# A PARAMETRIC STUDY ON WING DESIGN VARIABLES FOR TANDEM WING CONFIGURATION eVTOL AIRCRAFT


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## Abstract

The tandem wing configuration is an unconventional aircraft configuration with numerous beneficial aerodynamic and stability characteristics. It has recently gained popularity as a conceivable configuration for Electric Vertical Take-off and Landing (eVTOL) aircraft. An important factor in determining the performance of a tandem wing aircraft is the wing design that directly influences the aerodynamic efficiency and flying qualities. This paper examines this effect of multiple wing design parameters on the aerodynamics and stability for tandem wing aircraft through a parametric and optimization design study. A vortex lattice method model integrated with a Python code is used to perform the design study and facilitate the calculations of multiple specified cases in a short time. The mass properties are calculated automatically for different wing geometries and then an optimizer is used to improve the aircraft layout of the baseline tandem wing. The parametric study reveals the sensitivity of wing design parameters on the aerodynamic efficiency and flight stability. The study also highlights the interplay between wing structural weight and aerodynamic efficiency. The following optimization study provides a framework for maximizing the range of a tandem wing aircraft and generates interesting design concepts for a tandem wing eVTOL aircraft. It is demonstrated that the optimization framework provides an increase of at least 30% in the range parameter of a tandem wing aircraft with the inclusion of flight stability constraints. The study considers the interaction between aerodynamics, structural weight, and aircraft handling qualities. Overall, the results provide beneficial design insights to fixed wing and eVTOL aircraft designers.


## NOTATION

| | |
|---|---|
| AR | Aspect Ratio |
| $b$ | Wingspan, [m] |
| $C$ | Dimensionless Coefficient |
| $D$ | Drag Force, [N] |
| $e$ | Battery specific energy, [kWh/kg] |
| eVTOL | Electric Vertical Takeoff and Landing |
| $g$ | Gravitational acceleration, [m/s$^2$] |
| $L$ | Lift Force, [N] |
| $m$ | Mass, [kg] |
| MTGW | Maximum Takeoff Gross Weight, [kg] |
| $S$ | Area, [m$^2$] |
| $V$ | Airspeed, [m/s] |
| $W$ | Aircraft weight, [N] |
| $x$ | Longitudinal coordinate, [m] |
| **Greek** | |
| $\beta$ | Sideslip Angle, [deg] |
| $\Gamma$ | Dihedral Angle, [deg] |
| $\zeta$ | Damping Ratio |
| $\eta$ | Total efficiency |
| $\Lambda$ | Sweep angle, [deg] |
| $\lambda$ | Taper ratio |
| $\omega_n$ | Natural Frequency, [rad/s or Hz] |

**Subscripts**

| | |
|---|---|
| b | Battery |
| cg | Center of gravity |
| fw | Front wing |
| $l$ | Rolling moment |
| $m$ | Pitching moment |
| $n$ | Yawing moment |
| $q$ | Pitching rate |
| rw | Rear wing |
| spo | Short period oscillations |
| tot | Total aircraft |
| winglet | Winglet |

## 1. INTRODUCTION

In the past, aircraft designers have explored numerous different aircraft configurations. Among these configurations, the tandem wing configuration has not been a favoured design choice for a traditional aircraft. However, there are successful tandem wing aircraft in the past, for example, the Burt Rutan Quickie. In recent years, the tandem wing configuration has been gaining popularity especially in the design of eVTOL aircraft because the tandem wing configuration permits easier placement of many lift rotors for hover. Many prominent eVTOL aircraft such as the A$^3$ Vahana [1] and Volocopter Voloconnect [2] have incorporated the tandem wing aircraft configuration in their design. Some more notable examples of tandem wing eVTOL aircraft are shown in Figure 1.


**Figure 1 Notable Tandem Wing eVTOL Aircraft [3]–[11]**

An important aspect of the design of a tandem wing eVTOL are the independent wing parameters for both forward and aft wing surfaces. As the wing design choices directly influence the aircraft aerodynamic efficiency as well as the flight stability. With both performance aspects being vital for an eVTOL aircraft to achieve operational and commercial realization. The aircraft aerodynamic efficiency directly affects the achievable flight range while the flight stability affects the ease of piloting an aircraft and the comfort of passengers.

There have been a few notable studies in the past that have focused on elucidating wing design choices for a tandem wing aircraft. A study performed by Dikshit et. al. has provided a starting point for the study of wing design variables for a tandem wing aircraft [12]. The study compared the performance of the tandem wing aircraft configuration against a conventional wing-tail, canard, and flying wing configuration. The comparison study indicated that the tandem wing configuration has many potential benefits in terms of both aerodynamics and stability. The configuration showed excellent natural longitudinal stability characteristics and achieved good aerodynamic efficiency for a well-chosen wing design. The study has also provided an initial insight into the effects of wing design variables such as ratio of front wing and rear wing area, distance between wings and wingspan on the aerodynamics and stability characteristics of tandem wing aircraft.

Previous studies performed by Boling and Zha and Schoser et. al. have also illuminated the important aspects of the wing design for a tandem wing aircraft [10], [13]. Boling and Zha performed a numerical design and optimization study to improve the longitudinal stability of a tandem wing aircraft configuration. Their study concluded that a front wing with a smaller aspect ratio and larger incidence angle than the rear wing provided a marked improvement in the longitudinal stability of a tandem wing aircraft. A similar conclusion was reached for the wing aspect ratios in the study performed by Schoser et. al. which studied the stability and controllability of the Widgeon aircraft (Figure 1).

Another prominent study has been performed by Andrews and Perez to evaluate the longitudinal stability of dual wing aircraft configurations [14]. The authors identified an important conflict that arises between aerodynamics and stability in dual wing configurations. For dual wing configurations, there exists a specific centre of gravity position (c.g.) that provides the best aerodynamic efficiency. However, this c.g. position also makes the aircraft unstable. Such a conflict was also found by Dikshit et. al. in their study. Both Andrews and Perez explored the possibility of changing the front and rear wing areas to improve the stability of a dual wing configuration. They concluded that having a smaller front wing area could improve the aircraft longitudinal stability.

In addition to the existing parametric studies, it is also important to subsequently develop frameworks for the optimization of a tandem wing aircraft. Studies performed on the optimization of aircraft designs have shown that the design of an aircraft is impacted by the interplay between multiple variables and disciplines such as aerodynamics, structural weight, and propulsion mechanisms [15], [16]. For example, an increase in the wingspan could improves the aircraft aerodynamic efficiency but at the cost of an increased wing structural weight. In the context of an eVTOL aircraft, an increase in structural weight can be understood as a decrease in the available battery weight that can be carried within the allowable MTGW. As such, the aircraft performance might be affected by the reduction in battery weight even with improved aerodynamic efficiency. Hence, it is important to capture the interplay behaviour between the performance variables in the design of a tandem wing aircraft.

Past studies have only investigated a limited list of wing design variables and have not explored an exhaustive overview of the wing design of a tandem wing aircraft. As such, the first aim of this paper will be to provide a comprehensive parametric study of the wing design variables for a tandem wing eVTOL aircraft from the perspective of aerodynamics and flight stability. The comprehensive list of wing design variables covered within this study include wing aspect ratio, ratio of front wing to rear wing surface area, wing taper ratio, distance between the two wings, height difference between the front wing and rear wing, sweep angle for the rear wing, sweep angle for the front wing, dihedral angle, and placement of winglets.

Our tandem wing design studies have also focused on the interplay of design aspects such as aerodynamics, structural weight, and flight stability. It is important to highlight the interplay behaviour between these disciplines as it is an important factor in the determination of a feasible tandem wing design. Therefore, the second aim of the paper is to provide a multidisciplinary optimization framework for the aircraft design of a tandem wing aircraft based on aerodynamics, structural weight, and flight stability. Flight stability is a major discipline not covered in the other existing optimization studies and therefore, it is novel to include it here to perform a comprehensive optimization of the wing design.

# 2. DESCRIPTION OF PARAMETRIC AND OPTIMIZATION STUDY

This section describes the parametric and optimization study that is formulated to determine the best design choices for a tandem wing aircraft configuration.

## 2.1. Design of Baseline Tandem Wing Aircraft

A defined baseline tandem wing aircraft configuration was specified as a reference design for the parametric and optimization study. Figure 2 illustrates the complete geometry and surface model of the baseline aircraft that was used for vortex lattice method (VLM) calculations. The geometry comprises of two rectangular wings representing the most generic wing geometry for a tandem wing aircraft. The fuselage has been designed to resemble a generic cylindrical fuselage. The aircraft wings were designed with a NACA 2412 airfoil profile, while the vertical tail was designed with a NACA 0012 airfoil. These airfoils were chosen as the NACA airfoil profiles provided generality to the aircraft configuration used as a baseline. This baseline configuration has been defined with reference to the illustrated aircraft shown in Figure 1. The main interest is the effect of the aircraft wing design and therefore, it is sufficient to use the surface model to perform this study.

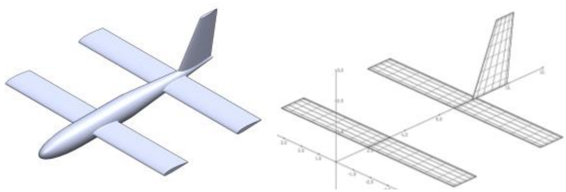


**Figure 2 Geometry and Surface Model of Baseline Tandem Wing Aircraft**

The baseline set of aircraft parameters are listed in Table 1. All aircraft dimensions are parameterized to easily perform different variations on the aircraft geometry and to investigate the effects on the aircraft performance. The baseline aircraft parameters were chosen to reflect realistic aircraft design choices for eVTOL aircraft. These choices were determined through previous trade studies on various eVTOL aircraft and guidelines set forth by aviation regulators.

**Table 1 Parameters of Baseline Tandem Wing Aircraft**

| Surface | Parameter | Values |
|---|---|---|
| Front wing | span, m | 10.00 |
| | aspect ratio | 7.69 |
| | area, $m^2$ | 13.00 |
| | incidence, deg | 3.256 |
| | airfoil | NACA2412 |
| | quarter chord $x_{fw}$, m | 2.1676 |
| Rear wing | span, m | 10.00 |
| | aspect ratio | 7.69 |
| | area, $m^2$ | 13.00 |
| | incidence, deg | 0.8891 |
| | airfoil | NACA2412 |
| | quarter chord $x_{rw}$, m | 7.1676 |
| | c.g. location $x_{cg}$, m | 3.5845 |
| | reference static margin | 0.25 |

The aircraft span was chosen to be 10 m to yield a compact aircraft dimension that can easily fit on all helipads. The reference MTGW was chosen to be 3175 kg which is in accordance with the recommendations in the EASA guidelines [17]. The total wing area was chosen to be 26 $m^2$ to reflect the typical wing loading of an eVTOL aircraft. A best range cruise speed ($V_{br}$) of 240 km/h was chosen to represent a competitive cruise speed for an eVTOL aircraft. The incidence angles of the front wing and rear wing were chosen to trim the aircraft at a typical angle of attack of 2.5° for an airspeed of 240 km/h.

The vertical tail was sized through a trade study of the ratio of vertical tail area and wing area from many existing conventional aircraft. The average ratio found from the study was 0.13 which was used in the design of the vertical tail. The other parameters of the vertical tail were chosen from statistical data provided in [18]. Further details on the method used for the geometry definition in this study can be found in [12].

An additional aircraft component defined in this study was the implementation of a pair of winglets. The design of an aircraft winglet is complex and may require a separate dedicated design study. However, for the purposes of this study, a simple winglet is designed with reference to the winglet design detailed in [19]. The root chord of the winglet is forced to be equal to the tip chord of the wing and employs the NACA 0012 airfoil profile. The winglet area is made a free variable to study its effect on the performance of a tandem wing aircraft. Table 2 summarizes the winglet design parameters, with a visualization of the employed winglet provided in Table 3.

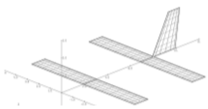
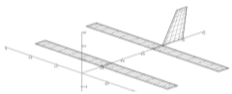
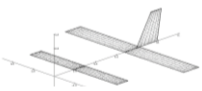
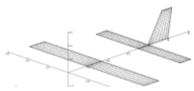
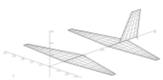
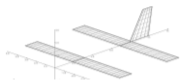
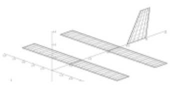
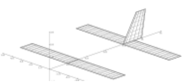
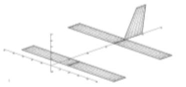
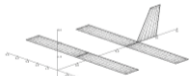
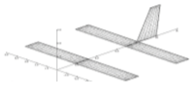
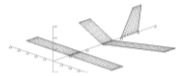
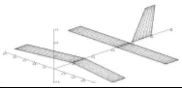
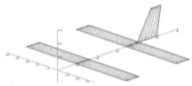
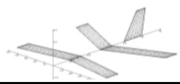
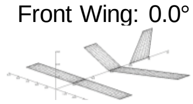
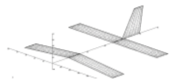
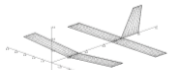
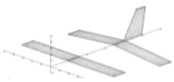
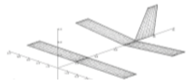
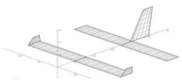
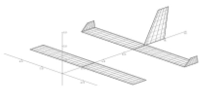
**Table 2 Parameters of Winglet Design**

| Parameter | Values |
|---|---|
| root chord, m | 1.30 |
| area, $m^2$ | variable |
| incidence, deg | -2.0 |
| airfoil | NACA0012 |
| taper ratio | 0.50 |
| quarter chord sweep, deg | 40 |
| cant angle, deg | 0 |
| twist, deg | 0 |

## 2.2. Wing Design Variables and Parametric Variations

A comprehensive set of wing design variables were chosen to elucidate the best design choices for a tandem wing aircraft. A summary of these design variables and variations studied are shown in Table 3. Each design parameter is varied independently to isolate the effect of each individual parameter on the aircraft aerodynamics and stability characteristics. When one parameter is varied, the other parameters are held at the baseline values given in Table 1.

**Table 3 Summary of Parametric Study Design Variables**

| Design Parameters | |
|---|---|
| **Lower Bound** | **Upper Bound** |
| Wingspan | |
| 9.0 $m$  | 13.0 $m$  |
| Front Wing Area Fraction (Ratio of front wing area to combined area of both wings) | |
| 0.35  | 0.65  |
| Wing Taper Ratio | |
| 0.2  | 1.0  |
| Horizontal Distance Between Wings (Distance between aerodynamic centres of the two wings) | |
| 4.0 $m$  | 6.0 $m$  |
| Height of Front Wing | |
| -0.5 $m$  | 0.5 $m$  |
| Rear Wing Sweep Angle | |
| 0.0°  | 25.0°  |
| Front Wing Sweep Angle | |
| -10.0°  | 0.0°  |
| Combined Sweep Angle (Rear Wing: 25.0°) | |
| Front Wing: -10.0°  | Front Wing: 0.0°  |
| Front Wing Dihedral Angle | |
| -5.0°  | 5.0°  |
| Rear Wing Dihedral Angle | |
| -5.0°  | 5.0°  |
| Winglets on Front Wing | |
|  | |
| Winglets on Rear Wing | |
|  | |

## 2.3. Optimization Method

The optimization study of the tandem wing aircraft focuses on a holistic approach to the design of an aircraft. The main parameter of interest to any eVTOL aircraft designer is the achievable flight range of the aircraft. The range of an electric aircraft is given by the equation given below.

$$R = e.\eta.\frac{L}{D}.\frac{m_b}{m_{tot}}.\frac{1}{g} \quad (1)$$

This equation clearly illustrates that the range of an electric aircraft is dependent on the aircraft parameters for the aerodynamic efficiency as well as the battery mass fraction. From a practical standpoint, the battery weight that can be carried by an aircraft is also dependent on the structural weight of the other components. Certain geometrical changes like increasing wingspan, improves the aircraft aerodynamics but at the expense of increased weight of the wing. This increase in wing weight reduces the amount of battery that can be carried onboard the aircraft for a MGTW of 3175 kg. This reduction in battery mass can cause the aircraft range to reduce even if the aerodynamic efficiency improves. This is the exact interplay of aerodynamics and weight that is intended to be captured by the optimization study of the tandem wing aircraft, to determine an optimal balance of both parameters.

Apart from aerodynamics and weight factors, another important aspect of the design is the aircraft flight stability characteristics. There are a set of aircraft requirements placed on the dynamic stability which provide standards for the handling qualities and comfortable for passengers. Adhering to these requirements will significantly improve the design of a tandem wing eVTOL aircraft.

With the above arguments in mind, an optimization framework was defined using the product of aerodynamic efficiency and battery mass fraction as the objective function. The optimization constraints were set based on the required handling qualities of a commercial passenger aircraft. These set of constraints will ensure that the optimized design meets the requirements for the aircraft flight stability characteristics.

# 3. COMPUTATIONAL METHODS

The sections that follow details the computational methods and models that were used in the study. The models include a vortex lattice model, a mass estimation model, and an optimization framework.

## 3.1. Vortex Lattice Method (VLM) Model

A vortex lattice method model was utilized to perform the parametric and optimization study. The vortex lattice method is an inviscid method which is well oriented towards calculating the lift and induced drag of a wing or tail surface [20]. Airfoil drag polars generated from XFOIL [21] are used to account for viscous effects within the vortex lattice method. Athena Vortex Lattice (AVL) employs the vortex lattice method with a solver developed by Mark Drela and Harold Youngren [22], [23]. In this study, AVL was used to perform the aerodynamics and flight dynamics calculations for the different wing designs and configurations. The model setup of the input files and the panel distributions used are similar to those used in [12].

To estimate the masses of various wing designs and configurations, the NASA – Flight Optimization System (FLOPS) model was utilized [24]. The mass distribution is defined by calculated point masses for the different wing designs and configurations to conduct the flight dynamics calculations using the AVL - Eigenvalue Analysis. This mass distribution was defined according to the method used in [12].

This setup and analysis method was validated against higher order computational fluid dynamics (CFD) calculations in [12] and deemed to be sufficiently accurate and effective to perform the parametric study of various aircraft configurations. The vortex lattice model is a lower order model which entails significantly lower computational time as compared to CFD models. The derived model is also able to realistically capture the flight dynamics characteristics of various aircraft configurations.

## 3.2. OpenMDAO Framework

The optimization framework of the study was setup using OpenMDAO [25]. OpenMDAO is an open-source Python based optimization framework which was used for prominent aerospace optimization studies in aircraft design. The optimization framework was coupled with both the AVL and FLOPS models to calculate the aerodynamic efficiency, handling qualities and aircraft mass.

The framework can be visualized using the diagram shown in Figure 3. The framework essentially allows an input of an initial aircraft wing geometry and a specified battery mass. It subsequently outputs the aerodynamic efficiency and masses of each surface in the initial design using the integrated AVL and FLOPS models.

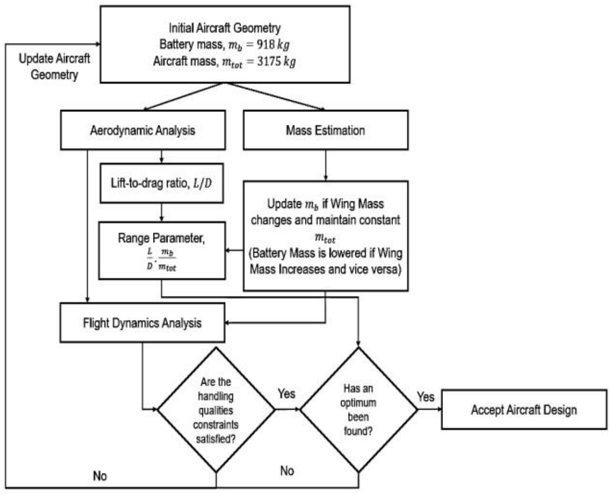



**Figure 3 Aero-Mass-Stability Optimization Framework**

Afterwards, it performs the flight dynamics analysis of the initial design and checks for the satisfaction of the set of specified flight stability constraints. The flight stability constraints used in the study are defined in the MIL-F-8785B standard [26]. The constraints used here pertain to the constraints set for a Class II aircraft for Category A and Category B flight phases. This is the most apt category of constraints for a typical eVTOL aircraft, given that flight conditions such as cruise, take-off and landing are more important for such aircraft.

An additional constraint is applied onto the static margin to ensure that the c.g. position obtained within the optimizer is viable. A summary of these constraints is provided in Table 4.

A relaxed constraint was used particularly for the phugoid mode damping ratio as the investigated models here resemble glider-like models with lower drag than actual aircraft. A lower drag value leads to a lower phugoid mode damping ratio. Hence, to allow for a more complete optimization of the wing aerodynamics, the phugoid mode damping was relaxed to prevent wing designs generated with superfluously high drag values.

**Table 4 Summary of Handling Qualities Constraints for Optimisation**

| Category | Variable | Lower bound | Upper bound |
|---|---|---|---|
| Longitudinal stability | Static Margin | 0.10 | 0.25 |
| | Short period thumbprint plot constraint $\frac{(\zeta_{spo}-0.7)^2}{0.28^2}+\frac{(\omega_{n_{spo}}-0.5)^2}{0.11^2}$ | - | 1.00 |
| | Phugoid damping ratio | 0.025 | - |
| Directional and roll stability | Dutch roll damping ratio | 0.08 | - |
| | Dutch roll damping | 0.15 | - |
| | Dutch roll natural frequency | 0.4 | - |
| | Roll constant | 0.00 | 1.40 |
| | Spiral time (s) | 20.0 | - |

Finally, the optimizer will proceed to check whether an optimum has been converged for the aircraft range parameter. If either of the above conditions is false, the framework continues to update the wing design and performs the subsequent calculations. Once an optimum is found within the constraints of flight stability, the optimizer displays the optimized design. The framework utilizes the COBYLA scheme to perform optimization [27]. COBYLA is a local optimization scheme and therefore, the optimization framework is unable to directly report a global optimum. Hence, several different initial conditions were used to verify a true optimal for the tandem wing design.

Multiple global optimization algorithms such as dual annealing and genetic algorithms were tested to provide a global optimum solution. However, the algorithms were unable to provide satisfactory results and therefore, the approach of using a local optimization algorithm with multiple initial conditions was implemented to find the optimal tandem wing designs within the study.

# 4. PARAMETRIC STUDY OF TANDEM WING DESIGN

This section discusses the results of the parametric study of a tandem wing aircraft. All calculations for this parametric study were performed under sea level conditions for a cruise speed of 240 km/h. Steady level flight conditions are assumed for all calculations with the pitching moment being trimmed for stability calculations by varying the incidence angle of the rear wing. The static margin was kept constant at 0.25 within this study. This was determined by the baseline aircraft study performed in [12]. The lift-to-drag ratio was used to compare the aerodynamic efficiency of the various aircraft designs while the stability derivatives were used to evaluate the flight stability.

## 4.1. Wingspan and Wing Aspect Ratio

The wingspan and wing aspect ratio are two of the main defining wing design parameters for any aircraft design. A parametric study of wingspan was performed and its effect on the lift-to-drag ratio and wing mass was investigated.

Figure 4 shows the variation of lift-to-drag ratio with wingspan. It shows that in general that increasing the wingspan improves the aerodynamic efficiency. This is because with a longer span, the downwash of the wing is distributed over a wider span which reduces the induced drag of the wing. It is observed that increasing the wingspan at fixed area shows the most significant increase in lift-to-drag ratio due to a significant increase in aspect ratio.

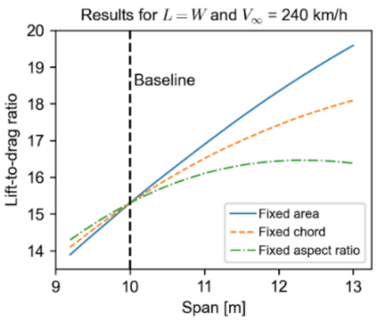



**Figure 4 Variation of Lift-to-drag ratio with Wingspan**

However, this increase in wingspan is accompanied by an increase in structural weight of the wing. Figure 5 shows the variation in wing mass with wingspan. It shows that as the wingspan was increased, the wing mass also increases. This highlights the explicit interdependence between aerodynamic efficiency and wing mass, whereby improving aerodynamic efficiency can come at the cost of increased weight.

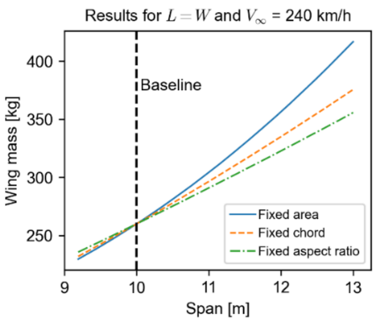



**Figure 5 Variation of Wing Mass with Wingspan**

To clearly show the interdependence between aerodynamic efficiency and wing mass, a parametric study of the range parameter $\left(\frac{L}{D} \cdot \frac{m_b}{m_t}\right)$ was performed. The wingspan and wing aspect ratio were varied, and the results are shown in Figure 6. The resulting plot was created using the values based on the wing and battery mass interdependence described in Section 2.3. The contour plot shows that as the wingspan and wing aspect ratio increases, the range parameter increases, reaches a maximum of 4.7626 and then reduces for any further increase.

This indicates that there is an optimal wingspan and wing aspect ratio which gives the best aircraft range. This trend occurs because with an increased in the wingspan and wing aspect ratio, the aerodynamic efficiency increases but at the cost of increased the wing structural weight, resulting in a reduction of the battery mass fraction. The optimal wingspan and wing aspect ratio represents the optimal condition where a balance is struck between an increase in aerodynamic efficiency and a reduction in battery mass fraction. Locating the middle ground between these two design variables is essential in obtaining the best possible flight performance.

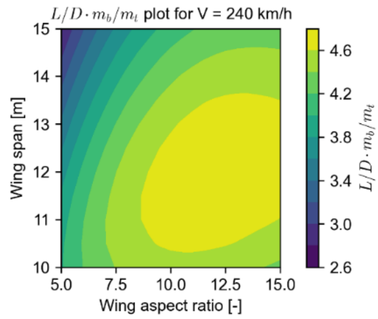



**Figure 6 Contours of Range Parameter with Variation in Wingspan and Wing Aspect Ratio**

## 4.2. Front Wing Area Fraction

It has been shown in several studies that the ratio of front and rear wing areas has a significant effect on the aerodynamics and stability of a tandem wing aircraft [12], [14]. As such, a parametric study of front wing area fraction was performed. The results of this study are shown in both Figure 7 and 8.

Figure 7 displays the variation of the front wing area fraction to lift-to-drag ratio. The results show that unequal wing areas provide a greater improvement in aerodynamic efficiency. This is because the aspect ratio of the individual wings was kept constant. Hence, as the area of one wing increases, the wingspan simultaneously increases and that reduces the value of the induced drag. Unequal wing areas also reduces the aerodynamic interference between the longer wing and the shorter wing surfaces. This reduced effect of the wing interference possibly also improves the aerodynamics of the aircraft. The aerodynamic insights gained here is in contrast with the best design choice for longitudinal stability which is illustrated with the following result.

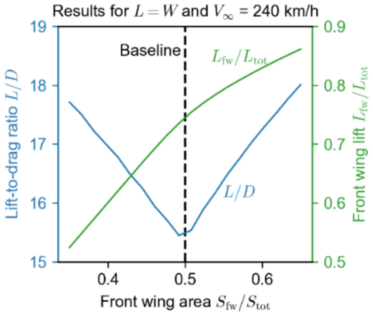



**Figure 7 Variation of Lift-to-drag ratio with Front Wing Area Fraction**

Figure 8 shows the results for the variation of pitch damping with front wing area fraction. It shows that the best pitch damping is achieved when the wing areas are equal. This is because, in the equal condition, it gives the largest tail surface or canard surface area which enhances the pitch damping of the aircraft. Under this condition, the largest and most effective stabilizing surfaces are created.

Figure 8 also illustrates the effect of front wing area fraction on the directional stability of the aircraft. The result shows that as the front wing area increases, the directional stability of the aircraft increases. This is because as the front wing area increases, the aerodynamic centre of the aircraft shifts forward accordingly. This allows for a more forward c.g. position as the static margin is kept constant. Therefore, a more forward c.g. increases the moment arm of the vertical tail and improves the directional stability of the aircraft [28]. The opposite effect happens when the front wing area is reduced.

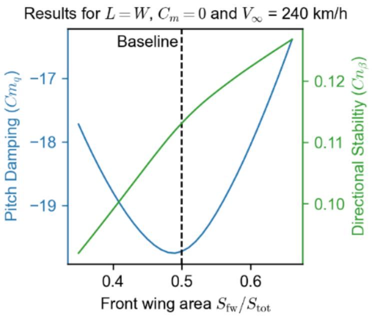



**Figure 8 Variation of Pitch and Directional Stability with Front Wing Area Fraction**

## 4.3. Wing Taper Ratio

The taper ratio of a wing affects the spanwise lift distribution which influences the induced drag generated by the wing. As the wing taper ratio reduces, the wing lift distribution shifts towards an elliptical lift distribution which generates lower induced drag. In the past, it has been determined that an optimal taper ratio is between 0.3 and 0.4 [20]. Taper ratios within this range results in a lift distribution closest to an ideal elliptical lift distribution and generate the lowest induced drag. However, this condition might differ for a tandem wing aircraft.

To study this effect, a parametric study of wing taper ratio was performed. Figure 9 shows the results of the variation of taper ratio to lift-to-drag ratio. It shows that the lift-to-drag ratio follows a parabolic variation with taper ratio although the variation is small. The optimal lift-to-drag ratio is achieved at a higher taper ratio of 0.5 instead of the typical taper ratio range. For a tandem wing configuration, this concludes that the best aerodynamic efficiency is achieved with a slightly higher taper ratio of 0.5 applied on both wings.

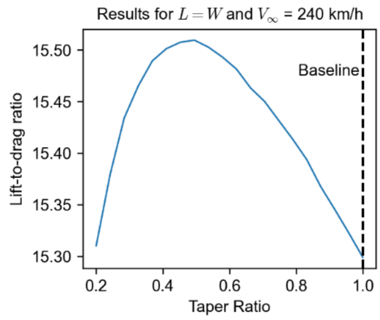



**Figure 9 Variation of Lift-to-drag ratio with Taper Ratio**

## 4.4. Horizontal Distance Between Wings

The horizontal distance between both wings affects the longitudinal stability of the aircraft. Figure 10 shows the variation of the horizontal wing distance to pitch damping. The result shows that the pitch damping of the aircraft improves with an increase in the horizontal wing distance. However, there exists an optimal pitch damping limit which provides the best longitudinal handling qualities. Hence, the horizontal wing distance should only be increased to the optimal pitch damping value.

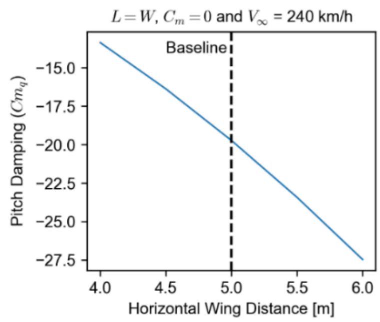



**Figure 10 Variation of Pitch Damping with Horizontal Wing Distance**

## 4.5. Height of Front Wing

The height between the front and rear wing influences the aerodynamic interaction for a tandem wing aircraft. Figure 11 shows the results of a parametric study of front wing relative height with regards to aerodynamic efficiency. The result shows that by placing the front wing above or below the position of the rear wing, an improved aerodynamic efficiency could be observed. This is because a non-planar system is created when the front wing height is changed which reduces the induced drag generated. In non-planar wing systems, the downwash from the front wing is spread vertically and thus, the induced angle of attack experienced by the rear wing is reduced which causes a reduction in induced drag. In terms of the momentum changes, due to the vertical spread of the downwash, a larger mass of air can be influenced with a lower average velocity change to generate the same amount of lift which produces less energy and lower drag [29]. The height effect is observed to be the same for raising and lowering of the front wing surface.

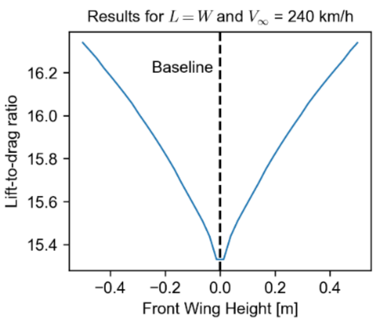



**Figure 11 Variation of Lift-to-drag ratio with Front Wing Height**

## 4.6. Wing Sweep Angle

The wing sweep angle mainly impacts the lateral stability of an aircraft and has a less dominant effect on the aerodynamic efficiency. However, the sweep angle impacts the directional stability of the aircraft through a shift of the aerodynamic centre of the aircraft. For example, as the aircraft front wing is swept forward, the aircraft aerodynamic centre shifts forward as well. This allows for a more forward c.g. position which increases the vertical tail moment arm and improves the directional stability of the aircraft. This directional stability argument is displayed in the results shown in Figure 12.

Only the results for the front wing sweep angle are shown as the changes in directional stability are not as pronounced for the rear wing sweep angle, as the aerodynamic centre is biased towards the front wing of the aircraft due to the prevailing lift curve slopes of the wings when they are of equal dimensions. This indicates that there is less significant change in the aerodynamic centre with change in rear wing sweep and therefore, less allowable shift in the c.g. position.

Figure 12 also shows the variation of directional stability under the condition of combined sweep of both the front and rear wings. In this case, the rear wing is swept back by 25° and it is held at this constant sweep angle while the front wing forward sweep angle is varied. This is also shown visually in Table 3.

The results show that under the condition of combined sweep applied to the tandem aircraft, the variation of directional stability per degree change in sweep angle is similar to the case where only the front wing sweep angle is varied. However, the directional stability was increased at each value of the front wing sweep angle. This improvement in directional stability under the combined sweep condition could be attributed to the reduction in lift curve slope of the rear wing after a rearward sweep angle is applied. As the rear wing lift curve slope reduces, the aerodynamic centre of the aircraft is pushed further forward towards the front wing, more so than the case when the rear wing was straight. This allows for a more forward c.g. location than the case where the rear wing is kept straight, hence improving the directional stability further.

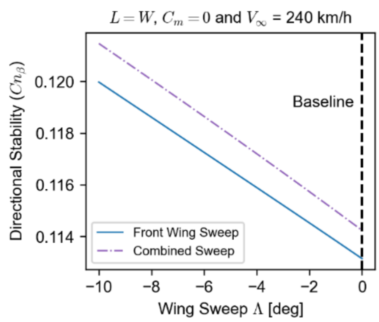



**Figure 12 Variation of Directional Stability with Front Wing Sweep Angle**

A forward sweep angle reduces the dihedral effect of the aircraft. This is because increasing forward sweep is akin to reducing the dihedral angle of the wing [30]. This is shown by Figure 13 whereby, as the forward sweep is increased, the dihedral effect of the aircraft reduces (becomes more positive). A reduction in the dihedral effect of the aircraft impacts both the stability for spiral and Dutch roll modes. An increase in dihedral effect improves the spiral mode stability while it reduces the Dutch roll stability of the aircraft. This poses a conflict to the designer while designing swept wings for an aircraft. Increasing forward sweep of the front wing leads to a reduction in spiral mode stability, making it difficult for a pilot to correct for the instability of this mode. On the other hand, increasing the rearward sweep of the rear wing improves the dihedral effect of the aircraft as increasing the rearward sweep is akin to increasing the dihedral angle of the wing. Adding a rearward sweep to the rear wing improves the spiral mode stability of the aircraft.

Under the condition of a combined sweep, the results indicate that as the front wing forward sweep is increased, the dihedral effect of the aircraft reduces, however, the dihedral effect value for each sweep angle is more negative than the case where the rear wing was straight. This indicates that the negative effect of a forward sweep on the front wing can be reduced by introducing a rearward sweep on the rear wing. In this sense, a tandem wing aircraft with opposite sweeps on both wings offers a good compromise to the aircraft designer. The forward sweep of the front wing allows for a more forward c.g. for better directional stability while the rearward sweep of the rear wing maintains the spiral mode stability of the aircraft. The use of combined sweep in tandem wing designs would aid the aircraft designer to resolve the conflict between Dutch roll stability and spiral mode stability.

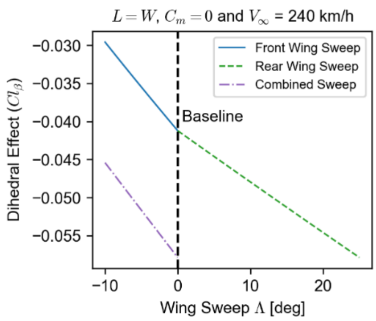



**Figure 13 Variation of Dihedral Effect with Wing Sweep Angles**

Wing sweep has a less dominant effect on the aerodynamic efficiency of the aircraft. Table 5 details the effect of wing sweep on aerodynamic efficiency for several selected data points in the sweep parametric variation. It was observed that a forward sweep on the front wing increases the aerodynamic efficiency slightly while a rearward sweep on the rear wing or a combined sweep reduces the aerodynamic efficiency. Overall, if the wings of a tandem wing aircraft are to be swept, they should be applied for the purposes of improving the aircraft stability instead of the aerodynamic efficiency.

**Table 5 Effect of Wing Sweep on Aerodynamic efficiency**

| Parameter | Wing | | Lift-to-drag ratio $L/D$ |
|---|---|---|---|
| | Front | Rear | |
| **Baseline** | - | - | 15.300 |
| **Sweep [deg]** | 0 | 25 | 15.162 |
| | -10 | 0 | 15.340 |
| | -10 | 25 | 15.206 |

## 4.7. Wing Dihedral Angle

A dihedral angle on the wing increases the dihedral effect of an aircraft whereas an anhedral angle reduces the dihedral effect of the aircraft. This is illustrated by Figure 14 which shows the variation of dihedral effect with the rear wing dihedral or anhedral angle. As discussed earlier, changes to the dihedral effect can affect the Dutch roll and spiral modes stability of an aircraft.

It was found that the dihedral angle does not affect Dutch roll stability as severe as to the spiral mode stability, whereby it can significantly improve the spiral mode stability. The results are similar for the variation of the front wing and rear wing dihedral angle.

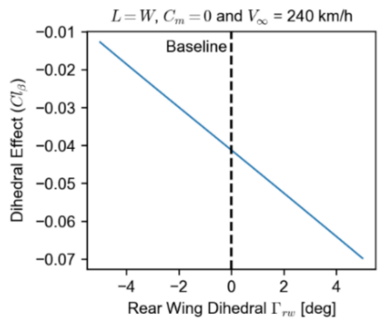



**Figure 14 Variation of Dihedral Effect with Rear Wing Dihedral Angle**

The wing dihedral angle also affects the aerodynamic efficiency of the tandem wing aircraft albeit the effect is less prominent than its effect on the aircraft stability. Table 6 displays the data for selected points within the parametric variation of dihedral angle. As shown by the results, increasing the dihedral or anhedral angles of the wings leads to an increase in the aerodynamic efficiency of the aircraft. This is because increasing the dihedral or anhedral angle creates a non-planar wing system. As discussed in Section 4.5, creating a non-planar system which reduces the induced drag while improves the aerodynamic efficiency of the aircraft.

**Table 6 Effect of Wing Dihedral Angle on Aerodynamic efficiency**

| Parameter | Wing | | Lift-to-drag ratio $L/D$ |
|---|---|---|---|
| | Front | Rear | |
| **Baseline** | - | - | 15.300 |
| **Dihedral [deg]** | -5 | 0 | 16.137 |
| | 5 | 0 | 16.201 |
| | 0 | -5 | 16.158 |
| | 0 | 5 | 16.169 |

## 4.8. Placement of Winglets

Winglets are a crucial part of the wing design for fixed wing aircraft to optimize aerodynamic efficiency. As such, the addition of winglets to both the front and rear wing of a tandem wing aircraft was investigated. Figure 15 shows the variation of lift-to-drag ratio with the winglet area of the front wing and rear wing. The results show that as the area of the winglet is increased the aerodynamic efficiency of the aircraft improves. The trend is similar for both the front wing and rear wing winglet area. This is because increasing the area of the winglet drives the wingtip vortices further away from the main wing surface, thus reducing the effect of downwash. Thereby, reducing the induced drag of the aircraft and improving the lift-to-drag ratio. However, it can also be expected that the aerodynamic efficiency might reduce if the winglet area is to be increased beyond a certain limit. This is because there will be a significant additional profile drag component leading to the reduction of the overall aerodynamic efficiency.

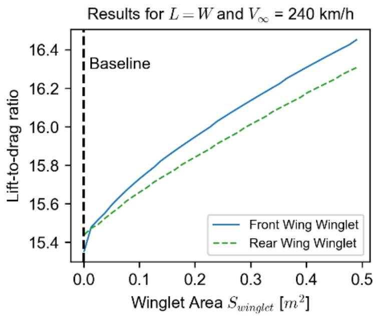



**Figure 15 Variation of Lift-to-drag ratio with Winglet Area**

An additional benefit of adding a winglet to the rear wing of the aircraft is shown by Figure 16. It shows that as the rear wing winglet area is increased, the directional stability of the aircraft increases. This is because a winglet behaves similar to an additional vertical tail surface, thereby improving the directional stability. In this regard, placing the winglet on the rear wing is more beneficial in maintaining the static and dynamic stability, as the c.g. of the baseline tandem wing aircraft is closer to the front wing than the rear wing. This results in a longer moment arm for a winglet placed on the rear wing granting greater directional stability, effectively behaving like a vertical tail surface.

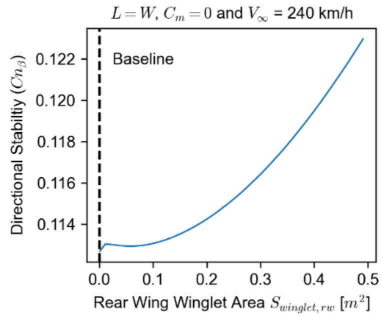



**Figure 16 Variation of Directional Stability with Rear Wing Winglet Area**

# 5. OPTIMIZATION OF A TANDEM WING DESIGN

A total of 4 different initial conditions were used to perform an optimization study of the baseline tandem wing aircraft design. These initial conditions are summarized in Table 7. The optimizations were performed for only wingspan, aspect ratio and taper ratio. However, the analysis can be extended further to include every other wing design variable. The c.g. position is also made a variable to account for effects of c.g. position on the aircraft performance. C.g. position plays an important role in aerodynamic efficiency and stability as seen in the past studies. The optimization study was performed at the same reference conditions as the parametric study.

**Table 7 Initial Conditions for Optimization**

| Case | Front Wing | | | Rear Wing | | | c.g. position |
|---|---|---|---|---|---|---|---|
| | $b$ [m] | AR | $\lambda$ | $b$ [m] | AR | $\lambda$ | Measured from nose of aircraft |
| **a** | 10 | 7.69 | 1.0 | 10 | 7.69 | 1.0 | 3.5845 |
| **b** | 12 | 7.69 | 1.0 | 8 | 7.69 | 1.0 | |
| **c** | 8 | 3.845 | 1.0 | 12 | 11.535 | 1.0 | |
| **d** | 12 | 7.69 | 0.5 | 8 | 7.69 | 0.5 | |

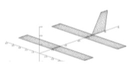
The optimizations were conducted under different constraint conditions and the results were studied to elucidate optimized tandem wing geometry with different design considerations. The results of the optimization study are discussed in detail in the following sections.

## 5.1. Aero-Mass Optimization

This section details the results of the optimization study pertaining to an aerodynamic and structural weight optimization of a tandem wing design. In this optimization paradigm, the handling qualities constraints are ignored, while the main objective of the tandem wing design is optimized purely for maximum flight range.

The results of this optimization strategy are shown in Table 8. The results show that depending on the initial conditions, there are two major optimum designs. One design resembles a conventional wing-tail aircraft while the other resembles a canard aircraft. The results show that both optimum designs have a higher range parameter than the baseline tandem wing aircraft. This indicates that the improved aerodynamic efficiency and weight efficiency can be seen for configurations with unequal wing areas. The taper ratio is also made to be as low as possible to provide further benefits to the aircraft weight and aerodynamics.

**Table 8 Results of Aero-Mass Optimization**

| Case | $\frac{L}{D}\cdot\frac{m_b}{m_{tot}}$ | Design Sketch |
|---|---|---|
| **a** | 6.5239 |  |
| **b** | 6.5220 |  |
| **c** | 6.4390 |  |
| **d** | 6.5235 |  |
| **Baseline** | 4.4235 |  |

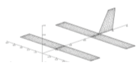
The previous parametric study performed here has also shown that the wing configurations with longer overall span and unequal wing areas tend to have higher aerodynamic efficiencies. Under this paradigm of aero-mass optimization, designs with longer spans and unequal wing areas might be preferred to purely increasing the aircraft range. However, it is also important to note that these designs may not possess adequate handling qualities and optimums will deviate when considering the subsequent constraints for handling qualities.

## 5.2. Aero-Mass-Stability Optimization

This section details the results of the optimization study with the inclusion of the handling qualities constraints mentioned in Section 2.3. In this study, the tandem wing was optimized for maximum flight range with additional requirements to achieve good flight stability characteristics.

The results of this optimization strategy are shown in Table 9. The optimum designs have changed from those obtained in the aero-mass optimization strategy. The conventional wing-tail designs remained the same between strategies conveying that these designs could achieve good handling qualities with a high range. However, several of the initial conditions produced designs with wings of comparable sizes indicating that such designs are preferred by the framework when handling qualities are considered. This concludes that a particular tandem wing variant could be a better design choice when simultaneously maximizing for flight range and improving the handling qualities. This is not to say that other configurations such as the canard aircraft are not able to achieve good handling qualities. The results presented here are just a consequence of the different initial conditions being defined and there are likely other factors to consider in aircraft design such as manufacturing and aeroelastic constraints.

**Table 9 Results of Aero-Mass-Stability Optimization**

| Case | $\frac{L}{D}\cdot\frac{m_b}{m_{tot}}$ | Design Sketch |
|---|---|---|
| **a** | 6.5170 |  |
| **b** | 5.5610 |  |
| **c** | 5.5702 |  |
| **d** | 6.5320 |  |
| **Baseline** | 4.4235 |  |

For the tandem wing configuration, it shows good pitch damping from the results of the parametric study and therefore, could be the preferred design choice while trying to adhere to the strict constraints of the short period mode. For the aero-mass-stability optimization, it provides an average increase of 1.6215 in the range parameter value while the aero-mass optimization saw an average increase of 2.0786 in the range parameter value. This highlights when imposing the handling qualities constraints, it restricts the improvements in the wing design of the aircraft. However, when considering these constraints, it led to designs that will be more realistic and practical for use in the eVTOL industry.

## 6. CONCLUSIONS

A parametric and optimization study of a generic tandem wing configuration has been performed. The study focused on improving the aerodynamic efficiency and flight stability of a tandem wing aircraft. A VLM model was used to perform the study along with the NASA FLOPS mass estimation model and OpenMDAO optimization framework. The study revealed important effects of the different wing design variables on the performance of a tandem wing aircraft.

Unequal wing areas provided a higher aerodynamic efficiency but with a lower pitch damping. Increasing the wingspan improves aerodynamic efficiency but also increases the structural wing weight. A taper ratio of approximately 0.5 provides the optimal performance for the tandem wing aircraft. Increasing the distance between the wings improved pitch damping of the aircraft. Creating a non-planar wing system through varying the wing height improves the aerodynamic efficiency. A rearward wing sweep angle improves the lateral stability while slightly reduces the aerodynamic efficiency of the aircraft. Increasing dihedral angle of the wing also improves lateral stability, however, a differential dihedral between both tandem wings creates a non-planar wing system and thus improves the aerodynamic efficiency. Finally, addition of winglets to either wing surface improves the aerodynamic efficiency, while the addition of winglets to the rear wing improves directional stability.

The optimization study has provided insights into ideal design choices with the consideration of the interplay of different and multiple design variables. The aero-mass optimization strategy revealed that several initial conditions led to an optimized design with unequal wing areas and reduced taper ratios. This indicated that such designs are aerodynamically and structurally more efficient. While the aero-mass-stability optimizations revealed that wings of similar sizes could be preferrable when considering the handling qualities constraints, but a configuration with unequal areas could perform just as well if not better. Several local optima were presented, based on the proposed aero-mass-stability optimization framework. This was done to verify the optimal tandem wing design as the optimization framework utilizes a local optimization algorithm. This is an attempt to present preferred design choices under different constraint conditions. The designs could still be tweaked for final performance enhancements beyond the suggestions of the optimization framework. This ultimately depends on the proposed mission of the aircraft and other requirements set by the aircraft designer.

It is hoped that the information gained from this parametric and optimization study will prove to be beneficial to eVTOL aircraft designers in improving future tandem wing eVTOL aircraft designs.

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