GAIN-SCHEDULING CONTROL OF HELICOPTER LONGITUDINAL FLIGHT

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Abstract

This paper summarises a recent study into the design of *MIMO* gain-scheduling controllers (in an *LPV* form) for the longitudinal flight control of helicopters. Based on Linear Matrix Inequalities (*LMIs*) and quadratic H^∞ performance objectives (Ref. 1), the study proposes a 2-degrees of freedom (*2-DOF*) gain-scheduling control configuration to achieve both good robustness and required flight handling qualities within a whole specified region of operation. Relevant issues such as the choice of the weighting functions are discussed. The paper also provides a description of a new technique for affine *LPV* system modelling and its application to helicopters.

1. Introduction

A helicopter system can be generally modelled in the following non-linear and parameter-dependent form:

Plant $P(t, \theta(t))$:

$$\begin{aligned} \dot{x}(t) &= f(x(t),u(t),w(t),\theta(t)), \\ z(t) &= h_z(x(t),u(t),w(t),\theta(t)), \\ y(t) &= h_y(x(t),u(t),w(t),\theta(t)). \end{aligned} \quad (1)$$

where $x(t) \in \Re^n$ the states, $u(t) \in \Re^m$ the plant inputs, $y(t) \in \Re^p$ the measurable plant outputs, $w(t) \in \Re^q$ the exogenous inputs including reference input $r(t)$, and $z(t) \in \Re^e$ is a measured error (system performance) output. The parameter variables are defined as an l-dimensional parameter vector, $\theta(t)$, which, in most cases, may just be of the state variables $x(t)$ and/or the system output variables $y(t)$.

With such a model, the control objective becomes to find a gain-scheduling control, defined as the control $u(t)$ from the parameter ($\theta(t)$)-dependent controller

Controller $K(t,\theta(t))$:

$$\begin{aligned} \dot{x}_k &= f_k(x_k(t),y(t),\theta(t)), \\ u(t) &= h_k(x_k(t),y(t),\theta(t)). \end{aligned} \quad (2)$$

which maintains performance throughout the whole operating region (See Fig. 1).

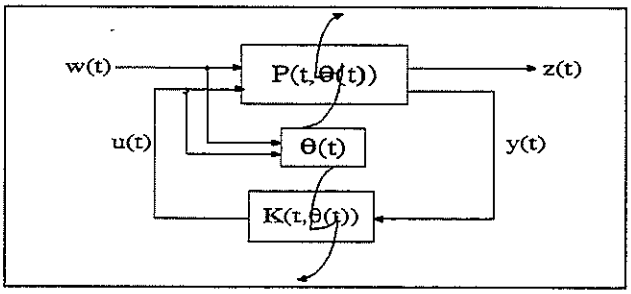


Fig. 1 Gain scheduling control --- general case

For a long time, the design of gain-scheduled controllers has mostly followed a classic two-step approach. First linear controllers are designed for linearised plants at frozen points (frozen θ) and then a schedule is designed which links the linear controllers normally by ad-hoc interpolation. Overall qualities such as stability and robust performance are then evaluated through simulation.

The classic synthesis of a gain-scheduled controller from a group of linear controllers has the advantage that a variety of up-to-date linear control methods can be used. However, the disadvantage is that there is no guarantee of satisfactory performance and robustness along all possible trajectories of the scheduling parameters $\theta(t)$.

During recent years, significant progress has been made in gain-scheduling control and a comprehensive survey study on the frameworks used has recently been made (Ref. 2). Among these so called 'one step synthesis' (simultaneous control and scheduling) methods for gain-scheduling control, there is the *Lyapunov* function/quadratic H^∞ performance approach based on a Linear Parameter-Varying (*LPV*) model of the plant (1).

2. Framework for *LPV* Model Based Gain-Scheduling Control

Generally, an *LPV* system is a linear time-varying system in which the state-space matrices are fixed functions of some vector of parameter variables, i.e. in state-space form,

LPV System $P(\theta)$:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\theta) & B_w(\theta) & B_u(\theta) \\ C_z(\theta) & D_{zw}(\theta) & D_{zu}(\theta) \\ C_y(\theta) & D_{yw}(\theta) & D_{yu}(\theta) \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} \quad (3)$$

where $A(\bullet)$: $\Re^l \to \Re^{n\times n}$, $B(\bullet)$: $\Re^l \to \Re^{n\times(q+m)}$, $C(\bullet)$: $\Re^l \to \Re^{(e+p)\times n}$ and $D(\bullet)$:$\Re^l \to \Re^{(e+p)\times(q+m)}$ are continuous, bounded functions of the parameters θ.

From *LPV* system modelling, an obvious choice for the structure of the associated gain scheduling controllers would be the *LPV* form of controllers, i.e., in state space form,

Controller $K(\theta(t))$:

$$\begin{aligned} \dot{x}_k &= A_k(\theta(t))x_k + B_k(\theta(t))y, \\ u &= C_k(\theta(t))x_k + D_k(\theta(t))y \end{aligned} \quad (4)$$

where $A_k(\bullet)$: $\Re^l \to \Re^{nk\times nk}$, $B_k(\bullet)$: $\Re^l \to \Re^{nk\times p}$, $C_k(\bullet)$: $\Re^l \to \Re^{m\times nk}$ and $D_k(\bullet)$:$\Re^l \to \Re^{m\times p}$ are continuous and bounded functions of θ.

In terms of Linear Matrix Inequalities, the quadratic H^∞ performance gain-scheduling control of *LPV* systems can be expressed as:

For the *LPV* plant $P(\theta)$ (3), find an integer $m \ge 0$, a matrix $X_c = X_c' > 0$, and a continuous and finite-dimensional (*nk*-states) controller $K(\theta)$(4), such that for all admissible parameters $\theta(t)$:

$$\begin{bmatrix} A_c'(\theta)X_c + X_cA_c(\theta) & X_cB_c(\theta) & C_c'(\theta) \\ B_c'(\theta)X_c & -\gamma I & D_c'(\theta) \\ C_c(\theta) & D_c(\theta) & -\gamma I \end{bmatrix} < 0 \quad (5)$$

which is sufficient to ensure that the closed-loop matrix function A_c is quadratically stable over the parameter domain $\mathcal{P}$ (the *Lyapunov* function $V(x) = x'X_cx$ gives global asymptotic stability) and the L_2-induced norm of the input/output map (w→z) is bounded by γ: $\|z\|^2 \le \gamma\|w\|^2$.

Here the matrix function $\begin{bmatrix} A_c(\theta) & B_c(\theta) \\ C_c(\theta) & D_c(\theta) \end{bmatrix}$ represents the closed-loop system from w→z. In terms of the conventional open-loop plant (3) with $D_{yu}(\theta)=0$, and a *nk*th-order scheduling controller $K(\theta)$ (4), we have

$$\begin{aligned} A_c(\theta) &= A^a(\theta) + B_u{}^a(\theta)K(\theta)C_y{}^a(\theta) \\ B_c(\theta) &= B_w{}^a(\theta) + B_u{}^a(\theta)K(\theta)D_{yw}{}^a(\theta) \\ C_c(\theta) &= C_z{}^a(\theta) + D_{zu}{}^a(\theta)K(\theta)C_y{}^a(\theta) \\ D_c(\theta) &= D_{zw}(\theta) + D_{zu}{}^a(\theta)K(\theta)D_{yw}{}^a(\theta) \end{aligned} \quad (6)$$

where:

$$A^a(\theta) = \begin{bmatrix} A(\theta) & 0 \\ 0 & 0_{nk\times nk} \end{bmatrix},$$

$$B_w{}^a(\theta) = \begin{bmatrix} B_w(\theta) \\ 0 \end{bmatrix}, \quad B_u{}^a(\theta) = \begin{bmatrix} 0 & B_u(\theta) \\ I_{nk\times nk} & 0 \end{bmatrix},$$

$$C_z{}^a(\theta) = \begin{bmatrix} C_z(\theta) & 0 \end{bmatrix}, \quad C_y{}^a(\theta) = \begin{bmatrix} 0 & I_{nk\times nk} \\ C_y(\theta) & 0 \end{bmatrix},$$

$$D_{zu}{}^a(\theta) = \begin{bmatrix} 0 & D_{zu}(\theta) \end{bmatrix}, \quad D_{yw}{}^a(\theta) = \begin{bmatrix} 0 \\ D_{yw}(\theta) \end{bmatrix}.$$

When compared to the conventional two-step synthesis framework for gain scheduling control (Ref. 2), the *LPV* description is clearly a good basis for one-step synthesis and the framework introduced here possesses a strong form of robust stability with respect to time-varying parameters and has the clear advantage over the others in exploiting the realness of the parameters, thus producing a less conservative design.

However, a principal difficulty in solving the *LMIs* problem appears to be the infinite number of constraints imposed by (5); a convex feasibility problem with an infinite number of constraints. For feasibility, one must normally resort to finding a grid of the parametric space $\mathcal{P}$ on which to solve for approximations to the infinite problem.

A particularly interesting case is the class of *LPV* plants where the state space matrices depend affinely on a time-varying parameter θ that varies in a polytope $\mathcal{P}$ of vertices $\omega_1, \omega_2, \ldots, \omega_r$, i.e. $\theta(t) \in Co\{\omega_1, \omega_2, \ldots \omega_r\}$ $:= \left\{\sum_{1}^{r} \alpha_i \omega_i : \alpha > 0, \sum_{i=1}^{r} \alpha_i = 1\right\}$. Under some feasibility assumptions on $D_{yu}(\theta), B_u, C_y, D_{zu}, D_{yw}$ and the pairs $(A(\theta(t)), B_u)$ and $(A(\theta(t)), C_y)$ (Ref. 1), the plant system matrix $P(\theta)$ can be defined to be in a matrix polytope with vertices $P(\omega_i)$, i.e.

$$P(\theta): \begin{pmatrix} A(\theta) & B_w(\theta) & B_u \\ C_z(\theta) & D_{zw}(\theta) & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{pmatrix}.$$

$$\in Co\left\{\begin{pmatrix} A_i & B_{wi} & B_{ui} \\ C_{zi} & D_{zwi} & D_{zui} \\ C_{yi} & D_{ywi} & D_{yui} \end{pmatrix} := \begin{pmatrix} A(\omega_i) & B_w(\omega_i) & B_u \\ C_z(\omega_i) & D_{zw}(\omega_i) & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{pmatrix}, i=1,\ldots,r\right\} \quad (7)$$

It therefore seems justified to design a polytopic form of the controller along the same projections of $\theta(t)$ on the vertices ω_i (with the same $\alpha_i, \{i=1,\ldots,r\}$, which are measured on-line):

$$K(\theta): \begin{pmatrix} A_k(\theta) & B_k(\theta) \\ C_k(\theta) & D_k(\theta) \end{pmatrix}$$

$$\in Co\left\{\begin{pmatrix} A_{ki} & B_{ki} \\ C_{ki} & D_{ki} \end{pmatrix}, i=1,\ldots,r\right\} := \sum_{i=1}^{r} \alpha_i K_i = \sum_{i=1}^{r} \alpha_i \begin{pmatrix} A_{ki} & B_{ki} \\ C_{ki} & D_{ki} \end{pmatrix} \quad (8)$$

Routine for gain-scheduling controller synthesis

The basic design routine for the affine *LPV* based quadratic H^∞ performance gain-scheduling controller comprises:

- Compute a single *Lyapunov* matrix $X_c = X_c' > 0$ satisfying all the r convex constraints (5) for the vertices $\omega_i (i=1,\ldots,r)$ of the parameter polytope;
- Define the *LPV* controller $K(\theta)$ as affine and therefore an 'interpolation' of the vertex controllers K_i. Once the *Lyapunov* matrix X_c has been determined, adequate vertex controllers $K_i \{i=1,\ldots r\}$ can be calculated (off-line) by solving the corresponding convex optimisation at each of the vertex points $\omega_i \{i=1,\ldots,r\}$, employing standard *LMIs* routines.
- The gain-scheduling control $K(\theta)$(4) is updated on-line in real time based on the measurement of parameter $\theta(t)$ and its decomposition (α_i), enforcing the expected quadratic performance over the entire parameter polytope $\mathcal{P}$ and along arbitrary parameter trajectories.

This particular control synthesis procedure is included in the recent *Matlab LMI Control Toolbox* (Ref. 3) from which some principal m-functions have been used in our design work.

A configuration for *2-DOF* gain-scheduling controller synthesis

In most cases, control synthesis is based on an augmented open-loop plant model plus various auxiliary weights, through which different closed-loop control strategies, e.g. model-tracking, *2-DOF* (Ref. 4, 5) control etc., can be realised through the optimisation of a designated input/output response.

For the affine model based quadratic H^∞ performance gain-scheduling controller design, one has to bear in mind that if a basic *LPV* plant model is affine in $\theta(t)$ (fortunately, this exists in many practical situations), the augmented open-loop configuration developed has normally to maintain the affiness property to achieve the polytope form of gain-scheduling control.

A control configuration suitable for *2-DOF* gain-scheduling control is shown in Fig. 2.

For $w\ (w_1\ w_2) \rightarrow z\ (z_1\ z_2)$, this set up has the following state-space (system) description:

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_d \\ \dot{x}_m \\ z_1 \\ z_2 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} A_p(\theta) & 0 & 0 & 0 & B_p(\theta) & B_p(\theta) \\ 0 & A_0(\theta) & 0 & \rho B_0(\theta) & 0 & 0 \\ 0 & 0 & A_m(\theta) & 0 & 0 & B_m(\theta) \\ \rho C_{sel} C_p(\theta) & -\rho C_0(\theta) & 0 & -\rho^2 D_0(\theta) & \rho C_{sel} C_p(\theta) & \rho C_{sel} D_p(\theta) \\ 0 & 0 & C_m(\theta) & 0 & 0 & D_m(\theta) \\ 0 & 0 & 0 & \rho I & 0 & 0 \\ C_p(\theta) & 0 & 0 & 0 & D_p(\theta) & D_p(\theta) \end{bmatrix} \begin{bmatrix} x_p \\ x_d \\ x_m \\ w_1 \\ w_2 \\ u \end{bmatrix} \quad (9)$$

where:

$W_d := \begin{bmatrix} A_0(\theta) & B_0(\theta) \\ C_0(\theta) & D_0(\theta) \end{bmatrix}$ is a tracking model,

$W_m := \begin{bmatrix} A_m(\theta) & B_m(\theta) \\ C_m(\theta) & D_m(\theta) \end{bmatrix}$ is an uncertainty weighting,

$W_e = \rho I$ is a performance weighting, *pilot* $= \rho I$, and C_{sel} is an output selection matrix.

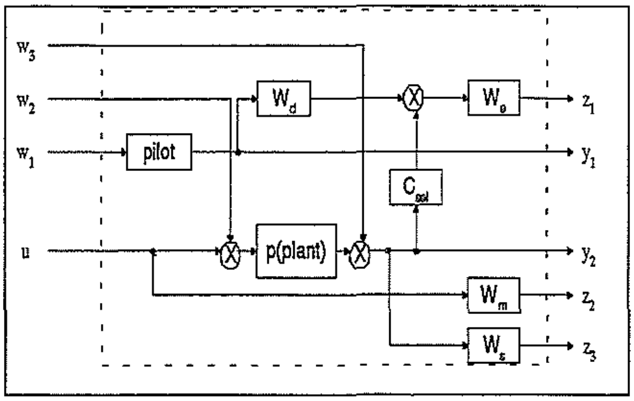


Fig. 2 *2-DOF* Control Configuration

As the description (9) reveals, when the performance weight W_e is chosen as a constant gain ρI (or some other parameter-independent dynamics), the augmented open-loop configuration is readily made affine, provided the basic plant model is affine. However, the modelling uncertainties may not be precisely incorporated into the model, (and hence into the synthesis), throughout the whole operating envelope, owing to the heuristic and constrained design of $W_m(\theta)$.

3. Affine *LPV* Modelling of Helicopters

Modelling of the helicopter longitudinal dynamics

Our study starts with a family of 6th-order (4th order plant + 2nd order actuator dynamics) linearised models representing helicopter longitudinal dynamics. The models are derived from a non-linear helicopter model of the Westland *Lynx* trimmed at a series of even-interval forward flight velocities (the scheduling variable U) throughout the flight envelope, ranging from 0 to 160 knots.

The family of linear models can be put into a parameter (U)-dependent model, which, in this particular case, has the form:

$$\begin{aligned} \dot{x} &= A_H(U)x + B_H(U)u \\ y &= C_H(U)x + D_H(U)u \end{aligned} \tag{10}$$

where the six states: $x=[u_b\ w_b\ q\ \vartheta\ \Theta_{0s}\ B1_s]'$, the two control inputs $u=[\Theta_0\ B1]'$, and u_b, w_b are the forward and vertical linear body velocities, respectively, q is the body pitch angular velocity, ϑ is the body pitch angle, Θ_0 is the main rotor collective input and Θ_{0s} is the state from its associated 1st-order actuator dynamics, $B1$ is the longitudinal cyclic control input and $B1_s$ is the state from its 1st-order actuator dynamics.

As expected, the introduction of the actuator dynamics in the two control channels makes the control matrix $B_H(U)$ parameter-independent. Suppose the actuator dynamics modelled for these two channels are parameter-independent with 1st-order models $\frac{a_{mc}}{s+a_{mc}}$ and $\frac{a_{lc}}{s+a_{lc}}$. Then $A_H(U)$ and $B_H(U)$ in (10) become:

$$A_H(U)=\begin{bmatrix} a_{11}(U) & a_{12}(U) & a_{13}(U) & a_{14}(U) & a_{15}(U) & a_{16}(U) \\ a_{21}(U) & a_{22}(U) & a_{23}(U) & a_{24}(U) & a_{25}(U) & a_{26}(U) \\ a_{31}(U) & a_{32}(U) & a_{33}(U) & a_{34}(U) & a_{35}(U) & a_{36}(U) \\ 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_{mc} & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_{lc} \end{bmatrix}.$$

$$B_H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ a_{mc} & 0 \\ 0 & a_{lc} \end{bmatrix}. \tag{11}$$

The matrices $C_H(U)$ and $D_H(U)$ in the output equation are apparently dependent on the choice of the output variables, for which two factors are taken into account: 1) the system should be detectable/observable from the output variables, 2) the output variables should comprise those to be controlled under a handling specification.

With reference to the handling qualities specification for rotorcraft, *ADS33C* (1989), for the basic handling mission modes in longitudinal flight [(1) Attitude Command with Attitude Hold (*ACAH*), (2) Rate Command (*RC*) and (3) Transitional Rate Command with Position Hold (*TRCPH*)] three principle output variables are selected: the vertical velocity w_b, the angular pitch rate p and the pitch angle ϑ. These define a simple and parameter-independent form of the output equation in (10):

$$C_H = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}; \quad D_H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{12}$$

An analysis of the un-augmented helicopter plant models reveals that:

- The helicopter has a natural instability in the longitudinal dynamics throughout the whole flight envelope.
- The open-loop plant has a considerably low transfer gain in the body pitch control channel.
- There is a considerable change in the entries of the parameter-dependent system matrix $A_H(U)$ as the scheduling variable U varies from 0 to 160 knots, which results in a large variation in the plant dynamics, in terms of the eigenvalues, across the operational region.

Affine *LPV* modelling

Here the aim is to determine an *LPV* model in the form of (11) which is 1) affine in the scheduling variable U, and 2) a good representation of, or a close approximation to, the family of linear *Lynx* models.

One general and direct method of affine modelling is to treat each of the parameter-dependent entries, $a_{ij}(U)$, in the matrix $A_H(U)$ (11) as an independent parameter variable with a bound $a_{ij\text{-bound}}$ defined on U, which results in the following affine *LPV* model with the parameter vector $\theta=[a_{11}\ a_{12}\ \ldots\ a_{ij}\ \ldots]\ (i=1\sim3,\ j=1\sim6)$:

$$A_H(P) = A_{H0} + [A_{11} A_{12} \ldots A_{ij} \ldots]\theta' \qquad (13)$$

where:

$$A_{H0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_{mc} & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_{lc} \end{bmatrix};$$

$$A_{ij} = \begin{bmatrix} 0 & \cdots & & 0 \\ & & j & \\ \vdots & & \downarrow & \vdots \\ & i \rightarrow 1 & & \\ 0 & \cdots & & 0 \end{bmatrix} (i = 1\sim3, j = 1\sim6).$$

The major advantage of this kind of modelling clearly lies in the exact match between the affine model and the original *LPV* model. However, a fundamental problem in practice is the fact that for large, or even reasonable, sized parameter-dependent systems, it produces a large number (of the order of 2^m, where m is the total number of the independent variables taken into account) of vertices upon which the polytope of the parameter vector is defined. Even in our example of a simplified longitudinal system, for the 18 parameters in the A_H this modelling process will bring about $2^{18}=262144$ vertices! Since this number is also that of the sets of *LMIs* involved in the convex optimisation process, the approach will inevitably result in a massive or even impractical computational task with current resources.

Also, for many practical *LPV* systems (such as helicopters) where the parameter variations are dependent on, or defined by a few parameter variables, the actual parameter variation domain can only form a very limited subspace in the convex hull of the vertices from the above modelling process. Therefore any ignorance of this special dependence or constraint will inevitably produce a conservative design. This has been seen in some of our earlier gain-scheduling designs where some designated handling quality objectives could hardly be reached. It would therefore seem sensible to reduce the number of independent parameters (from 18 in the helicopter example) to a reasonable level.

At the other extreme, if each of the dependent parameters can be put, or approximately put, into an affine function of the independent parameters, in our case for example, $a_{ij}(U)=K_{0ij}+K_{ij}*U\ (i=1\sim3,\ j=1\sim6)$, (11) will be transferred into a very simple affine model

$$A_H(U) = \begin{bmatrix} K_{011} & K_{012} & K_{013} & K_{014} & K_{015} & K_{016} \\ K_{021} & K_{022} & K_{023} & K_{024} & K_{025} & K_{026} \\ K_{031} & K_{032} & K_{033} & K_{034} & K_{035} & K_{036} \\ 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_{mc} & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_{lc} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} U \qquad (14)$$

where the only parameter variable is U and the number of vertices is 2, corresponding to the minimum (0 (knots)) and maximum (126 (knots)) bounds of U.

Clearly, the feasibility of this modelling approach will depend on the extent to which each U-dependent entry in $A_H(U)$ can be approximated by an affine function of U, assuming that the associated derivations can be tolerated by the robustness properties of the controller.

An examination of the feasibility of fitting each of the U-dependent entries in the state matrix $A_H(U)$ with a proper affine function was made through a specially developed *Matlab* m-function. It demonstrates that the majority of these entries can be reasonably approximated by linear/affine fittings.

Following this approach, a trial gain-scheduling design was made based on the affine *LPV* model of the helicopter with $A_H(U)$ (11). Due to the very simplified model, the control synthesis became feasible and effective. However, the resulting gain-scheduling design had very poor robustness with regard to the original plant model. It was observed that once the affine design model was replaced by its corresponding real plant model at an operating point, the closed-loop performance deteriorated and in some cases even went unstable.

The only explanation for this appears to be that the simplification went too far and the errors resulting from the modelling were beyond the tolerance allowed by the robust control. Actually, for some parameters, e.g. $a_{21}(U)$ (Fig. 3), use of linear fittings was indeed very risky and, as observed, contributed to the major errors in the modelling.

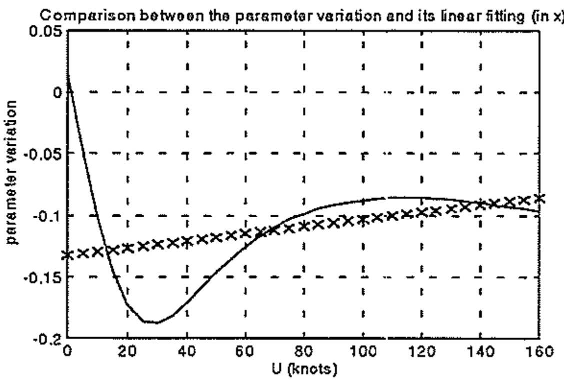


Fig. 3 Entry $a_{21}(U)$ and its approximation

Based on these studies and experience, a hybrid method for finding an affine *LPV* model is proposed. As a natural combination of the two approaches introduced above, it pursues an affine *LPV* model by fitting those of the matrix entries having, or approximately having, a linear dependence on the scheduling variables with affine functions, while taking the others which not only cannot be quite so fitted but also very influential, such as $a_{21}(U)$ and $a_{31}(U)$, as independent bounded parameter variables.

This has proven to be a good and effective modelling strategy for gain-scheduling control, and a useful compromise between feasible modelling for controller synthesis and accurate modelling. Following this approach for the helicopter plant where a_{21} was taken as an 'extra' independent parameter variable, making $m=2$ (U and a_{21}, 4 vertices), the affine model of the state matrix $A_H(U)$ becomes:

$$A_H(U)=\begin{bmatrix} K_{011} & K_{012} & K_{013} & K_{014} & K_{015} & K_{016} \\ 0 & K_{022} & K_{023} & K_{024} & K_{025} & K_{026} \\ K_{031} & K_{032} & K_{033} & K_{034} & K_{035} & K_{036} \\ 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_{mc} & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_{lc} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ 0 & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} U + \begin{bmatrix} 0 & \cdots & 0 \\ 1 & 0 \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} a_{21} \tag{15}$$

where U and a_{21} vary within: $U \in [0\ 160]$, $a_{21} \in [a_{21min}\ a_{21max}]$.

4. Synthesis for Gain-Scheduled Controllers

2-DOF gain-scheduling H^∞ control objective

A generalised gain-scheduling H^∞ performance control design based on *LPV* modelling can be described within the framework of Fig. 4 below:

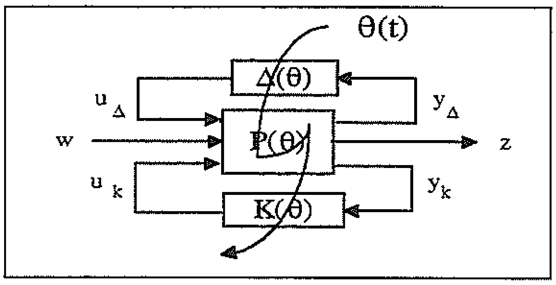


Fig. 4 H^∞ performance design framework

The control objective can be stated as the minimisation, over all possible *LPV* controllers, $K(s, \theta)$, of the H^∞ performance (the induced L^2-norm) of the closed-loop *LPV* system $T_{wz}(s, \theta)$, from w (the exogenous input) to z (the plant output), under the uncertainty perturbation block $\Delta(s, \theta)$ and over the whole compact parameter set θ upon which the *LPV* plant model, $P(s, \theta)$, is defined.

In the case when a *2-DOF* H^∞ performance control problem adopts the design configuration as in Fig. 2, the controllers consist of both a feedforward control, k_1, from y_1 (the pilot), and a feedback control, k_2, from the plant output y_2 (Fig. 5)

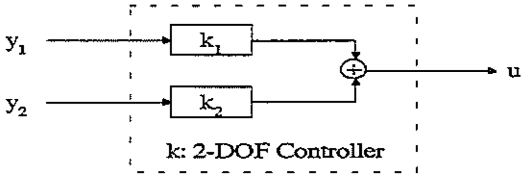


Fig. 5

And the control objectives at each frozen parameter vector, θ, can be further expressed in terms of the standard H^∞ norm optimisation:

$\|T_{wz}\|_\infty \to min.$ (θ is omitted for simplification) (16)

where one of the following modes of operation can for example be selected:

Mode 1. For $w\,(w_1\; w_2\; w_3) \to z\,(z_1\; z_2\; z_3)$:

$$T_{wz}=\begin{bmatrix} W_e(C_{sel}(I-Pk_2)^{-1}Pk_1-W_d)pilot & W_eC_{sel}(I-Pk_2)^{-1}P & W_eC_{sel}(I-Pk_2)^{-1} \\ W_m(I-k_2P)^{-1}k_1pilot & W_m(I-k_2P)^{-1}k_2P & W_m(I-k_2P)^{-1}k_2 \\ W_s(I-Pk_2)^{-1}Pk_1pilot & W_s(I-Pk_2)^{-1}P & W_s(I-Pk_2)^{-1} \end{bmatrix}$$

(W_s: the uncertainty weighting at the sensor point) (17)

This is an overall synthesis mode aiming for optimisation of model-following control ($w_1 \to z_1$), and robustness with respect to multiplicative uncertainties at both the actuator point ($w_2 \to z_2$) and at the sensor point ($w_3 \to z_3$).

Mode 2: For $w = (w_1\; w_2),\; z = (z_1\; z_2)$:

$$T_{wz}=\begin{bmatrix} W_e(C_{sel}(I-Pk_2)^{-1}Pk_1-W_d)pilot & W_eC_{sel}(I-Pk_2)^{-1}P \\ W_m(I-k_2P)^{-1}k_1pilot & W_m(I-k_2P)^{-1}k_2P \end{bmatrix} \quad (18)$$

aims at model-following control ($w_1 \to z_1$) and robustness with respect to multiplicative uncertainties at the actuator point ($w_2 \to z_2$).

Mode 3: For $w = w_1,\; z = (z_1\; z_2)$:

$$T_{wz}=\begin{bmatrix} W_e(C_{sel}(I-Pk_2)^{-1}Pk_1-W_d)pilot \\ W_m(I-k_2P)^{-1}k_1pilot \end{bmatrix} \quad (19)$$

is the mode for model-following control and a constraint on the control output.

Most of the uncertainties resulting from the modelling of helicopters are associated with the rotors and may be put into multiplicative uncertainties at the front actuating point, $P_H= P(1+\Delta)$. For this reason, and also for simplicity, mode 2 was used for defining the objectives of the gain-scheduling control. In this case, the exogenous inputs are w_1 (reference inputs) and w_2 (uncertainty perturbations), whilst the control outputs are z_1 (weighted tracking errors) and z_2 (weighted controller outputs). From formula (18), it can be seen that by appropriate choices of the sensitivity weighting, W_e, which balances the demands for desired handling and disturbance rejection, and the control weighting, W_m (on $(I-k_2P)^{-1}k_1pilot$ and $(I-k_2P)^{-1}k_2P$), the gain-scheduling controller generated will guarantee, in the sense of the H^∞ performance optimisation, a closed-loop system at each operating point which follows the desired performance requirements (in W_d), while maintaining guaranteed robustness in the face of modelling uncertainty within the plant.

Design of weighting functions

Weighting function W_d: For helicopter control, the open-loop interconnection for controller synthesis (Fig. 2) makes it possible for W_d to adopt directly the frequency-defined handling qualities specification that the closed-loop system should follow. These are given in *ADS33C* (Ref. 6) which formulates the specification as a series of transfer functions relating pilot inputs and vehicle responses of interest.

According to the qualities specification, for the *RC* and *TRCPH* handling modes, the desired transfer functions for the vertical velocity (*w*), roll rate (*p*), pitch rate (*q*) and yaw rate (*r*) can be modelled as first-order systems, while for the *ACAH* mode, the pitch attitude ϑ and the roll attitude ϕ are of great importance and normally presented as second-order models.

A typical example of the function W_d for control of the longitudinal flight, with output variables ($w_b\; q\; \vartheta$), is:

$$W_d=\begin{bmatrix} \frac{2.0}{s+2.0} & 0 \\ 0 & \frac{4.3}{s^2+2.93s+4.3} \end{bmatrix} \quad \text{or:} \quad W_d=\begin{bmatrix} \frac{2.0}{s+2.0} & 0 \\ 0 & \frac{4.0}{(s+4.0)} \end{bmatrix}$$

$(pair(w_b,\vartheta)\,for\,ACAH\&TRCPH)$ $(pair(w_b,q)\,for\,RC\&TRCPH)$

(20)

where a fast mode (λ_d = *4* (rad.s^{-1})) is assigned to pitch rate q for the demanded *RC* control. $\lambda_w = 2$ is assigned to the vertical velocity for Level 1 heave dynamics. For pitch angle ϑ, standard Level 1 parameters of $\omega_\vartheta = 2.071$ (rad.s^{-1}) and $\zeta_\vartheta = 0.707$ are used. The diagonal W_d structure also implies de-coupled model following control.

<u>**Weighting function W_m:**</u> This weight has the role of describing the model uncertainties and constraining the control outputs (refer to (18)), both of which generally require the weight to have a high-pass characteristic.

LTI controller synthesis suggests a weight with equal emphasis on the uncertainty description and the control constraint, e.g.

$$W_m = \begin{bmatrix} 0.5\frac{(s+0.1)}{(s+10)} & 0 \\ 0 & 0.5\frac{(s+0.1)}{(s+10)} \end{bmatrix} \quad (21)$$

This defines good robustness at the actuating point for all the *LTI* designs throughout the model range. But experience with this weight for *LPV* gain-scheduling control suggests a weight with much smaller gains, i.e. a more relaxed constraint on the control outputs, e.g.

$$W_m = \begin{bmatrix} 0.05\frac{(s+0.1)}{(s+10)} & 0 \\ 0 & 0.05\frac{(s+0.1)}{(s+10)} \end{bmatrix} \quad (22)$$

<u>**Weighting function W_e:**</u> this is the so-called performance weighting which is used to scale the model-following criteria. A dynamic form of the weighting was used in the synthesis to achieve a good trade-off between the requirements for model-following and disturbance rejection.

For the longitudinal control case, W_e adopts different bandwidths for the vertical velocity (6 rad.s^{-1}) and the pitch angle (10 rad.s^{-1}), respectively, to cope with the different tracking models. Two steps were involved in this particular weighting development for gain-scheduling control, step 1: search for a suitable weighting W_e for each of the linear models in the family, through use of the standard linear time-invariant (*LTI*) H^∞ control design and analysis for these 'frozen' models; then step 2: evaluate and, if necessary, modify the universal weighting from step 1 to generate an appropriate performance weighting for *LPV* gain-scheduling control.

For step 1, a typical performance weighting design for the *ACAH/TRCPH* mode is:

$$W_e = \begin{bmatrix} \frac{6}{(s+6)}\times 0.1 & 0 \\ 0 & \frac{10}{(s+10)} \end{bmatrix} \quad (23)$$

which, in view of the small singular value in the open-loop pitch control channel, has a relatively large gain in the second channel to bring about satisfactory control for all the *LTI* models throughout the operational region.

However, as expected, direct use of this same weighting in the *LPV*-model based gain-scheduling control design revealed that the effort to stabilise the plant within a much expanded polytopic space (owing to the introduction of some extra independent parameters) results in poor handling control in the pitch channel over the mid-frequency range of interest.

To reduce conservatism and to improve pitch handling, the weighting for the pitch was modified and, in particular, an extra pole and zero were introduced to make the weighting more centred and effective in the low/mid-frequency (0.1 rad ~ 10 rad.) range to boost performance matching. A typical example of a modified weight is:

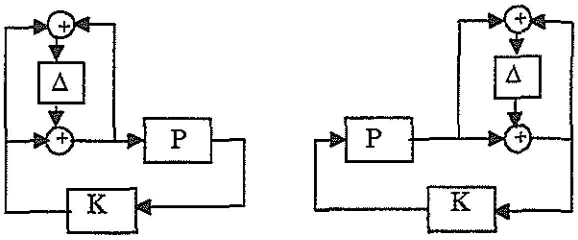
$$W_e = \begin{bmatrix} \frac{6}{(s+6)}\times 0.1 & 0 \\ 0 & \frac{10(s+0.001)}{(s+10)(S+0.01)} \end{bmatrix} \quad (24)$$

As the later results show, this produces satisfactory handling control in both the vertical and pitch manoeuvres.

<u>**Robustness evaluation**</u>

A robustness evaluation of a closed-loop *LTI* system with plant *P* and controller *K* can be achieved by use of singular value(σ) analysis, structured singular value (μ) analysis, and associated *MIMO* gain and phase margins (Ref. 7). From robustness indicators at various perturbation points of interest, guidelines for the refinement of the controller synthesis can be formulated.

Uncertainty Perturbation Structures. Two uncertainty perturbation structures of interest were considered. They are multiplicative uncertainty at the actuator side of the plant (Fig. 6(a)), and multiplicative uncertainty at the sensor side of the plant (Fig. 6(b)). The robustness evaluation considers the transfer functions 'seen' by the mixed feedforward/feedback multiplicative uncertainty blocks, i.e. transfer functions $M=(I-KP)(I+KP)^{-1}$ for Fig. 6(a) and $M=(I-PK)(I+PK)^{-1}$ for Fig. 6(b).



(a) (b)

Fig. 6 Multiplicative uncertainty perturbation structures

Synthesis routine and software development

Gain-scheduling controller designs were performed in *Matlab* using primarily the *LMI Control Toolbox* (Ref. 3). For transferring a practical helicopter control problem into the standard gain-scheduling controller synthesis module and incorporating the principles of *2-DOF* H^∞ control within the design, some specified and user-defined *Matlab* m-functions were developed. These together with some other auxiliary m-functions, made for setting up weightings, and various forms of system evaluation (including μ-analysis) etc., are used in the controller synthesis routine.

5. Application of Gain-Scheduling Control to Helicopter Longitudinal Flight

Following the various procedures introduced in the previous sections for the modelling, controller synthesis and closed-loop system evaluation and analysis, the *2-DOF* gain-scheduling control methodology was applied to the design of a longitudinal flight controller for a *Lynx* helicopter.

***2-DOF* configuration and controller synthesis — example**

Here a synthesis for the gain-scheduling control is presented, with the plant being modelled as (15), the open-loop control configuration being the *2-DOF* form as in Fig. 2 and the performance objective as (18). The relevant weightings were defined as (20) for W_d (*ACAH/TRCPH* mode), (22) for W_m and (24) for W_e. The pilot input gain matrix *pilot* was unity and the output selection matrix $C_{sel} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

The synthesis brings an optimal (minimum) solution for the H^∞ performance: $\gamma_{min} = 0.95$.

Evaluation and simulation

Remember the synthesis process actually produces a family of gain-scheduling controller vertices corresponding to the 2^n parameter vertices (corner vectors), $\omega_{i(i=1,\ldots n)} = \begin{pmatrix} A_{ki} & B_{ki} \\ C_{ki} & D_{ki} \end{pmatrix}$, see (8). The gain scheduling control, $K(\theta)$, is a polytope of these vertices and is formed/updated on line in real time along the same projections of the polytope of $\theta(t)$ measured.

***Case 1*:** evaluation of the time-varying gain-scheduled control system. This is based on a known/pre-defined time-varying trajectory $\theta(t)$, upon which both the plant $P(\theta)$ and controller $K(\theta)$ are defined. Time-domain simulation is mainly used for this case.

***Case 2*:** evaluation of the closed-loop system under gain-scheduling control at selected operating points, with an assumption of frozen θ at these evaluation points. This brings the convenience of incorporating any original *LPV* plant models and has the advantage that *LTI* models and analysis can be used.

Evaluation case 1 — time-varying gain-scheduled control

This was performed upon the linkup between the designed gain-scheduled controller and the family of *Lynx LTI* models. A *Matlab* function group, with the main function *PDSIMUT4.m*, was specially developed for this purpose.

Case description: suppose starting from a hover ($U=0$) state, the helicopter undergoes 20 seconds of constant acceleration (10.12ft/s^2), with a change of forward speed from hover (0 (knots)) to 120 (knots). During the process, two step inputs from the pilot, for vertical velocity (5 ft/s) and pitch angle (5 deg.), respectively, are imposed to the system for a 10 seconds period from the starting point of hover.

Fig. 7(a) shows the time-varying patterns of the two parameter variables, the forward speed $U(t)$ and the entry $a_{21}(t)$, which cover most of the parameter polytope (convex hull).

Fig. 7(b) and 7(c) show the control and stabilisation of the two major system variables, the heave velocity and pitch angle, from the time-varying gain-scheduling controller, with Fig. 7(b) for the step control input $w_1=[5\ 0]'$ and Fig. 7(c) the control $w_1=[0\ 5]'$.

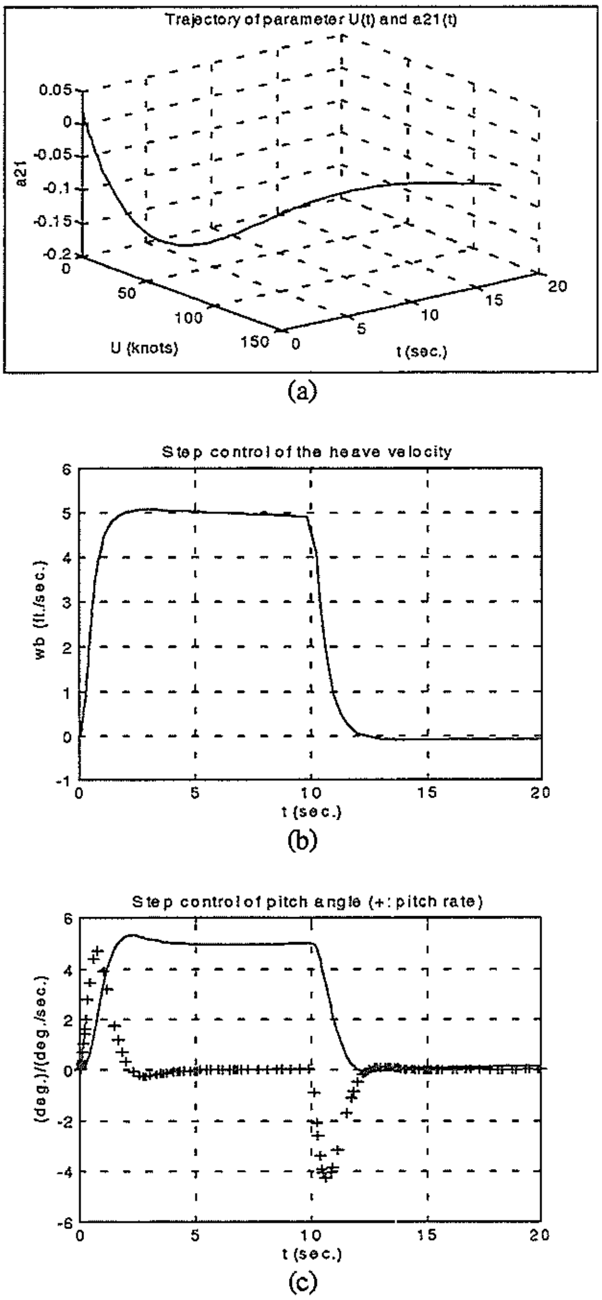


Fig. 7 Simulation of *LPV* gain-scheduling control

This can be viewed as an extreme evaluation, in view of the variations of the parameters and *LPV* model covers large parts of the polytope. The time-domain simulations demonstrate the ability and effectiveness of the gain-scheduling control on stability and almost perfect handling of the time-varying helicopter longitudinal dynamics. As far as the flight handling quality specification is concerned, both the vertical and pitch controls reach the *level-1* handling quality for the *ACAH/TRCPH* mode.

<u>Evaluation case 2</u> — gain-scheduling control at frozen operating points

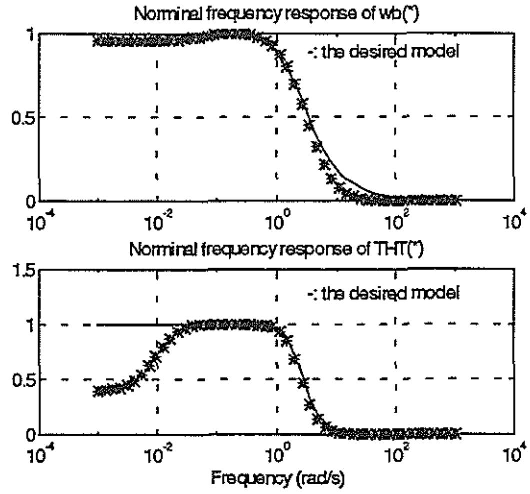
This was based upon the *LTI* closed-loop systems generated by interconnecting the original *LPV* plant models of the helicopter with the corresponding gain-scheduled controllers at a series of frozen operating points selected throughout the operational range. For example, for evaluation of the *LTI* closed-loop helicopter system at hover, firstly find θ at $U=0$, θ_0, then get the particular controller, $k(\theta_0)$, from the gain-scheduling controller polytope and link it with the linearised plant model for this point taken from the family of plant models given.

Evaluation of control: In response to pilot inputs, both the time- and frequency-responses of the two longitudinal output variables of most interest, the vertical velocity w_b ($\approx -\dot{h}$ for level-off flight) and the pitch angle ϑ, were evaluated at various operating points.

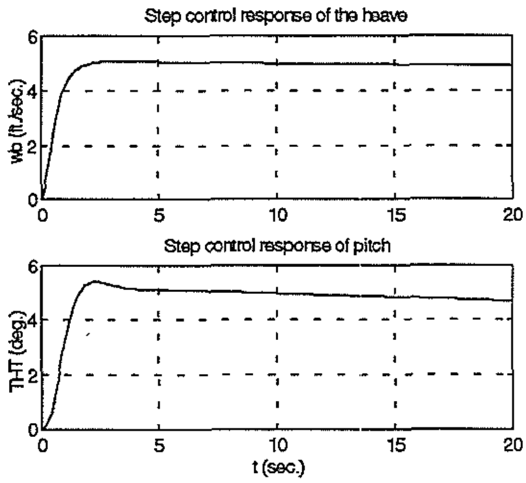
The evaluation reveals that due to the very small gain in the pitch control channel at low frequency, special measures such as the magnification of the performance weighting, W_e, as (24) are required to boost the pitch control effect.

Fig. 8 shows both the frequency and time-domain responses of w_b to step Θ_0 (main rotor collective control), and of ϑ to step B1 (longitudinal cyclic control). It can be seen from the frequency response of ϑ in Fig. 8(a) that its mid-range frequency response has been enhanced to match the desirable response (in '-'), bringing satisfactory handling control in both heave and pitch, see Fig. 8(b).

Fig. 9 and 10 show the same responses for a medium forward speed, $U=60$(knots), and a high speed, $U=120$(knots), respectively. Both present good handling control qualities of the two important output variables representing the *ACAH/TRCPH* mode. Systematic evaluation of the control from low speed to high speed also reveals that hover can be the most delicate state for control augmentation and demands some strong performance weighting to match the required performance objectives. However, along with the forward speed increase, there should ideally be a decreased performance weighting to match the increased plant gain in the pitch channel; this is clearly shown in the high speed case (Fig. 10) where some unexpected high gain response occurs at low frequency. This suggests the use of a parameter(U)-dependent weighting function to effect an improvement.



(a)



(b)

Fig. 8 Scheduled control at U= 0 (hover)

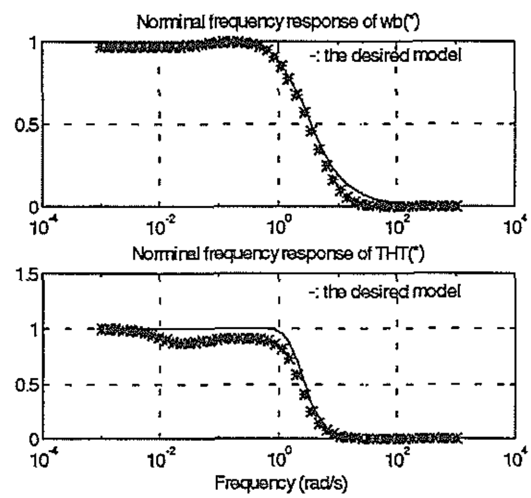


Fig. 9 Scheduled control at U= 60 (knots)

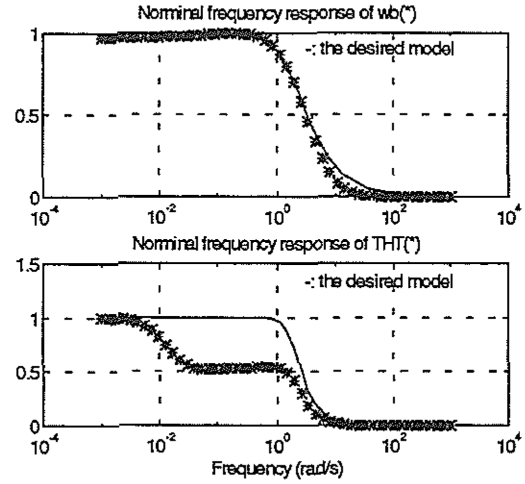


Fig. 10 Scheduled control at U= 120 (knots)

Evaluation of robustness: Finally, the robustness of the closed-loop system was examined, based on both singular value and μ analyses.

For robustness in the face of multiplicative uncertainty perturbations at the actuator side of the plant (Fig. 6(a)), and having the block (Δ) defined as: $\Delta := \{diag[\delta_1\ \delta_2]: \delta_i \in C\}$, the evaluation indicates good robustness, as expected from the design objectives (18), across the operating envelope in terms of the maximum stability tolerance for both structured and unstructured uncertainties. Results of analyses at some points of interest are shown below in Table 1, using a guaranteed *MIMO* gain and phase margin analysis (Ref. 7):

Table 1 Robustness at the actuator side.

U(knots)	$r_{\mu min}$ ($r_{\sigma min}$)	$GM_\mu(GM_\sigma)$ (±dB)	$PM_\mu(PM_\sigma)$ (±°)
0	0.77(0.63)	17.83(12.96)	75.36(64.65)
40	0.77(0.77)	17.67(17.64)	75.10(75.05)
80	0.76(0.75)	17.31(16.74)	74.47(73.44)
120	0.59(0.56)	11.83(10.99)	61.25(58.74)
160	0.37(0.33)	6.68(6.05)	40.25(37.02)

For the evaluation of robustness in the face of multiplicative uncertainty at the sensor side of the plant (Fig. 6(b)), the structured perturbation blocks for μ analysis are defined as $\Delta := \{diag[\delta_1\ \delta_2\ \delta_3];\ \delta_i \in C\}$. The results for the selected points are listed in Table 2.

In summary, the evaluation indicates that the designed H^∞ performance gain-scheduling controller enables the closed loop helicopter system to possess good robustness throughout the whole operational region. At the sensor side of the plant, although inspection of the singular values show relatively poor robustness, μ-analysis does suggest that the robustness can be much improved if the perturbations can be made de-coupled (i.e. confined to individual channels).

Table 2 Robustness at the sensor side.

U(knots)	$r_{\mu min}$ ($r_{\sigma min}$)	$GM_\mu(GM_\sigma)$ (±dB)	$PM_\mu(PM_\sigma)$ (±°)
0	0.59(0.028)	11.67(0.48)	60.75(3.17)
40	0.62(0.034)	12.62(0.60)	63.68(3.95)
80	0.62(0.05)	12.63(0.87)	63.69(5.70)
120	0.58(0.054)	11.39(0.93)	59.83(6.14)
160	0.41(0.018)	7.60(0.31)	44.75(2.04)

6. Concluding Remarks

This work appears to be the first to use *LPV/Lyapunov*-based quadratic H^∞ performance optimisation gain-scheduling control on a practical *MIMO* system, the helicopter.

A novel *2-DOF* control configuration was proposed and combined with gain-scheduling controller design. The roles of different weightings in the configuration were studied and valuable experience gained in weight selection for *LPV* gain-scheduling control. The resulting gain-scheduled flight control system possessed satisfactory handling qualities and robustness.

Affine modelling of *LPV* systems, in order to bring a practical design problem into the specified H^∞ performance gain-scheduling control framework, is another important issue. Although in many applications, *LPV* systems can be treated as affine, without considering the constraints of the parameters (e.g. the dependence of one parameter on another), the design will inevitably be conservative and inefficient. A contribution from this study has been the introduction and use of hybrid affine modelling of plants such as helicopters. This helps to bring a satisfactory compromise between the fidelity of the model and control effectiveness.

Application of the developed gain-scheduling methodology to longitudinal flight control of a *Lynx* demonstrated the effectiveness of the new approach to gain-scheduling control, and the promise it has for future use on large-scale *MIMO* control systems such as full 6-degrees of freedom helicopters.

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