A new appreciation of prescribed wake models for CFD analysis in view of aeroacoustic applications

Yoh MURAKAMI, Yasutada TANABE, Shigeru SAITO
Japan Aerospace Exploration Agency
yohm@chofu.jaxa.jp
and
Hideaki SUGAWARA
Ryoyu Systems Co., Ltd.

Abstract

This paper is related to an attempt to combine a generic CFD code with a relatively simple prescribed rotor wake model with an aim to reduce the computational time required for numerically simulating intricate flows around a helicopter rotor in view of acoustic applications such as BVI noise prediction. The base CFD code herein assumed is the *rFlow3D*, which has intensively been used for rotorcraft applications at JAXA (Japan Aerospace Exploration Agency) and its validity has firmly been upheld so far. For the prescribed wake model, the authors modified Beddoes' wake model so as to render a more realistic and detailed description of the rotor wake geometry, especially in descent, than the original model can, since acoustic analysis requires much more detailed description of the local aerodynamic condition on the rotor disc than flight dynamic analysis usually does. The hybridisation of the wake model with the CFD code has not been completed at this moment for the reasons stated later, however, it seems that the present hybrid model should arguably be promising to fulfill the above-mentioned primary objective. The details of the wake model and its striking features shall be focused and enunciated in the main body.

1. Introduction

Various CFD methods have been rapidly developed over last few decades, especially in the field of aerospace engineering. Rotorcraft engineering is, needless to say, one of the key branches of aerospace engineering in which CFD methods are extensively used. Due to the unsteadiness and complexity of a flow field around a rotor, numerical simulation of a realistic helicopter rotor had long remained as a thorny problem, in part because of the insufficient capability of contemporary computers. Early attempts to reduce the computational load include rigid and prescribed wake models, in which the assumed rotor wake geometry is analytically provided in advance, whereas the wake geometry in reality should be highly complicated and unsteady. Recent rapid development in computer science enabled aerodynamicists to adopt free wake methods with a sufficient number of grids, wherein the rotor wake can unsteadily develop with vortex elements conveyed by induced flow that vortex elements themselves induced in a time-marching manner. Nowadays, more exhaustive numerical methods based on the first principles such as the Euler and Navier-Stokes equations can also be directly executable even on a desktop computer with sufficient accuracy within a reasonable period of time. Since these methods such as the free wake method and the first principle methods generally require less number of conditions for approximation and thus can yield more realistic description of flow fields if the space is discretised into sufficiently fine meshes, less realistic prescribed wake methods once came to be almost off the mainstream of the first-line research scene. Yet, the computational time which the above-mentioned exhaustive methods invariably require remained as another problem, especially in those sorts of problems in which a sequence of test cases should be swept out, e.g., questing for the optimal phase of an active device or searching out in all directions the noisiest points around a rotor in BVI noise prediction. This led to the revival of prescribed models in a hybrid manner so that a relatively crude but speedy prescribed model should provide a *fairly reasonable* distribution of the induced flow with a background grid system in a CFD code. Note that the flow distribution can be offered to the CFD code as a starting condition, it should be better to provide the flow distribution around rotor blades as a local boundary condition for each step of numerical iterations, in view of minimising the computational time in total. In so doing, the present authors have been developing a comprehensive speedy simulation code, which was provisionally named *rPresto*. The base CFD code implemented in the *rPresto* is the *rFlow3D*, which has intensively been used for rotorcraft applications at JAXA and whose validity has arguably been sufficiently verified by now [1]. The *rFlow3D* is a highly versatile CFD code that can numerically simulate flows around rotorcraft in a wide range of Reynolds and Mach numbers under various conditions, but a single case may sometimes require a month or more to complete even with JAXA's cutting-edge supercomputer. The base of the prescribed wake model considered in this study is Beddoes' model, which was originally proposed by T. S. Beddoes [2] and then developed by many followers including B. van der Wall [3]. Note that, unlike most of flight dynamical applications where only six components of forces and moments of the rotor can often suffice for analysis, a more detailed local distribution of disc loading is indispensable for acoustic analysis, and therefore, the key to reducing the total computational time is the degree of reality that the prescribed wake model can bring in. It is vastly preferable that the prescribed wake model alone could provide a sufficiently realistic flow field as much as possible, and thus the Beddoes' original wake model should be as far improved as possible for the purpose above. A generic analytical code for rotorcraft flight dynamics based on wake theory, provisionally called *Aschenputtel*, has been developed solely by the first author at JAXA in a parallel manner to promoting theoretical improvements in the wake modelling, and thus the *rPresto* can be by and large considered as the hybrid of the *rFlow3D* and *Aschenputtel* in a loosely-coupling form. The further details of the newest wake model and the *rPresto* shall be enunciated in the following sections.

2. Description of the Models

2-1. The CFD Code

The *rFlow3D* is an Euler solver based on the moving overlapping grid method, developed chiefly by Y. Tanabe of JAXA [1] for simulating air flows around a helicopter rotor. This numerical code can conduct time-marching simulation by a dual-time stepping method, using either the LUSGS (Lower-Upper Symmetric Gauss-Seidel method) or DP-LUR (Data-Parallel Lower-Upper Relaxation) method. The FCMT (Fourth-order Compact MUSCL TVD) interpolation method and the tri-linear interpolation are used for data exchange between overlapped grid systems. Consequently, the *rFlow3D* ensures a fourth-order accuracy in the spatial resolution, with the favourable TVD (Total Variations Diminishing) property unimpaired. It should also be worth herein noting that the mSLAU scheme, which is an AUSM (Advection Upstream Splitting Method) scheme and developed from the SLAU scheme [4], is implemented in the *rFlow3D* to solve the non-linear terms of the Navier-Stokes equations, for handling low speed flows, whose Mach number can be as low as 0.01, on top of the ability of solving high speed flows where $M > 1$. Those readers who would like to know further details of the *rFlow3D* are encouraged to refer to Refs. [1] and [4].

2-2. The Prescribed Model

Generally speaking, prescribed wake models are related to several key issues such as; (i) how to describe the wake geometry, (ii) how to describe vortices, (iii) how to associate the wake geometry with the induced flow distribution, and (iv) how to appraise the results provided by the wake model. According to each of the highlighted aspects above, new features introduced in this study shall be illustrated hereinafter.

Regarding the wake geometry, the simplest description of a rotor wake should be a straight cylinder (Fig. 1, left). The second simplest may be a cylindrical model with the effect of wake contraction (Fig. 1, right) so that the wake tube radius, r, varies with the axial distance from the rotor disc, for example, as

$$r = r_v[\kappa + (1-\kappa)\exp(-g\Delta\Psi_v)], \qquad (1)$$

where r_v is the radial position where the pertinent vortex element is released, κ is the contract coefficient, $\Delta\Psi_v$ is the wake age and g is the damping coefficient. The contract coefficient, κ, can be predicted by momentum theory as 0.707, but Theodorsen and Landgrebe instead proposed the values of 0.816 and 0.780, respectively. The damping coefficient, g, can be empirically given as $0.145 + 27C_T$, [5].

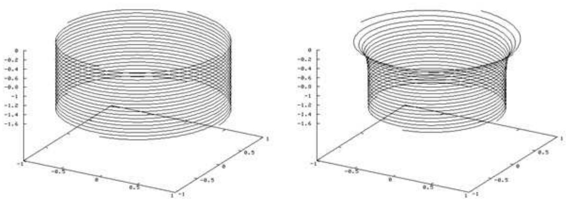


Fig. 1 Primitive wake models; without wake contraction (left); with wake contraction (right).

These cylindrical models may yield fairly reasonable predictions in hover, however, they are no longer realistic in forward flight, even if described as a skewed cylinder with a wake skew angle. Beddoes' original model is without doubt a significant milestone in the model development, which can beautifully describe the horseshoe-shaped roll-up of a rotor wake by following equations;

$$x = r_v \cos\Psi_v + \mu\Delta\Psi_v, \quad (2)$$

$$y = r_v \sin\Psi_v, \quad (3)$$

$$z = -\lambda_c \Delta\Psi_v - \int_0^{\Delta\Psi_v} \lambda_i d\psi, \quad (4)$$

where (x,y,z) indicates the position of the pertinent vortex element with the origin of the frame of reference on the rotor hub centre, where x-, y- and z-axes are defined positive aft, starboard and downwards, respectively. The second term on the right-hand side of Eq. (4) is to be explicitly spelt out in three different forms as follows;

(i) if $r_v \cos\Psi_v + \mu\Delta\Psi_v < -r_v \cos\Psi_v$ then

$$\int_0^{\Delta\Psi_v} \lambda_i d\psi = \lambda_0 \left\{ 1 + E\left(\cos\Psi_v + \frac{1}{2}\mu\Delta\Psi_v \right) - E\,|\,y^3\,| \right\} \Delta\Psi_v, \quad (5)$$

(ii) if $\cos\Psi_v > 0$ then

$$\int_0^{\Delta\Psi_v} \lambda_i d\psi = 2\lambda_0 \left(1 - E\,|\,y^3\,| \right) \Delta\Psi_v, \quad (6)$$

(iii) else

$$\int_0^{\Delta\Psi_v} \lambda_i d\psi = -\frac{2\lambda_0}{\mu} \left(1 - E\,|\,y^3\,| \right), \quad (7)$$

where μ is advance ratio, Ψ_v is the blade azimuth at the time point when the vortex element *was* released, λ_0 is the mean value for the induced velocity on the disc from momentum theory, and E is a coefficient related to the roll-up and is defined as $E = |\chi|$ [2] or $E = 0.5|\chi|$ [7] wherein χ is the wake skew angle, $\chi = \arctan[\mu/(\lambda_0 + \lambda_i)]$. Beddoes' model apparently yields quite a realistic wake geometry in forward flight for a reasonably large μ (Fig. 2, left), however, the author spotted a problem with the model that when in hover (i.e., $\chi = 0$), E becomes zero and hence Eqs. (5) and (6) are reduced down to $\lambda_0\Delta\Psi_v$ and $2\lambda_0\Delta\Psi_v$, respectively, resulting in an awkward uneven shape in which the front half of the rotor disc has the uniform induced flow of λ_0 while the rear half has $2\lambda_0$ (see Fig. 2).

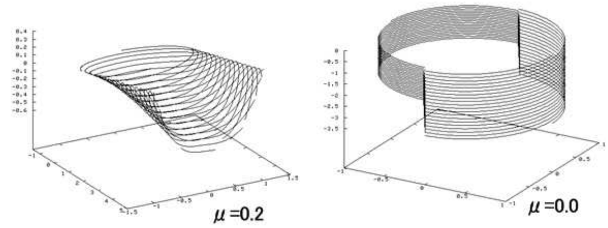


Fig. 2 Beddoes' wake model; $\mu = 0.2$ (left) and $\mu = 0.0$ (right).

In response to this problem, the first author proposed the following equations instead of Eqs. (5), (6) and (7) so that hover and forward flight cases should smoothly be bridged.

$$\int_0^{\Delta\Psi_v} \lambda_i d\psi = \lambda_0 \left\{ 1 + \frac{8E}{15\pi} + A(x) - 2\mu(y + B(y)) + E\left(\cos\Psi_v + \frac{1}{2}\mu\Delta\Psi_v \right) - E\,|\,y^3\,| \right\} \Delta\Psi_v, \quad (8)$$

$$\int_0^{\Delta\Psi_v} \lambda_i d\psi = \lambda_0 \left(2 - \exp(-\zeta\mu) \right) \left(1 + \frac{8E}{15\pi} + A(x) - 2\mu(y + B(y)) - E\,|\,y^3\,| \right) \Delta\Psi_v, \quad (9)$$

$$\int_0^{\Delta\Psi_v} \lambda_i d\psi = \lambda_0 \left[\frac{2(1 - \exp(-\zeta\mu))x}{\mu\Delta\Psi_v} + \exp(-\zeta\mu) \right] \left(1 + \frac{8E}{15\pi} + A(x) - 2\mu(y + B(y)) - E\,|\,y^3\,| \right) \Delta\Psi_v. \quad (10)$$

Note that $8E/15\pi$ and $2\mu y$ were added by van der Wall in Ref. [3] so that the model should align with momentum theory. Note also that control functions, $A(x)$ and $B(y)$, were further added by the author so that the twist and deflection of the wake tube should be considered in the mathematical modelling. These functions are related to pitching and rolling moments acting on the rotor and can thus be constants or linear functions in trim steady flight, but may also be quadratic, periodic or in a form of other functions when cyclic controls are imposed. Note that higher harmonic controls may also be possible to associate with the wake geometry by these control functions in the form of trigonometric functions.

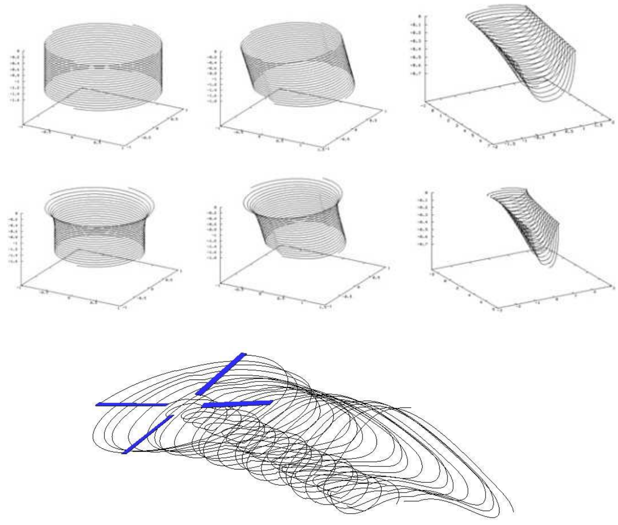


Fig. 3 Beddoes-Murakami's wake model with $\zeta = 10$ 'with wake contraction' (upper raw) and 'without wake contraction' (lower raw); hover case ($\mu = 0.0$, left column), transitional case ($\mu = 0.01$, middle column) and forward flight case ($\mu = 0.2$, right column), a closed-up view for quite a transitional case $\mu = 0.05$ with root vortices included.

It was underpinned that the new model expressed by Eqs. (8) – (10), that was provisionally named Beddoes-Murakami's model, albeit awfully audacious, ensures smooth transition between hover and forward flight. Note that this new model rapidly tends to Beddoes's model when μ is fairly large, though depending on the value of the decaying coefficient, ζ. The author further extended the model so as to implement the wake contraction as well (Fig. 3). In point of fact, the induced flow from momentum theory in Eqs. (8) – (10), λ_0, can be further extended in the following form instead of control functions, taking the first harmonics in the azimuthal variation into consideration;

$$\lambda_0 \to \lambda_0 + \lambda_{1c} \cos\Psi + \lambda_{1s} \sin\Psi, \qquad (11)$$

where the Fourier coefficients, λ_{1c} and λ_{1s}, may be determined by an iteration method so as to trim rotor moments. The *rPresto* is provided with both the bisectional iteration method and the Pitt and Peters' dynamic inflow model [8, 9] to determine the coefficients in relation to rotor moments. This method is convenient for flight dynamic applications. Also, the slight inclination of the whole wake geometry brought by λ_{1c} and λ_{1s} will really affect the distribution of induced flow, pressure, CnM2 and so forth on the rotor disc much more than a more drastic change such as twist and deflection in the wake shape in a far downstream will.

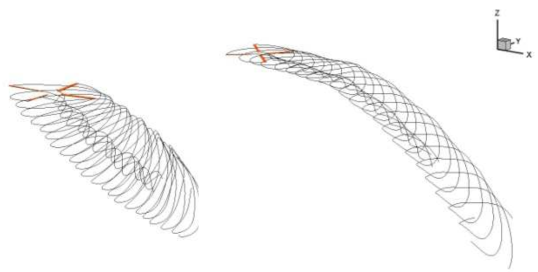


Fig. 4 Twisted wake (left) and deflected wake (right).

The problem with the control functions is that they may possibly distort the wake geometry to an unrealistic level. Another way to produce moments by deforming the wake geometry is to twist the whole shape or to deflect at a point in the downstream (Fig. 4). This type of deformation is more realistic to what will happen with the wake geometry in reality when a cyclic control is input in the cockpit. Nevertheless, the degree how much the rotor wake should be deformed remains as another problem. The author can recommend the deformation of the wake geometry, either by control functions, deflection or twist, should be provided from an experimentally realistic point of view, and the adjustment to λ_{1c} and λ_{1s} should be chiefly used to balanced the rotor moments.

With regard to the description of vortex elements, the classical Biot-Savart law, Eq. (12) below, is based on the assumption that vortices are infinitesimally thin;

$$d\mathbf{v} = \frac{\Gamma}{4\pi} \cdot \frac{d\mathbf{l} \times \mathbf{r}}{r^3}, \qquad (12)$$

where dl is the length of the pertinent vortex element and Γ is the circulation. Regarding the problem that the induced velocity will diverge when r tends to zero, the present author extended the expression of the circulation to the following form for fully formulating all three velocity components,

$$\Gamma = \Gamma_0 \frac{r_c^2}{1 + r_c^2}, \qquad (13)$$

where the base circulation, Γ_0, can optionally be described so as to gradually diminish with wake age due to the viscosity of air. Also, the base circulation may be so described that local variations in the lift should be reflected, e.g.,

$$\Gamma = \Gamma_0 R_v (1 + \mu \cos \Psi_v), \qquad (14)$$

where R_v and Ψ_v mean the radial position of vortex generation (% against the rotor radius) and the azimuth of the vortex element in question.

Regarding the determination of the value of the circulation, Beddoes suggested in Ref. [2] a rule of thumb, $\Gamma = 2.4 C_T / \sigma$, where σ is rotor solidity, but this evaluation is not theoretically derived and thus cannot universally be applied. The author thus suggests that the circulation should be determined in such an iterative manner that the average of the total axial flow induced by the wake model would coincide with that from momentum theory. r_c is the finite core radius of a pertinent vortex element. It is empirically known that r_c is usually 5 - 7% of the blade chord for a blade-tip vortex [6]. Note that the vortex core radius can optionally be described so as to gradually expand with the wake age. Leishman shows in Ref. [6] succinct but lucid reviews about the description of vortex, mentioning Scully's, Lamb's, Lamb-Oseen's and Vatistas' models. Other improvements and modifications implemented in the present new wake model include: (i) blade motions such as flapping and lead-lag are taken into consideration as azimuthally periodic displacement in the positions of vortex elements, (ii) Γ_0 periodically varies according to the local lift coefficient, (iii) self-twisting motion of vortices are taken into account, (iv) root vortices are also considered on top of tip vortices, and (v) Magnus effect of the mast and its influence on the wake geometry can optionally be considered so that the side force would drift the rotor wake sidewards.

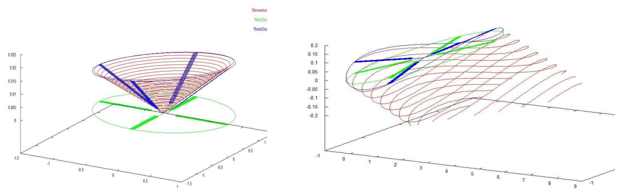


Fig. 5 Wake structure tracking flapping blades.

(The axial displacement is exaggerated.)

2-3. Interface

The *rFlow3D* and the wake model are sharing basically the same file format to describe a blade so that the necessary data should smoothly be transferred. The *rPresto* is supporting various coordinate systems including earth, body, hub, tip-path, no-feathering and ground coordinate systems so that it can widely be used for many possible applications. The wake model itself is able to solely conduct flight dynamic and acoustic analysis in combination with sub-programmes of blade element theory, acoustic analysis based on Farassat 1 and 1A Equations, and whole body trim programme. The hybridisation is now under the process for completion so that the wake model would provide the local distribution of induced flow with inner background grids, in which rotor blades are placed, as the boundary condition every step in the computation when the blades advance in a loosely-coupled manner. As the computational domain can be restricted to the inner background grid system, the total computational time is expected to hugely reduce down. A vital key to the success in the hybridisation for shortening the computation time required in total with a preferable accuracy lies chiefly in how much the wake model alone can predict the distribution of induced flow.

3. Interim Results

The validation commenced with experimental results from the JMRTS' (JAXA's Multi-purpose Rotor Test Stand) wind tunnel test database about some hover and forward flight cases, together with results of numerical simulation by the *rFlow3D*. Note that the authors have already reported the effect of modelling wake contraction, the number of rotor revolutions to trace back, swirl flow, and the coefficient of E in the 48th Aircraft Symposium [10] about hover cases, but forward flight cases are utterly new fruits this time.

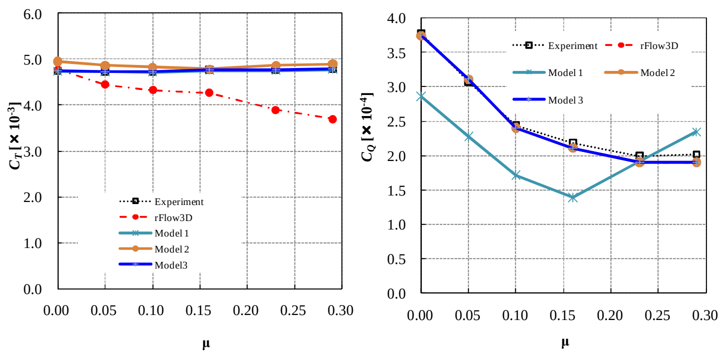


Fig. 6 C_T and C_Q against μ.

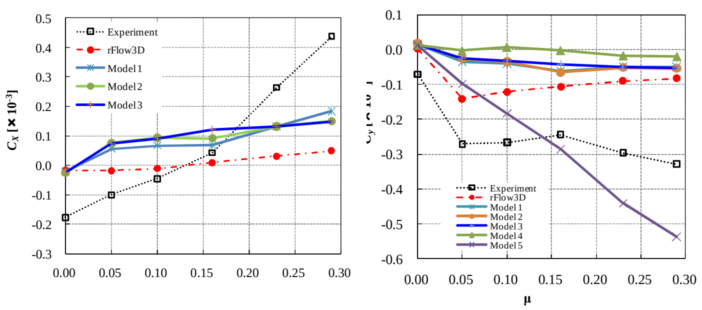


Fig. 7 CX and CY against μ.

Figures 6 and 7 show the rotor thrust, torque, drag and side force from the JMRTS, *rFlow3D*, and three wake models designated as Model 1, 2 and 3; Model 1 is based on Beddoes-Murakami's Model with linear approximation about the aerodynamic coefficients, Model 2 is the same as Model 1 but reading in the property of aerodynamic coefficients of NACA0012 from look-up tables, and Model 3 is the same as Model 1 but empirical correction factors are implemented in the blade element theory part. The diagram about the side force alone is added an additional line which represents the side force predicted by Model 3 plus an extra force due to Magnus effect of the rotor shaft. It can be seen that the results from wake models are closer to those from the *rFlow3D* than the experiment. Note that the wake structure is highly unstable when the advance ratio, μ, is fairly small. Hence, the fact that the wake models produced reasonable results around $\mu = 0.05$ and $\mu = 0.10$ indicate that the modification of smooth transition in the wake geometry in the present model functioned well as so intended. The rotor wake at the μ of 0.23 was visualised from the results of the *rFlow3D* and Beddoes-Murakami's Model as follows:

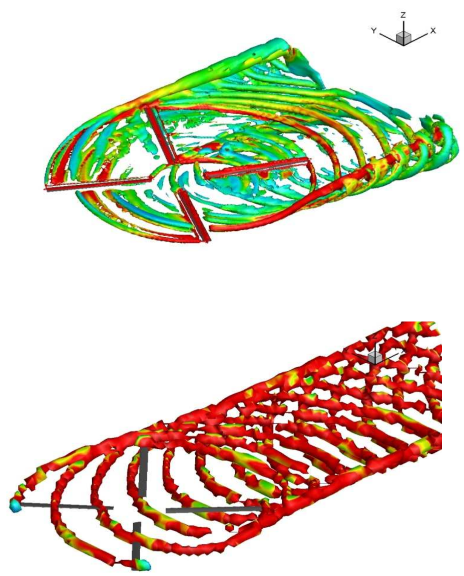


Fig.8 Rotor wake produced by the *rFlow3D* (upper) and the wake model (bottom) at the μ of 0.23.

Figure 9 shows the flow field induced by the wake model with two sections, each of which transects the hub centre laterally and longitudinally, with the rotor also depicted. Figure 9 shows the axial component of the induced flow on the rotor disc predicted by the wake model.

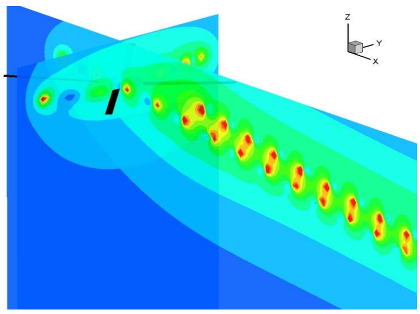


Fig. 9 Flow field induced by the wake model in the rotor wake domain.

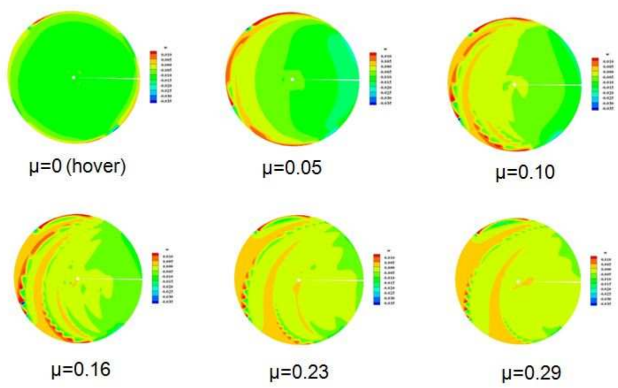


Fig. 10 Downwash distribution on the rotor disc.

Note that the upper edges of the discs in the diagram above are pointing star, the bottom edges port. The clear stripes recognised should indicate tip vortices below the discs. It can also be seen that the asymmetric distribution of downwash between advancing and retreating sides become obvious when the advance ratio tends to be large.

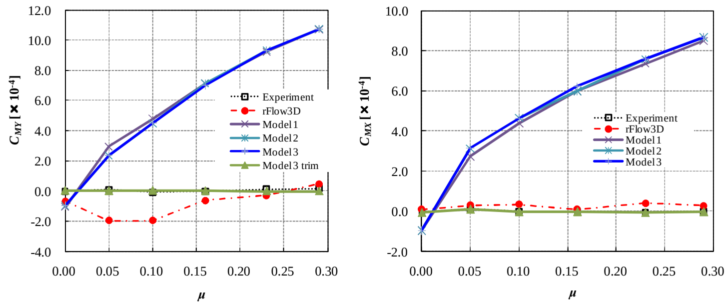
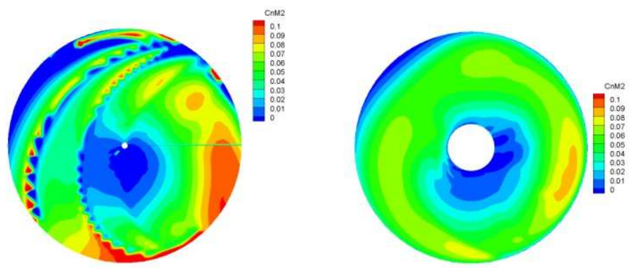


Fig. 11 Rolling and pitching moments against mu

Indeed, this is underpinned by the ever-increasing rolling and pitching moments unless the rotor moments are neatly trimmed. Figure 11 shows aerodynamic moments working on the rotor disc along x- and y- axes, C_{mx} and C_{my}, calculated by the *rFlow3D*, Model 1, 2, 3, and Model 3 plus moment trim mode, together with the JMRTS experimental result.

The fact that those moments calculated by Model 1, 2 and 3 nearly linearly increase against the advance ratio simply denotes that the lift on the advancing side increases, and that on the retreating side decreases, and this further denotes that the trim setting of cyclic control is not appropriately reflected on the wake geometry. With Model 3 plus trim function, the wake geometry was deformed so as to meet target values of moments. The result from the trim mode marvelously coincides with the experimental result. Since the result was produced in a target-driven manner, the wake geometry should be further developed so that appropriate wake geometry will automatically generated based solely on rotor controls in the future.



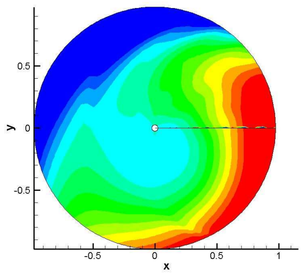


Fig. 12 Distribution of CnM2 at the μ of 0.23; wake model with a constant Γ for a fixed wake shape (upper left), CFD (upper right) and a varying Γ with wake models defined at each azimuth position for different blade positions (bottom)

In relation to acoustic analysis, the authors compared the distribution of CnM2 between the computational results from the *rFlow3D* and Beddoes-Murakami's model alone, Fig. 12. In the results shown in Fig. 12, one with a fixed wake mode for a particular blade position indicates a clear effect of the vortices below the disc, and the other about the induced flow distribution calculated with wake models generated at each blade position show a similar tendency to the CFD results in quality, though the wake model arguably over-predict the effect of induced flows in magnitude, which should likely indicate the excessively strong influence of the wake geometry.

4. Discussion about the present setback

The authors have rendered many a mathematical modification to Beddoes' wake model, and the new model showed ever so promising results about hover cases [10]. Yet, in forward flight cases, the asymmetric distribution of downwash between advancing and retreating sides became obvious, with the increasing advance ratio, resulting in sharp disagreement in rolling and pitching moments. For this problem, it was transpired that van der Wall's correction factors, which were mentioned in Section 2-2 alone was not of great help to balance out the moments. The authors thus deformed the wake geometry so that the rotor moments should settle down to anticipated values. Unless the wake model can adequately produced so that at least rotor forces and moments should indicate anticipated values, the wake model alone cannot be used for acoustic analysis. Also, the CnM2 distribution on the disc predicted by the wake model shows a little too large variations in CnM2, which will highly likely end up with over-predicting the BVI noise level. Still, it is encouraging that the overall tendency of the CnM2 distribution yielded by the wake model more or less agreed with that from the *rFlow3D*. Therefore, the next things the authors shall challenge should include (i) deterring too strong influence of vortices, (ii) modelling the rotor wake geometry so that trim condition of controls should correctly and automatically be associated with the geometry even in forward flight, that will in turn result in a reasonable distribution of the induced flow.

5. Conclusion

The conclusive remarks of this study can be summerised as follows;

- Beddoes' wake model was theoretically modified to smoothly describe the transitional state between hover and forward flight. The new model tends to Beddoes' model when advance ratio is fairly high;
- The Biot-Savart law is extended to incorporate the finite core radius of a vortex element together with the diffusion of the vortex, and the strength circulation diminishing, in the three-dimensional form;
- Wake tube contraction, swirl flow, blade motions, self-twisting motion of vortices and

root vortices were optionally implemented;

- The wake model so far produces reasonable results agreeing with experimental and simulational data in hover cases. Still, the discrepancies in rolling and pitching moments are obvious in forward flight cases unless trimmed on purpose. This forges further studies about how to adequately associate the rotor control with the geometry of rotor wake.

Possible further extensions of the wake model may include the effect of elastic deformation of blades, more detailed geometrical features of a rotor wake such as spiral-shaped roll-up, wake impingement upon the fuselage and tail surfaces and so forth.

6. References

1. Tanabe, Y., Saito, S. and Otani, I., "Application of SLAU Scheme to Helicopter CFD," 41st Fluid Dynamics Conference/Aerospace Numerical Simulation Symposium 2009, Chofu, Tokyo, June 18-19 2009 (In Japanese.)
2. Beddoes, T. S., A Wake Model for High Resolution Airloads, 2nd International Conference on Basic Rotorcraft Research, Triangle Park, NC, 1985
3. van der Wall, B. G., The Effect of HHC on the Vortex Convection in the Wake of a Helicopter Rotor, Aerosp. Sci. Technol. Vol. 4 (2000), pp. 321–336.
4. Shima, E. and Kitamura, K., “On New Simple Low-Dissipation Scheme of AUSM-Family for All Speeds,” 47th AIAA Aerospace Sciences Meeting, AIAA Paper 2009-136, Orlando, FA, January 5-8 2009.
5. Done G. and Balmford, D., "Bramwel's Helicopter Dynamics (second edition)," Butterworth Heinemann, Oxford, 2001
6. Leishman, J. G., "Helicopter Aerodynamics (second edition)," Cambridge University Press, Cambridge, 2006
7. Tanabe, Y. et al., "An Experimental Study of the Rotor/Fuselage Interaction," JAXA RR-10-003, Aug. 2010
8. Pitt, D. M. and Peters, D. A., “Theoretical Prediction of Dynamic Inflow Derivatives,” Vertica, Vol.5, No.1, pp. 21-34, 1981
9. Murakami, Y. and Houston, S. S., “Correction of the definition of mass flow parameter in dynamic inflow modelling,” Proceedings of the Institution of Mechanical Engineering, Part G, Journal of Aerospace Engineering, Vol. 223, Num. 7, pp. 1037-1040, 2009
10. Murakami, Y. et al., “Improved Hybrid Methods of CFD and Prescribed Wake Models for Rotorcraft,” 48th Aircraft Symposium, Shizuoka, Japan, 30th Nov. – 2nd Dec., 2010