COMPARISON OF OPTIMIZATION BASED INVERSE SIMULATION METHODS FOR HELICOPTER MANEUVERS

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Abstract

In a helicopter certification process, aviation safety agencies want to be sure that the helicopter can safely fly all maneuvers defined in its usage spectrum. Therefore, loads engineers carry out all these maneuvers for each appropriate combination of weight and center of gravity. Moreover, this maneuvering and load analysis process should be performed in the most efficient way. For this reason, this article works on two different algorithms, the gradient-based Symmetric Rank-One (SR1) and commercial optimization tool Siemens HEEDS, to perform desired helicopter maneuvers. In this study, helicopter pushover maneuver is carried out and the results for each algorithm are compared as an example. However, different maneuver results are also added to show applicability of the solution algorithms. Furthermore, rotorcraft simulation and modelling software FLIGHTLAB is used to simulate the maneuver. Rotorcraft is modeled as rigid and uniform inflow is used for the calculation of rotor aerodynamic loads.

ABBREVIATIONS

SR1	Symmetric Rank-One
$a_{x_{body}}$	Total acceleration component in body x axis
$a_{y_{body}}$	Total acceleration component in body y axis
$a_{z_{body}}$	Total acceleration component in body z axis
u	Velocity component in body x axis
v	Velocity component in body y axis
w	Velocity component in body z axis
$\dot{u}$	Acceleration component in body x axis
$\dot{v}$	Acceleration component in body y axis
$\dot{z}$	Acceleration component in body z axis
p	Angular rotation in body x axis
q	Angular rotation in body y axis
r	Angular rotation in body z axis
g	Gravity of Earth
$n_{z_{body}}$	Load factor in body x axis
SAS	Stability augmentation aystem
$\boldsymbol{B}^{(k)}$	Hessian matrix at k^{th} iteration
I	Identity matrix
f	Objective function
∇f	Gradient of objective function
$\boldsymbol{X}$	Design variable vector
α	Step size
ε	Perturbation constant
$\zeta_{t=t_o}$	Pilot collective cyclic input at time t_o
$\delta_{t=t_o}$	Pilot longitudinal cyclic input
γ	Blade flap angle
θ	Helicopter pitch angle
ϕ	Helicopter roll angle
P	Engine power
$c_{\#}$	Constraint number in objective function
$p_{\#}$	Penalty parameter in objective function

1. INTRODUCTION

During the design of rotorcraft, several different type of maneuvers are considered for the load calculation. These can be mainly categorized as limit and operational maneuvers. Limit maneuvers are used in the static structural sizing of the rotorcraft and described in the Certification Specifications[1]-[2]. On the other hand, operational maneuvers are defined in the usage spectrum of the rotorcraft and calculated for the fatigue sizing. For this reason, loads engineers carry out all the limit and operational maneuvers to calculate the critical loads needed for structural sizing of the helicopter. Each different maneuver has its own target/s and achieving of this/these target/s are either obtained by trial-error approach or some rational methods. In the trial-error, pilot control inputs are changed manually until all maneuver requirements are satisfied. Defects of this approach can be waste of time, high engineering and computational effort, increased cost due to repeated analysis, difficulty meeting maneuvering requirements. To achieve the most accurate results with minimum time, effort and the lowest cost, robust algorithms should be applied in performing helicopter maneuvers. Throughout history, there are variety of approaches to perform helicopter maneuvers. These attempts can be classified under three main headings as numerical differentiation approach[3], numerical integration approach[4] and global optimization method[5]. Although there are different approaches to find the control action required to perform flight maneuver, optimization based inverse simulation technique is one of the commonly used and practical

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algorithm[6]-[7]. In this study, inverse simulation methodology is implemented to perform the -1.0*g* pushover maneuver that is used in static structural sizing of rotorcraft[2]. Although this algorithm is generally used for tracking a particular trajectory[8], load factor and other constraints such as maximum flap angle, pitch angle and power are defined in the objective function. Trajectory of the pushover maneuver is the output of the inverse simulation algorithms applied. Two different algorithms which are gradient-based Symmetric Rank-One (SR1) and commercial optimization tool Siemens HEEDS will be used in the analysis and comparison will be made in terms computation time, convergence accuracy and efficiency. HEEDS uses a hybrid and adaptive algorithm called SHERPA.

2. DESCRIPTON OF PUSHOVER MANEUVER

Schematic representation of pushover maneuver and helicopter body axis system used in this study are shown in Figure 1 and Figure 2.

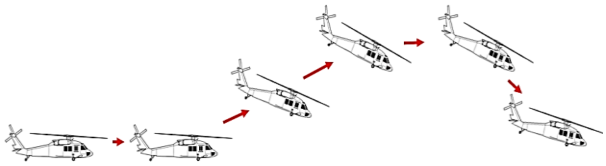

Figure 1. Flight Path of Helicopter Pushover Maneuver

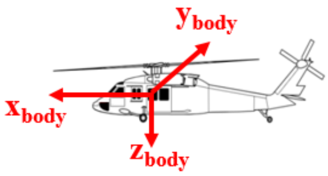

Figure 2. Helicopter Body Axis System

At the beginning of the simulation, rotorcraft is trimmed to the forward flight at constant altitude. In order to calculate helicopter load factors on body z-axis, it is necessary to obtain helicopter acceleration on this axis. According to Coriolis Transport Theorem, the acceleration values of the helicopter's center of gravity can be derived in body axis system as follows:

$$(1) \qquad \begin{bmatrix} a_{x_{body}} \\ a_{y_{body}} \\ a_{z_{body}} \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} x \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

where u, v and w represent helicopter velocity in body x-y-z axis, respectively. Similarly, p, q and r represent helicopter angular velocity about body x-y-z axis, respectively. Then, acceleration in body z axis can be obtained as

$$(2) \qquad a_{z_{body}} = \dot{w} + (p * v - u * q)$$

Since symmetrical pushover is considered, roll rate and sideward velocity should be close to zero. Thus, the acceleration formula is simplified as:

$$(3) \qquad a_{z_{body}} = \dot{w} - u * q$$

From the acceleration formula, the load factor in body z axis is calculated as:

$$(4) \qquad n_{z_{body}} = cos(\theta) * cos(\phi) - \frac{\dot{w} - u * q}{g}$$

From this equation, it can be concluded that negative pitch rate should be obtained to obtain negative load factor in body z axis.

3. METHODOLOGY

Optimization algorithms need three main elements which are called design variables, problem constraints and objective function. Therefore, for the solution of a helicopter maneuvering problem by using optimization algorithm, helicopter parameters are defined as:

- ✓ Pilot control inputs over time are defined as design variables.
- ✓ Maneuvering requirements within the scope of helicopter capability are defined as problem constraints.
- ✓ The solution accuracy for all the requirements expressed in the problem constraints is defined in the objective function.

Helicopters are controlled by four main pilot control inputs that are called longitudinal cyclic, lateral cyclic, collective cyclic and anti-torque pedal. However, in pushover maneuver, the optimization algorithms are applied by taking collective and longitudinal cyclic as time depended design variables[7]. Other control inputs are controlled by Stability Augmentation System (SAS) in FLIGHTLAB. In addition, the boundaries of pilot control inputs are defined in the FLIGHTLAB mathematical model.

SR1 Optimization Method Algorithm

Symmetric Rank-One method, one of the quasi-Newton methods, is a useful approach to solve non-linear optimization problems. In fact, it has been observed that SR1 is the most beneficial inverse simulation method for helicopter maneuver optimization among the algorithms studied in [7]. The main aim of SR1 method is to predict the approximated Hessian matrix value in quasi-Newton optimization algorithm because calculation of Hessian matrix causes increased time, cost and computational difficulty. The iterative formulation of quasi-Newton method can be shown as

$$(5) \qquad x^{(k+1)} = x^{(k)} + \alpha^{(k)} d^{(k)}$$

where $d^{(k)} = -\left[\boldsymbol{B}^{(k)}\right]^{-1} \nabla f\left(x^{(k)}\right)$ and also $\boldsymbol{B}^{(k)}$ represents the approximate the Hessian matrix for the objective function $f(x)$.

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SR1 method calculates subsequent approximated Hessian matrix ($\boldsymbol{B}^{(k+1)}$) by using computed approximated Hessian matrix $\boldsymbol{B}^{(k)}$ at k^{th} iteration and the gradient between these two sequential iterations. Formulation of approximated Hessian matrix calculation in SR1 method is given as

$$\boldsymbol{B}^{(k+1)} = \boldsymbol{B}^{(k)} + \frac{\left(q^{(k)} - \boldsymbol{B}^{(k)}\, p^{(k)}\right)\left(q^{(k)} - \boldsymbol{B}^{(k)}\, p^{(k)}\right)^T}{\left(q^{(k)} - \boldsymbol{B}^{(k)}\, p^{(k)}\right)^T p^{(k)}}$$

where $q^{(k)} = \nabla f\left(x^{(k+1)}\right) - \nabla f\left(x^{(k)}\right)$.

In this study, the Hessian matrix should be positive definite since the optimization algorithm aims to converge to the local minimum value[8]. Therefore, identity matrix is used as initial Hessian matrix value as $\boldsymbol{B}^{(1)} = I$. Thus, Hessian matrix in the next iteration is obtained with SR1 method by using Hessian matrix at current iteration.

General structure of the SR1 algorithm is described below:

Preparation:

→ Identification of time dependent design variable vector $\boldsymbol{X}^T$ from pilot control inputs

→ Identifications of step size α and perturbation constant ε values

Iteration:

→ Computation of the objective function $f\left(x^{(k)}\right)$ at k^{th} iteration

→ Computation of the objective function gradient $\nabla f\left(x^{(k)}\right)$ at k^{th} iteration

→ Computation of the approximate Hessian matrix $\boldsymbol{B}^{(k+1)}$ at $k^{(th+1)}$ iteration

→ Implementation of the line search algorithm

→ If the objective value convergence, which is the termination criterion, is met, the algorithm is finalized

The algorithm uses the Newton's Central Difference approximation in the gradient calculation of the objective function. In addition, fixed step size and perturbation constant values are used as 5 and 0.1, respectively[7]. After the search direction is obtained, the line search algorithm is implemented to achieve the minimum objective along the searched line. The line search algorithm can be formalized as

$$\alpha^{(k)} = \arg\min f\left(x^{(k)} + \alpha * d^{(k)}\right) \qquad (6)$$

where $d^{(k)} = -\left(B^{(k)}\right)^{-1} \nabla f\left(x^{(k)}\right)$.

To be able to apply line search step size correctly, the value of $d^{(k)}$ parameters is normalized with its maximum absolute value. Thus, while the design point of $x^{(k)}$ represents lower boundary of line search algorithm, $x^{(k)} + \alpha * d^{(k)}$ represents its upper boundary. Then, this boundary is separated eight equal intervals and objective values are obtained each design point. Afterwards, line search algorithm boundaries are updated according to position of minimum objective point and is repeated with these updated boundaries. Schematic representation of line search algorithm is given in Figure 3.

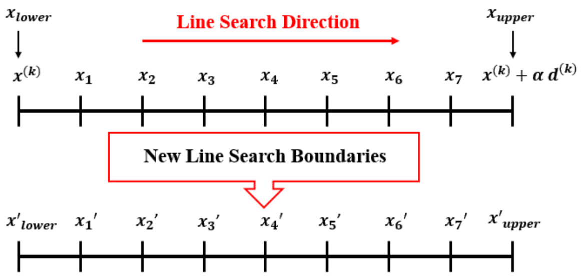

Figure 3. Schematic Representation of Line Search Algorithm

At the end of the line search algorithm, objective function reaches its minimum value at k^{th} iteration for defined step size and perturbation constant values. Thus, k^{th} iteration will be finalized and algorithm continues with following iterations until termination criteria is supplied.

HEEDS - SHERPA Algorithm

HEEDS uses a search strategy called SHERPA (Simultaneous Hybrid Exploration that is Robust, Progressive and Adaptive). This algorithm is a combination of global and local search strategies and uses multiple search methods simultaneously at any time ranging between two to ten[8]. There is no need to modify the tuning parameters of search methods, these are automatically determined by SHERPA. In the inverse simulation optimization problem, in addition to SR1 algorithm, commercial tool HEEDS is used and results are compared. Same constraints and objectives defined in SR1 algorithm are implemented to HEEDS. Schematic representation of HEEDS model for pushover inverse simulation problem is shown in Figure 4.

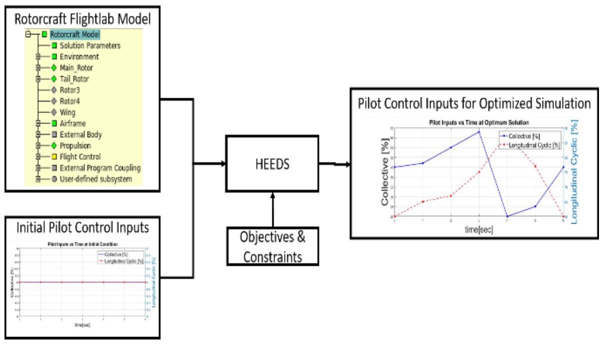

Figure 4. Schematic Representation of HEEDS for Inverse Simulation

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4. OPTIMIZATION PROBLEM STRUCTURE FOR PUSHOVER MANEUVER

For comparison of predefined approaches, it is desired to perform pushover as helicopter example maneuver. At the beginning of pushover maneuver, helicopter is trimmed to level flight at a certain speed and a fixed altitude. Then, helicopter start to climb to reach desired load factor. When the targeted load factor is obtained, helicopter ends its climbing motion and continue with diving motion. Schematic representation of pushover maneuver is shown in Figure 1.

In this study, it is aimed to perform pushover maneuver at -1.0*g* load factor in body z-axis. Before starting optimization process, helicopter is trimmed to the forward flight at 100 knot at 200 [ft] ISA 0º condition. Then, the optimization algorithm is applied by takin collective and longitudinal cyclic as time depended design variables. Other control inputs are controlled by Stability Augmentation System (SAS).

In the optimization problem, maneuver time is taken about 5.5 seconds and points of design variables, which are collective (ζ) and longitudinal (δ) cyclic, are modified at every half second. Therefore, the design variable vector has 2*11 = 22 design points as seen in Equation (7).

The optimization constraints consist of helicopter design limitations and pushover maneuver requirement. In pushover maneuver optimization, blade flap angle, helicopter pitch angle, engine power are defined as design limitations and load factor is defined as maneuver requirement. This study aims to reach the target load factor between 4.95 and 5 seconds of the maneuver. The formulation of each constraint is symbolized with $c_{\#}$ and formulated from Equation (8) to (11).

Design Variable Vector

(7) $$X^T = [\zeta_{t=0.5}, \zeta_{t=1}, \dots, \zeta_{t=5.5}, \delta_{t=0.5}, \delta_{t=1}, \dots, \delta_{t=5.5}\ ]$$

Constraint 1: Blade Flap Angle ($\boldsymbol{\gamma}$)

(8) $$c_1 = \begin{cases} -log(\gamma_{max} - max(\gamma)) - log(\gamma_{min} + min(\gamma)), \\ for\ \ max(\gamma) < \gamma_{max}\ and\ min(\gamma) > \gamma_{min}; \\ Infinity\ for\ otherwise \end{cases}$$

Constraint 2: Helicopter Pitch Angle ($\boldsymbol{\theta}$)

(9) $$c_2 = \begin{cases} -log(\theta_{max} - max(\theta)) - log(\theta_{min} + min(\theta)), \\ for\ \ max(\theta) < \theta_{max}\ and\ min(\theta) > \theta_{min}; \\ Infinity\ for\ otherwise \end{cases}$$

Constraint 3: Engine Power ($\boldsymbol{P}$)

(10) $$c_3 = \begin{cases} -log(P_{max} - max(P)) - log(P_{min} + min(P)), \\ for\ \ max(P) < P_{max}\ and\ min(P) > P_{min}; \\ Infinity\ for\ otherwise \end{cases}$$

Constraint 4: Load Factor ($\boldsymbol{n_{z_{body}}}$)

(11) $$c_4 = \int_{t_s=5.0}^{t_f=4.95} \left| n_{z_{body}} + 1 \right| \, dt$$

Constraint 1, 2 and 3 are represents the helicopter design limitations. In other words, these constraints should not be exceeded during the maneuvers due to safety. This means that these constraints should be restricted strictly. Hence, they are constrained with logarithmic barrier function shown in Figure 5.

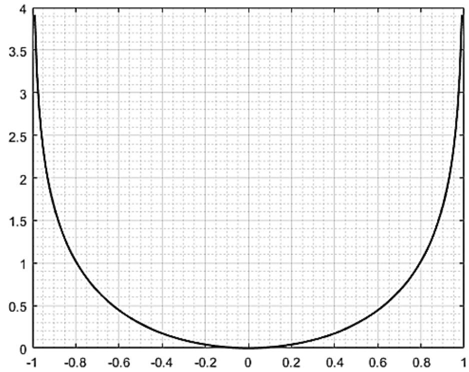

Figure 5. Logarithmic Barrier Function Representation

Figure 5 shows that logarithmic barrier function causes infinity objective value when constraint parameter approaches or exceeds the limits. Thus, it keeps constrained parameters away from their limits.

Finally, the objective function of this optimization problem is constructed by assigning penalty parameters ($p_{\#}$) to each constraint to regulate their importance. The mathematical formulation of the objective function is structured as:

(12) $$f(\boldsymbol{X}) = p_1 c_1 + p_2 c_2 + p_3 c_3 + p_4 c_4$$

5. RESULTS AND DISCUSSION

Pushover maneuver is defined as an optimization problem with the constraints in Chapter 4 and then carried out by applying SR1 and HEEDS methods.

Before making comparison of these two methods, optimization convergence results are investigated for each method separately. Then, final converged results are compared with each other.

In results figures, while the range of blade flap angle and pitch angle are scaled between -1 to 1, this range is from 0 to 1 for engine power. In addition, pilot control inputs ranges are represented in percentages. Other parameters do not have any scale or representation way.

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SR1 Results

For the pushover maneuver simulation, SR1 algorithm is used with two different constraint sets. Engine power, blade flap angle, load factor, pitch angle and altitude during the simulation are shown in Figure 6 for the converged solution.

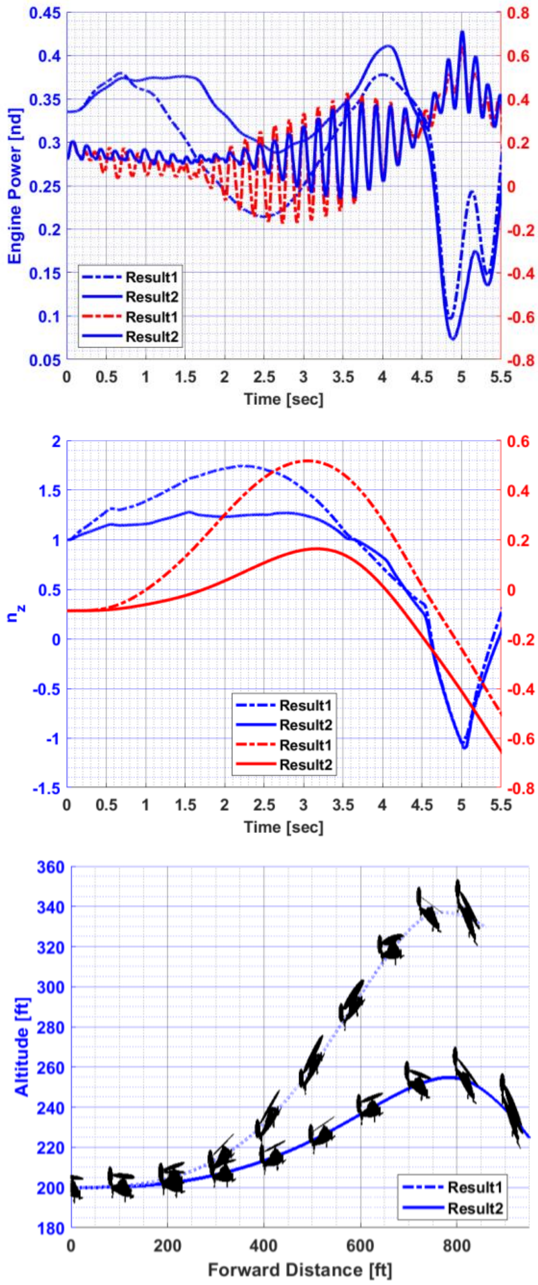


Figure 6. Converged Results of SR1 Method for Pushover Maneuver Simulation

First, analysis is performed using the constraints defined in Chapter 3 and the analysis results are labelled as "*Result1"*. Although all constraints are within their limits, load factor value reaches 1.73g at about 2.4th second. This is unexpected solution due to the nature of the maneuver. Therefore, the load factor value is additionally limited by the barrier function to keep it close to 1*g* until 4th second. In this way, SR1 method achieves its final solution as *"Result2"*.

Figure 6 shows that helicopter starts climbing until desired load factor achieved. Then, pitch angle is decreased and dive started to achieve the forward flight attitude.

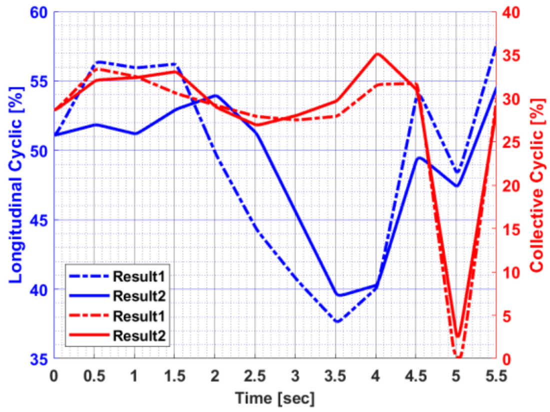


Figure 7. Design Variable Inputs of SR1 Method Analyses in Pushover Maneuver

Design variable parameters which are longitudinal cyclic and collective cyclic are also obtained throughout maneuvering and plotted in Figure 7. According to this figure, collective cyclic are reduced in order to reach -1.0*g* load factor at defined time interval. However, note that decrease of the collective is sharper than the longitudinal cyclic.

Objective value with respect to iteration number is plotted in Figure 8. This figure shows that Result1 converge a little bit faster than Results2. Since Results2 need to meet one more constraint, its convergence takes a little more time. However, at the end of the optimization process, almost same objective value is obtained.

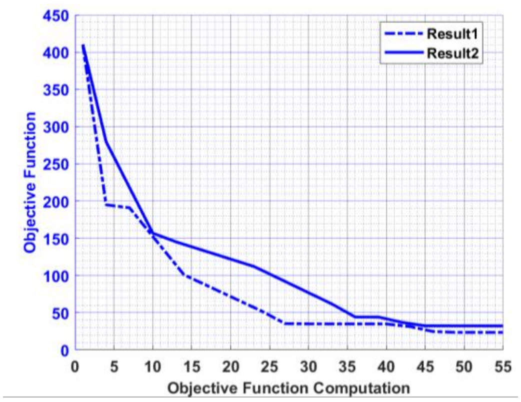


Figure 8. Change of Objective Function Value for SR1 Method Analyses

To conclude, SR1 method converges to obtain the aimed -1.0*g* pushover maneuver within the predefined constraints and time interval.

HEEDS - SHERPA Algorithm Results

In the application process of HEEDS – SHERPA algorithm, same constraint sets with SR1 method are used and results are plotted in Figure 9.

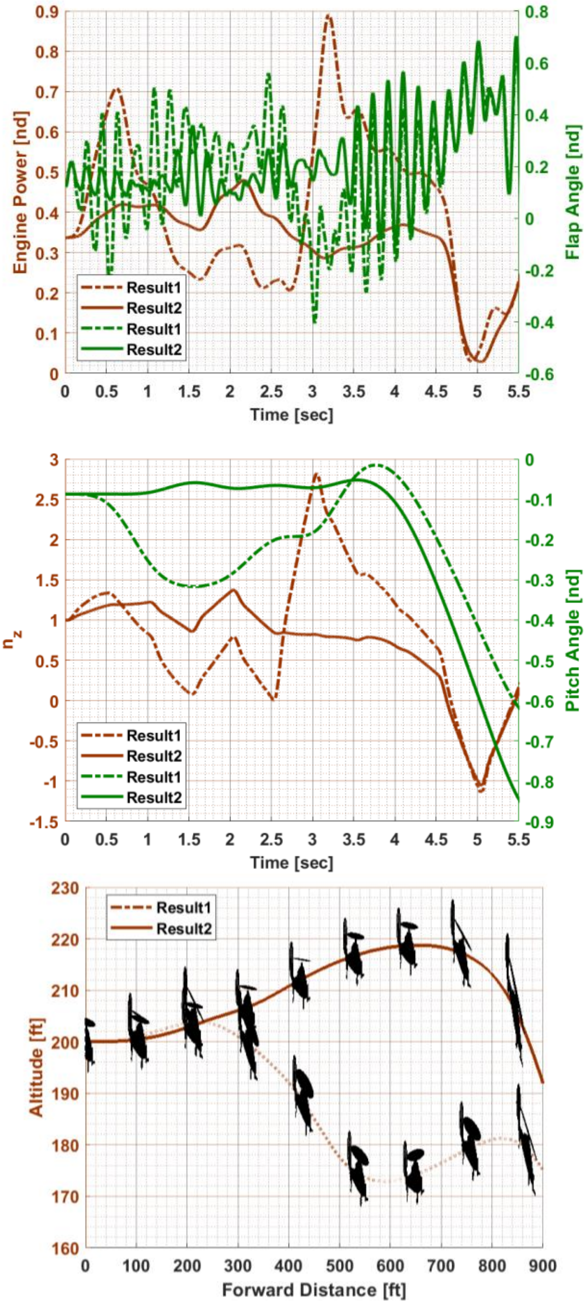


Figure 9. Converged Results of HEEDS – SHERPA Method for Pushover Maneuver Simulation

Constraint graphs in Figure 9 show that flap angle, pitch angle and engine power limitations are satisfied. Nevertheless, load factor and altitude graphs for Result1 reveal that helicopter behaves outside the definition of pushover maneuver. Therefore, same additional constrain with SR1 Result2 are also applied for this algorithm. According to the results, maneuver is performed as expected.

Figure 10 shows the converged values for design variables longitudinal cyclic and collective cyclic along time. It can be seen that there are abrupt applied pilot inputs for Results 1. This leads to increase of load factor before performing the Pushover maneuver. In addition, collective and longitudinal cyclic inputs are similar around the -1g.

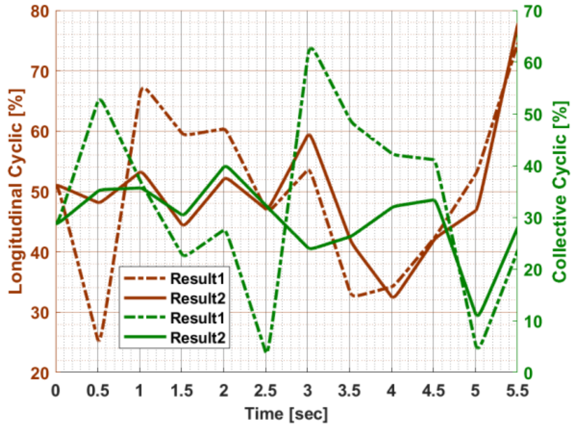


Figure 10. Design Variable Inputs of HEEDS – SHERPA Algorithm Analyses in Pushover Maneuver

Change of the objective function value throughout its computation is shown in Figure 11. According to this figure, Results2 converges to solution as the objective function is called approximately 1400 times. This number is much less than that of Result1. Therefore, it can be conclude that Result2 constraints are more effective in this algorithm for the solution of predefined pushover maneuver.

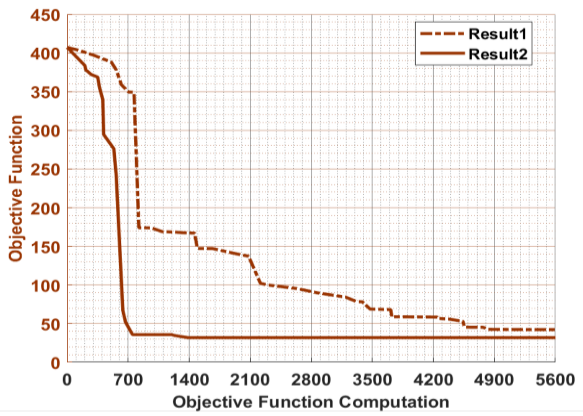


Figure 11. Change of Objective Function Value for HEEDS – SHERPA Algorithm Analyses

As a conclusion, HEEDS–SHERPA algorithm achieves the desired -1.0*g* pushover maneuver by meeting all constraints in Result2. Moreover, Result2 has more effective constraints.

Comparison of SR1 and Heeds-Sherpa Method

Final analysis results of SR1 method and HEEDS – SHERPA algorithm are plotted on same figure in Figure 12.

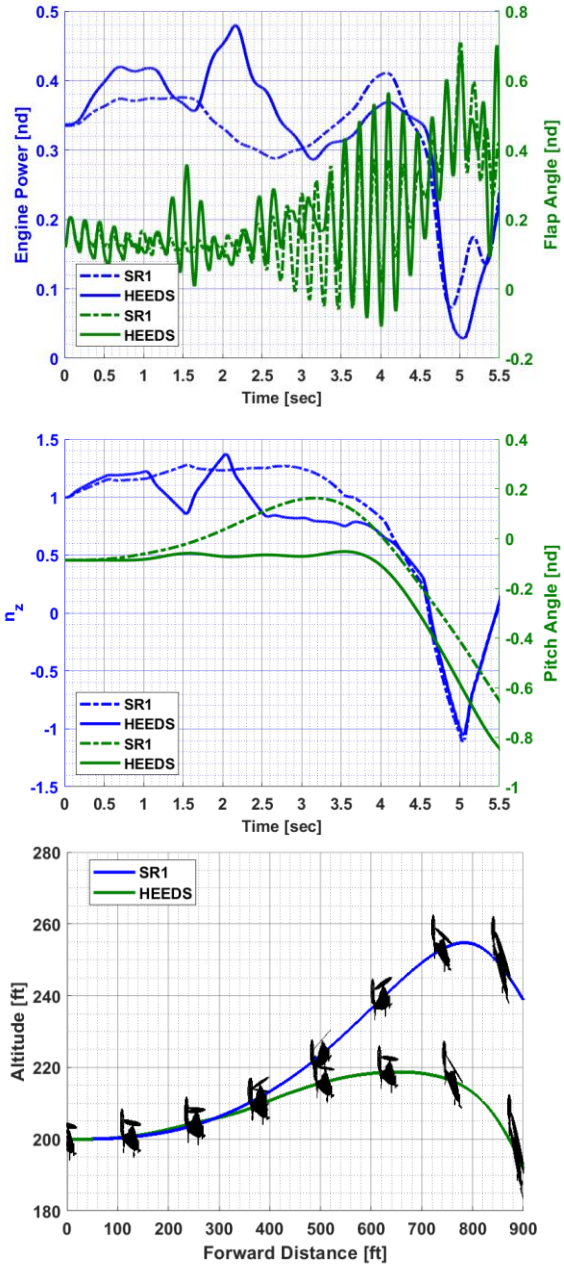


Figure 12. Optimization Results of SR1 Method and HEEDS–SHERPA Algorithm in Pushover Maneuver

According to Figure 12, the constraint result lines have different trends within the same constraint boundaries. This means that the predefined -1.0*g* pushover maneuver can be achieved with different helicopter behaviors. In other words, there is no unique solution to perform the -1.0*g* Pushover maneuver. Moreover, this variation in movement directly affects the attitude and altitude needed to reach the targeted load value.

Design variables collective and longitudinal cyclic are compared in Figure 13.

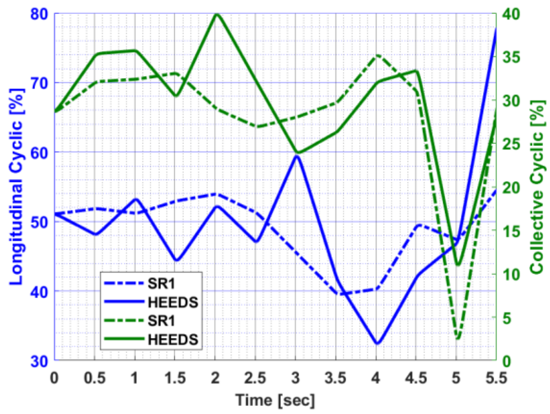


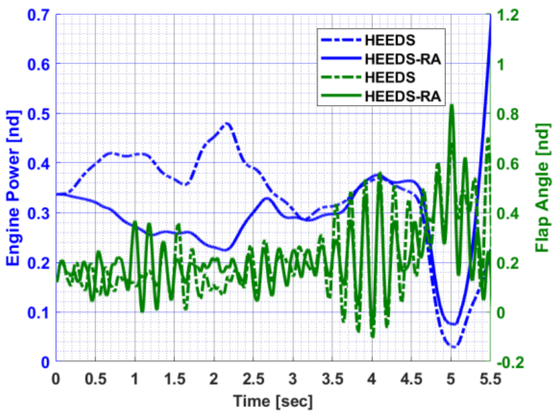
Figure 13. Design Variable Inputs of SR1 Method and HEEDS–SHERPA Algorithm Analyses

Figure 13 shows that both longitudinal cyclic and collective cyclic inputs are different for each optimization technique. These results prove that the predefined maneuver constraints can be satisfied with different pilot control inputs.

When the number of the objective function computations represented in Figure 8 and Figure 11 are compared, it can be concluded that SR1 method needs about 15 times less objective function calculations than HEEDS-SHERPA algorithm. This means that SR1 method is much faster and less costly.

HEEDS-SHERPA Algorithm Convergence

In order to view the convergence process of the Heeds-Sherpa algorithm, -1.0*g* Pushover maneuver is resolved in this algorithm using same constraints described earlier. Then, this repeated analysis results (HEEDS-RA) are plotted with the final HEEDS results given previous figures. The comparison figures are shown in Figure 14.



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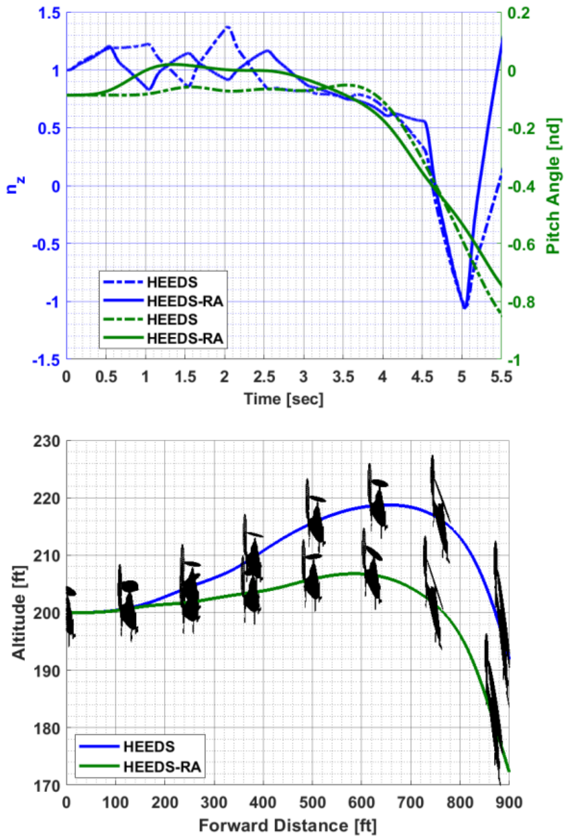

Figure 14. Comparison Results of HEEDS – SHERPA Algorithm Analyses in Pushover Maneuver

Collective and longitudinal cyclic inputs as required design variables are given in Figure 15.

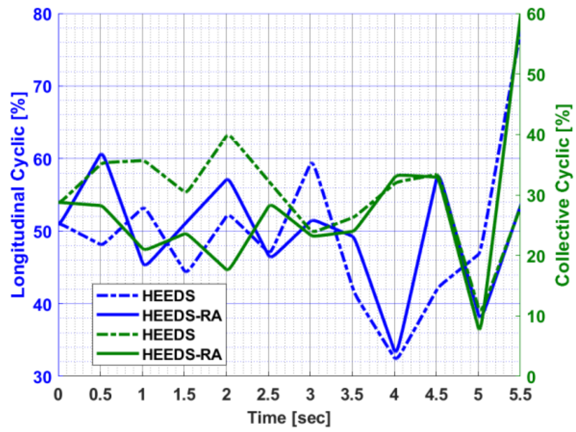

Figure 15. Design Variable Inputs of HEEDS–SHERPA Algorithm Analyses

Figure 14 and Figure 15 results show that although these two analyzes converge to the same maneuver under the same optimization inputs, their results are different. This is expected results since HEEDS-SHERPA algorithm converges to optimum solution by randomly searching for any points in the design variable set.

Hover to Forward Flight Maneuver Optimization

In order to investigate the applicability of the SR1 method and HEEDS-SHERPA algorithm for helicopter maneuvering, hover to 30 Knot forward flight maneuver at level altitude is performed with these approaches. Then, the results and control inputs are given in Figure 16.

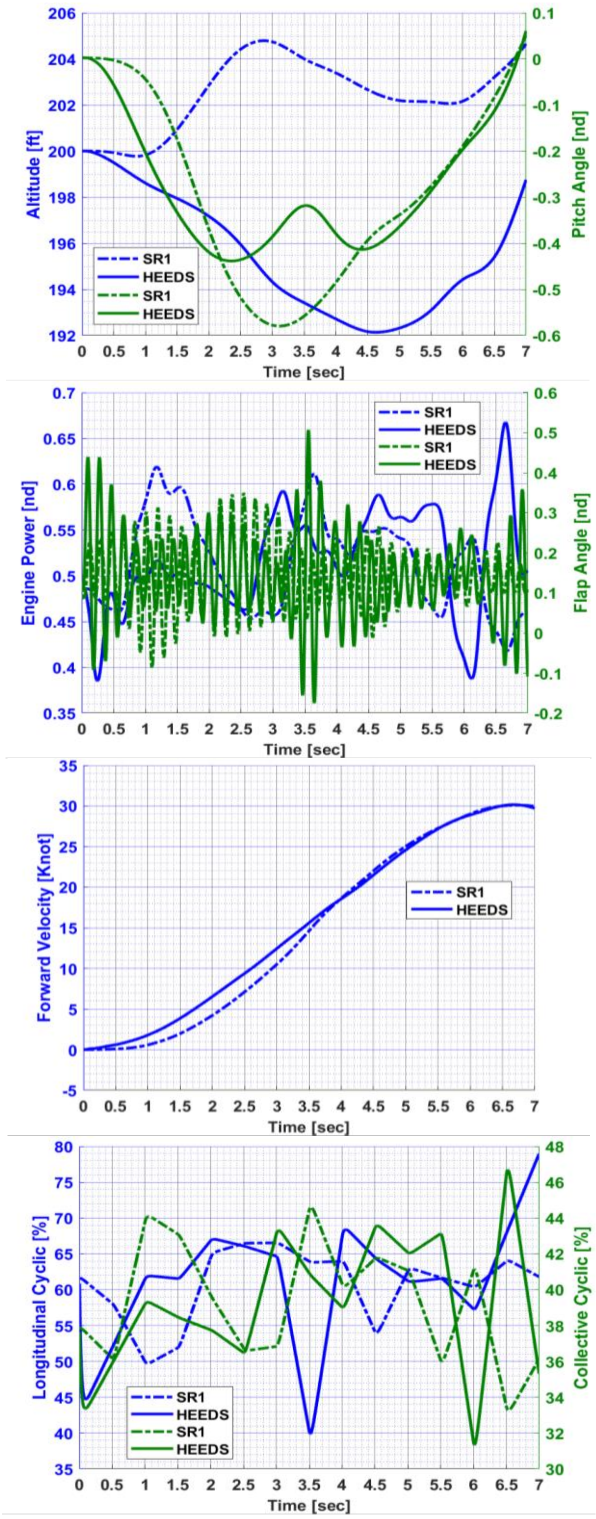

Figure 16. Optimization Results of Hover to Forward Flight Maneuver

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6. SUMMARY

This paper mainly demonstrates the comparison of Symmetric Rank-One method and HEEDS-SHERPA algorithm for helicopter maneuvering in terms of convergence time and accuracy to desired maneuver requirement. For this comparison, *-1.0g* Pushover maneuver at approximately 0.6V_{NE} is investigated as an example maneuver condition. Additionally, hover to 30 Knot forward flight maneuver is also investigated.

In the *-1.0g* Pushover maneuver analyses process, while the pilot control inputs of collective cyclic and longitudinal cyclic are defined as design variables, the maneuver requirements and helicopter design limitations are described as objective function. Afterwards, the maneuver results are compared for these two approach.

Maneuver results display that *-1.0g* Pushover maneuver can be performed with each approach. Nevertheless, when the objective function calculation numbers in optimization process are compared, it is deduced that SR1 method is much faster than HEEDS-SHERPA algorithm. Namely, SR1 method satisfies less costly solution to achieve desired maneuver. Therefore, it can be concluded that SR1 method is more useful approach than HEEDS-SHERPA algorithm for helicopter maneuvering optimization.

In order to certificate a helicopter, the helicopter must perform safely all maneuvers defined in helicopter usage spectrum. Therefore, these maneuvers should be carried out for certification requirements. In this paper, hover to 30 Knot forward flight maneuver is also targeted and obtained with SR1 method and HEEDS-SHERPA algorithm to prove the applicability of each approach for helicopter maneuvering.

To conclude, all analyses results show that both SR1 method and HEEDS-SHERPA algorithm are useful for helicopter maneuvering optimization. Unlike the traditional trial-error approaches, these optimization methodologies provide the more accurate results with minimum time, engineering effort and cost. However, SR1 method is more efficient than HEEDS-SHERPA algorithm in terms of these parameters. Finally, it is decided that Symmetric Rank-One method is the useful methodology to perform all helicopter maneuvers.

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