensional effect at blade tips.

c. a large negative pressure area on the inlet lip of Fenestron is formed and an additional thrust is generated.

Because of above attractive advantages of the shrouded propeller, it is considered for application to a helicopter as the shrouded tail rotor, e.g., on helicopeer Dauphin in France and helicopter Comanche in USA.

However, the existing publications on aerodynamic characterstics of the shrouded tail rotor are mostly the experimental results as well as the data of flight tests, and it seems that a relatively thorough analytical investigation hasn't been found. There is lack of the thrust division relationship between the rotor in shroud and the shroud itself. In this paper, a thrust division factor q is introduced and an analytical estimation on aerodynamic characteristics of Fenestron is developed. First, the thrust, power, and disk area of the shrouded tail rotors with the variation of q are analysed by slipstream theory. Then, expressions for numerical calculation are derived to evaluate the thrust and power again different pitchs by combining blade element theory and slipstream theory. Some conclusions are given.

Slipstream Theory of Shrouded Tail Rotor

Consider the slipstream of a shrouded rotor shown as Fig. 1. , and similar basic assumptions as for the slipstream theory of an isolated tail rotor are made to determine the relation of thrust, power, and flow velocity of a shrouded tail rotor.

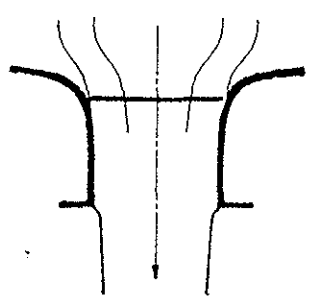

Figure 1. Schematic of slipstream through a shrouded tail rotor in axial flow state

Let flow velocity far upstream be V_0 , then, for a shrouded rotor,

$$\left.\begin{aligned} V'_1 &= V_0 + v'_1 \\ V'_2 &= V_0 + v'_2 \end{aligned}\right\} \tag{1}$$

Introducing the thrust division factor q

$$q = T'_S/T' \tag{2}$$

when $q = 0$, it corresponds to an isolated rotor.

Momentum equation gives the total thrust as

$$T' = T'_R + T'_S = \rho A'(V_0 + v'_1)v'_2 \tag{3}$$

and by Bernoulli's equation,

$$\begin{aligned} T' = T'_R + qT' &= \rho A'(2V_0 + v'_2)v'_2/2 \\ &+ q\rho A'(V_0 + v'_1)v'_2 \end{aligned} \tag{4}$$

Equating the right sides of Eqs (3) and (4), one obtains

$$q = 1 - \frac{V_0 + v'_2/2}{V_0 + v'_1} = 1 - \frac{V'_2 + V_0}{2V'_1} \tag{5}$$

The presence of a shroud ($q \neq 0$) substantially changes the slipstream downwards, and obviously $V'_1 > (V'_2 + V_0)/2$, rather than $V'_1 = (V'_2 + V_0)/2$.

In Figure 2, the variation of v'_2/v'_1 or V'_2/V'_1 with q is plotted, and it is seen that the induced velocity ratio v'_2/v'_1 or the flow velocity ratio V'_2/V'_1 decreases from greater than 1 to 1. But usually $\frac{v'_2}{v'_1} < 1$ or $\frac{V'_2}{V'_1} < 1$ is not desired (Ref. 2), otherwise the reverse pressure gradient flow and even flow separation would occur. So the magnitude of q should be restrained, namely, the contribution of the shroud should be restrained. For example, if $\frac{V_0}{v'_1} = 0, \frac{1}{2}, 1, \frac{3}{2}$ (or $\frac{V_0}{V'_1} = 0, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}$) (see Fig. 2), then what corresponds to $\frac{v'_2}{v'_1} = 1$ are $q = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$,and $\frac{1}{5}$.

In other words, the shroud is impossible to provide the thrust arbitrarily. The larger the $\frac{V_0}{v'_1}$ or $\frac{V_0}{V'_1}$, the smaller is the contribution of the shroud.

The power may be written by the energy equation

$$P' = \rho A'(V_0 + v'_1)\ (2V_0 + v'_2)\ v'_2/2 \tag{6}$$

Which is the same result as from the definition of power.

In order to clarify the gains of the shroud, comparisons between the shrouded rotor and the isolated one in three cases are made as follows:

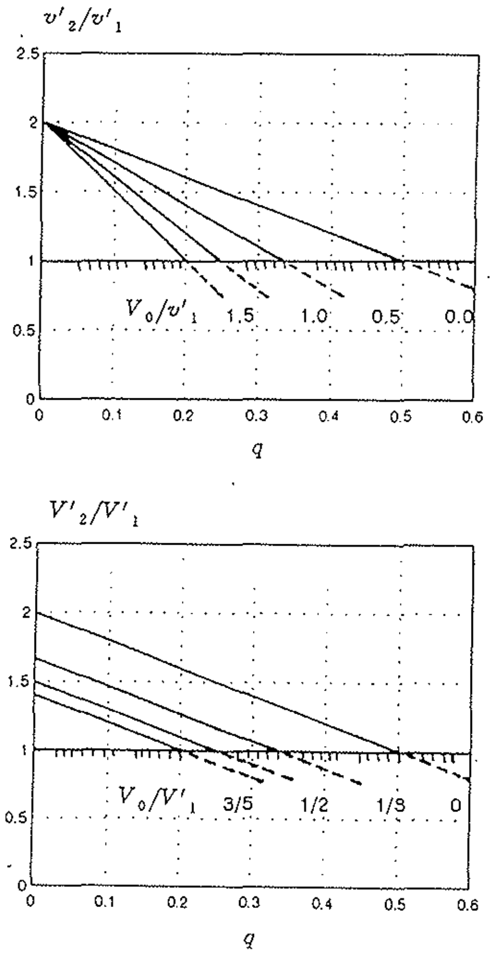

Figure 2. Variations of the induced velocity ratio v'_2/v'_1 and flow velocity ratio V'_2/V'_1 with the thrust factor q

a. Thrust comparison with equal power and same diameter:

Let $A' = A$ and $P' = P$, then

$$\left.\begin{aligned} \frac{T'}{T} &= \frac{V_2 + V_0}{V'_2 + V_0} \\ \frac{T'_R}{T} &= \frac{V_1}{V'_1} \end{aligned}\right\} \qquad (7)$$

Substituting Eq. (5) into Eq. (7), a cublic equation for T'/T is obtained.

$$(\frac{T'}{T})^3 - \frac{1}{1-q}\frac{V_0}{V'_1}(\frac{T'}{T})^2 - \frac{1}{1-q}(1 - \frac{1}{1-q}\frac{V_0}{V'_1}) = 0 \qquad (8)$$

Fig. 3 plots the variation of T'/T againat q for different $\frac{V_0}{v'_1}$. It is shown that, the thrust of a shrouded tail rotor is larger than that of an isolated tail rotor with same diameter under equal power. However, the increment of T'/T is decreasing with the increase of $\frac{V_0}{V'_1}$, so the contribution of the shroud is the greatest for static state, and the larger the V_0, the smaller is the contribution.

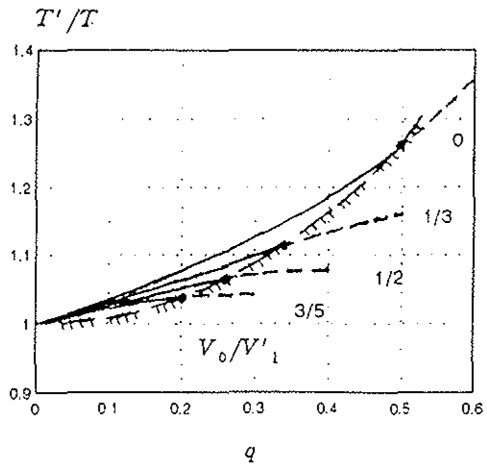

Figure 3. Variation of the thrust ratio T'/T with q

When $V_0 = 0$, i. e., in hover state, the thrust ratio can be written as

$$\frac{T'}{T} = \frac{1}{\sqrt[3]{1-q}} \qquad (9)$$

$$\frac{T'_R}{T} = \sqrt[3]{(1-q)^2} \qquad (10)$$

These indicate that always $T'_R < T$, and $T' > T$. When $q = 0.5$, $\frac{T'}{T} = \sqrt[3]{2} \approx 1.26$ and $\frac{T'_R}{T} = \sqrt[3]{\frac{1}{4}} \approx 0.63$.

b. Power comparison with same diameter and equal thrust

Let $A' = A$ and $T' = T$, then

$$\frac{P'}{P} = (1-q)\frac{V'_1}{V_1}$$

$$= \frac{2(1-q)}{(\frac{V_0}{V'_1}) + \sqrt{(\frac{V_0}{V'_1})^2 + 4(1-q-\frac{V_0}{V'_1})}} \quad (11)$$

The relation of P'/P against q is shown in Fig. 4. The required power of a shrouded rotor is smaller than of an isolated one in this case. Smallest power is required for static state and with the increase of $\frac{V_0}{V'_1}$, the decrement of P'/P is getting smaller.

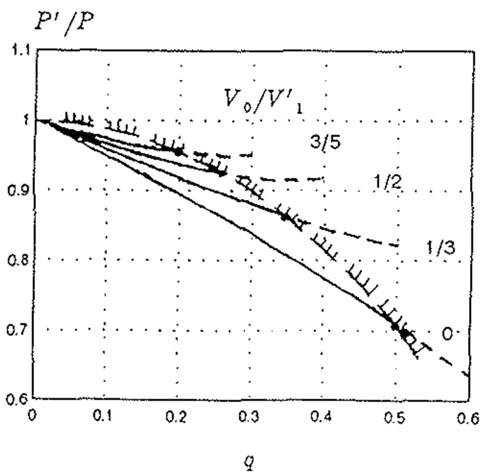

Figure 4. Variation of the power ratio P'/P with q

For the static state, i. e. $V_0 = 0$, Eq. (11) becomes

$$\frac{P'}{P} = \sqrt{1-q} \quad (12)$$

and $P'/P \approx 0.707$ for $q = 0.5$.

c. Disk area comparison with equal thrust and power:

Let $T' = T$ and $P' = P$, then

$$\frac{A'}{A} = \frac{V_1}{V'_1} = 1 - q \quad (13)$$

When $V_0 = 0$,

$$\frac{A'}{A} = \frac{v_1}{v'_1} = 1 - q \quad (14)$$

The above equations give the relation of A'/A against q both the same for axial and for static states.

Thus, the disk area of a shrouded rotor might be reduced to half of that of an isolatod one if the shroud is well designed to cause the total thrust equally divided between the shroud and the rotor ($q = 0.5$).

Aerodymanc Calculation of the Shrouded Rotor by Combined Blade Element Theory and Slipstream Theory

In order to evaluate the aerodynamic characteristics of a shrouded rotor for different blade pitch angle, the blade element theory is required further.

For a linear twist and constant chord blade, the blade element theory (Ref. 3) gives

$$C'_{T_R} = \sigma\int_{\bar{r}_0}^{1} a_\infty(\varphi_0 + \Delta\varphi\bar{r} - \beta_*)\cdot \sqrt{\bar{r}^2 + (\bar{V}_0 + \bar{v}'_1)^2}\bar{r}d\bar{r} - \sigma\int_{\bar{r}_0}^{1} C_x \sqrt{\bar{r}^2 + (\bar{V}_0+\bar{v}'_1)^2}\cdot (\bar{V}_0 + \bar{v}'_1)\, d\bar{r} \quad (15)$$

As the inflow angle β_* is usually larger for shrouded rotor over isolated one, it is approximately expressed here by a series of first three terms,

$$\beta_* = \frac{\bar{V}_0 + \bar{v}'_1}{\bar{r}} - \frac{1}{3}(\frac{\bar{V}_0+\bar{v}'_1}{\bar{r}})^3 + \frac{1}{5}(\frac{\bar{V}_0+\bar{v}'_1}{\bar{r}})^5 \quad (16)$$

Integrating Eq. (15) gives

$$C'_{T_R} \approx -\frac{1}{2}C_1[\sqrt{1+\bar{V}'^2_1} - \bar{r}_0\sqrt{\bar{r}_0^2 + \bar{V}'^2_1} + \bar{V}'^2_1 ln\frac{1+\sqrt{1+\bar{V}'^2_1}}{\bar{r}_0 + \sqrt{\bar{r}_0^2+\bar{V}'^2_1}}] + C_2(\sqrt{(1+\bar{V}'^2_1)^3} - \sqrt{(\bar{r}_0^2+\bar{V}'^2_1)^3}) \quad (17)$$

Where

$$C_1 = \frac{1}{3} a_\infty \sigma \bar{V}'_1 + c_x \sigma \bar{V}'_1 + \frac{1}{4} a_\infty \sigma \Delta\varphi \bar{V}'^2_1$$

$$C_2 = a_\infty \sigma [\frac{1}{3}\varphi_0 + \frac{1}{4}\Delta\varphi (1 - \bar{r}_0) - $$
$$- \frac{1}{3}\bar{V}'_1 (1 - \frac{1}{\bar{r}_0}) + \frac{3}{35}\bar{V}'^3_1 \cdot$$
$$(1 - \frac{1}{\bar{r}_0^3}) - \frac{5}{35}\bar{V}'^5_1 (1 - \frac{1}{\bar{r}_0^5})]$$

Uniform induced velocity is assumed in derivation because it is more reasonable for a shrouded rotor (Ref. 2).

Combining the expressions of slipstream theory and blade element theory, an equation including the thrust division factor and induced velocity can be obtained

$$f(q, \bar{v}'_1) = 0 \tag{18}$$

From Eq. (18), the induced velocity and thus the thrust and power of a shrouded tail rotor might be determined.

Sample Calculation

As an example, the thrust and induced power of Helicopter Z—9 shrouded tail rotor are calculated and compared with that of the corresponding isolated tail rotor.

In Fig. 5, the variation of the thrust coefficient ratio againat pitch angle at 0. 7R is drawn in hover state for different q (Note here the condition of equal power is not insisted). When the pitch is not so large, the shrouded tail rotor will produce less thrust than the isolated one with equal diameter. However, for larger pitches, it will provide greater thrust. The conclusion verifies experimental results for Gazelle shrouded tail rotor in Ref. 4.

Owing to the stall limitation, the maximum pitch of conventional tail rotor is lower (<30°) than that of Fenestron, the latter is 38° for Helicopter Z — 9. In fact, when the pitch of Fenestron approaches the maximum pitch, the increase of total thrust is getting more rapidly.

Fig. 6 shows a plot of the power ratio C'_P/C_P against pitch φ_7 in hover state for different q. Less power is required for Fenestron than for convertional tail rotor with same size.

If Fig. 5 and Fig. 6 are combined, a conclusion will be drawn that the improvement of aerodynamic characteristics for Fenestron is increasing with the increase of q.

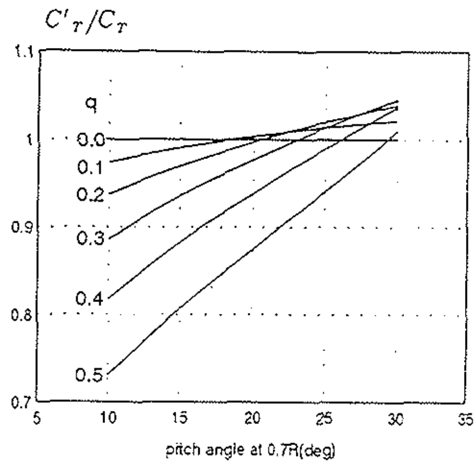

Figure 5. Variation of the thrust coefficient C'_T/C_T with the pitch φ_7(hover)

The variation of thrust coefficient ratio C'_T/C_T agant q and powes coefficient ratio C'_P/C_P againt q in a given axial flow state is plotted in Fig. 7 and Fig. 8. Comparing with hover flight, similar curves are obtaind but more rapid decrease of the ratios is seen for axial flow because of decreases of the section angle of attack.

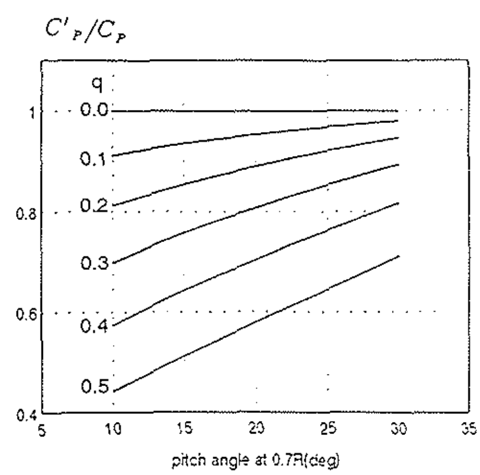

Figure 6. Variation of the power ratio C'_P/C_P with the pitch φ_7(hover)

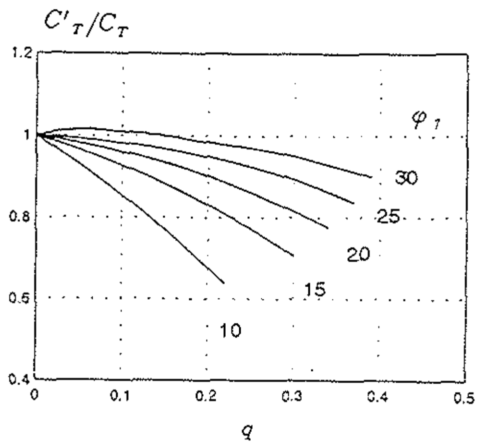

Figure 7. Variation of C'_T/C_T with q in axial flow state of shrouded tail rotor

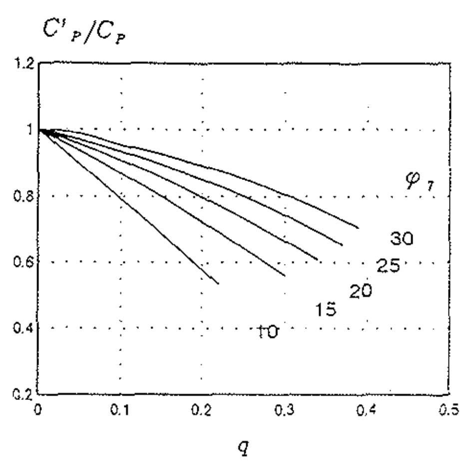

Figure 8. Variation of C'_P/C'_P with q in axial flow state of shrouded tail rotor

Conclusions

The following conclusions may be drawn through above analytical investigation on aerodynamic characteristics of shrouded tail rotor:

(1) The thrust division factor q relates to the flow velocity ratio V_0/V'_1 and V'_1/V'_2, and for a given V_0/V'_1, the q increases with the decrease of V'_2/V'_1, but usually $\frac{V'_2}{V'_1} < 1$ isn't desired, so q should be retrained, i. e., the contribution of the shroud is not unlimited.

(2) Under the condition of equal power, total thrust of a shrouded tail rotor is always greater than that of an isolated tail rotor with equal size, while the thrust of rotor in the shroud is always smaller than that of an isolated tail rotor, and the increment of total thrust is getting smaller with the increase of $\frac{V_0}{V'_1}$.

(3) With equal thrust, the required power of Fenestron is less than that of an isolated tail rotor and this decrement will be getting smaller with the increase of V_0/V'_1.

(4) With equal thrust and power, the ratio of the disk area between the shrouded and isolated tail rotor is decreasing linearly against q.

(5) For the same pitch, but not so large, the shrouded tail rotor will produce less thrust than the isolated one with equal diameter, while for larger pitch angles, the former will provide greater thrust.

(6) it is shown that the improvement of aerodynamic efficiency of shrouded tail rotor increases with q.

References

1. D. M. Black, H. S. Wainauski and C. Rohrbach. Shrouded propellers — a comprehensive performance study. Paper No. 68—994 presented at AIAA 5th Annual Meeting and Technical Display, Oct. 1968.

2. R. Mouille. The "Fenestron" a shrouded tail rotor concept for helicopters. Proceedings of the 42nd Annual Forum of the American Helicopter Society, May 1986.

3. Wang Shicun. Helicopter aerodynamics. China Aviation Industry Press, 1985. (In Chinese)

4. R. Mouille. The "Fenestron" shrouded tail rotor the SA. 341 Gazelle. Journal of the American Helicopter Society, Oct. 1970, pp. 31—37.

5. R. A. Wallis. Axial flow fans and ducts. John Wiley & Sons, 1983.

6. D. Kuchemann and J. Weber. Aerodynamics of propulsion. First Edition, McGraw-Hill,

New York, 1956.

7. W. H. Meier, W. P. Groth, D. R. Clark. Flight testings of a fan-in-fin antitorque and directional control system and a collective force augmentation system. AD/A—013407, 1975.

8. A. Pope, J. J. Harper. Low-speed wind tunnel testing. John Wiley & Sons, 1983.

9. J. K. Davidson, C. T. Havey and H. E. Sherrieb. Fan-in-fin antitorque concept study. AD 747806, 1972.

10. A. R. Kriebel. Theoretical stability derivatives for a ducted propeller. AIAA Paper No. 64—170, 1964.