

## FLIGHT PATH GENERATION FOR A HELICOPTER IN TAIL ROTOR FAILURE CONDITION

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### Abstract

The objective of this study is to generate viable flight trajectories for a helicopter with a failed tail rotor. A generic helicopter model using the FLIGHTLAB software is utilized for tail rotor failure simulation. For a conventional helicopter, there is a certain forward flight speed threshold, above which the vertical tail is sufficient to generate the required anti-torque during forward flight. Tail rotor failures above and below this threshold speed are treated in simulations. Optimal trajectories that brings the helicopter from autorotation to level flight for low speed failures; and from level flight to a predefined location for high speed failures are found. The path generation process is performed in the MATLAB environment with an open-source trajectory optimization tool, OptimTraj. The trajectories are obtained for different scenarios by using linear system dynamic models as constraints.

### 1. INTRODUCTION

Tail rotor is the system that provides the necessary anti-torque to balance the main rotor torque in single main rotor helicopters. By changing the tail rotor collective, the anti-torque can be adjusted; and, the helicopter can be controlled in the yaw axis. In case of a mechanical or structural failure, or aerodynamic phenomenon, the loss of tail rotor effectiveness, the control power in yaw axis could be lost. This study assumes a tail rotor failure (TRF) and that the tail rotor is totally lost, i.e. no forces or moments from the tail rotor are transmitted to the aircraft.

Commercial helicopters suffered 470 accidents because of a TRF between 1963 and 1997 [1]. According to FAA, TRF is the third major cause of fatal helicopter accidents between 1996 and 2007 [2].

When TRF occurs, the recommended emergency procedure is to go into autorotation and to land the helicopter in a safe zone as soon as possible. There are numerous studies to find and follow a flight trajectory during autorotational flight. For example, Abbeel et al. developed a controller to track an autorotative flight path which was previously generated by autorotation flight tests of an RC helicopter flown by an expert pilot [3]. Yomchinda et al. generated autorotation flight paths and applied path following control [4]. Taamallah et al. presented a trajectory planning and trajectory tracking for a small-scale helicopter in autorotation [5]. Bibik & Narkiewicz generated autorotation trajectory using linear quadratic control methodology [6]. In this study, however, it is assumed that when TRF occurs, the pilot cannot perform autorotation for landing due to lack of a suitable landing area.

In this paper, this situation is investigated considering two different possibilities: TRF at high forward speed at which the vertical fin is sufficient by itself to provide required anti-torque, and TRF at low forward speed. For high forward speed, the helicopter can fly without yaw control. However, the sideslip angle would be high so that the vertical fin could create sufficient moment. Directional control can be obtained with the lateral cyclic. Hence, the helicopter might succeed in arriving at a specified location. Figure 1 shows such a case. It is possible for a helicopter with TRF to achieve the desired destination with the remaining controls. This scenario is investigated by targeting two different positions in space.

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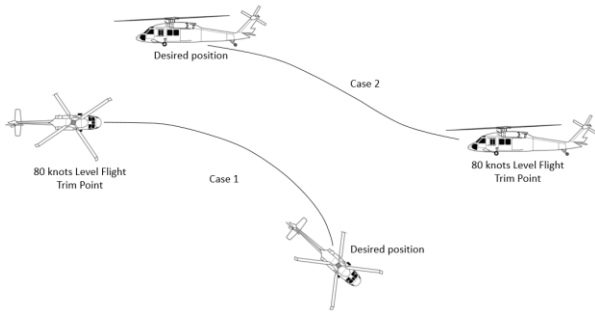


Figure 1: Illustration of high speed failure scenarios

For TRF at low forward speed, autorotation is the viable option in order to eliminate the need for anti-torque. That way, acceleration to a higher speed in level flight may be possible. Figure 2 illustrates such a situation. This scenario is investigated by targeting two different equilibrium points in state-space.

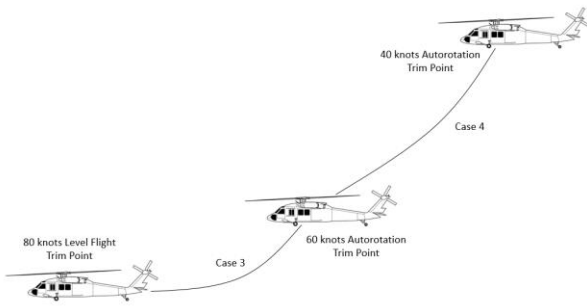


Figure 2: Illustration of low speed failure scenarios

Both cases are basically optimization problems that aim to change the state of the helicopter from an initial condition to a final condition by minimizing a predefined cost function while satisfying some constraints.

Other examples for trajectory generation in literature show optimal flight paths for ship landing [7][8], for operations with a slung load system [9], etc. Similar methods are handled in aircraft and spacecraft trajectory generation studies [10][11][12]. Since the aircraft dynamics and the path constraints include nonlinearities and high number of states, the optimization problem can often not be solved analytically. Therefore, numerical methods are developed for this purpose [13].

In this study, an open-source trajectory optimization tool, OptimTraj [14], is utilized to solve the problem. Trajectory optimization problems can be solved by direct collocation methods by defining an objective function, system dynamics,

and path & boundary constraints using OptimTraj. In order to reduce the computation time, linear helicopter models are used by assuming that the helicopter would be in the neighborhood of its equilibrium point. The linear helicopter models are created by linearizing the generic model in FLIGHTLAB [15] around the trim points in level flight, in 20 degree left & right bank turns at 80 knots, and in autorotation at 40 & 60 knots conditions. In addition to conventional pilot inputs, the main rotor acceleration is added as a control to linear models. The optimal trajectories are obtained for different cases by introducing different objective and constraint functions into OptimTraj.

## 2. HELICOPTER MODEL

In this study, a generic nonlinear model similar to Sikorsky UH-60A in FLIGHTLAB is utilized in order to find trim condition and generate linear helicopter dynamics. To reduce complexity and to be able to focus on the open-loop system, horizontal tail controller, SAS and pitch bias controller are switched off and the mechanical mixing of pilot controls is eliminated from the default nonlinear model. Tail rotor failure can be injected in the generic model by removing all force & moments coming from the tail rotor.

First, level trim analyses are performed in order to find the minimum speed at which the helicopter can maintain direction in case of TRF. The helicopter can be trimmed in level flight with some sideslip angles for 80 knots and above. The vertical tail is sufficient by itself at and above 80 knots and as the velocity increases, the required sideslip angle reduces.

Since the recommended autorotation speed is 80 knots for UH-60A [16] and maneuvering from level flight at 80 knots to autorotation at 80 knots would be reasonable, linearization for level flight is chosen at this velocity. For the high speed case, the aim is to navigate the helicopter to a final position. Therefore, turning maneuvers will be required to arrive at the specified location. Hence, 20 degrees left & right coordinated turns at 80 knots are included to the linear model set.

In this work, the trajectory that brings the helicopter from autorotation to level flight condition is investigated too. For this, autorotation at 40 & 60 knots models are created and added to the linear model set.

For the linear model, the states and controls are given as,

$$(1) x = [\Omega \ \phi \ \theta \ u \ v \ w \ p \ q \ r]^T$$

$$(2) u = [\delta_{coll} \ \delta_{lon} \ \delta_{lat} \ \delta_{ped} \ \dot{\Omega}]^T$$

Note that pedal control has no effect in the TRF condition. Therefore, main rotor acceleration is added into the controls and the main rotor speed is added into the states so that the directional control of the helicopter could be enhanced.

### 3. TRAJECTORY GENERATION

A single-phase continuous time trajectory optimization problem is considered in this study. The general formula of the problem is given as,

$$(3) \min_{t_0, t_F, x(t), u(t)} J_B(t_0, t_F, x(t_0), x(t_F)) + \int_{t_0}^{t_F} J_P(\tau, x(\tau), u(\tau)) d\tau$$

The constraints of the optimization problem can be defined with:

System dynamics,

$$(4) \dot{x} = f(t, x(t), u(t))$$

Boundary constraint,

$$(5) C_B(t_0, t_F, x(t_0), x(t_F)) \leq 0$$

Path constraint,

$$(6) C_P(t, x(t), u(t)) \leq 0$$

Bounds on state, control and time,

$$(7) x_l \leq x(t) \leq x_u$$

$$(8) u_l \leq u(t) \leq u_u$$

$$(9) t_l \leq t_0 < t_F \leq t_u$$

Solving the trajectory optimization problem analytically for a helicopter is not trivial. Therefore, numerical methods are implemented for solving complex optimization problems.

Direct collocation method is one of the numerical methods that can be used to solve trajectory optimization problems.

The expressions in the objective function or in the system dynamics can be approximated by different discretization methods. In the study, trapezoidal rule is selected for the discretization of Lagrange term in the objective function and for the defects calculation with system dynamics.

$$(10) \int_{t_0}^{t_F} J_P(\tau, x(\tau), u(\tau)) d\tau \approx \sum_{k=0}^{N-1} \frac{(J_{P_k} + J_{P_{k+1}})}{2} (t_{k+1} - t_k)$$

$$(11) \zeta_k = x_{k+1} - x_k - \frac{(f_k + f_{k+1})}{2} (t_{k+1} - t_k)$$

The constraints can be collected as,

$$(12) c(x) = [C_{B_0} \zeta_0 C_{P_0} \zeta_1 C_{P_1} \dots \zeta_N C_{P_N} C_{B_F}]^T$$

For the equality constraints,

$$(13) c(x) = 0$$

For the inequality constraints,

$$(14) c(x) \leq 0$$

Hence, the trajectory optimization constraints are represented as nonlinear programming constraints, then, the problem can be handled by parameter optimization solvers.

The system dynamics used in the optimization problem are provided by interpolating the linear models. For each time step, the linear models are scheduled based on the current states' distance (Euclidean second norm) to the equilibrium points of linear models in state space. Thus, the validity of the system dynamics is expanded so that different flight conditions can be covered during optimization.

The state constraints in OptimTraj are introduced differently based on the low speed and high speed scenarios. For high speed cases, the states are bounded along the trajectory as an added precaution for trajectory optimization. The terminal position constraint was introduced separately since positions are not included in the system states. For low speed cases, on the other hand, terminal constraints can be defined as state constraints since desired terminal conditions can be represented by the states themselves. The same added precaution about bounding the states along the trajectory is used for low speed cases as well.

In this study, the position information does not appear explicitly in the helicopter states. The position is calculated from the body states by using rigid body equations. Next, the function that computes the distance from the point, which is desired for the helicopter to go, to the current position is introduced to OptimTraj as an equality constraint. Hence, for feasible solutions, the final position of the helicopter must be on the specified point.

For position navigation in level flight, the minimum control objective function is implemented. For the transition cases from autorotation to level flight, minimum altitude descent is implemented as the objective function; hence, the minimum allowable height from the ground for TRF can be estimated.

For minimum control,

$$(15) \min_{t_0, t_F, x(t), u(t)} \int_{t_0}^{t_F} u(\tau)^2 d\tau$$

For minimum descent,

$$(16) \min_{t_0, t_F, x(t), u(t)} \int_{t_0}^{t_F} z_i(\tau, x(\tau), u(\tau))^2 d\tau$$

## 4. RESULTS AND DISCUSSION

In this study, flight path of a helicopter under TRF is calculated for four example cases. It is assumed that landing immediately is not possible and a landing area is far away. If the helicopter has sufficient forward speed, it is shown that it can fly in level with sideslip. For this scenario, a point in space is specified to navigate the helicopter and the flight trajectory is obtained for minimal input use. If the forward speed is low, the trajectory that brings the helicopter from autorotation to level flight is obtained. For this case, minimum altitude descent is aimed. The investigated flight scenarios are summarized in Table 1.

Table 1: Summary of investigated flight scenarios

Case	Failure Cond.	Linear Models	Objective Function	Constraints
Case 1	High speed	80kts level flight 80kts right turn 80kts left turn	Minimum control	Final Position Sys. Dynamics State Bounds
Case 2	High speed	80kts level flight 80kts right turn 80kts left turn	Minimum control	Final Position Sys. Dynamics State Bounds
Case 3	Low speed	60kts autorot. 80kts level flight	Minimum descent	Sys. Dynamics State Bounds
Case 4	Low speed	40kts autorot. 60kts autorot.	Minimum descent	Sys. Dynamics State Bounds

In the first and second cases, the trajectory for a helicopter in TRF initially in level flight condition is found for a desired position while minimizing the control inputs. In the first case, the helicopter is initially at  $[0, 0, -3000]$  ft and the terminal position is specified as  $[5000, 2500, -3000]$  ft in order to demonstrate the lateral maneuver capability of the helicopter in TRF. In the second case, in order to demonstrate the climb maneuver capability of the helicopter in TRF, the terminal position is specified as  $[5000, -500, -3200]$  ft in the NED coordinate system.

In Figure 3, the position trajectory solution is shown for the first case. Note that the last values are highlighted in the figure and the helicopter is close to the desired point in the final. Figure 4 indicates that the helicopter will roll at the beginning of the trajectory and stay in a positive roll for a while. Then, it will take a negative roll angle again, and complete the path at which the final states are close to initial values in level. It can be noticed that the heading angle changes slowly until it coincides with the desired point. Figure 5 shows the main rotor speed values through the trajectory. The algorithm changes the

rotor speed to reach the desired position. The problem is solved for minimum control effort. The controls are shown in Figure 6. Figure 7 indicates the linear model transition throughout the trajectory. The vertical axis of the graph states which equilibrium point the helicopter is close to. 0, 1 and 2 are the defined equilibrium point values that indicate 20-degree left bank turn trim, level flight at 80 knots, and 20-degree right bank turn trim, respectively. Intermediate values are achieved by interpolating the linear models.

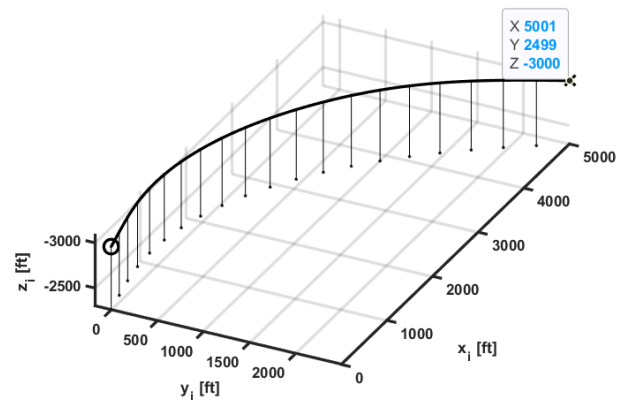


Figure 3: Case 1 - Position in NED coordinate system

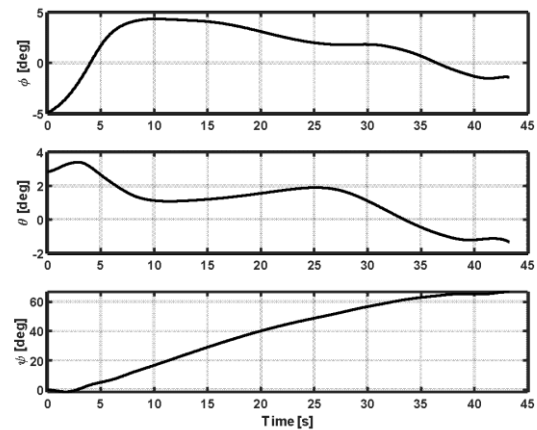


Figure 4: Case 1 - Euler angles

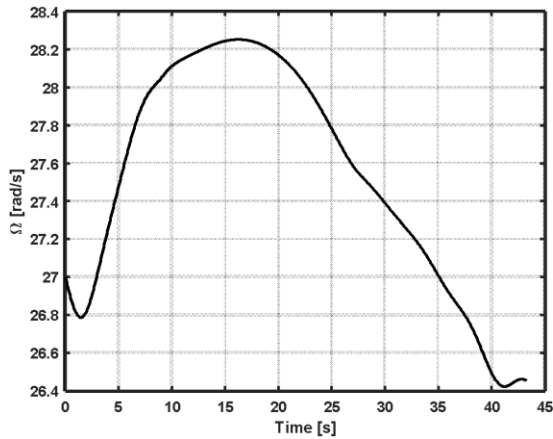


Figure 5: Case 1 - Main rotor speed

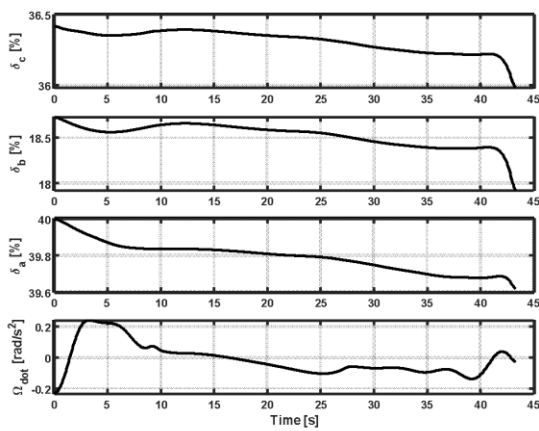


Figure 6: Case 1 - Control inputs

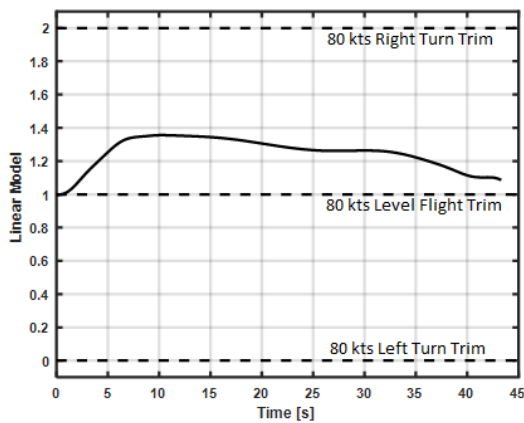


Figure 7: Case 1 - Linear model transition

In the second case, a trajectory for a different terminal position is obtained. In Figure 8, the position trajectory solution is shown for the second case. In Figure 9, it is shown that the helicopter will roll to its left, and without changing its heading angle too much, it will move to the left. Figure 10 shows the main rotor speed values through the path. The rotor speed control will be

utilized at its limit in order to manage the directional control. Since control inputs and linear model transition values are consistent with the mentioned figures, they are not shown for this case.

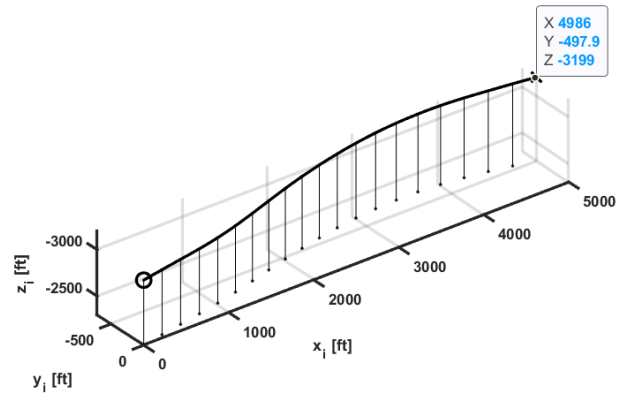


Figure 8: Case 2 - Position in NED coordinate system

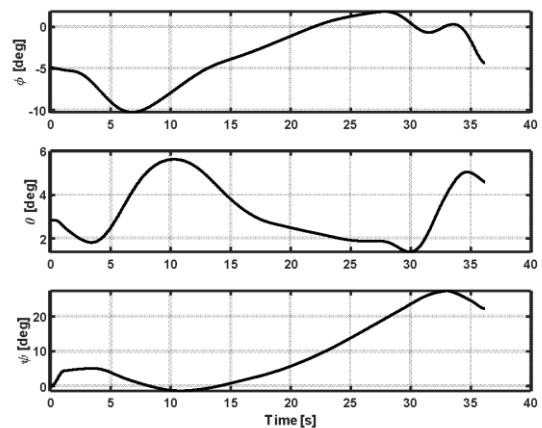


Figure 9: Case 2 - Euler angles

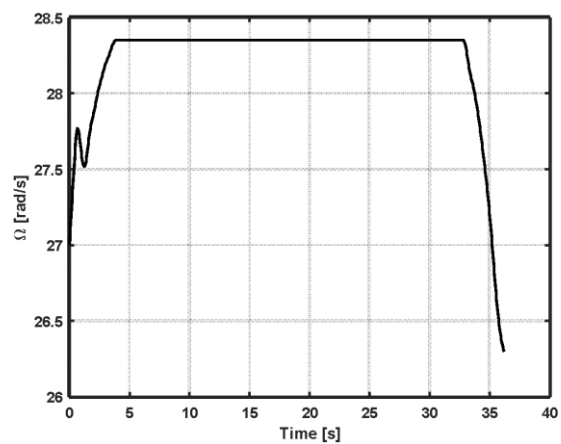


Figure 10: Case 2 - Main rotor speed

In the third case, the TRF is assumed to occur at a low speed and the helicopter is brought from autorotation at 60 knots to level flight at 80 knots while minimizing the altitude descent. Here, the initial states belong to the autorotation trim while the final states belong to the level flight trim.

The position trajectory of the helicopter is shown in Figure 11. Note that the problem is solved for minimum altitude descent and it shows at least 160 ft of altitude loss is required for transitioning from the 60 knots autorotation condition to level flight condition in TRF. Figure 12 indicates the airspeed parameters of the helicopter. As the helicopter approaches the level flight condition, the total velocity increases and the angle of attack decreases indicating that the descent rate reduces. In addition, the sideslip angle increases because the helicopter is no longer in autorotation and an anti-torque is required by the vertical tail. Figure 13, shows the maneuver is initiated by pitching down the nose of the helicopter. After reaching the necessary forward speed for the level flight, a flare maneuver is performed by a pitch up. The model transition can be followed in Figure 14. The linear model transforms from 1 to 2 indicating that the helicopter in autorotation trim at 60 knots completes the trajectory by achieving level flight trim at 80 knots.

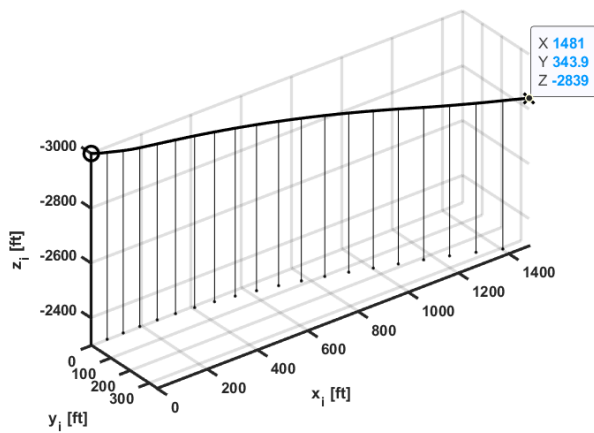


Figure 11: Case 3 - Position in NED coordinate system

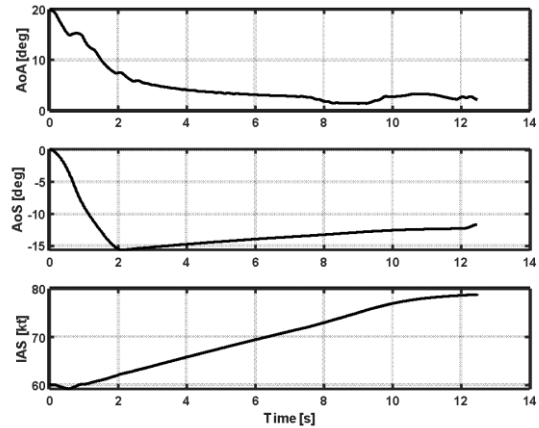


Figure 12: Case 3 - Aerodynamic parameters

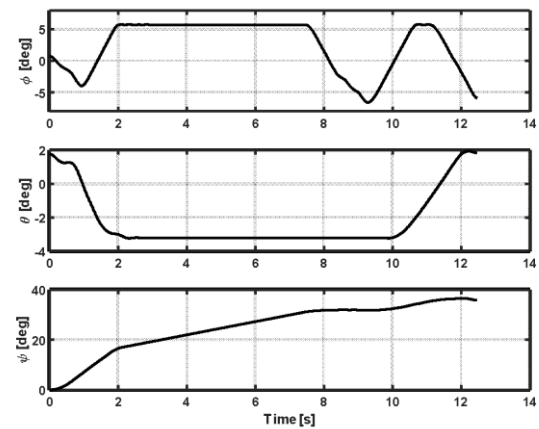


Figure 13: Case 3 - Euler angles

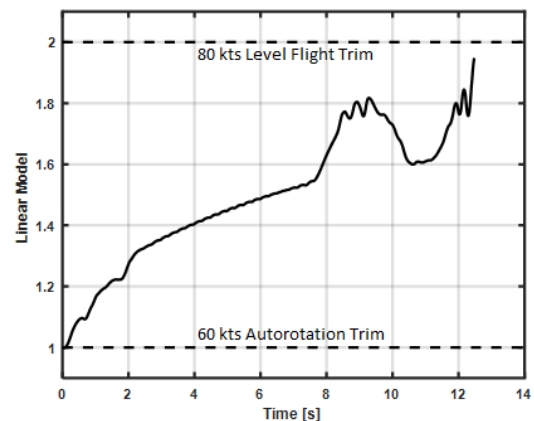


Figure 14: Case 3 - Linear model transition

In the fourth case, the TRF is assumed to occur at a much lower forward speed and the helicopter is brought from autorotation at 40 knots to autorotation at 60 knots while minimizing the altitude descent.

The position trajectory of the helicopter is shown in Figure 15. Note that the problem is solved for minimum altitude descent and a 450 ft altitude

loss is required to transition from 40 knots autorotation condition to 60 knots autorotation condition in TRF. Figure 16 indicates the airspeed parameters of the helicopter. As the helicopter approaches the 60 knots autorotation condition, similar changes happen as in the previous case. The total velocity increases and the angle of attack decreases indicating that the descent rate reduces. Figure 17, illustrates that the linear model transforms from 1 to 2 indicating that the helicopter in autorotation trim at 40 knots completes the maneuver by achieving autorotation trim at 60 knots.

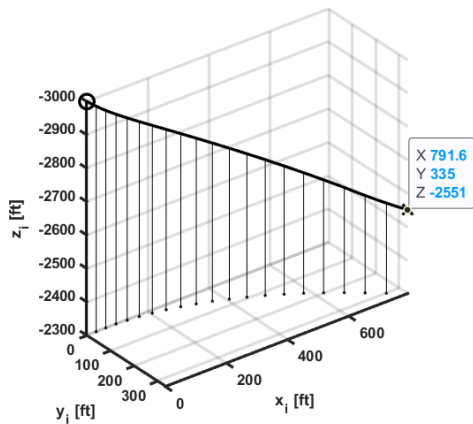


Figure 15: Case 4 - Position in NED coordinate system

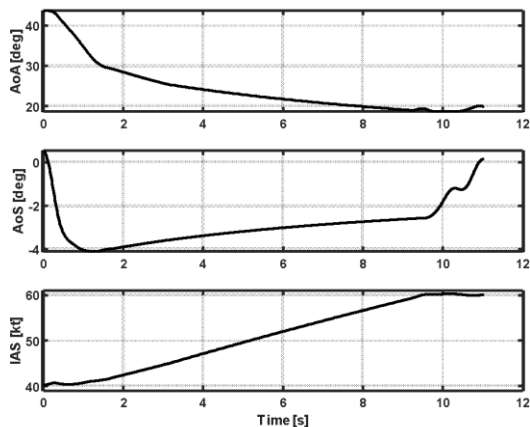


Figure 16: Case 4 - Aerodynamic parameters

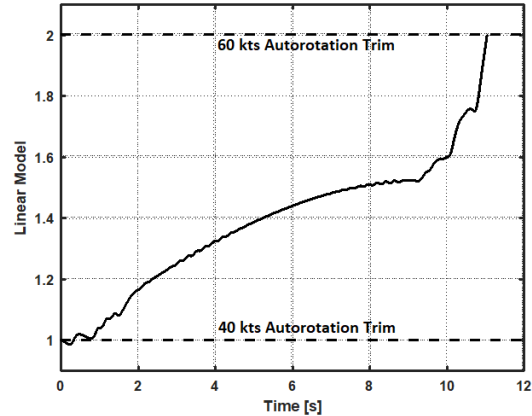


Figure 17: Case 4 - Linear model transition

## 5. CONCLUSION

The solutions of level flight guidance and trim condition transition are presented with four example cases. In the first and the second cases, TRF occurs in level flight condition and the helicopter is guided to a final position while minimizing the control inputs. For both cases, the final positions are captured with accepted tolerances. In the third and the fourth cases, the trim transition of the helicopter in TRF is performed while minimizing the altitude loss. In both cases, the desired trim conditions are achieved with minimum required altitude loss.

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