ROBUST GAIN SCHEDULING IN HELICOPTER CONTROL

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Abstract

This paper is about gain scheduled multivariable control laws for advanced rotorcraft control systems. A robust control law based on H_∞ optimisation is used as a baseline for the control law development. It is shown that the enchancement of linear controllers via current gain scheduling practices may not give the desired robustness or performance. A simple optimisation approach is employed to determine a class of nonlinear functions such that the closed loop performance stays within a prespecified tolerance.

1. Introduction

Linear controller design techniques are the most commonly used tool in industry. They are easy to use and the control solution is fairly visible to the systems' engineers. However, for helicopters with large operating envelopes quite often linear designs are driven beyond their limits. The assumptions regarding small deviations from nominal conditions are no longer satisfied. Airspeed dependent dynamics and different loading configurations may degrade significantly the guaranteed performance.

Over the last decade research in multivariable control laws seems to have tackled partially the problem of deviations from nominal conditions by improving the robustness of the control laws. Indeed, guaranteeing robustness against modelling errors and excursions from the design point proves a very effective tool in reducing the number of the linear designs accross the flight envelope. However, it can be argued that some sort of scheduling strategy for linear control laws will always be necessary. Therefore, the controller has to possess a clear structure and relatively low order. In the authors' opinion H_∞ optimisation in conjuction with μ - analysis offers, so far, one of the most attractive solutions to these requirements.

In the UK, several ground-based studies on the Large Motion Simulator (LMS) at DRA Bedford [16],[15] have shown that good stability margins alongside high performance requirements [2] are achievable. In [15] it was demonstrated that a two-degrees-of-freedom (2DOF) approach to the Loop Shaping Design Procedure (LSDP) provides an elegant framework for high bandwidth control law design. The design used a linear function to blend between two adjacent controllers. However, there is no guarantee that a linear schedule between two controllers guarantees closed loop stability let alone satisfactory performance. In practice engineers have to do extensive time domain simulations across the flight envelope to ensure that stability and desirable

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performance are guaranteed.

The theoretical background on the analysis and synthesis of scheduled systems is only in its infancy. Recent work has been concentrated on μ-analysis and Linear Parameter Varying (LPV) methods most notably in [14, 3, 4]. Useful guidelines from [14] alongside μ-analysis, in a multivariable context, have been used very succesfully in fixed wing areas eg. [13]. The key element of the above research was the uncertainty the designers were trying to compensate for. In the special case of polytopic plants it was possible to link the uncertainty with Lyapunov functions (see [3, 4]). However, Lyapunov functions are inherently a very conservative tool for control systems synthesis. It is not surprising that, so far, only small state dimension problems have been solved. Additionally the nonlinear plant description has to be converted into a LPV representation, which must depend affinely on the scheduling variable.

In this paper we show on an example that a linear gain schedule does not give the desired performance. Instead, there appears to be a class of nonlinear scheduling functions providing good closed loop stability margins. A simple optimisation approach is also proposed which enables the designer to choose an appropriate scheduling function.

2. Background

The starting, and probably the most important, point in any control law is the choice of the models to be used for linear controller design. It is essential that the linearisations are good representations of the plant, capturing as many nonlinearities as possible. Controlling a hovering helicopter presents the most challenging problem for the control laws as the unaugmented plant is unstable, highly nonlinear and cross-axis coupled. Therefore, the use of a low speed linearisations for controller design seems justified. However, good models in the hovering regime are hard to obtain. Airspeed, angle of attack and sideslip are typical signals that cannot be measured accurately. A robust multivariable controller would ensure that good disturbance rejection and command tracking are achievable in real flight.

Having justified the need for a robust controller we have a variety of methods to choose from. All the H_∞ techniques have their origins in the small gain theorem [17]. The designer is called to minimise ∞-norms (i.e. maximum gains) of different transfer functions, which in turn lead to different types of uncertainty. LSDP is compatible with additive perturbations to the normalised coprime factors and as it was shown in [6] the method encompasses the most general type of uncertainty. Additionally, there are other advantages making LSDP a powerful design tool for the helicopter control problem. We refer to the most important ones:

- The controller is designed using classical loop shaping ideas. The open-loop plant is shaped with frequency dependent weights. The weights typically are $P+I$ elements that specify the desired bandwidths.
- The controller is calculated exactly and the achievable cost function is also a measure of robust stability. Recall that the cost function as introduced in [12] reads the relationship

$$\gamma \triangleq \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \right\|_\infty \leq \frac{1}{\epsilon}. \quad (1)$$

  For SISO systems the maximum stability margin ϵ is equivalent to gain and phase margins [GM,PM] via the formula

$$GM \geq (1+\epsilon)/(1-\epsilon), \quad PM \geq 2\arcsin(\epsilon).$$

- The controller has equal dimension to the shaped plant and there are no pole-zero cancellations between the controller and the shaped plant.
- Gap-metric and μ-analysis can be employed to assess the robustness against perturbations on the plant and/or the controller. The transition from a controller K_α designed at an operating point α to a controller K_β designed at an operating point β can be performed, in the simplest way, by interpolating the gains of the control laws. In the case of loop shaping controllers K_β can be viewed as a perturbation of K_α along the trajectory of the scheduling variable. Similar arguments can be stated for the plant model used for the design of controller K_β. In view of the ν-gap theory (see [7]) we can have an estimation of the degraded performance when both plant and controller are perturbed to a certain distance, as viewed by the metric. More precisely the stability margin is degraded by no more than $arcsin(\epsilon_\beta) \geq arcsin(\epsilon_\alpha) -$

$arcsin\delta_\nu(G_\alpha, G_\beta) - arcsin\delta_\nu(K_\alpha, K_\beta)$, where $\delta_\nu(G_\alpha, G_\beta), \delta_\nu(K_\alpha, K_\beta)$ is the gap- metric between the plants and the controllers respectively.

- The controller can be written as an exact observer and implemented in the feedback loop. The state feedback uses rotor states within the augmentation loop and therefore it may be used for high bandwidth control as pointed in [5].

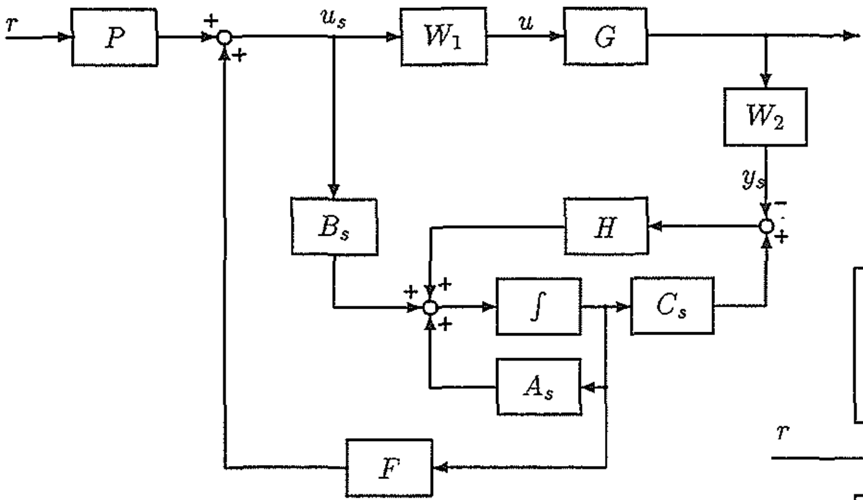

Figure 1: H_∞ controller written as an observer

3. "Intelligent" interpolation

Consider a loop shaping controller written in an observer form as in figure 1. The basic stabilisation gains are the control and output injection matrices H and F respectively. It was assumed that the plant and the controller can be written as convex functions of the form:

$$\begin{aligned} P &= (1-\beta)P_a + \beta P_b \\ K &= (1 - f(\beta))K_a + f(\beta)K_b \end{aligned} \quad (2)$$

where $\beta \in [0, 1]$ is the normalised speed (serving as scheduling variable) and $f(\beta) \in [0, 1]$ is the speed-dependent controller scheduling function. Here, convexity ensures that for $\beta = 0$ and $\beta = 1$ the controller corresponds to hover and high speed designs respectively. Clearly the nonlinear behaviour of the helicopter across the flight envelope has been divided into spaces where the model can be regarded as Linear Time Invariant (LTI). An LTI system with internal stability requirements alongside H_∞ bounds such as (1) guarantees closed loop stability only at frozen operating design points. To ensure full envelope performance we need to replace the infinite number of constraints imposed, with a μ-performance test. In other words the set of LTI plants alongside the LTI controllers have to be represented in a Linear Fractional Transformation (LFT) form as in figure 2. Here, r is the exogenous disturbances, q the vector of the signals to be minimised, u the control inputs and y are the outputs to be fed back to the controller.

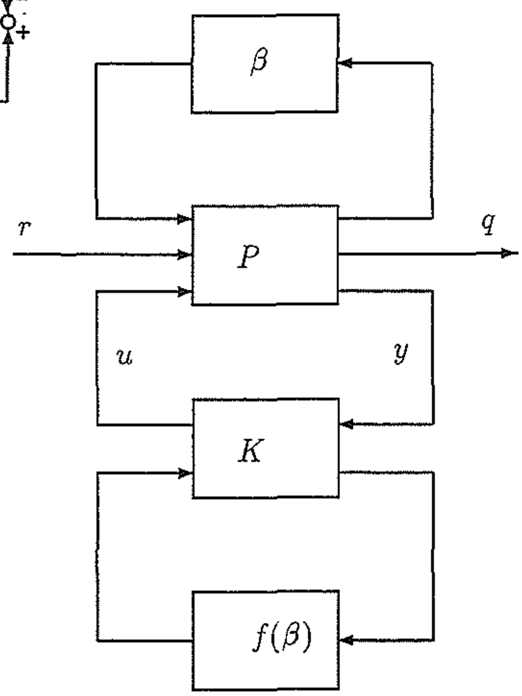

Figure 2: Linear Fractional Transformation of the gain scheduled system

In this case both plant and controller are approximated with high order polynomials (or with rational functions) and a standard μ-analysis test is carried out. Alternatively, a search over all the possible trajectories of the scheduling variable can be performed from which the designer is able to choose the scheduling law he wishes. More precisely, it is proposed to solve (3) $\forall i = 0...n$ where n is the number of the grid points.

$$\min_{\beta, f(\beta)} \left\| \begin{bmatrix} K_i \\ I \end{bmatrix} (I - G_i K_i)^{-1} M_i^{-1} \right\|_\infty \leq \frac{1}{\epsilon_i}. \quad (3)$$

4. Example

The helicopter under investigation is the Canadian B205 fly-by-wire research vehicle operated by the Flight Research Laboratory, Institute of Aerospace Research, National Research Council, Ottawa, Canada. Recently, an H_∞ ACAH [2], controller was designed using a 2DOF approach [10] and succesfully flight tested according to the ADS-33C requirements. Now we show that a linear gain schedule would not ensure performance over the entire flight envelope. The model used for this study is the quasi-static model found in [8]. The measurements selected for the feedback stabilisation loop are

- Vertical velocity (w)
- Pitch rate (q)
- Roll rate (p)
- Yaw rate (r)

The design of the frozen point controllers (one at hover and one at 120 knots) can be found in [1]. Figure 3 shows the cost function (3) evolution over the entire flight envelope. From the plot it can be deduced that if the hover controller was operating at speeds above 80 knots then a dramatic deterioration of the stability margins would be encountered. For the pair of the two designed controllers the scheduling function ensuring that the performance is less than a prespecified level has the form of figure 4. In other words the loop shapes that the designer specified at the frozen point designs remain compatible with robust stability requirements. There seems no reason why this process should converge for an arbitary distance between two adjacent operating points of the flight envelope. However, it seems to work well in practice, as demonstrated by the previous example. Any constrained optimisation method can be used to find the optimal / robust scheduling law.

[2] Attitude-Command Attitude-Hold

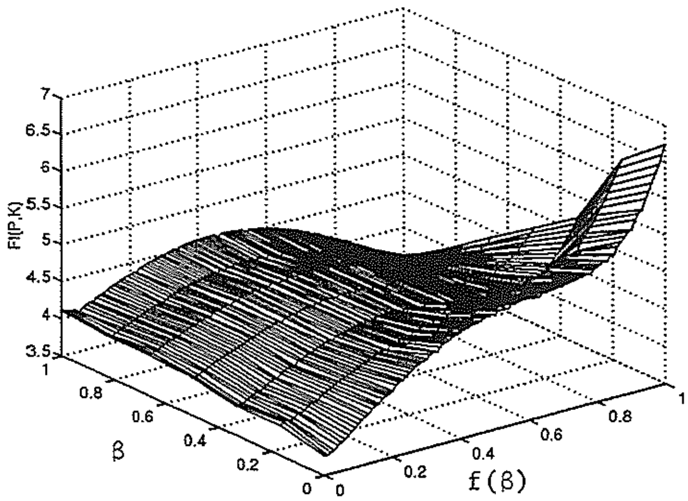

Figure 3: Cost function across the operating envelope. $f(\beta)$ - controller scheduling function, β - normalised forward speed, $\mathcal{F}(P,K)$ - cost function

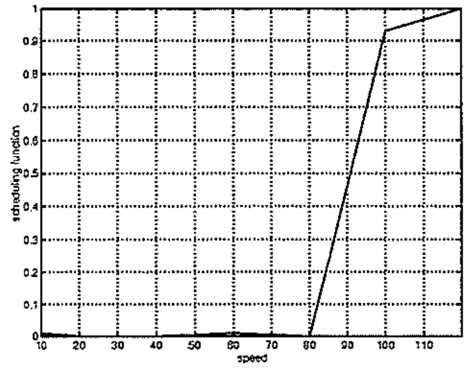

Figure 4: Scheduling function vs forward speed

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