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REDUCING VIBRATION BY STRUCTURAL MODIFICATION

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Summary

The philosophy underlying the proposed method as an aid in ultimately reducing vibration of a helicopter fuselage is described. An outline of the associated analysis is given and the results obtained so far for a simple structural model of a helicopter are presented.

1 Introduction

Rotor induced vibration of a helicopter fuselage is an ever present problem, and as far as possible in the design stage the vibration levels are kept to a minimum, particularly in the crew and passenger area. This may be achieved by using vibration absorbers mounted on the rotor head or in the fuselage, or by designing the gearbox and engine mountings so that the transmission of oscillatory forces is suppressed. One possibility that is not often considered is to design the fuselage structure itself so that vibration response in the important areas is minimised.

To be able to do this in an ideal situation would involve optimising the separate elements of the fuselage structure; it would be a complex problem, bearing in mind that it is now common for a fuselage structure to be represented in mathematical model form by hundreds or even thousands of elements. To treat each element as a variable would be beyond the scope of the average optimisation computer program. What is needed is some means of choosing the best few elements that can be treated as variables. One basis for choosing would be to pick those elements that have potentially the greatest effect on reducing the vibration. The present paper suggests a group of criteria for making the choice, these criteria being based on a little known property of linear structures that is described in Section 2.

Another, and possibly more important reason for being able to pinpoint sensitive elements or parts of the structure is in the case of the vibration performance of a newly assembled helicopter not achieving its specification. Again, there is the possibility of introducing vibration absorbers and also modification of the fuselage structure. The latter, if it is done, would most likely be done in a fairly ad hoc manner because of the pressure to get results quickly. It would clearly be advantageous to know beforehand which are the most sensitive areas of the fuselage structure with regard to affecting vibration response in the crew and passenger area.

The present method used for finding this is based on a property of linear structures noticed by Vincent [1] of Westland Helicopters Ltd. If a structure is excited by a sinusoidal force while either the mass at a point or the stiffness between two points (as represented by a spring) is continuously varied, then the response in the complex plane at some other point is seen to trace out part of a circular locus. Furthermore, it two parameters are varied, an area of "feasible response" is formed in the complex plane, from which it is possible to decide if the desired response is feasible for the combination of parameters chosen.

Another method that is being used in an exploratory way is that of Sciarra [2]. It is common practice to design, as far as it is possible, the fuselage such that the natural frequencies of the major normal modes are well separated from the rotor forcing frequency. Sciarra uses an energy density principle for defining which structural elements should be altered to achieve a better positioning of the natural frequencies.

In the following sections, a brief account of the analysis will be given with a description of the mathematical properties. The bases on which the sensitivity criteria rest will be described and the application to a simplified sixty degrees of freedom helicopter fuselage structure will be outlined.

2 Structural Response Theory

The object of this section is to establish the manner in which the response at a point on the fuselage is affected by altering one of the structural parameters. This may be generalised to examining the response at a point on any linear structure when either a stiffness between two points is altered, or the mass at a point is varied. The variation of damping is not considered as it is felt not to be a practical way of altering structural characteristics.

The amplitude vector $\underline{x}$ for a structure subject to a sinusoidal forced excitation of amplitudes $\underline{F}$ is given by

$$\underline{x} = \underline{G}\,\underline{F} \tag{1}$$

where

$$\underline{G} = \left[\underline{K} - \underline{M}\omega^2 + i\omega\underline{C}\right]^{-1} \tag{2}$$

and $\underline{M}$ is the mass matrix, $\underline{K}$ the stiffness matrix, $\underline{C}$ the (viscous) damping matrix and ω the circular frequency of the exciting forces. The matrix $\underline{G}$ is a matrix of complex receptances between all the points concerned.

Consider the structure shown diagramatically in Fig. 1. It is considered to have many degrees of freedom, and the response at material point q due to the single forced excitation at point p is to be examined.

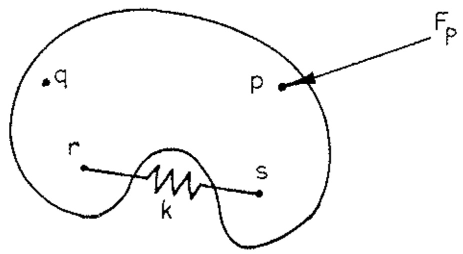

Fig. 1 Structure and Variable Stiffness Spring

A simple structural modification is made by inserting a linear spring k between two points r and s that have mutually compatible degrees of freedom. The spring is adjusted so as to exert zero force when the system is in equilibrium. When the original structure is considered as a free body, the forces exerted on it at points r and s by the spring are F_r and F_s, respectively, which have the relationship

$$F_r = k(x_s - x_r) = -F_s \tag{3}$$

The forcing vector $\underline{F}$ in equation (1) contains three non-zero elements, F_r, F_s and F_p, whereas the elements of immediate interest in the response vector $\underline{x}$ are x_q, x_r and x_s. Partitioning and expanding equation (1) yields

$$x_q = G_{qp}F_p + G_{qr}F_r + G_{qs}F_s \tag{4}$$

$$x_r = G_{rp}F_p + G_{rr}F_r + G_{rs}F_s \tag{5}$$

$$x_s = G_{sp}F_p + G_{sr}F_r + G_{ss}F_s \tag{6}$$

in which G_{ij} is the complex receptance providing the displacement at point i due to a force at point j.

The forces F_r and F_s may be expressed in terms of x_r and x_s by using equation (3) and subsequently elimination of x_r and x_s gives

$$\frac{x_q}{F_p} = G_{qp} + \frac{k(G_{sp} - G_{rp})(G_{qr} - G_{qs})}{1 + k(G_{rr} + G_{ss} - G_{rs} - G_{sr})} \tag{7}$$

This is now the modified complex receptance between points q and p in terms of k and the original complex receptances G_{ij}. It can be shown that as k varies from $-\infty$ to $+\infty$ the locus of the tip of the complex receptance vector traces out a circle in the complex plane. The relevant mathematics is given in Done and Hughes [3], which follows the theory originally established by Vincent [1].

If two spring elements of stiffnesses k_1 and k_2 are inserted into the structure, then the response is not constrained to follow a single curve. Let k_2 be temporarily held constant while k_1 is varied. A circular response locus will be produced. When k_2 assumes another value, another circular response locus is obtained as k_1 varies (see Fig. 2).

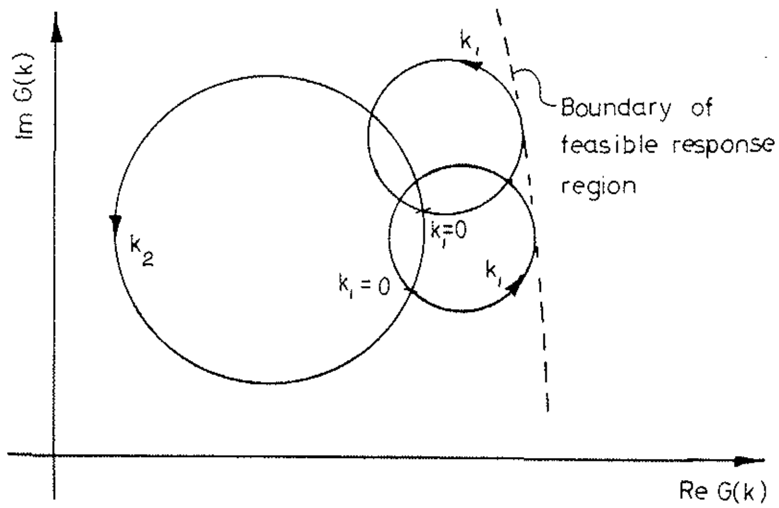

Fig. 2 Response Circles for Two Standard Parameters

If the k_1 circle is "started" at the same value of k_1 each time, then these starting points lie on the response locus of the system when k_2 alone is varied. By covering all possible combinations of k_1 and k_2 a region in the complex plane if formed inside which the response at point q due to an oscillatory force at p must lie. This is referred to as a "feasible response region". The boundaries of this region are given when the Jacobian $\partial(\zeta,\eta)/\partial(k_1, k_2)$ is zero, i.e.

$$\begin{vmatrix} \partial\zeta/\partial k_1 & \partial\zeta/\partial k_2 \\ \partial\eta/\partial k_1 & \partial\eta/\partial k_2 \end{vmatrix} = 0 \tag{8}$$

where ζ and η are the real and imaginary parts of the response per unit force x_q/F_p in equation (7).

Some examples of feasible response regions are shown in Fig. 3.

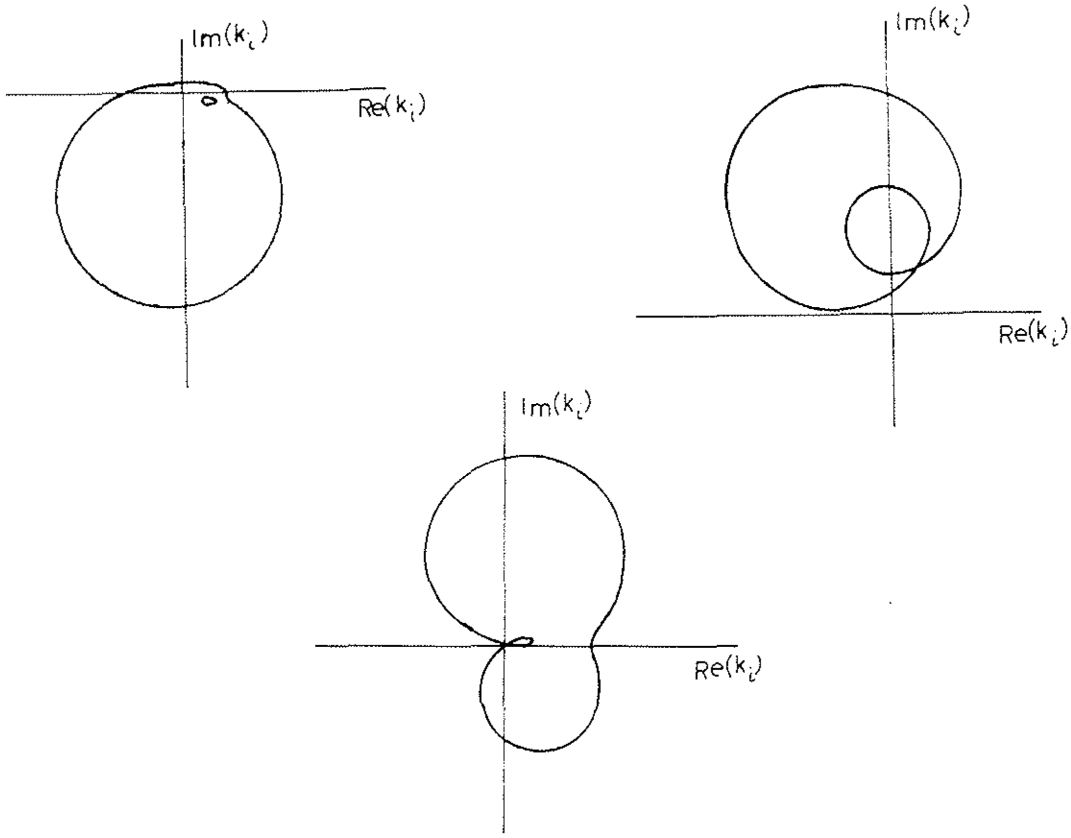

Fig. 3 Typical Feasible Response Regions

If, instead of using a simple spring, the structure is modified by changing a point mass, it can be shown that the expression for response is given by a degenerate form of equation (7). In fact, the form of the equation is exactly similar to that for a spring attached at one end to the structure and at the other end to ground. The direction of the locus as mass increases is clockwise (i.e. opposite from that for a stiffness increase.)

3 Application to Helicopter Fuselage Model

The foregoing analysis is now applied to the problem of determining which part of the fuselage structure of a particular helicopter is most effective in reducing the rotor induced vibrational response in the region of the pilot's seat. A simple two-dimensional model of the fuselage of the Westland Lynx comprising 25 elements and 60 degrees of freedom (two translational and one rotational at each node) is used. A sketch of the structural model layout appears in Fig. 4, and it may be noted that elements on the structure forming a reasonably well defined substructure are numbered consecutively. This is to allow an easy visual interpretation of the results described below. Both vertical and horizontal responses at the pilot's seat have been computed, but for the sake of brevity only the latter are plotted and discussed. The excitation on the structure is an oscillatory couple of frequency 21.7Hz applied at the top of element 1 as shown in the figure. Changes of stiffness between adjacent nodes are considered, as well as changes of point mass at the nodes. The best criteria for seeking out the most sensitive parts of the structure for achieving the desired minimum response will, it is hoped emerge as experience is gained; in the meantime different criteria are used as outlined below.

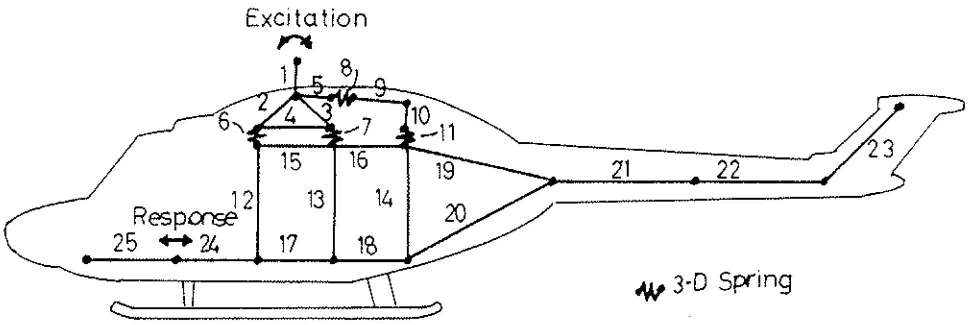

Fig. 4 Structural Model

Firstly, the diameters of the response circles for stiffness changes to each element are computed and tabulated in decreasing order of size. To obtain the relative effectiveness, which in this case is best described as the relative ability to change the response, the total range of circle diameters is normalised onto the range zero to unity. The final normalised values are shown in Fig. 5 with two ordinates appearing against each element. The shaded ordinate refers to the maximum of the three diameters associated with the element (there are three degrees of freedom per node and hence three ways a direct stiffness can be inserted into an element) and the unshaded ordinate shows the average.

The second measure of sensitivity adopted arises from choosing stiffnesses in pairs and examining to see if a pair can produce zero response at the pilot's seat (i.e. if the zero response point falls within the feasible region). Many pairs satisfy this criterion, so the number of times a given element occurs in a successful pair can be tabulated in descending order. The results showing relative effectiveness appear in Fig. 6. These are, as before, normalised on the range 0 to 1 and could be interpreted as providing the relative effectiveness of a given element to actually produce zero response in conjunction with another undefined element.

Thirdly, the actual minimum response that can be achieved for a single element stiffness change was computed for all possible changes. This is not strictly a measure of sensitivity, more a simple statement of the ability of a single element of the unmodified structure to achieve a desired zero response. As before, two ordinates are given for each element corresponding to the overall smallest minimum response, and the average of the three associated minimum responses. Because the orders of magnitude of the responses cover a large range, the logarithms of the inverses have been used with subsequent normalisation to compute the ordinates in order to achieve an even spread of results. These are given in Fig. 7.

Finally, response circle diameters for point masses introduced at the structure nodes are normalised and plotted in Fig. 8, again with the two ordinates per point applying as before.

The C.P.U. time for the separate operations was as follows: formation of eigenvalues and eigenvectors (mainly for checking purposes)(60 seconds), inversion and transformation to provide receptances (15 seconds), tabulation to produce

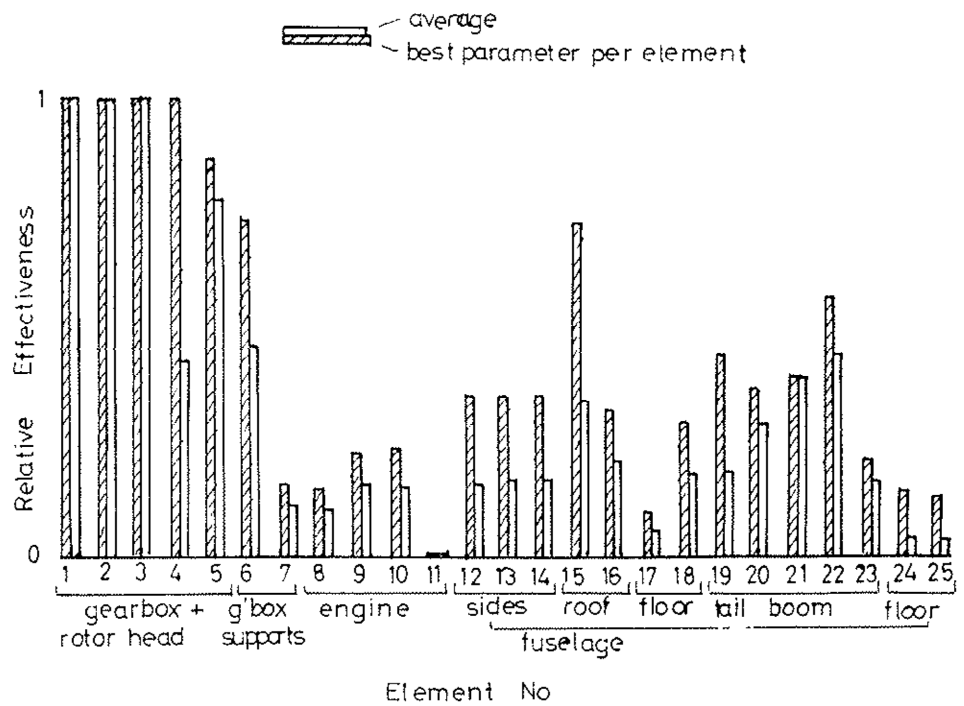

Fig. 5 Relative Effectiveness of Changing Stiffness Based on Response Circle Diameter

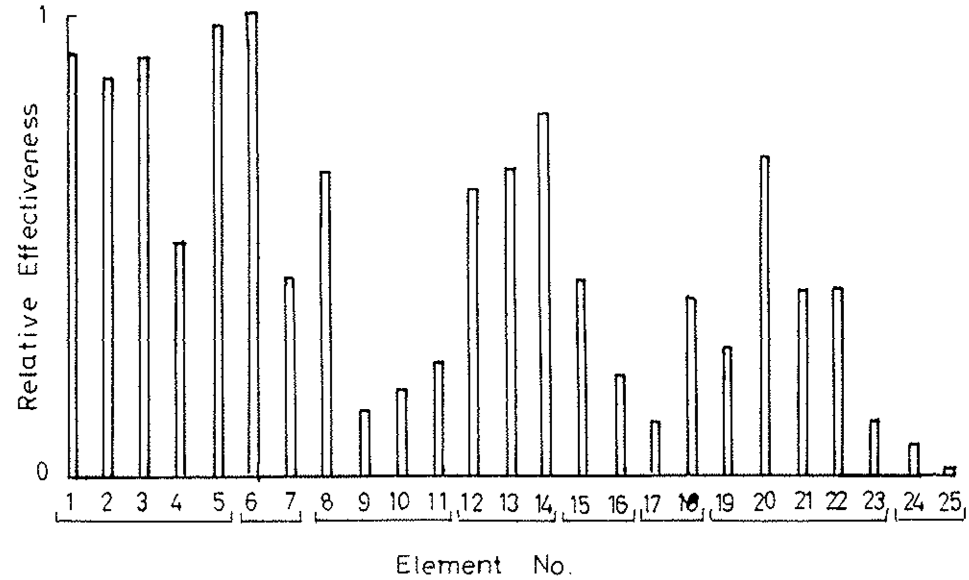

Fig. 6 Relative Effectiveness of Changing Stiffness Based on Zero Response Occurrence

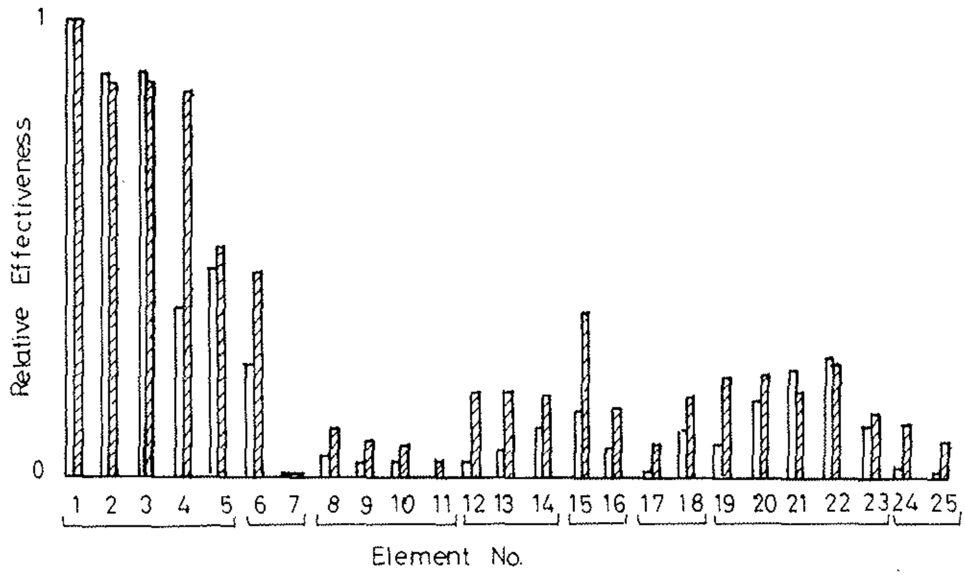

Fig. 7 Relative Effectiveness of Changing Stiffness Based on Minimum Response Achieved

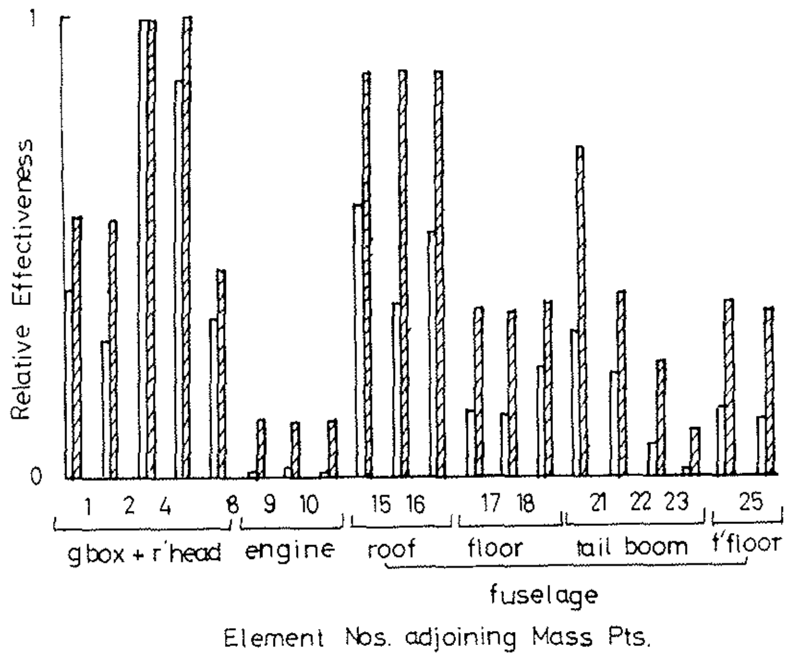

Fig. 8 Relative Effectiveness of Changing Mass Based on Response Circle Diameter

circle diameter and minimum response for 75 parameters (2.7 seconds), occurrence listing, which involved testing 2850 pairs of parameters for zero response (7.9 seconds). It should be noted that the receptance matrix in equation (2) has only to be worked out once for a given structural model at one forcing frequency; the remaining computations are straightforward arithmetic operations on the elements of the receptance matrix and require no iterative procedures.

4 Discussion

It is already known that, for the helicopter under examination, the gearbox and tail cone substructures are suitable areas for modification for the purpose of alleviating vibration in the crew area. This knowledge was gained through a process of trial and error guided by an unavoidably limited amount of theoretical analysis, and it resulted in the tail cone being stiffened between elements 22 and 23 and the gearbox mounting being considerably stiffened. The present method was not available at the time the modifications were made.

The results in Figs. 5, 6 and 7 for stiffness changes are mutually consistent, although different criteria for effectiveness are used, and the message provided is broadly in agreement with the known facts for this helicopter. The grouping of the longer ordinates suggests clearly that the gearbox stiffnesses play the most important part in controlling vibration in the crew area, and to a slightly lesser extent so do those of the tail cone structure. The fuselage sides form the next substructure of importance. What is most interesting is that the same conclusions are reached regardless of the criterion used. The gearbox being important is intuitively obvious as it is close to the source of the excitation; the fuselage sides are sensitive mainly in shear and this again is intuitively reasonable. The tail cone is maybe not so obviously important, so that it is particularly encouraging that the analysis has highlighted it.

The results for the relative effectiveness of introducing mass show that mass changes at the gearbox mounting points are important. When considered with the results for stiffness changes this appears to reinforce a current philosophy that the gearbox and its mounting merits a good deal of attention in the design stages if a satisfactory vibration performance is to be achieved.

It is not intended that anything more than these tentative conclusions should be drawn from the data presented. Refinements could be introduced into the computer programme so that the "raw" effectiveness is weighted by a factor depending on the extent to which a parameter remains within the bounds of practical reality; alternatively, mass and stiffness changes could be made interdependent, but it is not obvious with a simple model such as the one presently used that much extra information would be gained by doing either of these. It is anticipated that an analysis in greater depth on a more complicated and more representative structural model would normally follow, with attention directed at those areas or substructures known to be relatively sensitive. It is at this stage in the proceedings that formal optimisation can properly be considered as a means of finally crystallising a design or re-design.

Acknowledgements

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