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ROBUST ROTORCRAFT CONTROLLER DESIGN FOR TURBULENCE REJECTION

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ABSTRACT

This paper describes the results of an on-going investigation into controller design for the rejection of atmospheric turbulence. Two significantly different control system design approaches were applied to the problem of reducing the effects of gusts on the helicopter. In the first method the gust disturbance is considered as an extra input to the well-known H_∞ Loop Shaping Design Procedure (LSDP) design formulation. The second method utilizes a sliding mode controller. Results from the nonlinear simulations are presented, and both methods show significant improvement in the helicopters performance.

1 Introduction

The reduction of the effects of gusts is very important in reducing the pilot's workload, and enabling aggressive manoeuvres to be carried out whatever the weather conditions. As a consequence of decreased buffeting, the airframe and component lives will be lengthened, and passenger comfort increased. It has also been suggested (Hall and Wereley (1)) that turbulence could be a factor in the random nature associated in the vibration of the rotors. The design of rotorcraft flight control systems, which will maintain system stability and performance, has been receiving attention for many years now. Recent methods include H_∞ robust optimization ([Yue and Postlethwaite (2)], [Walker and Postlethwaite (3)]), eigenstructure assignment ([Manness and Murray-Smith (4)], [Samblancatt *et al* (5)]), sliding mode control (Foster *et al* (6)), and H_2 design (Takahashi (7)). The H_∞ frequency domain controller designs have been particularly successful (Walker *et al* (8)), and have proved themselves in piloted simulations. These design procedures use frequency information about the disturbances to limit the system sensitivity. In all of these designs there has not been implicit consideration of the effect that atmospheric turbulence would create. The first methodology to enable improved handling qualities in the face of gusts was made by including gust information in the traditional H_∞ Mixed Sensitivity formulation (Postlethwaite *et al* (9)). It was seen that by incorporating practical knowledge about the disturbance characteristics, and how it affects the real helicopter, improvements to the overall performance could be made.
The nonlinear helicopter model used for simulation purposes was developed at the Defence Research Agency (DRA), Bedford and is known as the Rationalised Helicopter Model (RHM). A turbulence generator module is included in the most recent version. This enables controller designs to be tested, in this case, for their disturbance rejection qualities at an off-line stage. The performance of particular new designs in minimizing the effect of gusts is assessed by comparison to a baseline H_∞ Loop Shaping Design Procedure

(LSDP) designed controller (McFarlane and Glover (10)). For the first designs, the gust was modeled as a perturbation on the velocity states of the helicopter model, and the disturbance included as an extra input signal to the Standard Compensation Configuration (SCC). The second design method uses the inherent robustness properties of particular sliding mode control laws. This sliding mode controller has been verified on the DRA flight simulator, and been shown to give high levels of performance and handling qualities (Foster *et al* (11)). Therefore to further examine the potential for improvements to performance, when using sliding mode control system design methods, it was important to test the turbulence rejection qualities. The implemented sliding mode control law formulation has the desirable property, amongst others, of being insensitive to matched uncertainty (i.e. uncertainty entering the system through the input distribution matrix). This property gave confidence to the gust rejection performance, since it was found in ([9]) that the turbulence could be practically considered as perturbing the velocity states and entering through the input.

2 Turbulence Model

A major topic in itself is the modeling of turbulence, which can be approached from the mathematical individual blade/blade element techniques ([Houston and Hamilton (13)],[Riaz *et al* (14)]) or from an engineering analysis of real flight velocity data and how a gust is propagated (Turner (15)). Two different applicable methods can be applied to the representation of the velocity fluctuations: Power Spectral Density (PSD) and Statistical Discrete Gust (SDG) models (Dahl and Faulkner (16)).

The PSD approach is to represent the turbulence record as a collection of continuous sine waves at different frequencies. There are many different functions that have been selected to fit the power/frequency distribution of the turbulence (Jones (17)). However, there is a disadvantage in PSD techniques, since for each sine wave an 'average' amplitude is calculated at that frequency to represent all the fluctuations. It is therefore possible to obtain identical power/frequency graphs for very different turbulence records. Also the lack of information about the rate of occurrence of large discrete events is often critical in aircraft or control system design. It is the distribution of changes in turbulence velocities, rather than of absolute velocities, which influence aircraft dynamic quantities such as acceleration, pitch rate and structural loads (Foster (18)).

The SDG method, however, uses a simple ramp gust model instead of a sine wave, where the ramp itself can be of any length or magnitude (i.e. scale or intensity). These are determined by an exponential joint probability distribution, where the parameters are obtained from experimental data. A further parameter called intermittency, is required in the formulation and relates to the continuous nature of the turbulence record. Pockets of fluctuations embedded in a background of relatively low activity would be considered as highly intermittent, as compared to a more continuous record which would have a low intermittency. It is therefore possible to represent relatively unusual large scale events which may have an important effect on aircraft response. Due to the above features, SDG theory has been seen by British industry to be more representative than PSD theory of the features which actually occur in the atmosphere. From the point of view of aircraft gust loads, rather than handling qualities verification, the PSD method has been argued to be more realistic (Hoblit (19)). The SDG method has been verified against a large amount of real turbulence data at the Defence Research Agency (DRA), Bedford, who have now included a SDG gust model in their nonlinear helicopter model.

3 Helicopter Model

The RHM covers the full working flight envelope of a Lynx-like, high performance military helicopter (Padfield (23)). This helicopter model can be linearized about a particular operating point in the flight envelope. A twenty knot linearization was taken for the designs, and represented by the following state space expression:

$$\dot{x}(t) = Ax(t) + B_o u(t) \quad (1)$$

$$y = Cx(t) \quad (2)$$

The eight state rigid body vector x is tabulated as follows:

State	Description
Θ	Pitch Attitude
Φ	Roll Attitude
p	Roll Rate
q	Pitch Rate
r	Yaw Rate
u	Forward Velocity
v	Lateral Velocity
w	Vertical Velocity

1 TABLE - State Vector

and the outputs to be controlled are:

Con. O/p	Descrip.	Pilot I/p	Units
$\dot{H}$ (y1)	Heave Vel.	Coll.	Ft/s
Θ (y2)	Pitch Att.	Long.	Rad.
Φ (y3)	Roll Att	Latt.	Rad.
$\dot{\Psi}$ (y4)	Head. Rate	Pedal	Rad/s

2 TABLE - Output Vector

4 H_∞ Loop Shaping Design Procedure

The loop shaping design procedure (LSDP) ([10]) is used to obtain performance/robustness trade-offs, whilst the robust stabilization procedure is a means of guaranteeing closed-loop stability. Due to the conflicting requirements of performance (tracking and disturbance rejection) requiring high gain, and robustness (sensor noise attenuation) requiring low gain, that there must be a frequency separation of these objectives. An acceptable compromise comes from performance being typically most important at low frequency, and robust stability at high frequency.

The procedure of the LSDP consists of three main stages:-

1. Loop Shaping. The singular values of the nominal plant are shaped, using filters $W1$ and $W2$ to give a desired open-loop frequency shape. The nominal plant, and the shaping weights are combined (figure. 1) to form the shaped plant, where $G_s = W_2 G W_1$.

Figure 1: The Shaped Plant Model

2. Robust Stabilization. (a) Calculate $\epsilon_{max} = (\gamma_o)^{-1}$. If $\epsilon_{max} \ll 1$, return to (1) and adjust the shaping weights. (b) Choose $\epsilon \leq \epsilon_{max}$, and synthesize a feedback controller, K∞ (Figure 2), which robustly stabilizes the normalized left coprime factorization of the plant G.

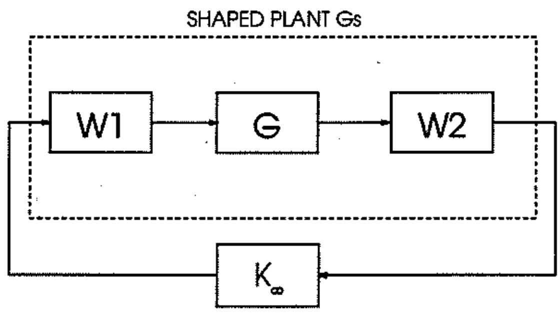

Figure 2: H∞ Robust Stabilization

3. The final feedback controller, K, is then constructed (Figure 3) by combining the H∞ controller, K∞, with the shaping weights, that is:-

$$K = W_1 K_\infty W_2. \tag{3}$$

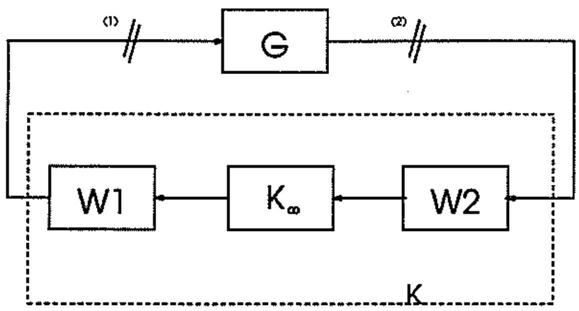

Figure 3: Final Controller

The ϵ_{max} can be thought of as an indicator of the success of the loop shaping, since if ϵ_{max} is small, then the desired performance is incompatible with robust stability requirements. The weights used in the design were:-

$$W_1 = Diagonal\,[\,\tfrac{s+4}{s}, \tfrac{s+4}{s}, \tfrac{s+4}{s}, \tfrac{s+4}{s}\,] \tag{4}$$

$$W_2 = Diagonal\,[1,1,1,1,0.15,0.25] \tag{5}$$

5 Disturbance Rejection Design

The effect of the turbulence disturbance was modeled as gust velocity x, y, z components perturbing the helicopter's velocity states. Therefore the disturbed system can be expressed as:

$$\dot{x}(t) = Ax(t) + B_o u(t) + B_g d(t) \tag{6}$$

$$y = Cx(t) \tag{7}$$

This is shown diagrammatically below (Figure 4).

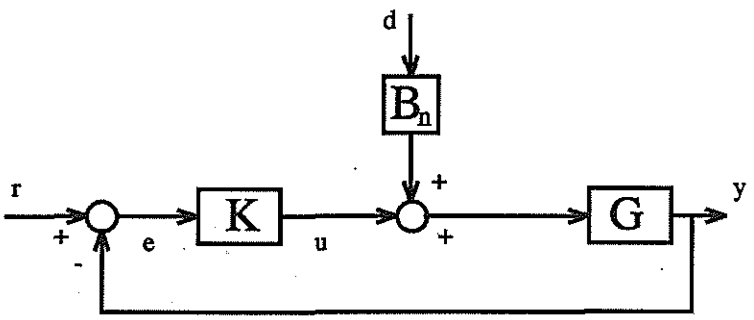

Figure 2 : Disturbance Rejection (Bn) Formulation

Where G_i is the state space of the plant but with its input distribution matrix set as the identity matrix. If the pseudo-inverse of the plant distribution matrix B_o was implemented instead, then the disturbance could be considered as entering at the plant input. In block diagram form this would be presented as below (Figure 2), where $B_n = (B_o)^{\dagger} B_g$.
The relation of the various loop transfer functions of the disturbance rejection design, to the structure involved with the H_∞ LSDP design can be seen by setting the disturbance input d to zero. Obviously the turbulence could not be represented as a plant output disturbance, since it would be unstable and continually growing in magnitude, due to the plant itself being unstable.
Selecting the constant gain matrix B_g from elements in the plant state matrix, the effect of the turbulence directly perturbs the helicopter velocity states. Choosing other representations of the gust, as maybe affecting the aircraft's rate states, are possible by changing the entries in B_g. A scaling of B_g was used to emphasize this element in the optimization procedure.
The same weighting functions, W_1 and W_2, as used in the baseline H_inftY controller design were implemented in this disturbance rejection design. Also since B_g is a constant gain matrix there is no increase in the order of the controller, as compared to the H_∞ LSDP design.

6 The Sliding Mode Control Law

Consider the following state space description of an uncertain plant:-

$$\dot{x}(t) = Ax(t) + Bu(t) + F(t, x, u) \tag{8}$$

where $x \in \Re^n$and $u \in \Re^m$ represent the usual state and input, B is full rank, $n > m$ and (A, B) is a controllable pair. The unknown function F represents system nonlinearities and model uncertainties in the system (Davies and Spurgeon [20]). An associated linear model which has ideal response characteristics is defined by:

$$\dot{w}(t) = A_m w(t) + B_m r(t) \tag{9}$$

where $w \in \Re^n$, $r \in \Re^p$, are the state vector of the model and the reference input respectively. It is assumed that the ideal model is stable so that the poles of the system, equation (9), have negative real parts. The associated control system design problem is thus that of determining a feedback strategy whereby the output variables of the plant, equation (8), faithfully follow those of the model. The following tracking error state is thus defined

$$e = x - w \tag{10}$$

Differentiating equation (10) with respect to time and substituting the plant and model dynamics from equation (8) and equation (9), the following model error dynamics are obtained.

$$\dot{e} = A_m e + (A - A_m)x + Bu - B_m r + F(t, x, u) \tag{11}$$

To satisfy the well-known model matching conditions for the nominal error system which will ensure asymptotic decay when $F(\cdot) \equiv 0$, the following structure is imposed upon the model.

$$A_m = A + BL_x \tag{12}$$

$$B_m = BL_r \tag{13}$$

The model is thus defined by a constant gain feedback matrix (L_x) for the nominal plant, and an input-output tracking precompensator gain matrix (L_r).
Note that if the control input, u, is defined by

$$u_1 = L_x x + L_r r \tag{14}$$

the nominal error dynamics are asymptotically stable. However, for a very nonlinear, uncertain system the problem of maintaining the tracking performance in the presence of a broad class of uncertainty contributions $F(\cdot)$ is particularly pertinent. The design of an augmenting control effort to counteract the uncertainty $F(\cdot)$ is now considered. The methodology employed has its roots in the well known sliding mode approach to controller design, where the error state is constrained to lie on certain surfaces in the error state-space. This method possesses certain inherent robustness properties, and with appropriate switching surface selection, enables the designer to prescribe desired error transient behaviour. A set of switching surfaces are defined to be fixed hyperplanes in the error space passing through the origin

$$s = Ce \tag{15}$$

where $C \in \Re^{m \times n}$ is a constant design matrix which determines the ideal rate of decay of the error states.
A sliding mode is achieved when the error states are constrained to the intersection of the hyperplanes (equation (15))

$$s = \{e : Ce = 0\} \tag{16}$$

The control required to achieve the desirable sliding mode condition, equation (16), was traditionally discontinuous in nature which was clearly undesirable for many applications. However, there are now well-established continuous nonlinear controllers which ensure equation (16) is satisfied in a completely robust fashion ([Ryan and Corless (21)], and [12]). Here the control effort equation (14) is augmented by

$$u_2 = L_e e + \rho(t, x, e, r)\frac{Ne}{\|Me\| + \delta} \tag{17}$$

so that

$$u = u_1 + u_2 \tag{18}$$

Here $L_e \in \Re^{m \times n}$ is an error-feedback to prescribe the rate of decay of the error states onto the switching surfaces. The matrices $N \in \Re^{m \times n}$ and $M \in \Re^{m \times n}$ are directly determined from the choice of switching surface C. The parameter $\delta > 0$ is a smoothing constant; for $\delta = 0$ an undesirable relay type control action would result. The block diagram (Figure.5) below shows the structure of the complete variable structure controller implemented, for example, on the non-linear helicopter model. This was built up in the SIMULINK environment, which was found to be a flexible environment to build up the entire design.

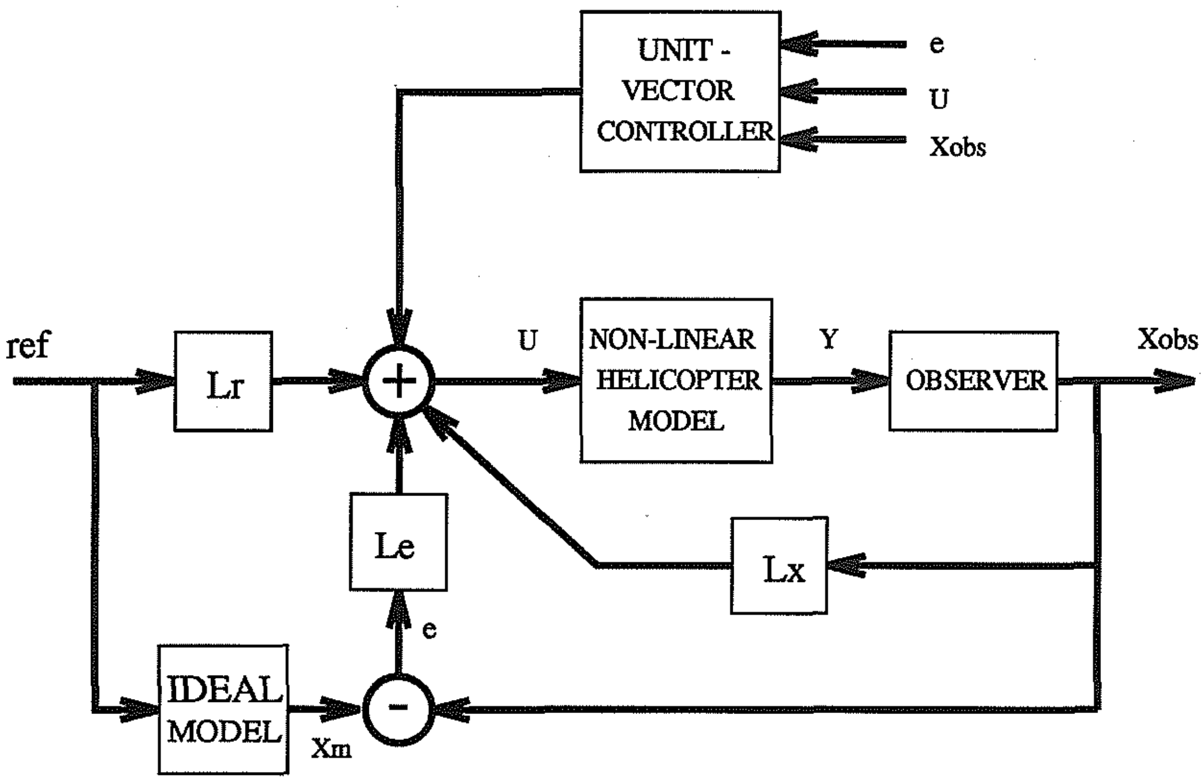

Figure 4: Figure 5: Block Diagram of Controller Structure

The design requirements of a statefeedback matrix L_x and a state observer can be satisfied by exploiting the structure of the H_∞ LSDP controller, since it can be split into a statefeedback matrix and Kalman filter structure (Hyde and Glover [22]), such as:

$$\dot{\tilde{x}} = A\tilde{x} + H(C\tilde{x} - y) + Bu \tag{19}$$

$$u = F\tilde{x} \tag{20}$$

where [A,B,C] is the state-space realization of the weighted plant, $H = -ZC^*$, and $F = B'(\gamma^{-2}I + \gamma^{-2}XZ - I)^{-1}$ where X and Z are the associated control and filtering algebraic Riccati equation solutions.

7 Frequency Domain Results

The following Bode frequency plots demonstrate that the original qualities of the H_∞ LSDP design are conserved in the new disturbance rejection design. The relative attributes of the sliding mode controller will not be examined here, since they are investigated at length in another report ([11]).

The sensitivity plot ($(I + GK)^{-1}$) of the mixed sensitivity design indicates the ability of the system to minimize the effects of an additive disturbance on the plant output.

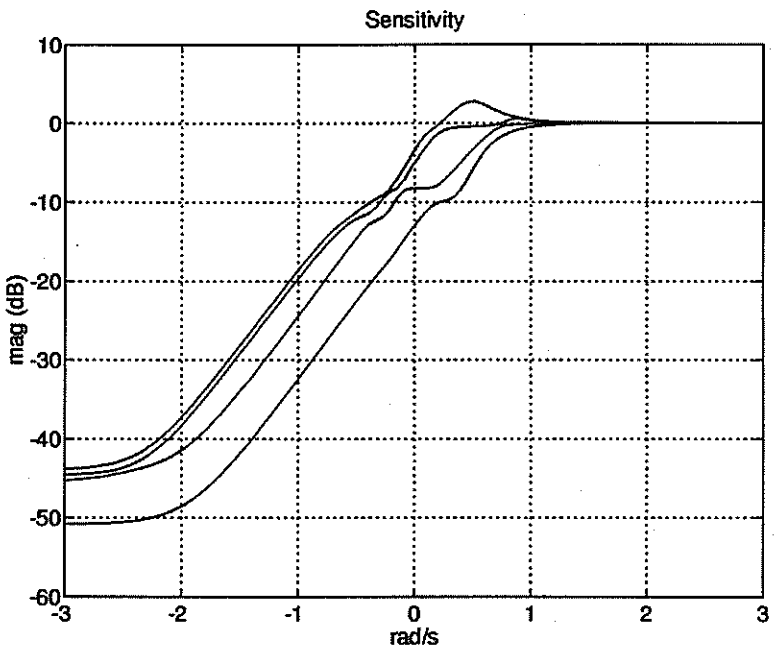

Figure 6 : LSDP Design

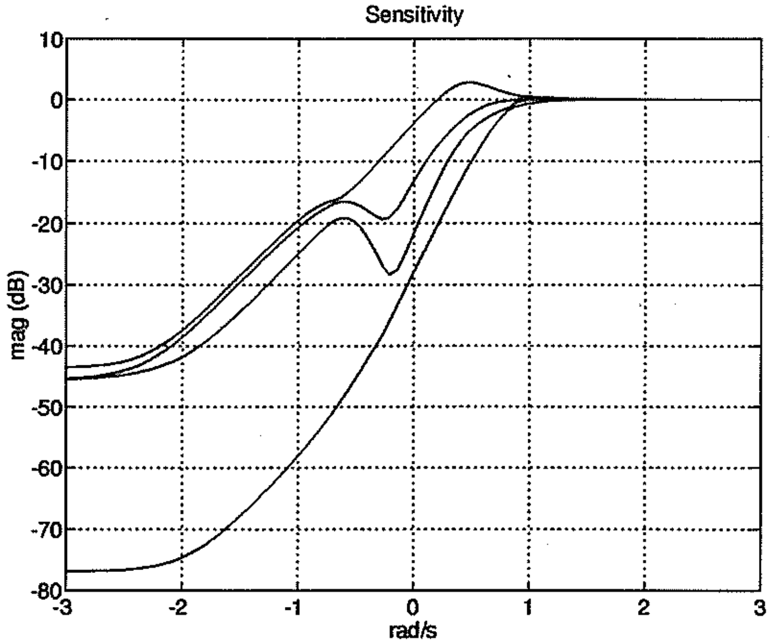

Figure 7 : Disturbance Rejection Design

The insensitivity capabilities are shown to be maintained through the use of the disturbance rejection formulation (Figures 6 & 7). The frequency weighting functions W_1 and W_2 were kept identical in both designs.

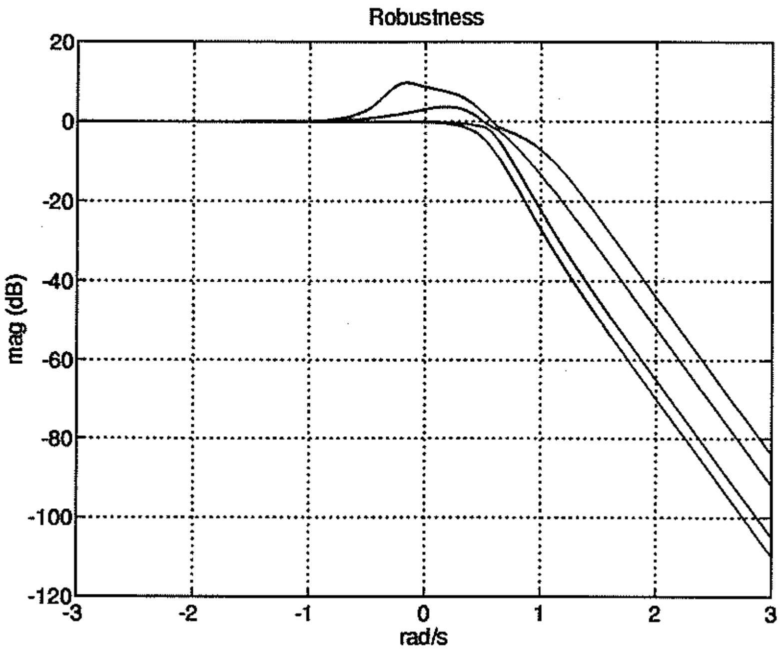

Figure 8 : LSDP Design

Also when the robustness to sensor noise plots ($GK(I+GK)^{-1}$) are compared (figures 8 & 9), then negligible degradation is seen. The helicopter was trimmed at a height of 100 feet, which must be noted, since the RHM includes the information that the scale lengths of the velocity components are shortened as altitude increases.

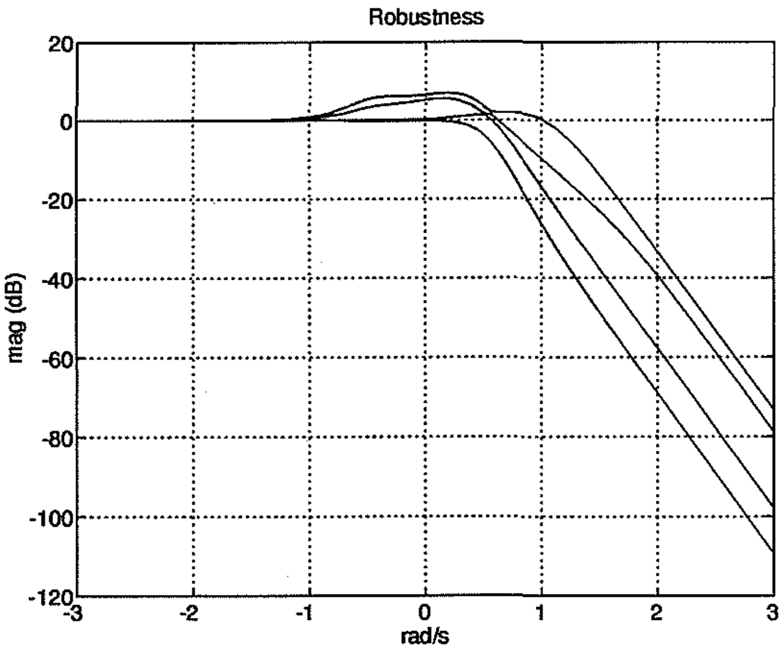

Figure 9 : Disturbance Rejection Design

8 Non-linear Simulation Results

The nonlinear model was trimmed at the 20 knot forward flight speed, and the effect of the turbulence on the four controlled outputs observed. New designs were tested with turbulence of varying parameters and correspondingly changing characteristics. The following plots (Figure 10) show one such gust that was to have its effect on the aircraft minimized.

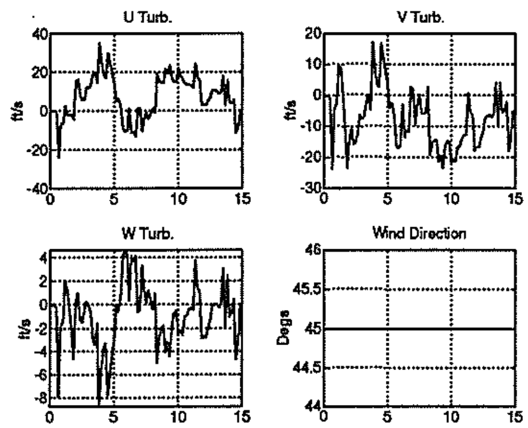

Figure 10 : Velocity Components of Turbulence (15secs)

The injection of a gust into the helicopter model's equations of motion is automatically carried out by the RHM, and the output response calculated (Figures 11, 12, 13 & 14). The controller in this case is being required to keep the aircraft flying forward at twenty knots.

8.1 Method 1. H_∞ Disturbance Rejection Design

The main reductions in the turbulence effects were seen in the pitch attitude and heave velocity, as illustrated in figures (11 & 12).

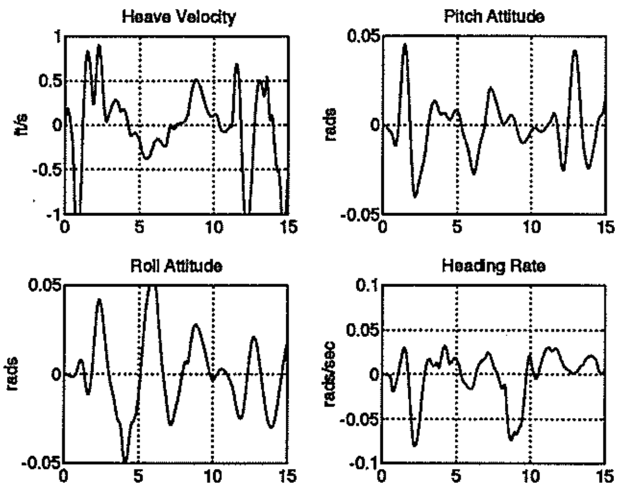

Figure 11 : LSDP Design Output Responses (15secs)

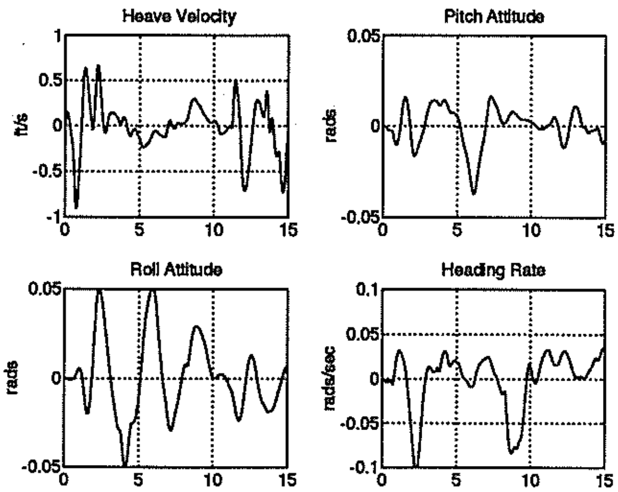

Figure 12 : Disturbance Rejection Design Output Responses (15secs)

8.2 Method 2. Sliding Mode Control Design

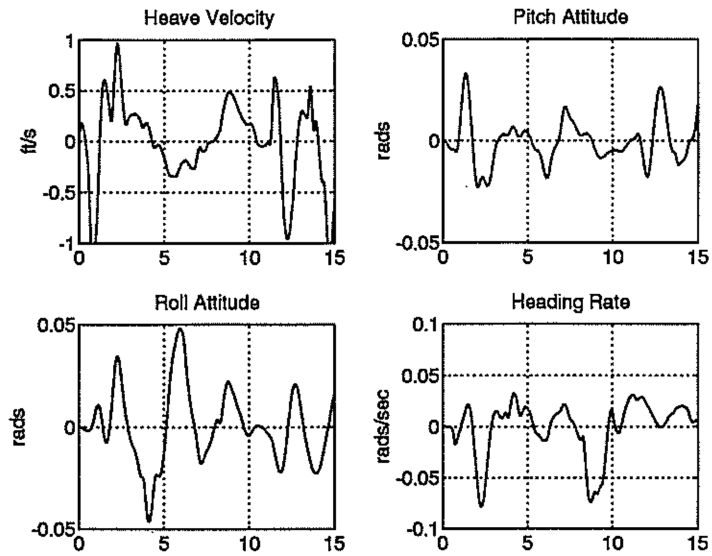

Figure 13 : Baseline LSDP Design Output Responses (15secs)

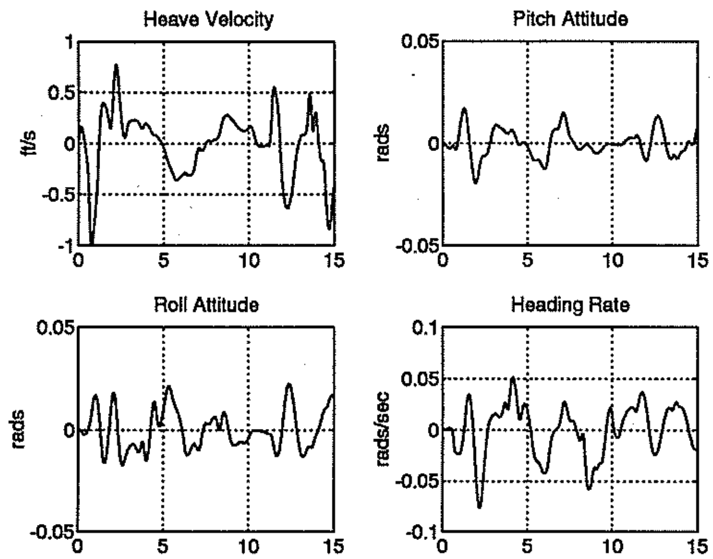

Figure 14 : Sliding Mode Controller Output Responses (15secs)

To help give more insight as to the workings of the sliding mode controller, the following plot shows the switching states (Figure.15) , which gives a measure of how close the error states are to the ideal switching surface. The fact that the switching states are all at small values shows that the controller is operating at near the specified performance. This specified performance depends on the ρ, δ, and ideal model characteristics chosen in the design stage.

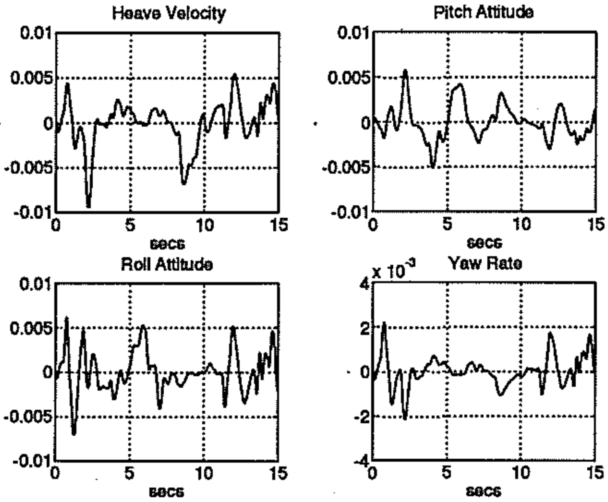

Figure 15 : Sliding Mode Controller Switching States (15secs)

The sliding mode controller significantly reduced the turbulence effect on pitch attitude and roll attitude. There was only a small change in the effect on heading rate. These figures show that in these main axes the helicopter would be easier to control for the pilot.

9 Conclusions

By incorporating knowledge about turbulence activity into controller design, a substantial reduction in the effect on handling qualities can be made. For the first H_∞ design method the final controller was the same order, and the singular value bode frequency plots showed no deterioration in the sensitivity and robustness qualities, when compared to the well-known H_∞ LSDP design formulation. The yaw channel disturbance rejection properties still require further investigation.

The sliding mode controller showed that as a consequence of its inherent robustness and disturbance rejection properties, it had significant turbulence rejection qualities.

Also because of the reduction of turbulence effects in heave velocity, pitch attitude, and roll attitude the pilot's workload would be considerably reduced, allowing more aggressive manoeuvres to be carried out with a higher degree of precision. Also passenger comfort and safety would be increased.

Future work will involve an investigation into different measures of performance. Also further enhancements to the sliding mode controller will be examined by exploiting the possible statefeedback/Kalman Filter structure of the H_∞ disturbance rejection controller.

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11 REFERENCES

1. Hall,R.S., and Wereley,M.N., 1993,
J Guid Control and Dyn, 16, 793-797

2. Yue,A., and Postlethwaite,I., 1990,
IEE Proc., Vol.137, Part D, No. 3. '

3. Walker,D.J., and Postlethwaite,I., 1991,
'Discrete time H_∞ control laws for a high performance helicopter', University of Leicester, Report 91 (3 January).

4. Manness,M.A., and Murray-Smith,D.J., 1992,
J Am Helicopter Soc, 18-32

5. Samblancatt,C., Apkarian,P., and Patton,R.J., 1990
Euro Rotorcraft Forum, No. 2.3.1.

6. Foster,N.P., Spurgeon,S.K., and Postlethwaite,I., 1993,
Euro Rotorcraft Forum, Paper No. H4

7. Takahashi,M.D., 1993,
J Guid Control and Dyn, 16, 579-584

8. Walker,D., Postlethwaite,I., Howitt,J., and Foster,N., 1993,
J Am Helicopter Soc/NASA Conf.

9. Postlethwaite,I., Foster,N.P., and Walker,D.J., 1994,
Control'94), 1284-1289.

10. McFarlane,D., and Glover, K., 1990,
'Robust control system design using normalized coprime factor plant descriptions', Lecture notes in control and information sciences, Springer Verlag.

11. Foster,N.P., Spurgeon,S.K., and Postlethwaite,I., 1994,
'Full envelope sliging mode control system design with application to an ACT rotorcraft', To be Published.

12. Spurgeon,S.K., and Davies,R.
Int. J. Control, 57, pp. 1107-1123.

13. Houston,S.S., and Hamilton,R.A., 1993,
Euro Rotorcraft Forum, Paper No. G18

14. Riaz,J., Prasad,J.V.R., Schrage,D.P., and Gaonkar,G.H., 1993,
J Am Helicopter Soc, 84-88

15. Turner,G.P., 1993,
'Discussion of some issues relating to the prediction of Rotorcraft response to turbulence', DRA Working Paper FSB WP(93)055.

16. Dahl,H.J., and Faulkner,A.J., 1979,
Vertica, 65-78

17. Jones,J.G., 1978,
'Review of atmospheric turbulence and models', RAE Technical Memorandum FS 186.

18. Foster,G.W., and Jones,J.G., 1989,
J Aero Sci, 162-176

19.Hoblit,F.M., 1988,
'Gust loads on Aircraft: Concepts and Applications', Am Inst. Aero/Astro., ISBN 0-930403-45-2

20. Davies,R., and Spurgeon,S.K., 1993,
IFAC, Sydney, Vol.10, 43-46.

21. Ryan,E.P., and Corless,M., 1984,
IMA Journal of Mathematics and Control Information, 1, 223-242.

22. Hyde,R.A., and Glover,K., 1990,
Conference on Decision and Control, Honolulu.

23. Padfield, G.D., 1981,
'Theoretical model of helicopter flight mechanics for application to piloted simulation', RAE TR 81048.