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DESIGN METHODOLOGY

FOR

MULTIVARIABLE HELICOPTER CONTROL SYSTEMS

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SUMMARY

This paper describes the results obtained during Bell Helicopter Textron's (BHTI) evaluation of the Model Based Compensator (MBC) as a proposed design methodology for multivariable control systems. The highlights of the theoretical foundation will be presented, but the rigorous mathematical development of the MBC will not be covered. The modifications to existing BHTI flight simulation programs will be described.

The multivariable control system design methodology will be illustrated by designing an MBC for a tiltrotor aircraft in airplane mode. Selected computer outputs are presented to show the flow of the design process.

INTRODUCTION

The design of a flight control system for contemporary helicopters is one of the most challenging tasks facing the helicopter industry. There are several reasons for this difficulty, including: wide flight envelope to include hover and high speed forward flight; the importance of the oscillatory airloads acting on the rotor; nonlinearities including, stall, compressibility, and aeroelastic effects; and finally, the limited resources available to the helicopter industry for developing improved design methodologies.

Shown in Figure 1 is a diagram of the closed loop controller/helicopter system. For a given helicopter, the control system design task is to produce a controller that can do the following:

a. Stabilize the system.

b. Provide for good command following for pilot inputs.

c. Eliminate the undesirable cross coupling.

d. Be insensitive to modeling errors, high frequency excitation, and measurement noise.

e. Be robust to changes in the flight envelope, gross weight, c.g., speed, etc.

The advent of digital computers, both for the control system design process and for the airborne implementation, has had a profound effect on the design process. Previous design techniques were based on low order single-input/single-output transfer functions that were then enclosed with feed-forward, feed-back, and shaping filters to produce an acceptable control system. This was then followed by the helicopter flight test, where the system was "tuned" to obtain its best performance. One of the biggest problems associated with the conventional design process is the lack of a clearly defined design methodology making it very difficult to train the next generation of control system design engineers.

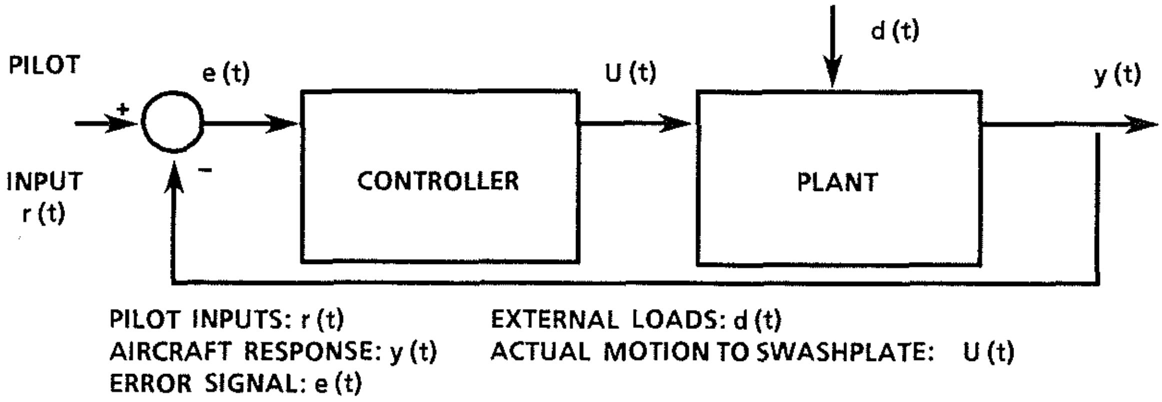

Figure 1. Closed Loop System.

DESIGN METHODOLOGY

The design of a helicopter control system can be broken into the following steps:

1. Determine the equilibrium flight condition corresponding to the specified forward speed, rate of climb, gross weight, c.g. position, and atmospheric properties.

2. Render a linearized model of a helicopter that represents the nonlinear characteristics of the system over some region.

3. Identify the control strategy to be used (rate command, attitude hold, etc.).

4. Develop a control system that will meet the desired design objectives.

5. Evaluate the behavior of the linear control system at equilibrium flight condition.

6. Investigate the robustness of the controller through evaluation of its performance at off-design flight conditions.

Subsequent steps would test the control system in a real-time flight simulator for pilot evaluation prior to the actual flight test of the system.

Developing a control system that will meet the desired design objectives (step 4) is the most difficult. Many control systems have been designed using a simplified helicopter model and a set of single-input-single-output (SISO) transfer functions. Those classical techniques in the hands of experienced control system designers were adequate in the past. However, as the demands increase on both the pilot and the aircraft, new design methodologies must emerge. New techniques must capitalize on recent advances in theoretical formulations, and the ever increasing availability and capacity of digital computers, for use both in the design process and the airborne implementation of the control system.

As part of the BHTI Independent Research and Development Program a multivariable control system design methodology was developed. The theoretical basis of the design methodology is an MBC. The Comprehensive Program for the Theoretical Evaluation of Rotorcraft (COPTER) program is used to describe the helicopter; and the Ctrl-C program is used to implement the design process.

Shown in Figure 2 is the flowchart of the design methodology. The items involving the use of COPTER are done on the IBM 3090 mainframe computer. The design of the MBC is done on the VAX 8800 computer. A hyperchannel is used to transfer files between the two computers.

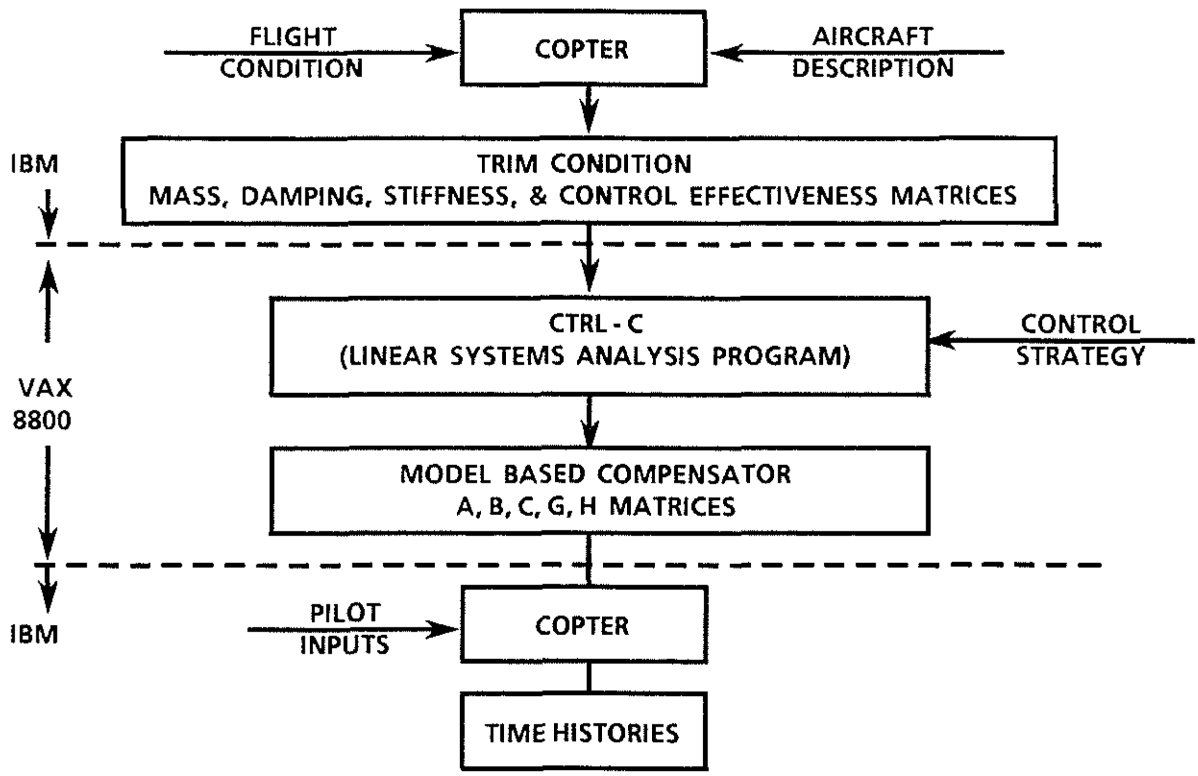

Figure 2. Design Methodology for Multivariable Control System.

The first six steps of the design methodology are based on the COPTER program (Reference 1) and the use of the Ctrl-C (Reference 2) control system design and analysis program. COPTER is BHTI's second-generation rotorcraft flight simulation program, which is an outgrowth of the Rotorcraft Flight Simulation Program C81 (Reference 3). The COPTER program can simulate a large variety of rotorcraft configurations (conventional, tandem, side by side, coaxial, and tiltrotors) with a uniform level of analytical texture in the two aeroelastic rotor analyses (for teetering, gimballed, articulated, or hingeless hubs with two to seven blades); the fuselage and fixed surface aerodynamics; and elastic pylon motion. The aeroelastic rotor analysis is based on the modal approach using up to 20 fully coupled (beam, chord, torsion) mode shapes for each rotor. The aerodynamic model of the rotor is based on strip theory with bi-variant table look up for specified airfoils to define the stall and compressibility effects. COPTER includes unsteady aerodynamic effects, as well as sophisticated wake analyses, e.g., the CAMRAD (Reference 4) free wake analysis.

The first operation of COPTER is to define the equilibrium flight condition for the specified flight condition. By assuming that the tip path plane associated with rigid blades can be defined by the fore-and-aft and lateral flapping angles, the determination of the equilibrium flight condition consists of an iterative solution of ten equations of motion (summation of forces and moments on the fuselage; and the fore and aft and lateral moment balance for two rotors) considering 11 independent variables (four pilot controls, three fuselage Euler angles, and the two flapping angles on the two rotors). Usually one of the Euler angles is specified to produce ten nonlinear algebraic equations in ten unknowns. A modified Newton Raphson technique is used to solve the system of equations using a Jacobian matrix obtained by numerical differentiations. The Jacobian matrix can be obtained by considering only the steady and one/rev response of the elastic rotor modes; or by a time history integration following each perturbation of the state variables. In a very similar manner the control system's effectiveness is determined. Following this trim process, COPTER can proceed along two distinct paths. The first path is a linearization of the math model in the stability analysis (STAB) portion of the program. The second path is a time history integration of the equations of motion to define the helicopter's response to control inputs, weapon firing, and gusts.

The most common mode of operation for STAB is to treat the helicopter/rotor system as having six rigid body degrees of freedom, which is analogous to stating that the following times for the rotor are much shorter that those for the flight modes. Using the conventional state variables (u forward velocity, w vertical velocity, q pitch rate, v sideward velocity, p roll rate, and r yaw rate), STAB produces a 6×6 mass, damping, and stiffness matrix for the particular helicopter at the specified flight condition. The forces and moments produced by unit pilot control inputs are also produced. This concludes the first two steps in the design process.

The design of the MBC (Reference 5) is done by a program based on the Ctrl-C program on the VAX 8800 computer. The first operation is to convert the COPTER generated mass, damping, and stiffness matrices to state space notation. Since the stiffness matrix has only two nonzero rows and columns, the state space equations become

$$\underset{8\times1}{X} = \underset{8\times8}{A}\ \underset{8\times1}{X} + \underset{8\times4}{B}\ \underset{4\times1}{U} \qquad (1)$$

where

$$X^T = [u, w, q, v, p, r, \theta, \phi] \qquad (2)$$

and

$$U^T = [Collective, F/A\ Cyclic, Lateral\ Cyclic, Pedal] \qquad (3)$$

The observed quantities, Y, in standard notation are

$$\underset{4\times1}{Y} = \underset{4\times8}{C}\ \underset{8\times1}{X} + \underset{4\times4}{D}\ \underset{4\times1}{U} \qquad (4)$$

Using s as the standard Laplace operator, the open loop plant can be expressed as

$$\underset{4\times1}{Y} = \underset{4\times4}{C[sI - A]^{-1}B}\ \underset{4\times1}{U} + \underset{4\times4}{D}\ \underset{4\times1}{U} \tag{5}$$

Contained within equation 5 are all sixteen individual SISO transfer functions between the four selected observed quantities and the four pilot inputs. Equation 5 can be written as

$$Y(s) = G_p(s)\ U(s) \tag{6}$$

where $G_p(s)$ is the open loop plant.

For the multivariable equation 6, singular values are used to condense the information in the multi-input multi-output (MIMO) transfer function. The singular values of $G_p(s)$, corresponding to a particular value of s are obtained by taking the positive square root of the eigenvalues of the matrix formed by the product of the complex conjugate transpose of $G_p(s)$ times the matrix $G_p(s)$. Thus, there will be four singular values for each value of $s = j\omega$, but the most important singular values are the maximum (σ_{max}) and minimum (σ_{min}) singular values as a function of frequency. It can be shown that, at a particular value of $s = j\omega$, the magnitude portion of all sixteen SISO transfer functions must be bounded by the maximum and minimum singular values at the value of $s = j\omega$. The resulting singular value plots do not contain any phase angle information.

The goal of the multivariable design process is to obtain a controller $K(s)$, shown in Figure 3, that will have the desired attributes for the closed loop system. The controller should meet the following requirements:

a. Stabilize the system.

b. Provide for good command following in the low frequency regime for the specified control strategy.

c. Eliminate or reduce the cross coupling between the controls.

d. Be insensitive to modeling errors, high frequency excitation, and measurement noise.

e. Be robust, so that the controller will perform satisfactorily for a broad range of operating conditions.

BHTI has focused on the described MBC as the most promising multivariable design methodology. The MBC combines the best features of the Linear Quadratic Regulator (LQR) and the Kalman Filter. The LQR serves to define the control inputs required to bring a system to a desired condition with a minimum control input. The Kalman Filter produces a "best" estimate of the state variables based on a set of noisy measurements.

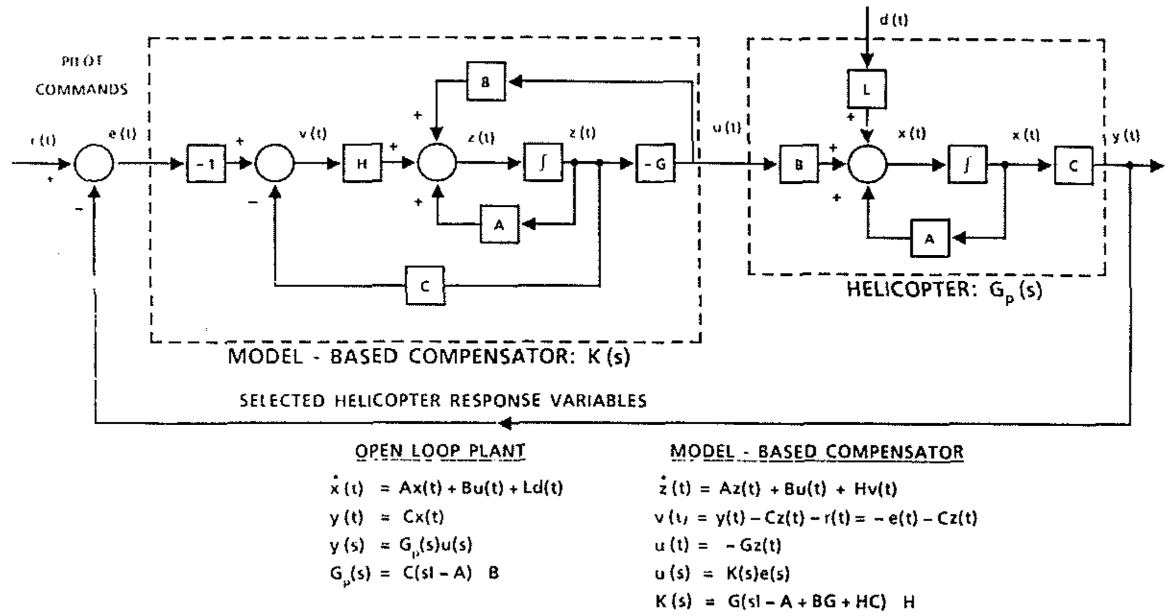

Figure 3. Model Based Compensator.

The LQR seeks a control input

$$U = -GX \tag{7}$$

that will minimize the cost function

$$J = \int_{t_o}^{t_f} \{X(t)^T QX(t) + U(t)^T R\ U(t)\}dt \tag{8}$$

For the system,

$$\dot{X} = AX + BU \tag{9}$$

where Q is the weighting matrix for the state variables and R is the relative cost of the control input. The G matrix of equation 7 is obtained from the steady state Ricatti equation. Combining equations 7 and 9 gives the closed loop LQR description

$$\dot{X} = [A - BG]X \tag{10}$$

as shown in Figure 4. Two very important points must be made concerning the LQR. First, the LQR is a regulator designed to maintain the current values of the state variables without regard to its ability to control, i.e., change the state variables to a set of desired values. Second, the LQR is a full feedback system which requires that all the state variables in the system be available for use in the control law determination. Subject to the observability and controllability requirements, the LQR will produce a stable system. BHTI built and tested a

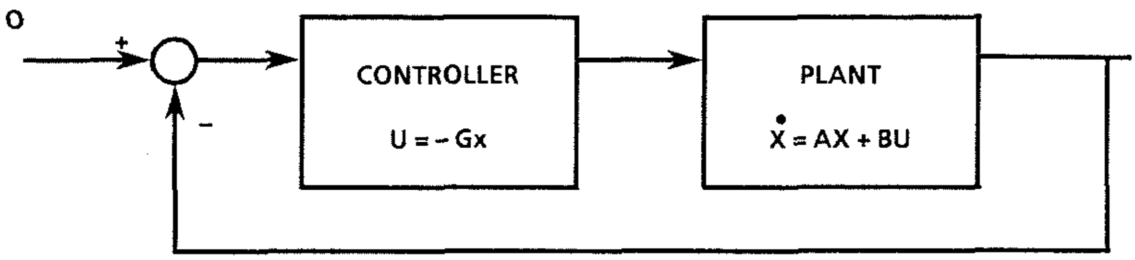

Figure 4. Linear Quadratic Regulator.

double inverted pendulum shown in Figure 5 to demonstrate the LQR methodology.

In order for the MBC to determine the command inputs to control the helicopter the MBC must have the "best estimates" of the true values of the state variables of the helicopter. These "best estimates" of X can be obtained by solving in real time:

$$\dot{\hat{X}} = A\hat{X} + H[y - C\hat{X}]$$

where y is the measured values (corrupted by random measurement errors) of the helicopter state variables and H is the Kalman Filter gain matrix. The H matrix is obtained from the steady state solution of the Ricatti equation. The block diagram of this Kalman Filter is shown in Figure 6.

The block diagram for the MBC is shown in Figure 3. The linear time invariant plant model $G_p(s)$ is expressed by the A, B, C, and D matrices produced by the COPTER program. The MBC requires that the G and H matrices be determined to complete the design of the system. One of the most attractive features of the MBC is that there exists a logical procedure for obtaining the G and H matrices.

The first design step is to obtain the H matrix by considering the open loop Kalman filter expressed by

$$Y(s) = C[sI - A]^{-1} H \qquad (11)$$

The H matrix is obtained by solving the algebraic Ricatti equation for Σ in the equation

$$O = A\Sigma + \Sigma A^T + LL^T - \frac{1}{\mu}\Sigma C^T C\Sigma \qquad (12)$$

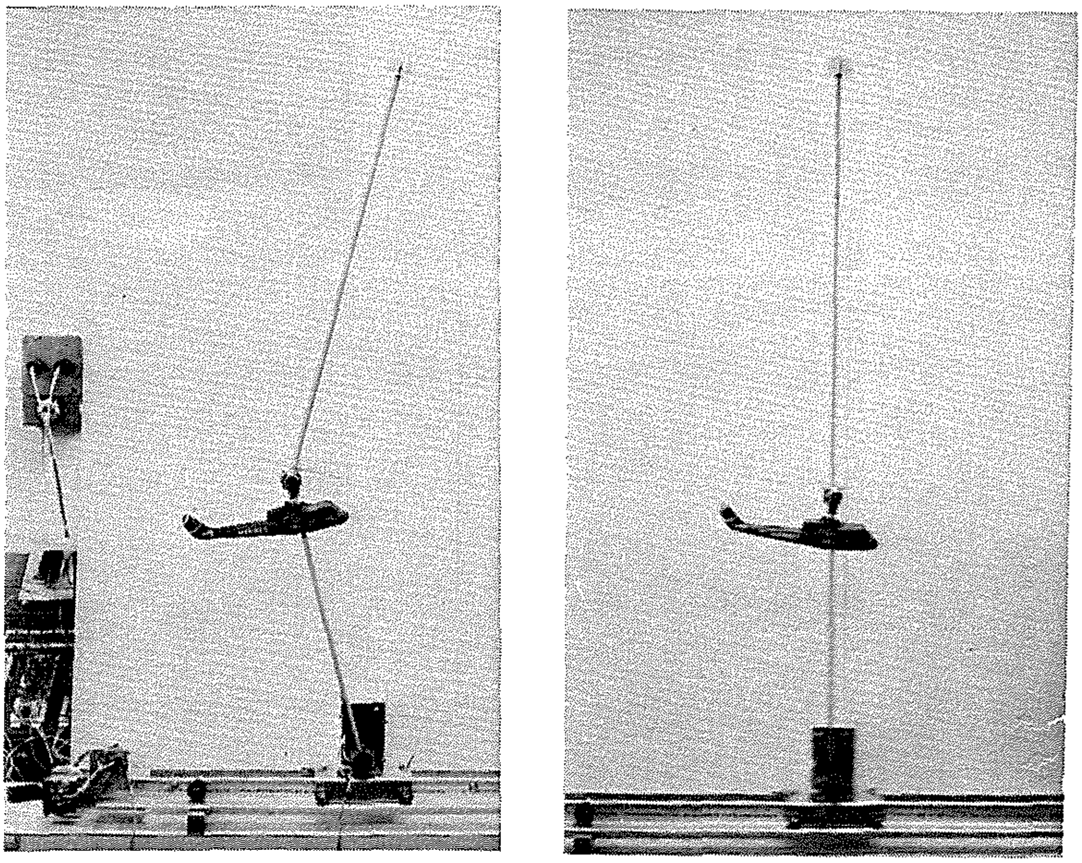

Figure 5. Double Inverted Pendulum Stabilized by Linear Quadratic Regulator.

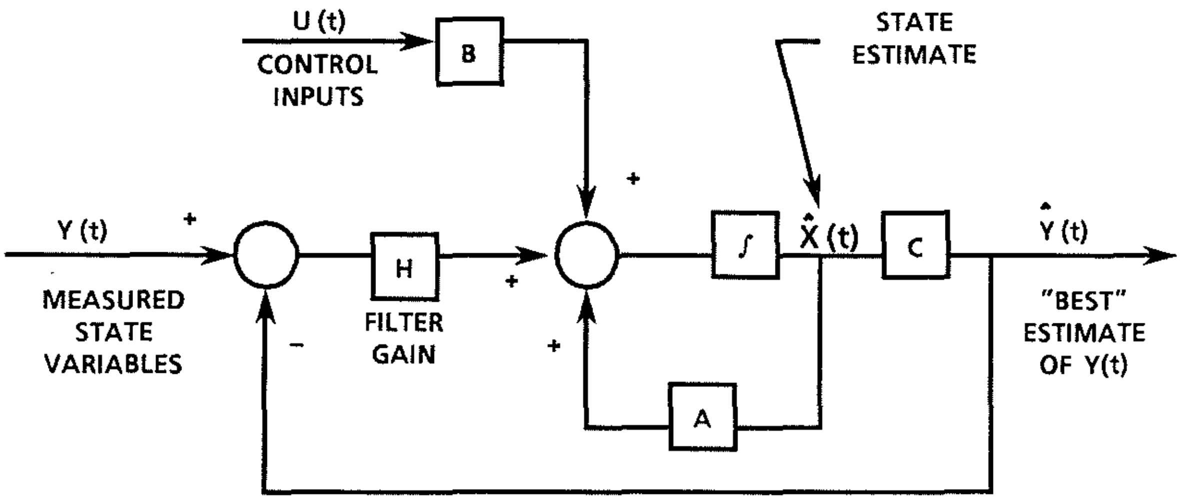

Figure 6. Closed Loop Kalman Filter.

where the matrix L and the scalar μ are the design variables. The matrix H in the MBC is then given by

$$H = \frac{1}{\mu} \Sigma C^T \tag{13}$$

If integral control is being used to eliminate steady state error to command input, then by selecting

$$L = \begin{bmatrix} -(CA^{-1}B)^{-1} \\ C^T(CC^T)^{-1} \end{bmatrix} \tag{14}$$

all of the singular values will be equal in the very low frequency regime. The numeric value of the equal singular values is determined by the scalar parameter μ in equation 12. Figure 7 contains the singular values of the "Kalman Filter" portion of the MBC. The singular values of the open loop Kalman Filter will serve as the goal for the open loop MBC. Thus, by changing the scalar, μ will change the band width goal for the MBC.

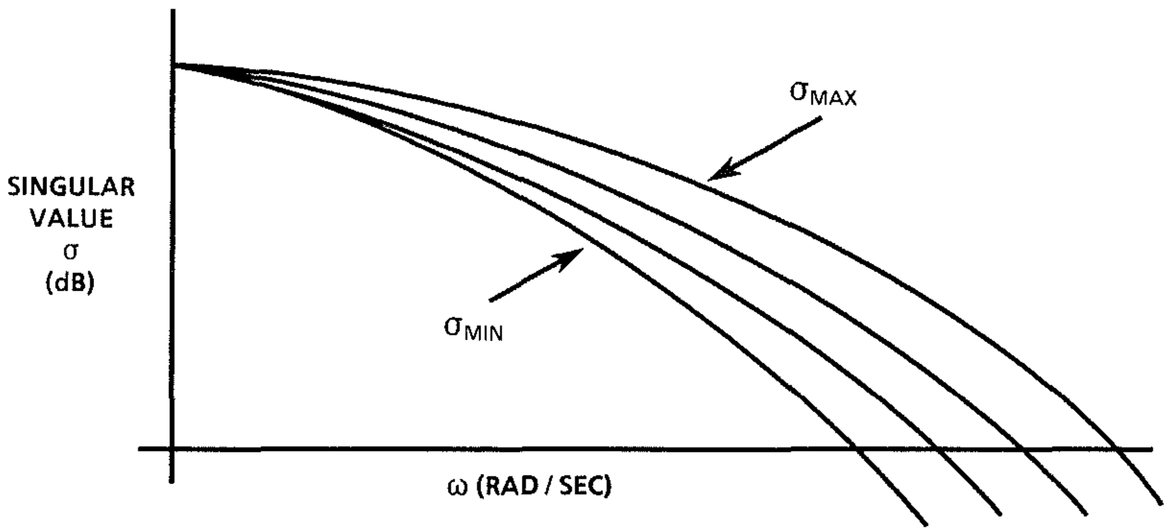

Figure 7. Target Singular Values.

The final step in the design of the MBC is to determine the G matrix shown in Figure 3. The G matrix is obtained by solving the LQR problem associated with the open loop plant in the Ricatti equation (see equation 8).

$$\dot{X} = AX + BU \tag{15}$$

$$Y = CX + DU \tag{16}$$

$$\text{with} \quad Q = C^T C \ \text{and} \ R = \frac{1}{\rho} [I] \tag{17}$$

The algebraic Ricatti equation for the LQR requires finding the symmetric matrix K such that

$$0 = -KA - A^TK - Q + KBR^{-1}B^TK \tag{18}$$

The control gain matrix G is given by

$$G = -R^{-1}B^{T}K \tag{19}$$

The primary design variable is the scalar ρ in equation 17. Once the G matrix is found it is possible to investigate the performance of the coupled MBC and plant (see Figure 1). It can be shown that in the limit as $\rho \rightarrow o$ (equation 17), the singular value plot of the open loop coupled MBC and plant would approach the singular value plot (especially in the low frequency region) for the related Kalman Filter problem given by equations 11, 12, 13, and 14.

Once the plant model has been defined and the control strategy specified, the MBC design process centers on two major tasks:

a. Adjusting μ in the "Kalman Filter" problem to identify a desired open loop singular value plot

b. Adjusting ρ to obtain a control gain matrix G that will recover the desired open loop singular value plot

The resulting MBC will be stable around the design point. If the plant is non-minimum phase, i.e., has transmission zeros in the right half plane, the "degree of recovery" will be reduced but the system will be stable.

The steps for designing a MBC can be summarized as:

a. Determine the A, B, C, and D matrices for the open loop helicopter.

b. Select the control strategy identifying which state variable will be controlled by each of the four inputs.

c. Augment the plant matrices for the integral control added to each input channel.

d. Determine the target singular values for the open loop compensator plus plant using equations 11 through 14 by adjusting the scalar μ. The system bandwidth is often used as a guide for selecting μ.

e. Solve the LQR problem for the plant for small values of ρ (equation 17) to determine the G matrix in Figure 3.

f. For each G matrix, calculate the singular values of the coupled MBC plus plant to see the singular values approach those obtained in step d.

g. Calculate the closed loop singular values to assess the performance of the system.

EXAMPLE USE OF DESIGN METHODOLOGY

The design methodology will be illustrated by designing and evaluating a MBC for a proposed tiltrotor aircraft flying without a stability augmentation system in airplane mode. Corresponding to the step one, as described in the previous section, the trim flight parameters determined by the COPTER program are as shown in Table 1. The yaw angle was required to be zero during the trim process.

TABLE 1. EQUILIBRIUM FLIGHT CONDITION.

Tiltrotor Aircraft in Airplane Mode

Airspeed	236 kn	Fuselage Angles			
Air density			Yaw	0.0	deg
Ratio	0.9272		Pitch	0.078	deg
			Roll	0.0	deg

Total Horsepower 1585.0 hp

Pilot Control Positions (Percent)		Blade Angles (deg)	Right Rotor	Left Rotor
Collective	50.88	Collective	75.3	75.3
F/A Cyclic	54.56	F/A Cyc	0.0	0.0
Lat Cyc	50.0	Lat Cyc	0.0	0.0
Pedal	50.0	F/A Flapping	0.03	0.03
		Lat Flapping	-0.09	-0.09

Gross Weight 13590 lb

Cg Station 295.54

The STAB portion of COPTER then generates the mass, damping, stiffness, and control effectiveness derivative matrices as shown in Table 2. Contained in Table 3 are the stick fixed eigenvalues for the particular flight condition.

These four matrices defined by the COPTER program become the input data for the DESIGN program. The DESIGN program uses the Ctrl-C linear systems analysis package, and is executed on a VAX 8800 computer. The first operation of DESIGN is to generate the minimum order A and B matrices, which are shown in Table 4. The next step in the design process is to identify the control strategy: i.e., what response state variables will be controlled by what pilot input. Table 5 shows the selected control strategy for this example problem.

TABLE 2. PLANT MATRICES

MASS MATRIX

442.4	.0	.0	.0	.0	.0
.0	442.4	.0	.0	.0	.0
.0	.0	.1791E+05	.0	.0	.0
.0	.0	.0	442.4	.0	.0
.0	.0	.0	.0	.4867E+05	-1234.
.0	.0	.0	.0	-1234.	.6025E+05

DAMPING MATRIX

14.40	-27.29	-814.0	.8239	-878.0	-874.6
68.13	512.5	-.1636E+06	-4.393	-204.3	-233.2
-43.55	603.1	.4205E+05	-158.3	206.	2361.
-.3414E-01	.1463E-01	1.645	149.7	22.71	.1639E+06
-.5048	.2254	22.84	905.1	.5021E+05	-70.01
.7087	.7048	70.24	-935.4	.1394E+05	.7205E+05

STIFFNESS MATRIX

.0	.0	.1359E+05	.0	.0	.0
.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.1359E+05	.0
.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0

COLLECTIVE	F/A	LATERAL	PEDAL	
8155.	-154.9	-96.60	-256.9	X FORCE
-11.38	-1093.	-.9115E-01	-.3385	Z FORCE
-.1748E+05	-.2372E+05	174.6	878.6	PITCH MOM
.4411E-01	.2225E+05	65.61	1442.	Y FORCE
-.2507	-.2865	.1956E+05	4983.	ROLL MOM
-.1504	-.9505	4096.	-.3297E+05	YAW MOM

TABLE 3. STICK FIXED EIGENVALUES

-0.0868 + 0.0000i
-0.0158 + 0.1294i
-0.0158 - 0.1294i
-1.3445 - 0.0000i
-0.5789 - 2.4072i
-0.5789 + 2.4072i
-2.1724 - 3.3328i
-2.1724 + 3.3328i

TABLE 4. PLANT A AND B MATRICES

A MATRIX

								STATE VARIABLE
-0.0341	0.0646	0.0336	-0.0020	0.0363	0.0361	-0.5615	0.0000	U
-0.1606	-1.2084	6.7323	0.0104	0.0084	0.0096	-0.0008	0.0000	W
0.1581	-1.7884	-3.1340	-3.5052	-0.1161	-0.1329	0.0001	0.0000	Q
0.0001	0.0000	-0.0001	-0.3544	-0.0009	-6.7717	0.0000	0.5615	V
0.0006	-0.0003	-0.0005	-1.0436	-1.0380	-0.0289	0.0000	0.0000	P
-0.0007	-0.0007	-0.0012	0.8682	-02526	-1.1964	0.0000	0.0000	R
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	θ
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	φ

B MATRIX

19.3063	-0.3667	-0.2287	0.6082
-0.0268	-2.5772	-0.0002	0.0008
-55.9212	-75.5872	0.5586	-2.8110
0.0001	0.0000	0.1553	-3.4138
-0.0003	-0.0004	23.1391	-5.0742
-0.0001	-0.0009	4.3694	31.2518
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

TABLE 5. CONTROL STRATEGY

PILOT INPUT	CONTROLLED STATE VARIABLE
Collective	Forward Velocity (U)
F/A Cyclic	Fuselage Pitch Angle (θ)
Lateral Cyclic	Fuselage Roll Angle (ϕ)
Pedal	Sideward Velocity (V)

At this state the plant A matrix, A_p, is 8 × 8; the plant B matrix, B_p, is 8 × 4; the plant C matrix, C_p, is 4 × 8; and the plant D matrix, D_p, is a null 4 × 4 matrix. Integral control is now added to all four pilot inputs to obtain the design matrices A_D, B_D, C_D, D_D of the following form:

$$A_D = \underset{12\times12}{\left[\begin{array}{c|c} O & O \\ \hline B_p & A_p \end{array}\right]} \tag{21}$$

$$B_D = \underset{12\times4}{\left[\begin{array}{c} I \\ \hline O \end{array}\right]} \tag{22}$$

$$C_D = \underset{4\times12}{[O \mid C_p]} \tag{23}$$

$$D_D = \underset{4\times4}{[0]} \tag{24}$$

The design of the MBC begins with the determination of the H matrix (see equations 12, 13, and 14) based on the selected L matrix (equation 14). Since the design plant has integral control, the L matrix can be selected to insure that all the singular values in the low frequency region will have the same value, and that value is controlled by the μ parameter. The bandwidth of the target system is therefore a function of μ. Contained in Figure 8 are target singular values for μ = 0.01. Notice that the very low frequency singular values are equal, and the band width of the target system is approximately 4 rad/sec. The corresponding H matrix is shown in Table 6.

The G matrix shown in Table 7 is obtained by using equations 17, 18, and 19 with ρ = 0.00001. The singular value plot of the open loop coupled MBC/plant is shown in Figure 9, which demonstrates that the coupled system recovers the desired plot of Figure 8.

TABLE 6. H MATRIX

0.0203	-0.0883	-0.0007	0.0025
-0.0327	0.0622	0.0070	0.0238
-0.0026	-0.0057	0.0445	0.1099
0.0009	0.0048	-0.0298	-0.1047
3.3077	-0.5034	-0.0107	-0.0197
-0.5034	3.1288	-0.0128	-0.0389
-0.0107	-0.0128	3.3274	0.5700
-0.0197	-0.0389	0.5700	5.0453
0.2608	0.1156	-0.1393	-0.6090
0.2158	0.0221	-0.0261	0.0863
-0.0440	-0.0736	0.6985	1.2921
0.0110	0.0577	-0.2520	-1.3269

TABLE 7. G MATRIX

Starting at row 1, columns 1 through 8:

110.7320	1.1094	-0.9137	1.2974	312.3417	-50.7051	3.9927	6.3391
1.1094	54.2927	-0.0063	0.0837	-47.6376	-311.5027	0.0068	0.8039
-0.9137	-0.0063	38.3607	3.2150	-3.4394	0.4365	315.0449	-18.7565
1.2974	0.0837	3.2150	82.7201	6.4704	-2.2250	-24.0031	-310.3715

Starting at row 1, columns 9 through 12:

0.4334	-1.8328	0.5992	-1.6542
1.1143	-19.3137	0.0063	-0.4264
-0.0041	0.0144	30.6710	7.7304
0.0061	-0.0810	-1.6978	75.3553

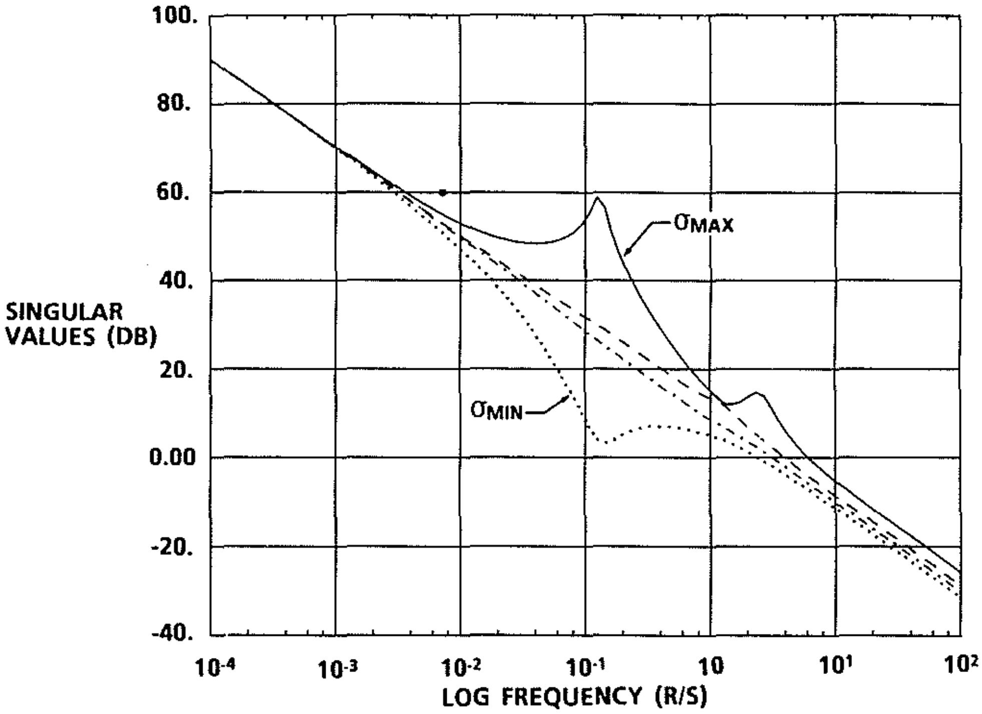

Figure 8. Target Singular Values.

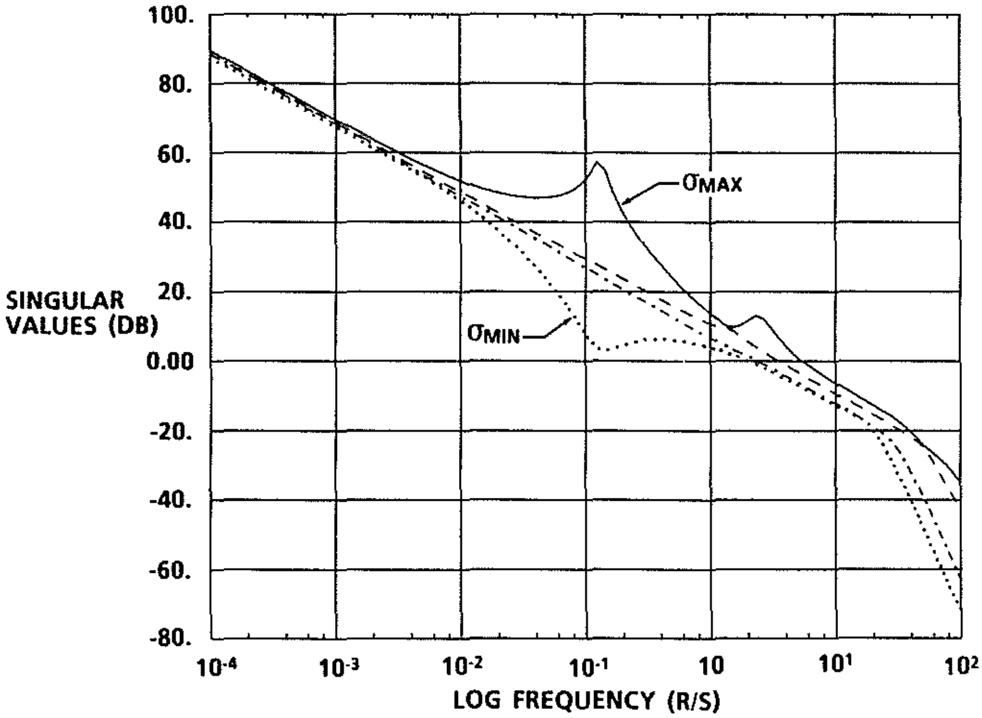

Figure 9. Recovered Singular Values.

EVALUATION OF PROPOSED MBC

The evaluation of the MBC begins by evaluating the eigenvalues of the closed loop system, shown in Table 8, which shows the system to be stable. However, the MBC has added additional dynamics to the system, which increases the number of closed loop poles.

The final analytical evaluation of the MBC must be done in the COPTER program to evaluate the effects of nonlinearity. Referring to Figure 2, the time history portion of the COPTER program was modified to simulate the MBC. That modification required that the pilot control inputs be summed with the appropriate aircraft response variables to serve as the input to the MBC. The 12 linear differential equations of motion of the MBC were numerically integrated in harmony with the existing equations of motion in COPTER. The output from the MBC was then used as the actual swashplate input to the aircraft. The COPTER program has long had the ability to calculate the open loop response due to pilot input; but with the modifications just described, the "control input" actually becomes the commanded response of the state variable in accordance with the selected control strategy. All rigid body response state variables (u, w, q, v, p, r, ψ, θ, and ϕ) are fed back to the MBC, and the observability matrix C_p (see equation 4) is used to select which four will be controlled by the four pilot inputs.

The time history response of the closed loop coupled MBC/plant system to a command roll attitude is shown in Figures 10, 11, and 12. Shown in Figure 10 is the commanded and achieved angular response of the aircraft. The roll rate trace shown in Figure 11 implies a time constant of 0.26 second. Figure 11 shows the time history of the lateral velocity, or sideslip angle. Since there was no pilot command to the pedals, the lateral velocity was held at the trim value. The MBC then regulated the lateral velocity as well as fuselage pitch angle (controlled by F/A cyclic) and forward velocity (controlled by the collective input). Thus a lateral cyclic input is a command to perform a coordinated turn. The control inputs to the aircraft to perform the desired maneuver are shown in Figure 12.

ROBUSTNESS

The robustness of a control system is a measure of its ability to perform satisfactorily at off-design conditions and in the presence of plant nonlinearities, random excitation, measurement noise, and unmodeled dynamics. There is not a rigorous manner of defining the region of robustness for the control system. Several methods can be suggested. Referring to the MBC block diagram (Figure 3), one technique would be to change the A and B plant matrices corresponding to the new flight condition, while holding fixed the A and B within the MBC. A root locus plot as a function of forward velocity, or some other flight parameter, could then be achieved. A more strenuous examination would perform a time history integration at off-design points using the COPTER program as shown in Figure 2. Figure 13 shows the commanded and achieved roll angles as calculated by COPTER for a forward velocity of 275 knots using a MBC designed around a forward velocity of 236 knots. Similar results using a flight speed of 200 knots are shown in Figure 14.

TABLE 8. CLOSED LOOP EIGENVALUES

-1.8200	+	0.0000i
-0.0248	+	0.1243i
-0.0248	-	0.1243i
-1.1545	+	0.0000i
-1.4298	+	0.0982i
-1.4298	-	0.0982i
-3.1729	-	0.0000i
-3.2045	-	0.5514i
-3.2045	+	0.5514i
-2.4214	+	2.7372i
-2.4214	-	2.7372i
-2.1795	+	3.3315i
-2.1795	-	3.3315i
-9.7619	+	16.9210i
-9.7619	-	16.9210i
-19.5819	+	0.0000i
-28.8930	-	0.0000i
-14.4502	-	25.0986i
-14.4502	+	25.0986i
-38.0130	+	0.0000i
23.2462	-	35.6098i
23.2462	+	35.6098i
-55.2559	+	55.2868i
-55.2559	-	55.2868i

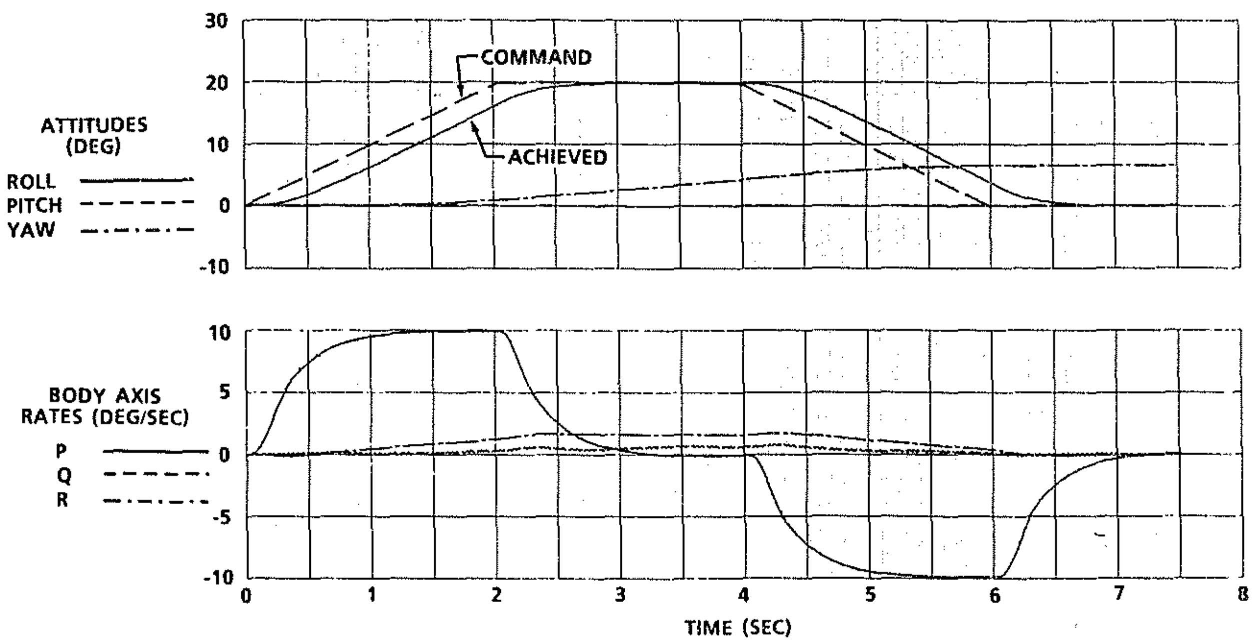

Figure 10. Commanded and Achieved Response at V = 236 Kn.

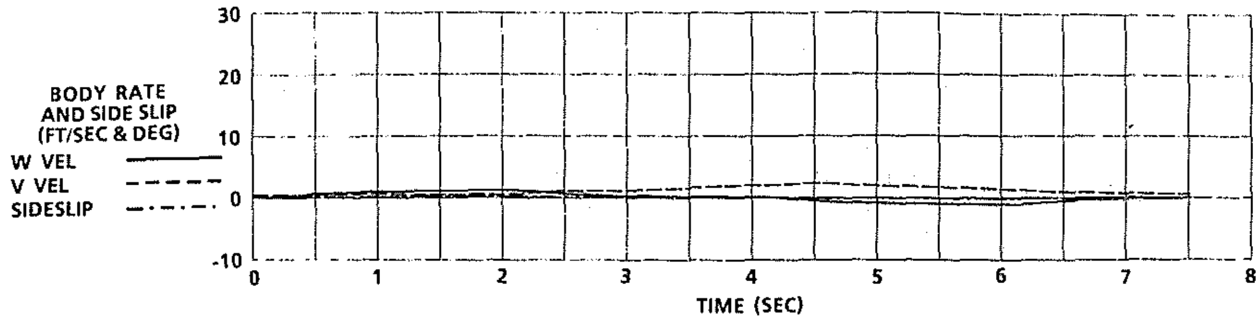

Figure 11. Body Rates and Sideslip at V = 236 Kn.

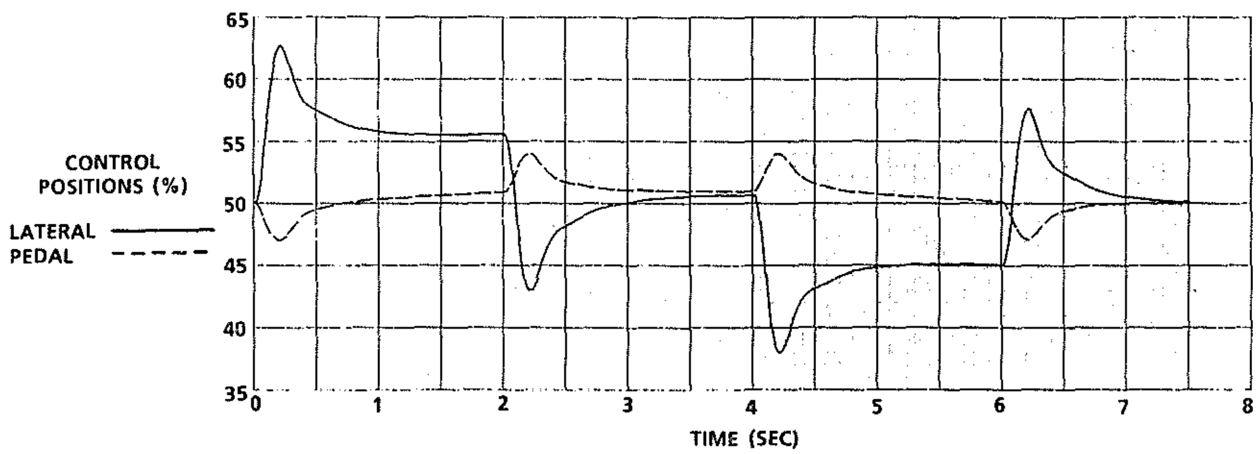

Figure 12. Control Inputs to Aircraft at V = 236 Kn.

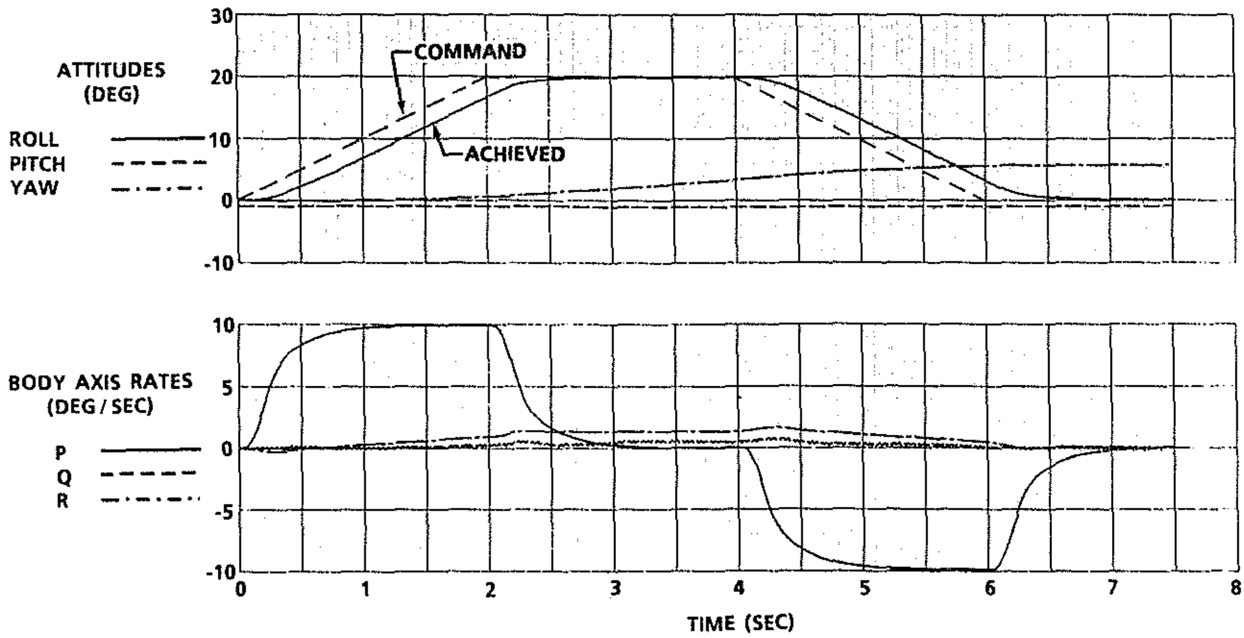

Figure 13. Commanded and Achieved Response at V = 275 Kn.

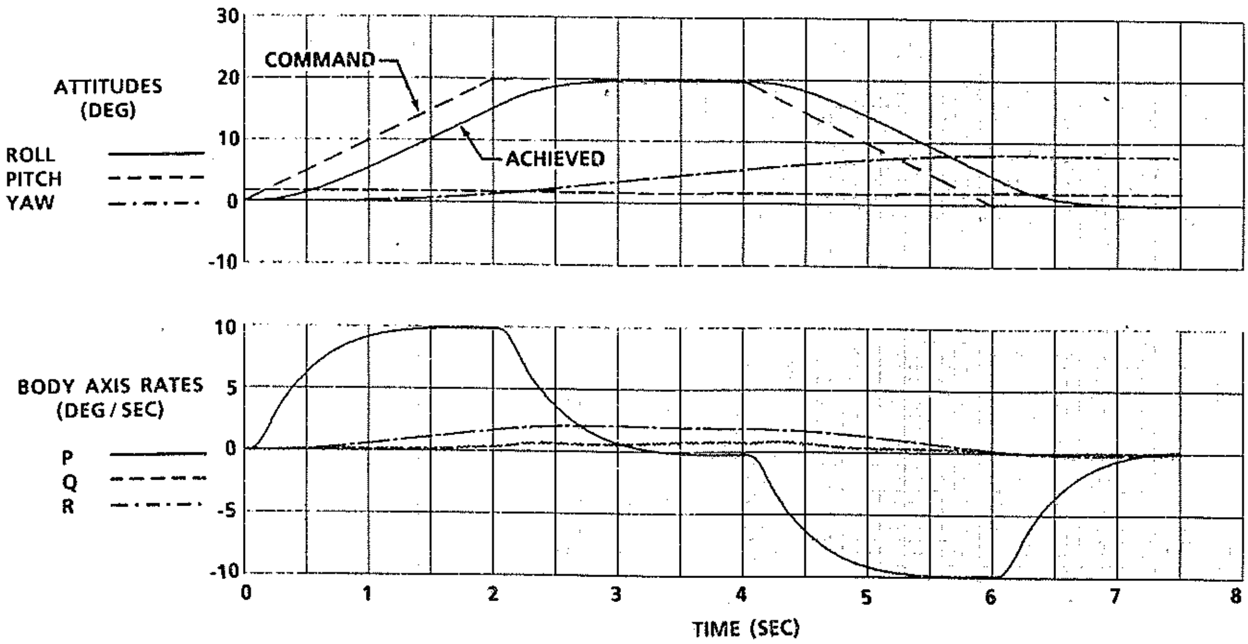

Figure 14. Commanded and Achieved Response at 200 Kn.

In the event that a single system does not have the desired robustness, two options are open; 1) perform linear interpolation on the previously computed and stored G and H matrices; 2) make a step change in the G and H matrices at a common point in the robustness region. The latter method was investigated by Murphy et al. (Reference 6) for a higher harmonic control of helicopter vibrations.

IMPLEMENTATION

The use of an MBC on an aircraft will require a significant digital computer capacity. Referring to the MBC block diagram shown in Figure 3, it can be seen that the inputs to the MBC must include the pilot commands, and aircraft state variables. The twelve linear differential equations in the MBC must be integrated numerically in real time to produce the actual command to the swashplates. The real time integration problem may be trivial when compared with the robustness problem. There are several potential approaches. The first would be an online, real time definition of the A and B matrices for the aircraft which could then be used to recalculate the G and H matrix for the MBC. Such an approach is beyond the state of the art at the present time.

Other approaches are based on gain scheduling for ensuring the performance of the MBC over the total flight envelope. Gain scheduling requires the design of MBC's at a large number of discrete flight conditions throughout the flight envelope. Each MBC could then be represented by a stored 12×12 matrix. Linear interpolation could then be used to select the appropriate 12×12 matrix for the current flight condition.

A second gain scheduling approach would require the knowledge of the stability region for each of the previously calculated MBC's. Assuming then that these regions overlap, it becomes possible to make a step change in the 12×12 matrix as the flight condition changes. This latter approach was suggested by the work of Murphy et al. during their work on higher harmonic control.

CONCLUDING REMARKS

The most attractive feature of the MBC design process is the reduction of the design process to the choice of two scalar parameters μ (equation 12) and ρ (equation 17). However, this feature of only needing to manipulate two variables may also limit the designer's flexibility. Scaling the state variables will influence the MBC, but the practicing control system designer will have some difficulty in using scaling values as design parameters. A possible solution is to combine a nonlinear programming (NLP) algorithm to seek the "optimum" scaling.

The MBC design has the tremendous advantage of guaranteeing a stable system at the design point, but the slight disadvantage that additional dynamics associated with the MBC can produce lightly damped stable roots, causing "ringing." The use of an NLP might solve this problem by formulating an optimal design problem as follows: 1) the constraints would be the requirement of a specified minimum damping ratio for each mode, 2) the design variables would be the significant components in the G and H matrices of the MBC, and 3) the NLP might even be used to control the time constraints of the system.

FUTURE DEVELOPMENT

The next major step in BHTI's evaluation of the MBC is to combine an MBC with a real time man-in-the-loop simulator. The primary purpose of the man-in-the-loop simulation is to obtain the pilots' evaluation of the MBC. Since the MBC design process has an explicit control system design strategy, it may be possible for the pilot in the simulator to evaluate various strategies quickly.

Future analytical/theoretical efforts must include techniques for including the system nonlinearities in the design process.

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