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NON-LINEAR FLIGHT MECHANICS OF A HELICOPTER ANALYSIS BY APPLICATION OF A CONTINUATION METHODS

BY

K. SIBILSKI
MILITARY UNIVERSITY OF TECHNOLOGY, POLAND

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NON-LINEAR FLIGHT MECHANICS OF A HELICOPTER ANALYSIS BY APPLICATION OF A CONTINUATION METHOD

Krzysztof SIBILSKI
Military University of Technology
Warsaw, Poland

Abstract

Recent development in the field of numerical analysis of non-linear differential equations created a class of computer algorithms known as continuation methods. These methods follow solution curves of non-linear equations of motion represented by functions of a number of variables and parameters, respectively. This approach was successfully demonstrated for aircraft flight dynamics analysis.

The aim of this paper is to present some significant results obtained in helicopter flight dynamics, when this methodology is applied to a real helicopter, is the Polish PZL "Sokół" ("Falcon"). The paper is essentially related to oscillatory motions and to analysis of a kind of rotorcraft intense spiral glide motion, called also as "a spin of the helicopter". The equations of motion used in these investigations assumed a realistic "individual blade" helicopter model. Aerodynamic model includes also a region of higher angles of attach including deep stall phenomenon.

1. Introduction

There are many problems associated with flight dynamics for modern and advanced helicopters, which are not solved (or solved rather unsatisfactory) with traditional tools. A list of such problems includes among others flight control for modern combat helicopters, which flies close to the ground to utilise the surrounding terrain, vegetation, or manmade objects. The obstacle-avoidance manoeuvres are repeatedly realised in extreme, limiting flight conditions. Such manoeuvres are joined with a number of singularities, including "unexpected" helicopter motion. As the result of them, there is dangerous of faulty pilot's actions. Therefore, it is need to investigate rotorcraft dynamics phenomena associated with flying in extreme conditions.

Non-linear problems are central to several important rotorcraft motions. Inertial coupling is the most important non-linearity in roll-coupling instability, while stall phenomena of the main rotor blades involve both non-linear aerodynamics and inertial coupling. Standard linearized mathematical model cannot be here applied. The primary aim of the paper is to discuss capabilities of dynamical system theory methods as the tools to analyse such phenomena. Recent development in the field of numerical analysis of non-linear equations created class of computer algorithms knows as *continuation methods*. Dynamical system theory has provided a powerful tool for analysis of non-linear phenomena of helicopter behaviour. In the application, the bifurcation theory [1], [2] and numerical continuation methods [3], [4] have been used to study roll-coupling instabilities and stall/spin phenomena of a number of aircraft models. Results of great interest have been reported in several papers (it can be mentioned papers by Jahnke and Culick [5], Carroll and Mehra [6], Chuitteau [7], or the recent study by Avanzini and de Matteis [8]). Continuation methods are numerical techniques for calculating the steady states of systems of ordinary differential equations and can be used to study helicopter motion instabilities.

Carroll and Mehra [6] were the first to use a continuation technique to calculate the steady states of aircraft. They determined the steady states of a variable sweep aircraft and the F-4 fighter aircraft. By studying the steady states of these two aircraft they explained that wing rock appears near the stall angle of attack due a Hopf bifurcation of the trim steady state. They also calculated the steady spin modes for the aircraft and predicted the control surface deflections at which the aircraft would undergo stall/spin divergence. They have calculated the resulting steady state of the aircraft and have developed recovery techniques using their knowledge of the steady spin modes for the rotorcraft.

Guicheteau [7] has used continuation methods and bifurcation theory in analysis of non-linear dynamics of aircraft model that includes unsteady aerodynamic coefficients. He has analysed the effects of a lateral offset of the cg. and the influence of gyroscopic momentum of rotating engine masses on spin entry recovery.

Jahnke and Culick [5] have recalled the theoretical background of dynamical system theory and bifurcation technique and they have comprehensive review of the relevant investigations in this field. They have studied the dynamic of the F-14 fighter aircraft by determining the steady states of the equations of motion and seeking bifurcations. Jahnke and Culick have shown that continuation method is very useful in analysis of the wing rock instability, spiral divergence instability and spin dynamics.

The present paper is continuation of the previous works of the author [12], [19], [20]. The non-linear an "individual blade" helicopter model is applied to determine the helicopter motion. It is shown [9], [10], [11] that an "individual blade" model, including a correct representation of the rotor-engine drive train, is required to adequately predict rotorcraft response for aggressive manoeuvres. It is assumed that the helicopter fuselage is a rigid body and the motion of rigid blades about flap, lead-lag, and axial hinges is considered, while the tail rotor is a linear model using strip/momentum theory with a uniformly distributed inflow. Simplified model of vortex field is applied and spatial structure of tip vortex trajectories is taken into consideration. Unsteady aerodynamics for prediction of rotor blade loads is included, and the ONERA type stall model is used ([13], [23]).

The purpose of this approach is to evaluate the risk of helicopter control loses using the continuation methods and bifurcation theory. After a brief description of the methodology and associated procedures, certain post stall flight event is studied by means of checking the stability characteristics related to unstable equilibria. The results from dynamical systems theory are used to predict the nature of the instabilities caused by the bifurcations and the response of the helicopter after bifurcations is encountered. Numerical simulation of the helicopter spin is used to observe chaos phenomenon in aggressive manoeuvres.

2. Theoretical background

Dynamical systems theory provides a methodology for studying systems of ordinary differential equations. The first step in analysing a system of non-linear differential equations, in the dynamical system theory approach, is to calculate the steady states of the system and to investigate their stability. Steady states of a system can be found by setting all time derivatives equal zero and solving the resulting set of algebraic equations. The Hartman-Grobman theorem ([1], [2]) provides that the local stability of a steady state can be determined by linearizing the equations of motion about the steady state and calculating the eigenvalues. A steady state is locally stable if the real parts of all the eigenvalues of the linearized system are negative. If the real part of any eigenvalue of the linearized system is positive, the steady state is locally unstable. In the neighbourhood of a steady state the system will be attracted to the steady state if the steady state is stable and repelled from the steady state if the steady state is unstable.

When the linearized system is non-singular, the implicit function theorem proves that the steady states of the system are continuous function of the parameters of this system [2]. Thus it can be stated, that the steady states of the equations of motion for the helicopter are continuous functions of the swash plate deflections, the main rotor collective pith and the tail rotor collective pitch. Stability changes can occur as the parameters of the system are varied in such a way that the real parts of one or more eigenvalues of the linearized system change the sign. Changes in the stability of a steady state lead to qualitatively different responses for the system and are called bifurcations. Stability boundaries can be determined by searching for steady states, which have one or more eigenvalues with zero real parts. There are many types of bifurcations and each has different effects on the rotorcraft response. Qualitative changes in the response of the helicopter can be predicted by determining how many and what types of eigenvalues have zero real parts at the bifurcations point. Bifurcations for which one real eigenvalue is zero lead to the creation or destruction of two or more steady states. Bifurcations for which one pair of complex eigenvalues has zero real parts can lead to the creation or destruction of periodic motion. Bifurcations for which more than one real eigenvalue or more than one pair of complex eigenvalues has zero real parts lead to very complicated behaviour.

2.1. Bifurcation Theory

For steady states of rotorcraft motion, very interesting phenomena appear when even if one negative real eigenvalue crosses the imaginary axis when control vector varies. Two cases can be considered.

1. The steady state is regular, i.e. when the implicit function theorem works and the equilibrium curve goes through a limit point. It should be noted that a limit point is structurally stable under uncertainties of the differential system studied.

2. The steady state is singular. Several equilibrium curves cross a pitchfork bifurcation point, and bifurcation point is structurally unstable.

If a pair of complex eigenvalues cross the imaginary axis, when control vector varies, Hopf bifurcation appears ([14], [15]). Hopf bifurcation is another interesting bifurcation point. After crossing this point, a periodic orbit appears. Depending of the nature of nonlinearities, this bifurcation may be sub-critical or supercritical. In the first case, the stable periodic orbit appears (even for large changes of the control vector). In the second case the amplitude of the orbit grows in portion to the changes of the control vector.

Other domain of interest concerns the behaviour of the system when periodic orbits loose their stability (Fig.1). Three possibilities can to be concerned in this case:

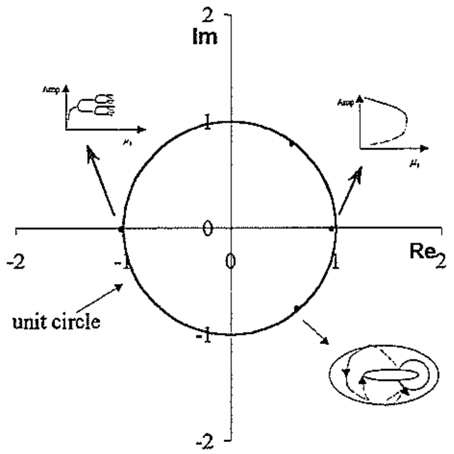

Fig. 1 Bifurcations of period orbits

1. A real eigenvalue crosses the point +1. There appear periodic limit points in this case.

2. A real eigenvalue crosses the point −1. There occurs a period doubling bifurcation in this case. In the vicinity of this point, the stable periodic orbit of period T becomes unstable, and a new stable periodic orbit of period $2T$ appears. This type of stability loss conduct to chaotic motion.

3. Two conjugate eigenvalues leave the unit circle. Motion lines on stable or unstable tours surround the unstable orbit after this case of bifurcation.

Guckenheimer and Holmes [1], Ioos and Joseph [2], Keller [4], Crawford [16], Troger and Steindl [17] have provided a thorough introduction to the Bifurcation Theory.

2.1. Continuation Methods

Continuation methods are a direct result of the implicit function theorem, which proves that the steady states of a system are continuous functions of the parameters of the system at all steady states except for steady states at which the linearized system is singular. The general technique is to fix all parameters except one and trace the steady states of system as a function of this parameter. If one steady state of the system is known, a new steady state can be approximated by linear extrapolation from the known steady state ([3], [4]). The slope of the curve at the steady state can be determined by taking the derivative of the equation given by setting all time derivatives equal to zero. If two steady states are known, a new steady state can be approximated by linear extrapolation through the two known steady states. The stability of each steady state can be determined by calculating the eigenvalues of the linearized system. Any changes in stability from one steady state to the next will signify a bifurcation.

3. Non-linear equations of motion

The purpose of this work has been to use dynamical system theory and continuation methods to analyse the equations of motions of a helicopter. The coupled equations of motion which involve the main rotor and body degrees of freedom are solved simultaneously, as present in detail in Ref. [19]. The blade element rotor model, in addition to represent non-linear, unsteady aerodynamics, enables correct representation of the flight dynamics of helicopter. Equations of dynamic equilibrium of forces and moments are determined in the system co-ordinates fixed with the fuselage and the systems of co-ordinates fixed with the rotor blades. Detailed way of determining of these equations can be found in [18], [19], [21], and [22].

The full set of non-linear equations can be also developed by LaGrange approach using the well-known MAPLE® symbolic processing software for expanding the equations. The Lagrangian approach and MAPLE® symbolic processing software can be systematically applied to make possible the full error-free expansion of the equations of coupled rotor-fuselage motion. The symbolic processing software can be utilised to convert the equations of motion into Fortran source code that is formatted specially for numerical solving.

The helicopter motion is described by the set of 10+2n (n-number of main rotor blades) non-linear differential equations with periodic coefficients, which can be presented in the form [19]:

$$\mathbf{A}(\mathbf{x}(t))\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)), \qquad \mathbf{x}(0) = \mathbf{x}_0 \quad (1)$$

Where **x** is the state vector:

$$\mathbf{x} = [u, v, w, p, q, r, \dot{\beta}_1, .. \dot{\beta}_n, \zeta_1, \Omega, \psi_1, .. \psi_n, \beta_1, .. \beta_n, \zeta_1, .., \zeta_n, \Theta_e, \Phi_e, \Psi]^T \quad (2)$$

u, v, w are linear velocities of the centre of fuselage mass in the co-ordinate system fixed with the fuselage, p, q, r are angular velocities of the fuselage in the same co-ordinate system; Θ_e, Φ_e, Ψ_e are pitch, roll and yaw angles of the fuselage, β_i - i-th blade flap motion about flap hinge ζ_i - i-th blade lead-lag motion about lead-lag hinge. **u** is the control vector:

$$\mathbf{u} = [\theta_0(t), \kappa(t), \eta(t), \phi_T(t)]^T \quad (3)$$

Where: θ_0 is angle of collective pitch of the main rotor; κ is control angle in the longitudinal motion; η is control angle in the lateral motion, and ϕ_T is angle of collective pitch of the tail rotor.

The set of ordinary differential equations (1) was solved using the continuation and bifurcation software AUTO97 (available at *ftp://ftp.cs.concordia.ca/pub/doedel/auto*). This very useful free software gives all desired bifurcation points for different values of control vector components (for example swash plate deflections).

4. Aerodynamic forces and moments

The requirements for method on aerodynamic load calculations stem both from flow environment and from algorithms used in analysis of helicopter flight. The airframe model consists of the fuselage, rotor blades, horizontal tail, vertical tail and landing gear. The fuselage model is based on wind tunnel test data (as function of angle of attack α and slip angle β). The horizontal tail and vertical tail are treated as aerodynamic lifting surfaces with lift and drag coefficients computed from data tables as functions of angle of attack α and slip angle β. The tail rotor is linear model using strip-momentum theory with an uniformly distributed inflow. The effects of rotor wash on the airframe are included in the model. The technique used provides the essential effects of increased interference velocity with increased rotor load and decreased interference as the rotor wake deflects reward with increased forward speed [19], [21]. Aerodynamic data is for a NACA 23012 airfoil in the angle of attack range +/-23° and the compressibility effects have been included there. The data have been blended with suitable low speed data for the remainder of the 360° range to model the reversed flow region and fully stalled retreating blades. Semi-empirical methods, that use differential equations, have been used to predict the unsteady aerodynamic loads and dynamic stall effects. The basic model was developed by ONERA [23]. The ONERA model is a semi-empirical, unsteady, non-linear model which uses experimental data to predict aerodynamic forces on an oscillating airfoil which experiences dynamic stall ([13]). State variable formulations of aerodynamic loads to allow use existing codes for rotorcraft flight simulation

5. Results of numerical analysis

All the results presented in this section refer to a PZL „Sokół" helicopter in forward, near service ceiling flight.

Continuation methods require a known steady state as a starting point for the continuation procedure. It is usually easy to determinate steady states that are at low and moderate values of collective pith. Determining the steady states intensive spiral glide motion ("helicopter spin") modes for rotorcraft is more difficult task and it is usually not possible to be certain all the steady states for a particular helicopter have been determined.

The approach used to find the spin modes in this work was to guess an initial spiral glide mode as a starting point for the continuation method algorithm, then let the algorithm run until either a true steady spin was determined or the algorithm ran into numerical problems.

Steady spin modes can also be obtained from figs 2-7. Recall that the steady states for $\eta\in$<-10.3°, -9°>, and $\eta\in$<1.8°, 3.1°>, there are regions of unstable spin modes. Figures 2-7 show the steady states of the PZL "Sokół" as a function of lateral swash plate deflection η for a collective pitch of 20^0, longitudinal swash plate deflection of –2.5°, and tail rotor collective pitch of 24°. Those figures show, that multiple steady states exists for most lateral swash plate deflections. For example, a vertical line representing -4^0 of swash plate deflection intersects three steady states. All of them are stable, so the helicopter could exhibit any of these three steady states. One stable steady state at -4^0 lateral swash plate deflection represents the horizontal flight trim configuration ($p=q=r=\Theta=\Phi=0$). The other two stable steady states represent steady spiral-glide (or steady "helicopter spin") trim conditions. The segment of unstable steady states contains the trim conditions between the lateral swash plate deflections at -10.3^0 and -9^0; -2.5^0 and -1.6^0; 1.9^0 and 3.1^0 because of six Hopf or saddle-node bifurcation. Those bifurcations occur at $\eta=-10.3^0$ (saddle-node bifurcation); η=-9° (Hopf bifurcation); η=-2.5° (saddle-node bifurcation); η=-1.6° (Hopf bifurcation); η=1.90 and η=3.1° (Hopf bifurcations).

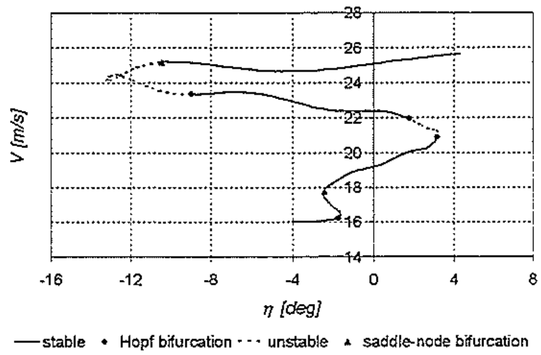

Fig. 2 Steady states for lateral manoeuvres with κ=-2.5°, Θ_0=20°, ϕ_T=24° – variation $V(\eta)$

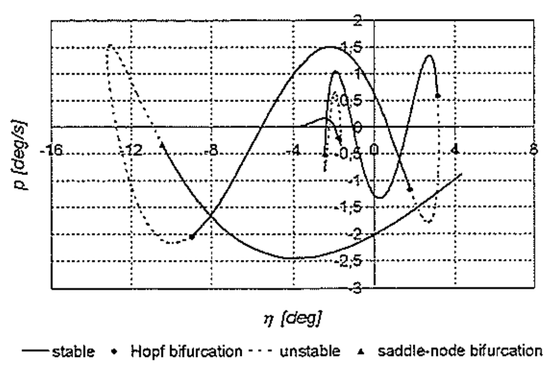

Fig. 3 Steady states for lateral manoeuvres with κ=-2.5°, Θ_0=20°, ϕ_T=24° – variation $p(\eta)$

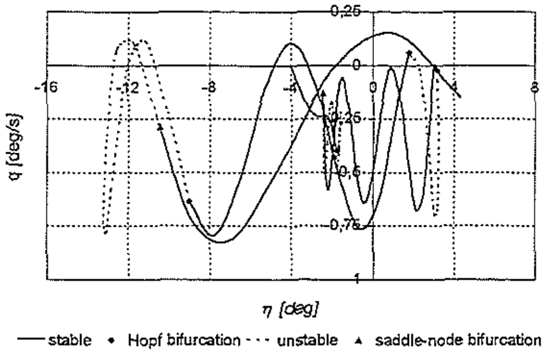

Fig. 4 Steady states for lateral manoeuvres with κ=-2.5°, Θ_0=20°, ϕ_T=24° – variation $q(\eta)$

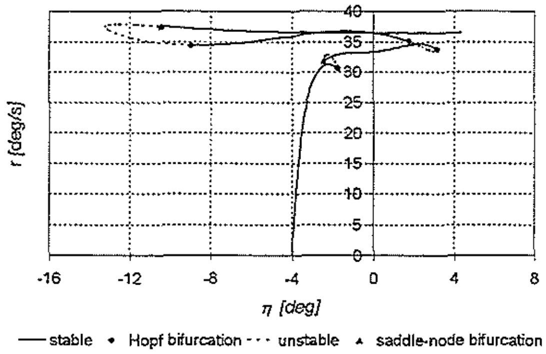

Fig. 5 Steady states for lateral manoeuvres with κ=-2.5°, Θ_0=20°, ϕ_T=24° – variation $r(\eta)$

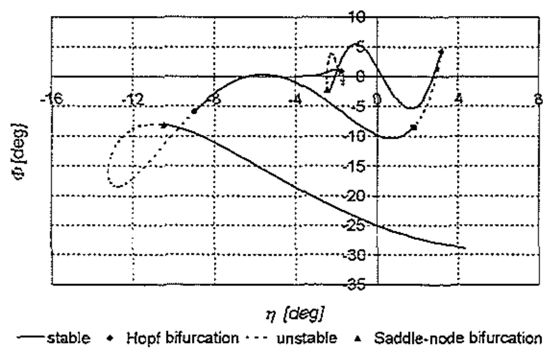

Fig. 6 Steady states for lateral manoeuvres with κ=-2.5°, Θ_0=20°, ϕ_T=24° – variation $\Phi(\eta)$

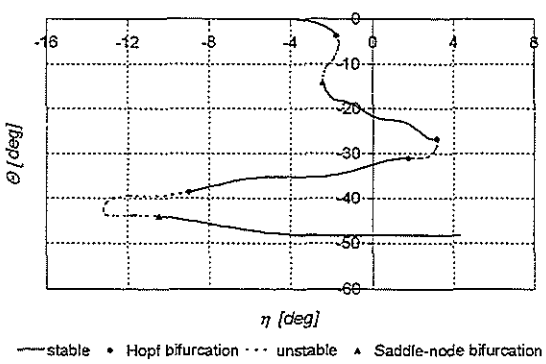

Fig. 7 Steady states for lateral manoeuvres with κ=-2.5°, Θ_0=20°, ϕ_T=24° – variation $\Theta(\eta)$

Figures 8-18 shows an attempted spin entry using only the lateral swash plate deflection. A small perturbation of swash plate deflection (near the initial value at η_o=1.8°) causes the rotorcraft to enter spin with a positive roll rate. During the spin all flight parameters increases its values. In terms of continuation methods, the spin is unstable because of Hopf bifurcation, that occurred at η_T=1.8°.

The Poincare maps of selected state parameters are shown in Figs. 15-18. It can be stated that taking into consideration unsteady rotor-blade aerodynamic model and hysteresis of aerodynamic coefficients, one counters significant irregularities in solution of equations of motion, that are characteristic for chaotic motion. When the condition for the onset of chaotic motion is satisfied, both flapping and pitching motions appear to have chaotic oscillations (see Tang and Dowell [24], [25], [26], [27]).

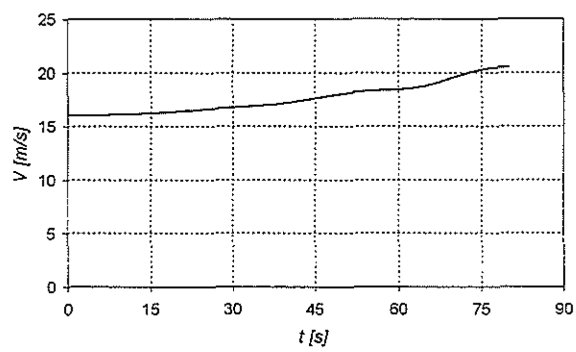

Fig. 8. Simulation of helicopter spin - variation of the airspeed

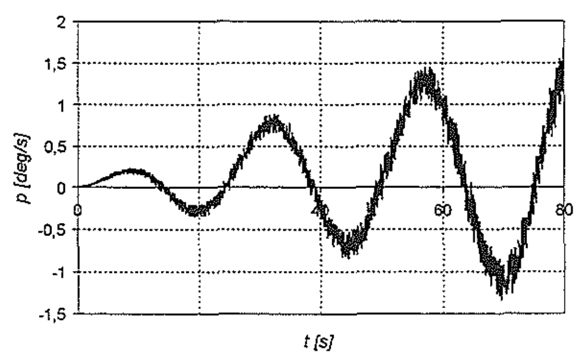

Fig. 9 Simulation of helicopter spin - variation of roll rate

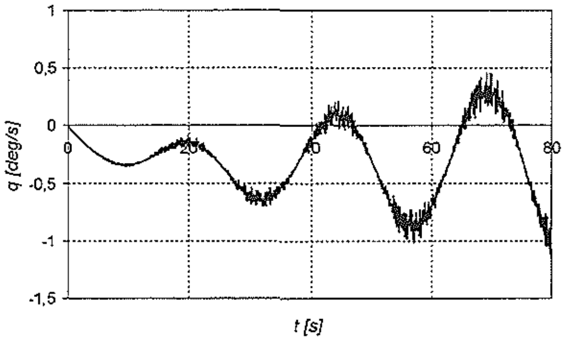

Fig. 10 Simulation of helicopter spin
- variation of pitch rate

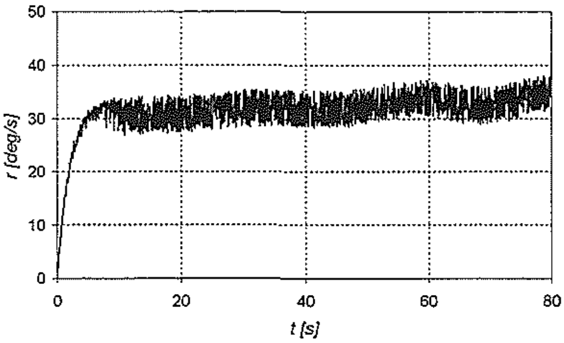

Fig. 11 Simulation of helicopter spin
- variation of yaw rate

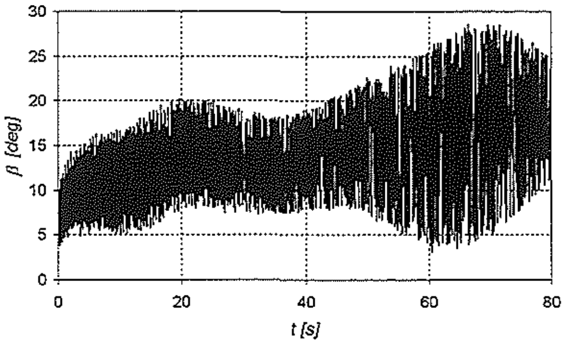

Fig. 12 Simulation of helicopter spin
- variation of flap angle

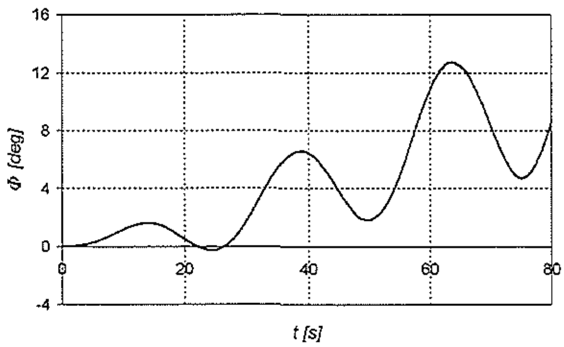

Fig. 13 Simulation of helicopter spin
- variation of roll angle

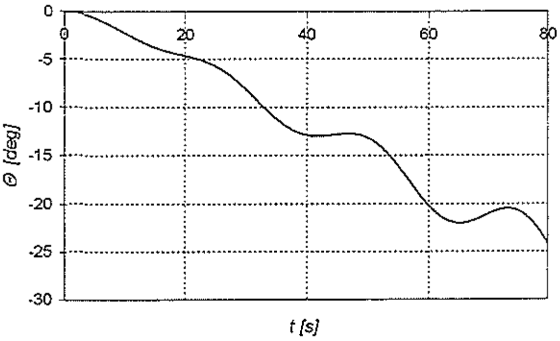

Fig. 14 Simulation of helicopter spin
- variation of pitch angle

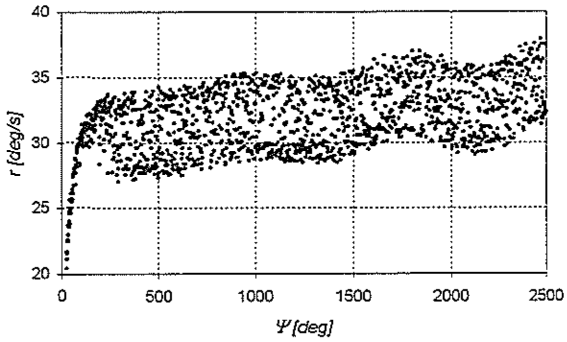

Fig. 15 Simulation of helicopter spin.
Poincare map – variation of $r(\Psi)$

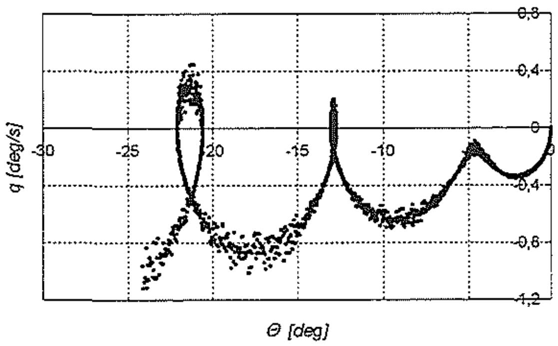

Fig. 16 Simulation of helicopter spin.
Poincare map – variation of $q(\Theta)$

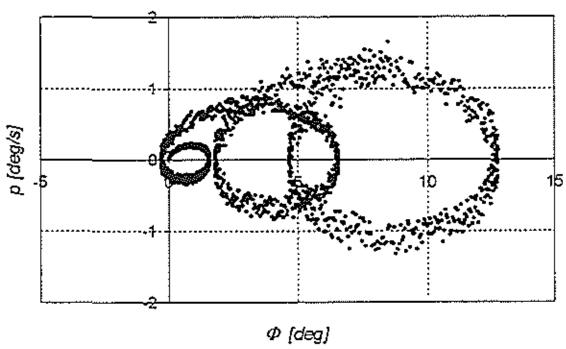

Fig. 17 Simulation of helicopter spin.
Poincare map – variation of $p(\Phi)$

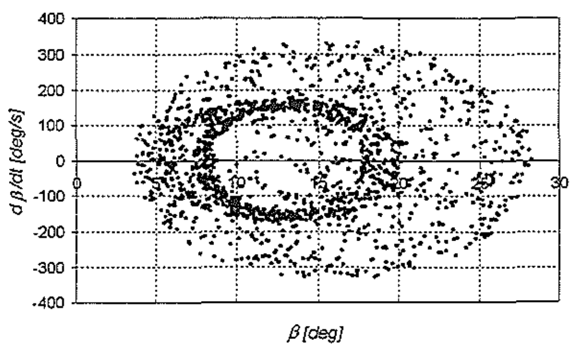

Fig. 18 Simulation of helicopter spin.
Poincare map – variation of $\dot{\beta}(\beta)$

6. Conclusions

The above results show the value of using of continuation methods and dynamical systems theory for analysing the equations of motion for a helicopter. The efficiency of the method makes it possible to analyse complicated aerodynamic models using the complete equations of motion for the entire range of collective pitch and swash plate deflections.

The method presented has great potential for designing control laws. Simple feedback control system ([20], [21]) can also be included in the helicopter model to determine the effects of control systems on various instabilities. Knowledge of the swash plate deflections that cause bifurcations can also be used to escape from motions caused by a jump in the state of the helicopter. Continuation methods can be extended to determine rotorcraft motion as functions of the parameters of the system.

Acknowledgements

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