24 th EUROPEAN ROTORCRAFT FORUM
Marseilles, France - 15th-17th September 1998

Référence : DY14

# Validation of Deutsch Stability Criteria for Helicopter Ground Resonance

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ABSTRACT In this article, we revisit the stability criteria for the well-known helicopter ground resonance problem. The exact Routh's criterion is derived in symbolic form for a simple three-DOF model. It is demonstrated how Routh's criterion can be reduced to Deutsch's stability criterion with a number of approximations. The latter is also evaluated numerically against Routh's criterion for several configurations. By calculating a correction factor to be applied to Deutsch's criterion in order to satisfy Routh's criterion, it is demonstrated that Deutsch's criterion is conservative for some helicopter configurations while deficient for others. Finally, Routh's criterion is applied to a four-DOF ground resonance model where the helicopter has two translational degrees of freedom. Again, results are compared against Deutsch's criterion for one of the two instability regions and for a range of body frequencies.

## 1. INTRODUCTION

The phenomenon of helicopter ground resonance has been studied extensively by many researchers over the last five decades. The original work by Coleman and Feingold, summarized in [1], attributed ground resonance to mechanical coupling between horizontal hub displacements and lead-lag blade oscillations; it laid the basis for analyzing this instability for articulated rotors. Subsequently, the research on the topic shifted from articulated rotor configurations to hingeless rotors [2, 3] and later to bearingless rotor systems [4]. Much of this work has focused on the effects of rotor configuration, aerodynamics modelling [5] and various rotor/blade design parameters on the instability. A number of investigations conducted in the last two decades provided experimental data to support theoretical analysis of the ground resonance phenomenon [6, 7]. More recently, researchers have incorporated nonlinear dampers (in landing gear and rotor) in the ground resonance model [8, 9]. In [10], the full nonlinear motion equations are simulated in time to determine the response of the hub and the blades in ground resonance. Comprehensive reviews of the literature on the subject can be found in several articles dealing with helicopter aeromechanical stability [11, 12, 13] as well as a recent review in [14] specifically on ground resonance.

The advent of computers, numerical analysis and symbolic manipulation software have enabled helicopter designers to use more powerful techniques to analyze helicopter behaviour in ground resonance. As evidenced by the articles cited, much effort has been made towards improving the aerodynamic and blade modelling and to study how they affect the predictions of the ground resonance models. By contrast, one central aspect of

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this problem—explicit stability criteria—has received relatively little attention in the recent investigations of the ground resonance phenomenon. A well-known stability criterion proposed by Deutsch over 50 years ago [15] has remained essentially unchanged since its invention. Because of its simplicity, it is still widely used to check for ground resonance instability of helicopters.

In the present paper, the authors investigate the application of Routh's criterion for studying the stability of linear dynamics systems to the ground resonance problem. Our motivation is to develop a more general and possibly a more accurate criterion for the ground resonance instability. In the process, we bring to light the approximate nature of Deutsch's criterion. A numerical validation of Deutsch's criterion against Routh's criterion is conducted which reveals that the former is conservative for some helicopter configurations, while not sufficient for others. Finally, the applicability of Deutsch's criterion to the case of multiple regions of instability is investigated.

Towards these objectives, the paper is organized as follows. We begin with a concise summary of Deutsch's criterion and its variations and extensions in the literature. Subsequently, Routh's criterion is defined and employed to determine a stability criterion for a three-DOF helicopter/rotor model where the helicopter has a single translational degree of freedom (DOF). Section 4 establishes the relationship between the two criteria, both from the analytical perspective and by numerically testing Deutsch's criterion against Routh's criterion. Both criteria are then applied and compared for a four-DOF model of the helicopter which exhibits two regions of ground resonance instability.

## 2. Deutsch's Stability Criterion and Variations

### 2.1 Deutsch [1946]

In 1946, Deutsch [15] proposed a simple stability criterion for the ground resonance problem. His derivation was based on a helicopter model which included the effective mass and the natural frequency of the hub, the basic inertia and geometric properties of the rotor blades, as well as body and blade damping. Deutsch considered two cases: (I) where the hub frequencies are the same in all directions and (II) where the hub has one degree of freedom in the plane of rotation. For a given configuration and mechanical properties, Deutsch's criterion determines the minimum amount of body damping and blade damping required to overcome the ground resonance instability. Using Deutsch's original notation, the criteria for case I and II configurations are stated, respectively, in the following forms:

$$\lambda_p \lambda_\phi > \frac{\Lambda_3}{p-1} \qquad (1)$$

$$\lambda_p \lambda_\phi > \frac{1}{2}\frac{\Lambda_3}{p-1} \qquad (2)$$

where

$$\lambda_p = \frac{C}{(M+Nm)\omega}, \quad \lambda_\phi = \frac{C_\zeta}{I_\zeta \omega}, \qquad (3)$$

$$\Lambda_3 = \frac{1}{2}\frac{Nm}{(M+Nm)}\frac{ml^2}{I_\zeta} \qquad (4)$$

and $p = \frac{\Omega}{\omega}$ is the rotor speed at the center of instability nondimensionalized by the hub frequency $\omega$ (using conventional notation.) The other symbols used in the above are: $C$ and $C_\zeta$ denote the hub and blade damping respectively, $N$ is the number of rotor blades, $M$ is the *effective* mass of the hub, $m$ and $I_\zeta$ are the blade mass and moment of inertia about the drag (lead-lag) hinge and $l$ is the distance from the drag hinge to the center of mass of the blade. Substituting the above definitions and introducing $S_\zeta = ml$ as the blade first moment of inertia about the drag hinge, gives the commonly cited dimensional form of Deutsch's criterion. It is written for case II configuration as:

$$CC_\zeta > \frac{NS_\zeta^2}{4}\frac{\omega^3}{\Omega - \omega} \tag{5}$$

## 2.2 Done [1969]

Over 20 years later, Done [16] studied the ground resonance problem by using a simplified model similar to Deutsch's second configuration where the hub is constrained to have one translational ($x$-) degree of freedom. In particular, Done's model consists of a chassis of mass $M$ concentrated at the hub, and blades, each of mass $m$ concentrated at a distance $b$ from the drag hinge. After transforming the three-DOF model to the two-DOF model for bi-normal coordinates, Done derives a stability criterion in the form (using notation adopted in this paper):

$$\bar{C}_x\bar{C}_\zeta > \frac{\lambda\omega_x^3}{2(\Omega - \omega_x)} \tag{6}$$

where $\lambda$ is the ratio of total blade mass to twice the overall mass:

$$\lambda = \frac{1}{2}\frac{Nm}{(M + Nm)}, \tag{7}$$

$\bar{C}_x$ and $\bar{C}_\zeta$ are the damping coefficients defined as follows:

$$\bar{C}_x = \frac{C_x}{(M + Nm)}, \quad \bar{C}_\zeta = \frac{C_\zeta}{mb^2} \tag{8}$$

By substituting the above definitions and the first moment of inertia of the blade about the drag hinge, $S_\zeta$, for the product $mb$, Done's criterion can be rewritten in the form identical to Deutsch's criterion of Eq. (5) for the one-DOF hub motion, with the correspondence $\omega = \omega_x$, $C = C_x$.

## 2.3 Johnson [1980]

In his book, Johnson [17] considers the stability criterion for a chassis model with two hub degrees of freedom—the longitudinal and lateral displacements of the hub, $x$ and $y$. This model results in the 8th-order characteristic equation of the system which cannot be solved analytically for the exact stability boundary. Johnson obtains an approximate stability criterion by making the following assumptions in his derivation:

1. Terms of order higher than $O\left((\frac{S_\zeta}{I_\zeta})^2\right)$ are neglected.
2. Terms of order higher than two in the damping coefficients are neglected.
3. The stability criteria are derived for the centers of the two corresponding instability regions. These in turn are defined by the frequency coalescence conditions which in dimensionless form are:

$$1 - \bar{\nu}_\zeta = \bar{\omega}_i, \quad i = x, y \tag{9}$$

With the above assumptions and considering the nonisotropic case, that is $\omega_x \neq \omega_y$, Johnson derives two stability criteria for the instability with each of the two body degrees of freedom, that is for the point of coalescence of the regressing lead-lag frequency with either one of the two chassis frequencies. These are written in the dimensional form as [17, p. 683]:

$$\frac{C_iC_\zeta}{\omega_i^2} > \frac{N}{4}\frac{1 - \bar{\nu}_\zeta}{\bar{\nu}_\zeta}S_\zeta^2, \quad i = x, y \tag{10}$$

and can be reduced to Deutsch's criterion for case II configuration with the substitution of the frequency coalescence conditions (9). Furthermore, by considering the isotropic case ($\omega_x = \omega_y$), Johnson rederives Deutsch's stability criterion for case I configuration which states that this case requires twice the damping of the anisotropic case.

From Johnson's development for the two-DOF model of the chassis one may conclude that Deutsch's criterion (5) can be used to surpress the instability with any body mode, and is valid independently of the number of body modes (or degrees of freedom) included in the model. This would imply that the coupling between different degrees of freedom of the craft is either negligible or does not affect the characteristics of the individual instabilities.

## 3. Application of Routh's Criterion to Ground Resonance Instability

### 3.1 Routh's Criterion Briefly

Routh's criterion provides a means for determining the stability of a linear time-invariant system without explicitly calculating the eigen-values (poles) of the system. For a system represented by the characteristic equation of the form $Q(s)$,

$$Q(s) = b_n s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \cdots + b_1 s + b_0 = 0 \quad (11)$$

the criterion can be summarized in the following two statements:

1. A necessary condition for stable roots is that all the coefficients in the characteristic polynominal be positive.

2. The number of roots of the characterisitc equation with positive real parts is equal to the number of changes of sign of the coefficients in the first column of the Routhian array [18].

The aforementioned Routhian array is a triangular array defined as:

$$\begin{array}{l|lllll} s^n & b_n & b_{n-2} & b_{n-4} & b_{n-6} & \cdots \\ s^{n-1} & b_{n-1} & b_{n-3} & b_{n-5} & b_{n-7} & \cdots \\ s^{n-2} & c_1 & c_2 & c_3 & \cdots & \\ s^{n-3} & d_1 & d_2 & \cdots & & \\ \cdots & \cdots & \cdots & \cdots & & \\ s^1 & j_1 & & & & \\ s^0 & k_1 & & & & \end{array}$$

where the constants $c_i$, $d_i$, etc. are calculated according to the following pattern, until they are equal to zero:

$$\begin{aligned} c_1 &= \frac{b_{n-1}b_{n-2} - b_n b_{n-3}}{b_{n-1}} \\ c_2 &= \frac{b_{n-1}b_{n-4} - b_n b_{n-5}}{b_{n-1}} \\ \vdots & \qquad \vdots \\ d_1 &= \frac{c_1 b_{n-3} - b_{n-1} c_2}{c_1} \\ d_2 &= \frac{c_1 b_{n-5} - b_{n-1} c_3}{c_1} \\ \vdots & \qquad \vdots \end{aligned}$$

The labor in evaluating the array can be significantly reduced by making use of the following *theorem:*

**Theorem** The coefficients of any row may be multiplied or divided by a positive number without changing the signs of the first column [18].

As described in the following subsection, a symbolic Routhian array (RA) for the ground *resonance model* was obtained with Maple symbolic manipulation program. The calculation of rows 3 through $n+1$ of the array was implemented in Maple with the following concise code:

```
for i from 3 to (n+1) do
   i1:= i-1;
   for j from 1 to n/2 do
      j1:= j + 1;
      RA[i,j]:=(RA[i1,1]*RA[i-2,j1]
      - RA[i-2,1]*RA[i1,j1])/RA[i1,1];
   od;
od;
```

The previously stated theorem was used wherever possible to simplify the symbolic expressions for the entries in rows of the Routhian array.

### 3.2 Three-DOF Ground Resonance Model

We now derive Routh's criterion in symbolic form for the simple three-DOF ground resonance model comprising the $x$-translation of the helicopter *center of mass* and two cyclic lead-lag motions of the rotor. This model, defined in Eqs. (12-14) below, is adopted from the ground resonance model described in [19]:

$$\mathbf{T}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} + \mathbf{S}\mathbf{u} = 0 \tag{12}$$

where

$$\mathbf{T}=\begin{bmatrix} M & Nm & 0 \\ \frac{m\,b^2}{2} & I_\zeta & 0 \\ 0 & 0 & I_\zeta \end{bmatrix};\quad \mathbf{D}=\begin{bmatrix} C_x & 0 & 0 \\ 0 & C_\zeta & 2\,I_\zeta\,\Omega \\ 0 & -2\,I_\zeta\,\Omega & C_\zeta \end{bmatrix}; \tag{13}$$

$$\mathbf{S} = \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_\zeta + (m\,ba - I_\zeta)\,\Omega^2 & C_\zeta\,\Omega \\ 0 & -C_\zeta\,\Omega & K_\zeta + (m\,ba - I_\zeta)\,\Omega^2 \end{bmatrix} \tag{14}$$

and $\mathbf{u} = [x \;\; \zeta_c \;\; \zeta_s]^T$. Following [19], the symbol $a$ denotes the radial offset of the drag hinge from the hub. It is noted that the present model and accordingly, the corresponding stability criterion do not require the somewhat ambiguous concepts of "hub effective mass" used by Deutsch and Johnson ($M$ in the above is the total helicopter mass). Furthermore, it can be directly extended to include up to six degrees of freedom of the helicopter.

The characteristic equation for the three-DOF ground resonance model is:

$$Q_3 = b_6 s^6 + b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0 = 0 \tag{15}$$

The coefficients $b_i$, $i = 1 \ldots 6$, derived from the model (12-14) are simplified by introducing six positive nondimensional parameters:

(i) the body frequency $\bar{\omega}_x = \sqrt{K_x/M}/\Omega$,

(ii) the nonrotating blade frequency $\bar{\omega}_\zeta = \sqrt{K_\zeta/I_\zeta}/\Omega$,

(iii) the rotating blade frequency $\bar{\nu}_\zeta^2 = \bar{\omega}_\zeta^2 + \frac{aS_\zeta}{I_\zeta}$,

(iv) the blade parameter $\bar{p}_\zeta = NS_\zeta^2/(2I_\zeta M)$,

(v)-(vi) the damping ratios:

$$d_x = \frac{C_x}{2M\omega_x}, \quad d_\zeta = \frac{C_\zeta}{2I_\zeta\omega_\zeta}$$

We observe that the parameter $\bar{p}_\zeta$ also appears in Johnson's derivation of the dimensionless form of the stability criterion and typically, $\bar{p}_\zeta << 1$. By defining $\bar{p}_\nu = \bar{\nu}_\zeta^2 - 1$ in addition to the above, the coefficients of the characteristic equation (15) take the following form:

$$b_0 = (\bar{p}_\nu^2 + 4\,d_\zeta{}^2\bar{\omega}_\zeta^2)\,\bar{\omega}_x^2\Omega^2$$

$$b_1 = 2\,\Big(d_x\,\bar{\omega}_x\,(\bar{p}_\nu^2 + 4\,d_\zeta{}^2\bar{\omega}_\zeta^2) + 2\,\bar{\omega}_x^2 d_\zeta\,\bar{\omega}_\zeta\,(\bar{p}_\nu + 2)\Big)\,\Omega$$

$$b_2 = \bar{p}_\nu^2 + 4\,d_\zeta{}^2\bar{\omega}_\zeta^2(1 + \bar{\omega}_x^2) + 2\,\bar{\omega}_x^2(\bar{p}_\nu + 2) + 8\,\bar{\omega}_x\,\bar{\omega}_\zeta\,d_x\,d_\zeta(\bar{p}_\nu + 2)$$

$$b_3 = 4\,\Big(d_x\,\bar{\omega}_x\,(2d_\zeta{}^2\bar{\omega}_\zeta^2 + \bar{p}_\nu + 2) + d_\zeta\,\bar{\omega}_\zeta(\bar{\omega}_x^2 + \bar{p}_\nu + 2)\Big)\,/\Omega$$

$$b_4 = \Big(4\,d_\zeta{}^2\bar{\omega}_\zeta^2 + 4 + (2 - \bar{p}_\zeta)\,\bar{p}_\nu + 8\,d_x\,\bar{\omega}_x\,d_\zeta\bar{\omega}_\zeta + \bar{\omega}_x^2\Big)\,/\Omega^2$$

$$b_5 = \frac{2\,(d_x\,\bar{\omega}_x + (2 - \bar{p}_\zeta)\,d_\zeta\,\bar{\omega}_\zeta)}{\Omega^3}$$

$$b_6 = \frac{1 - \bar{p}_\zeta}{\Omega^4}$$

It is evident that all $b_i$'s are positive definite for $\bar{p}_\zeta < 1$.

The first column of the Routhian array contains 7 elements which were derived in Maple and are listed below:

$$\begin{aligned} RA[1,1] &= b_6 \\ RA[2,1] &= b_5 \\ RA[3,1] &= \frac{b_5 b_4 - b_6 b_3}{\mathrm{num}(RA[2,1])} \\ RA[4,1] &= \frac{b_3(b_5 b_4 - b_6 b_3) + b_5(b_1 b_6 - b_2 b_5)}{\mathrm{num}(RA[3,1])} \\ \vdots &= \vdots \\ RA[7,1] &= b_0 \end{aligned}$$

After expanding in terms of $b_i$'s, it can be demonstrated that all entries above are positive definite with the exception of the sixth element, $RA[6,1]$. Hence, Routh's criterion ($RC$) for stability of the three-DOF ground resonance model becomes:

$$RC := RA[6,1] > 0 \tag{16}$$

With some symbolic manipulation and simplifications, one can express $RC$ as a finite power series in $d_\zeta$ and $d_x$ which, using the order notation, takes the following form:

$$RC = \sum_{i=0}^{3} O(d_\zeta^{3-i} d_x^i) + \sum_{i=0}^{4} O(d_\zeta^{5-i} d_x^i)$$
$$+\sum_{i=0}^{5} O(d_\zeta^{7-i} d_x^i) + \sum_{i=1}^{5} O(d_\zeta^{9-i} d_x^i) + \sum_{i=3}^{5} O(d_\zeta^{11-i} d_x^i) > 0 \tag{17}$$

If one retains terms up to fifth order in the damping ratios $d_\zeta$ and $d_x$, $RC$ simplifies to:

$$\begin{aligned} RC \approx\ & O(d_\zeta^3) + O(d_\zeta^2 d_x) + O(d_\zeta d_x^2) + O(d_x^3) \\ & +O(d_\zeta^5) + O(d_\zeta^4 d_x) + O(d_\zeta^3 d_x^2) \\ & +O(d_\zeta^2 d_x^3) + O(d_\zeta d_x^4) > 0 \end{aligned} \tag{18}$$

The above clearly exposes the approximate nature of Deutsch's criterion, which in terms of the nondimensional parameters employed here can be stated as:

$$O(d_\zeta d_x) - O(1) = d_\zeta d_x - \frac{\bar{p}_\zeta \bar{\omega}_x^2}{8\bar{\omega}_\zeta (1 - \bar{\omega}_x)} > 0 \tag{19}$$

### 3.3 Significance of Blade and Body Damping

One important conclusion that follows from Deutsch's stability criterion is that ground resonance cannot be stabilized without the presence of *both* rotor and body damping. Although intuitively appealing, this does not immediately follow from Routh's criterion of Eq. (17) because of the presence of the "single-damping" terms (e.g., $O(d_\zeta^3)$ and $O(d_x^3)$). This conclusion was tested by applying the symbolic Routh criterion to two models: one with $d_x = 0$ and the other with $d_\zeta = 0$. In the latter case, the 6-th entry in the Routhian array reduces to:

$$RC|_{d_\zeta=0} = RA[6,1]|_{d_\zeta=0} = \frac{4\bar{p}_\nu^5 \bar{p}_\zeta}{\text{num}(RA[5,1])} \tag{20}$$

where $\text{num}(RA[5,1]) > 0$. Accordingly for stability, we require

$$\bar{p}_\nu = \bar{\nu}_\zeta^2 - 1 > 0 \tag{21}$$

which represents the condition that the nondimensional rotating frequency of the rotor be larger than unity. This corresponds to the well-known fact that the ground resonance instability does not exist for stiff-inplane rotors.

In the case when $d_x = 0$, stability is governed by the coefficients of the blade-damping terms: $O(d_\zeta^3)$, $O(d_\zeta^5)$ and $O(d_\zeta^7)$ in (17). Upon their examination, we were able to show that for a soft in-plane rotor ($\bar{\nu}_\zeta < 1$) and typical parameter values, these terms are negative definite and hence stability is not possible in the absence of body damping.

## 4. Relationship between Deutsch's and Routh's Criteria

### 4.1 Analytical Derivation

The general form of Deutsch's stability criterion can be obtained from Routh's criterion (17) by retaining one of the two candidate sets of terms in the series: (i) $O(d_\zeta^3)$ and $O(d_\zeta^4 d_x)$ or (ii) $O(d_x^3)$ and $O(d_\zeta d_x^4)$ terms. Interestingly, these are not the lowest order terms in (17) but are the only choice which can yield Deutsch's general form in Eq. (19). Starting with the simpler case (ii), as it requires no additional approximations, the resulting Routh's criterion is:

$$\begin{aligned} (RC)_{ii} &\approx O(d_x^3) + O(d_\zeta d_x^4) \\ &= d_x^3 \left( d_\zeta d_x - \frac{\bar{p}_\zeta (1 - \bar{\nu}_\zeta^2)^3}{16(\bar{\nu}_\zeta^2 + 3)\bar{\nu}_\zeta^2 \bar{\omega}_\zeta \bar{\omega}_x} \right) > 0 \end{aligned} \tag{22}$$

or equivalently,

$$d_\zeta d_x > \frac{\bar{p}_\zeta(1-\bar{\nu}_\zeta^2)^3}{16(\bar{\nu}_\zeta^2+3)\bar{\nu}_\zeta^2\bar{\omega}_\zeta\bar{\omega}_x} \qquad (23)$$

The above is clearly different from Deutsch's criterion, even after the substitution of the frequency coalescence condition. For case (i), if we impose the frequency coalescence condition and retain terms to first order in the blade parameter $\bar{p}_\zeta$ (similarly to Johnson), we obtain:

$$\begin{aligned}(RC)_i &\approx O(d_\zeta^3) + O(d_\zeta^4 d_x) \\ &= 16(\bar{\omega}_x - 1)(\bar{\omega}_x - 1)^3 d_\zeta \bar{\omega}_\zeta d_x \bar{\omega}_x \\ &\quad +2(\bar{\omega}_x - 1)^3\bar{\omega}_x^3\bar{p}_\zeta > 0 \qquad (24)\end{aligned}$$

or

$$8(1-\bar{\omega}_x)\bar{\omega}_\zeta d_\zeta d_x - \bar{\omega}_x^2\bar{p}_\zeta > 0 \qquad (25)$$

which is identical to Deutsch's criterion (19). It is worth noting that satisfaction of Eq. (25) does not ensure satisfaction of the criterion (23) derived above and the relative significance of the two is weighted by $d_\zeta$ and $d_x$, respectively. Accordingly, the accuracy of Deutsch's criterion depends on the particular values of the damping ratios, as well as the other nondimensional parameters. The numerical evaluation presented in the next subsection corroborates this analysis.

### 4.2 Numerical Evaluation

The validity of Deutsch's approximation is now evaluated by testing it against the exact Routh's criterion (17) for a typical range of values for parameters $\bar{\omega}_x$, $\bar{\omega}_\zeta$ and $\bar{p}_\zeta$ and one of $d_\zeta$ or $d_x$. Based on the review of the literature for different helicopter configurations, we found that the normalized body and blade frequencies typically lie in the range 0.1 to 0.8; the blade parameter $\bar{p}_\zeta$ may vary from 0.001 to 0.05, depending on the inertia properties of the blades and the craft. Finally, the body and blade damping ratios are usually: $d_\zeta \in [0.005, 0.05]$ and $d_x \in [0.01, 0.1]$. The procedure used to conduct the evaluation can be summarized as follows.

For a given set of parameter values, $\bar{\omega}_x$, $\bar{\omega}_\zeta$, $\bar{p}_\zeta$ and, for example, $d_\zeta$, the damping ratio $d_x$ is calculated according to Deutsch's criterion, in particular:

$$d_x = D\frac{\bar{p}_\zeta\bar{\omega}_x^2}{8d_\zeta\bar{\omega}_\zeta(1-\bar{\omega}_x)} \qquad (26)$$

with $D = 1.01$. The resulting value together with the other parameters are then tested according to Routh's criterion of Eq. (17). This procedure is repeated for 100 values of $\bar{\nu}_\zeta$ in the vicinity of the center of instability as defined by Eq. (9) with $i = x$. Depending on whether Deutsch's criterion is found conservative or not sufficient, we adjust the factor $D$ on the right-hand side of (26) until stability, as per Routh's criterion, is just violated or ensured. Following the terminology in [10], we refer to the aforementioned factor $D$ as Deutsch's number. The results are summarized by plotting $D$ as a function of two parameters, typically the other damping ratio ($d_\zeta$ in the present case) and one of the body or blade frequencies.

Two representative plots are included in Figure 1 for $\bar{\omega}_\zeta = 0.2$ and two values of $\bar{p}_\zeta$. These results were generated for 10 values of blade damping ratio, $d_\zeta = 0.005\ldots0.05$ and 13 values of the nondimensional body frequency, $\bar{\omega}_x = 0.1,\ 0.15\ \ldots\ 0.7$. The plane at $D = 1$ corresponds to Deutsch's criterion satisfied exactly. These graphs demonstrate that Deutsch's criterion is conservative ($D = 0.2$) for some parameter values and is inadequate ($D = 2$) for others. Similar results for other values of the nonrotating blade frequency $\bar{\omega}_\zeta$ lead to the following conclusions. Deutsch's criterion provides good estimates ($0.8 < D < 1$) for the damping required to overcome the instability for low values of the nondimensional body frequency ($\bar{\omega}_x \approx 0.1$-$0.3$) and low blade parameter ($\bar{p}_\zeta \approx 0.001$). Indeed, it is conservative for these values and when the blade damping is also low. The accuracy of Deutsch's criterion deteriorates as the body frequency and the blade parameter values increase. For high values of these parameters, the criterion predicts damping ratios ($d_\zeta$ or $d_x$) larger than one and hence, is not practical. We also observed that the performance of Deutsch's criterion is more sensitive to the variations in the body frequency than the nonrotating blade frequency.

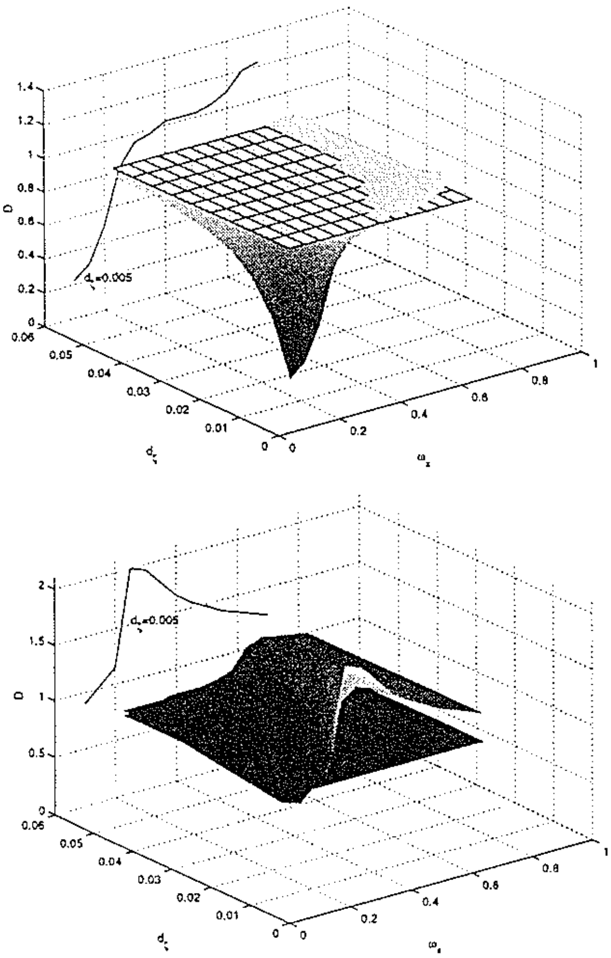

Figure 1: $D$ for $\bar{\omega}_\zeta = 0.2$ and
(a) $\bar{p}_\zeta = 0.001$ (top),
(b) $\bar{p}_\zeta = 0.03$ (bottom)

It is worth noting that in [10], a comparison was made between Deutsch's predictions and results obtained with the nonlinear simulation of the ground resonance instability. Based on our findings, we suggest that the discrepancy between the two is not necessarily due to the nonlinear terms in the simulation, but may be as well due to the approximate nature of Deutsch's criterion for the *linear* ground resonance model. In his textbook, Bielawa [20[ observes that Deutsch's criterion is good when the body and blade damping levels are of comparable order of magnitude. We were unable to confirm this conclusion nor to make any other generalizations along these lines.

## 5. Extension to Four-DOF Ground Resonance Model

The above analysis is extended to a four-DOF helicopter/rotor model which includes the $x$- and $y$-translations of the body center of mass. The characteristic equation for this model is of 8th order and its coefficients $b_i$ can be expressed in terms of the previously defined nondimensional parameters and two new parameters—the body frequency $\bar{\omega}_y$ and the corresponding damping ratio $d_y$.

As for the three-DOF model, Maple was employed to determine the symbolic form of the Routhian array. However, the additional degree of freedom made further symbolic analysis intractable and hence, we proceed directly to the numerical evaluation of Deutsch's criterion. The main goal here is to assess how the coupling between the two body motions affects the validity of Deutsch's criterion at the two instability regions. Indeed, the fact that characteristics of the instability with one body mode are affected at least by the frequency of the other mode is implicit in Deutsch's original criterion. This is evident from the fact that the two criteria—one for configuration II and the other for the isotropic configuration I—are different by a factor of two.

### 5.1 Numerical Results for Four-DOF Model

To illustrate our findings on this issue, we present a series of numerical results similar to those obtained for the three-DOF model. For purposes of conciseness as well as for comparison with the previous results, we concentrate here on the instability resulting from the frequency coalescence with the $x$-mode of the craft. Thus, as for the results in the preceeding section, the evaluation is conducted in the vicinity of $\bar{\nu}_\zeta = 1 - \bar{\omega}_x$.

The plots in Figures 2 and 3 are analogues of Figures 1(a) and 1(b) and are obtained for the same values of $\bar{\omega}_\zeta$ and $\bar{p}_\zeta$, respectively. Each of the two figures contains six plots calculated for three values of the $y$-DOF body frequency ($\bar{\omega}_y = 0.2,\ 0.5,\ 0.8$) combined with one of the two values of the $y$-mode damping: (1) $d_y = 0.001$ to represent the minimal damping case and (2) $d_y = 0.1$ to represent a practical maximum damping value. As in Figure 1, the value of $D$ is evaluated over the grid of parameter values created by $d_\zeta \in [0.005, 0.05]$ and $\bar{\omega}_x \in [0.1, 0.7]$. In each plot, we have also indicated the location of the isotropic manifold, $\bar{\omega}_x = \bar{\omega}_y$ (dashed lines). Also shown is the curve at the particular blade damping value ($d_\zeta = 0.005$) and, where appropriate, the isotropic case projection ($\bar{\omega}_x = \bar{\omega}_y$, solid line). It is noted that the isotropic case for $\bar{\omega}_y = 0.8$ is outside of the $\bar{\omega}_x$ values considered.

5.2 Discussion

From the plots in Figures 2 and 3, we can draw the following conclusions. Qualitatively, our results are in agreement with Deutsch's predictions, in particular, that more damping is required in the isotropic case. As one approaches the isotropic condition, the value of $D$ increases sharply. This trend is particularly pronounced for low value of $y$-mode damping (see left columns of Figures 2 and 3). Our results indicate, however, that Deutsch's factor of two predicted to stabilize the isotropic configuration is by far insufficient. In fact, for all cases considered here, Routh's stability criterion required values of $D$ higher than six. (These correspond to the truncated ridges or ridges exceeding the scale of the plots). The four-DOF model results also indicate that once sufficiently away from the "isotropic region," the results approach those calculated for the three-DOF model. This is clearly visible in Figures 2(c) and 3(c) where the isotropic condition lies outside of the range of $\bar{\omega}_x$ frequencies considered (compare (c) plots with Figure 1(a)).

Interesting conclusions follow from the results calculated with a high value of damping $d_y$ (right columns in Figures 2 and 3). For example, in the case of Figure 2(d), the isotropic condition does not require more damping and the results in this plot again look very similar to Figure 1(a). This is likely because the high damping of the $y$-mode reduces its contribution to the dynamics response of the craft and hence, the instability characteristics in this case are very similar to those predicted with the three-DOF model. The same was not observed for the high value of $\bar{p}_\zeta$ (see right column of Figure 3) where it appears that high damping of the $y$-motion may actually worsen the instability with the $x$-mode (compare Figures 3(b) and 3(e)). This demonstrates that the damping of the "other" mode has complex and subtle effects on the ground resonance instability characteristics with a given body mode.

Finally, upon comparison of Figures 2 and 3, we can observe that the effect of increasing the blade parameter $\bar{p}_\zeta$ is to widen the isotropic band. Johnson gives a definition of an isotropic support as one where the frequencies $\bar{\omega}_x$ and $\bar{\omega}_y$ are of $O(\bar{p}_\zeta)$ apart. He proceeds to note that this being an extremely small difference, "the isotropic case is not important except when the rotor support structure is truly axisymmetric" [17, p. 684]. The results presented here are in partial concurrence with Johnson's statements since the isotropic region increases with $\bar{p}_\zeta$. Quantitatively, however, our findings indicate a significant isotropic band which may exist for any helicopter configuration.

## 6. Conclusions

In this paper we have investigated the stability criteria for the ground resonance phenomenon. Routh's criterion was applied to this problem and its damping requirements were compared against those predicted by Deutsch. Starting with the simple three-DOF model, where the craft has only one translational degree of freedom, Routh's criterion was derived in symbolic form as a function of six nondimensional parameters. This analytsis revealed the approximate nature of Deutsch's criterion. A numerical investigation showed that for some configurations, characterized by low body frequency, low blade parameter and low blade damping, Deutsch's predictions for body damping required for stability were conservative. On the other hand, for other configurations, they were insufficient and up to twice the amount of damping was required to ensure stability of the system.

For the ground resonance model with two body degrees of freedom, the qualitative predictions of Deutsch's criterion were confirmed. In particular, Routh's criterion also requires significantly more damping for the isotropic helicopter configuration. However, the increase in damping by a factor of two, as suggested by Deutsch, is completely inadequate in the isotropic cases. We also observed that the damping requirements for the instability with one body mode are affected by the frequency and damping of the other mode. Our investigation also revealed that the isotropic region, where significantly higher damping values are necessary, is not small, contrary to earlier findings.

## References


1. Coleman, R.P. and Feingold, A.M. [1958]. "Theory of Self-Excited Mechanical Oscillations of Helicopter Rotors with Hinged Blades," NACA Report 1351.

2. Lytwyn, R.T., Miao, W. and Woitsch, W. [1971]. "Airborne and Ground Resonance of Hingeless Rotors," *Journal of the American Helicopter Society*, Vol. 16, No. 2, pp. 2–9.

3. Ormiston, R.A. [1977]. "Aeromechanical Stability of Soft Inplane Hingeless Rotor Helicopters," *Proc. Third European Rotorcraft and Powered Lift Aircraft Forum*, Paper No. 25, France.

4. Hooper, W.E. [1986]. "A Parametric Study of the Aeromechanical Stability of a Bearingless Rotor," *Journal of the American Helicopter Society*, Vol. 31, pp. 52–64.

5. Friedmann, P.P. and Venkatesan, C. [1986]. "Influence of Unsteady Aerodynamics Models on Aeromechanical Stability in Ground Resonance," *Journal of the American Helicopter Society*, Vol. 31, pp. 65–74.

6. Bousman, W.G. [1981]. "An Experimental Investigation of the Effects of Aeroelastic Couplings on Aeromechanical Stability of a Hingeless Rotor Helicopter," *Journal of the American Helicopter Society*, Vol. 26, No.1, pp. 46–54.

7. Weller, W.H. [1994]. "Variation in Hover Aeromechanical Stability Trends with Bearingless Main Rotor Design," *Journal of the American Helicopter Society*, Vol. 39, No. 1, pp. 70–79.

8. Tang, D.M. and Dowell, E.H. [1986]. "Influence of Nonlinear Blade Damping on Helicopter Ground Resonance Instability," *Journal of Aircraft*, Vol. 23, No. 2, pp. 104–110.

9. Gandhi, F. and Chopra, I. [1994]. "An Analytical Model for a Nonlinear Elastomeric Lag Damper and Its Effect on Aeromechanical Stability in Hover," *Journal of the American Helicopter Society*, Vol. 39, No. 4, pp. 59–69.

10. Robinson, C.S., Wood, E.R. and King, R.L. [1998]. "Full Nonlinear Simulation of Coupled Rotor/Fuselage Response Using Symbolically Derived Equations of Motion," *Proc. American Helicopter Society 54-th Annual Forum,* Washington.

11. Ormiston, R.A. [1983]. "Investigations of Hingeless Rotor Stability," *Vertica,* Vol. 7, No. 2, pp. 143–181.

12. Chopra, I. [1990]. "Perspectives in Aeromechanical Stability of Helicopter Rotors," *Vertica,* Vol. 14, No. 4, pp. 457–508.

13. Ormiston, R.A. [1991]. "Rotor-Fuselage Dynamics of Helicopter Air and Ground Resonance," *Journal of the American Helicopter Society,* Vol. 36, No. 2, pp. 3–20.

14. Sharf, I. [1995]. "Literature Survey on Helicopter Ground Resonance," Contract Report.

15. Deutsch, M.L. [1946]. "Ground Vibrations of Helicopters," *Journal of the Aeronautical Sciences,* Vol. 13, No. 5, pp. 223–234.

16. Done, G.T.S. [1974]. "A Simplified Approach to Helicopter Ground Resonance" *Aeronautical Journal* , Vol. 78, pp. 204–208.

17. Johnson, W. [1980]. Helicopter Theory, Princeton University Press.

18. D'Azzo, J.J. and Houpis, C.H. [1995]. Linear Control System Analysis and Design: Conventional and Modern, New York: McGraw-Hill, p. 185.

19. Nahas, M.N. [1984]. "Helicopter Ground Resonance-A Spatial Model Analysis," *Aeronautical Journal,* Vol. 88, pp. 299–307.

20. Bielawa, R.L. [1992]. Rotary Wing Structural Dynamics and Aeroelasticity, American Institute of Aeronautics and Astronautics Inc., p. 351.

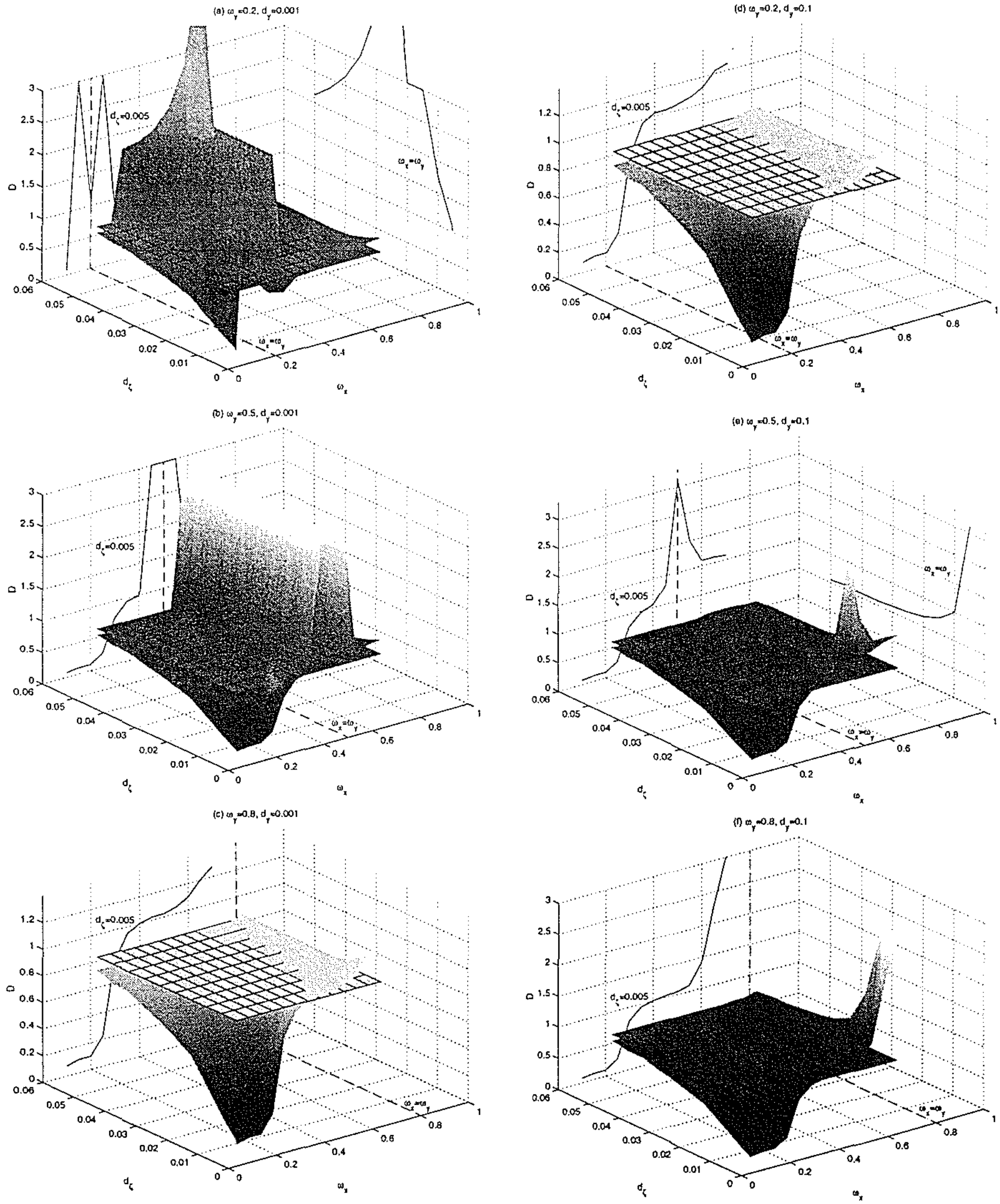


Figure 2: $D$ for $\bar{\omega}_\zeta = 0.2$ and $\bar{p}_\zeta = 0.001$

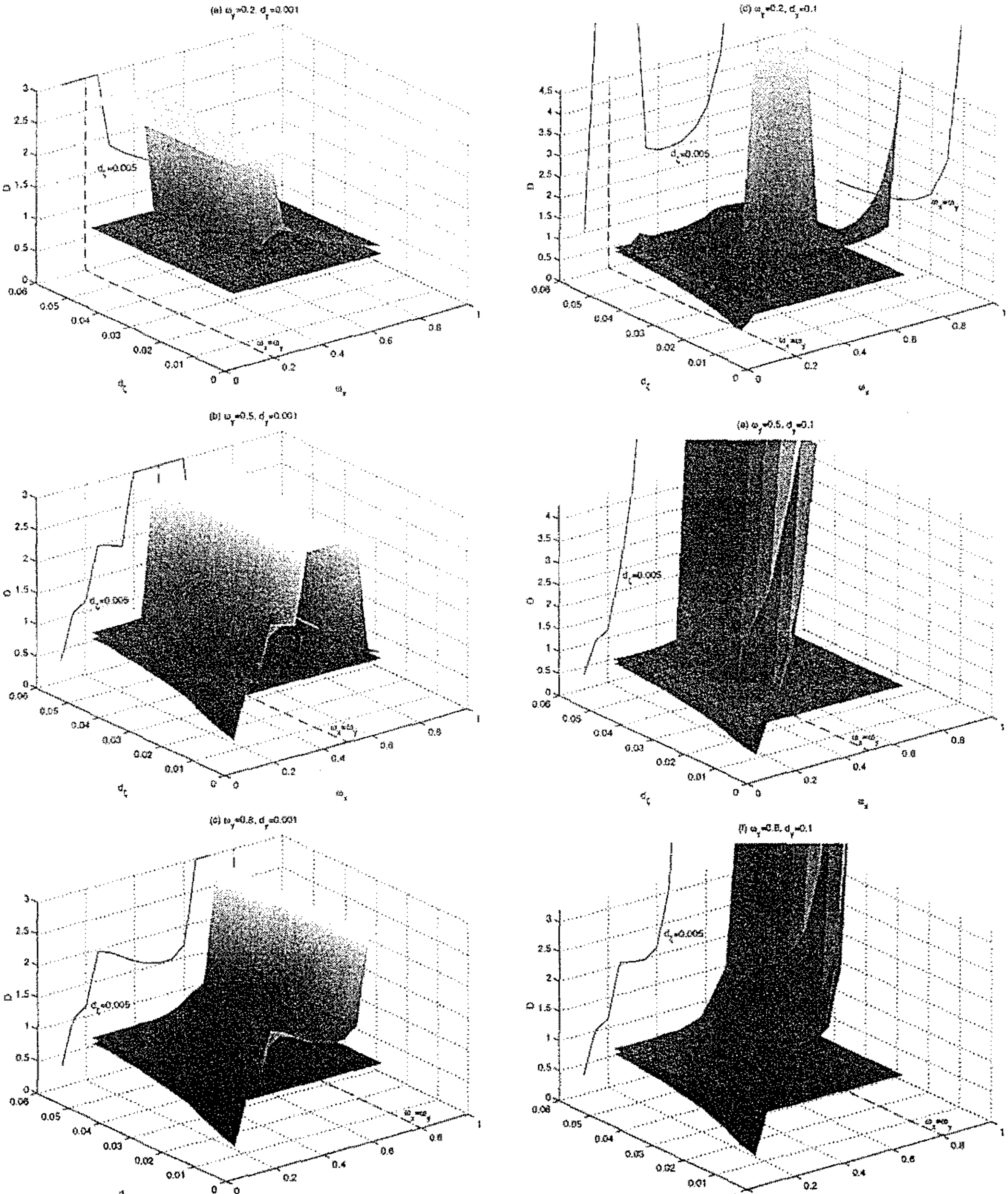


Figure 3: $D$ for $\bar{\omega}_\zeta = 0.2$ and $\bar{p}_\zeta = 0.03$