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FINITE DIFFERENCE TECHNIQUES AND ROTOR BLADE AEROELASTIC PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

Since the early works of Hubolt and Brooks, many nonlinear formulations have been introduced for rotor blade aeroelastic studies by Hodges and Dowell, Kaza and Kvaternik, Rosen and Freidmann, and Jonnalagadda and Pierce. In all these studies, small strains, finite slopes and quasisteady aerodynamics have been used. Efforts have also been made to include lifting line theory and unsteady effects with relatively simple structural dynamic models (e.g. CAMRAD). In all these studies, the partial differential equations of motion have been reduced to a set of differential equations by using global Galerkin methods, Galerkin finite element methods and finite element methods based on energy principles. In this paper two step explicit finite difference techniques have been used to directly integrate the partial differential equations to obtain transient and steady-state responses as well as to evaluate the stability of the system. A significant benefit of such a solution procedure is in utilizing improved aerodynamics other than the quasisteady aerodynamics.

I. INTRODUCTION

Results of continuous efforts to develop new rotor blades with less complex structure, lower maintenance requirements, improved reliability and performance have resulted in the introduction of hingeless and bearingless rotor blades. The modeling of the resulting motion of a hingeless rotor blade is usually based on moderate rotation and small strains. This introduces geometric nonlinearities in structural, inertia and aerodynamic operators.[1 4] Besides, forward flight conditions introduce periodic coefficients to the problem. Usually as a first step in the solution procedure, the spatial dependency of the nonlinear, coupled rotor blade equations is eliminated by using global Galerkin methods[4 6] or by using finite element methods.[7 10] For an aeroelastic stability analysis, these equations have been linearized about an appropriate equilibrium or steady-state response and stability boundaries are obtained by eigen-data analysis. For the hover case, this eqilibrium position is the nonlinear static deflection of the rotor blade. In forward flights the steady-state position is periodic in time and depends on the over all trim state of the helicopter.

Generally fourth order Runge-Kutta and Hamming's predictor corrector methods have been used for generating time history solutions.[4 6 9 11] Panda and Chopra[12 13] have solved nonlinear rotor blade equations. By using an iterative procedure for flap-lag-torsion motions in forward flight. A somewhat similar type of quasilinear procedure has been used by Freidmann and Kottopalli.[7 14] Another approach for response calculations has been introduced by Jonnalagadda [4] where a periodic shooting technique is used by utilizing the Floquet transition matrix through on iterative scheme. Recently Bori[15] has used a time finite element approximation method to calculate the steady response of an articulated rotor blade. A similar finite element method in time has been formulated on the basis Hamilton's principle in weak form by Panda and Chopra[16] to solve for the response of a composite rotor blade. Izadpanah[17] has introduced a p-version time element method in space-time domain problems with applications for the flapping response of an articulated rotor blade. In Reference 17 a bilinear formulation which originates from the variational form of Hamilton's principle of varying action has been introduced.

Transient Response

Available methods for transient and steady-state responses involve approximations and linearizations at certain stages of the analysis. When a system is subjected to transient changes such as gust or maneuver one needs to generate time history solutions. Finite element methods in time domain are still at a developing stage and need further research. On the other hand time marching techniques by finite difference methods are well established by the efforts and studies in the field of computational fluid mechanics.[18 19]

In this paper a finite difference method for transient and steady-state response solutions of rotor blades has been introduced. The objective is to use two-step finite difference schemes for the solution of rotor blade dynamical partial differential equations in space and time. Long range objective of this study is to use computational unsteady aerodynamics solutions (CFD) to response calculations through a compatiable finite difference mesh distribution along spatial and time coordinates.

Finite Difference Methods

During the past two decades, [18 19] many different finite difference methods have been developed and tested for solutions of hyperbolic and parabolic partial differential equations. In this paper, a two-step expilict, conditionally stable finite difference scheme has been selected for the solution of rotor blade dynamical partial differential equations. With this numerical scheme the governing equations are discretized in both spatial and time coordinates. As applied to beam problems, Abyhankar and Hanagud[20 21] have used a two-step numerical solution technique for the solution of forced, nonlinear vibrations of a buckled beam problem and the associated chaotic response problem. In this paper this numerical solution technique has been first applied to the solution of flap-lag bending partial differential equations of Reference 7 in hover and forward flight conditions. As a second step the same method of solution has been applied to flap-lag torsion partial differential equations given in Reference 7. In order to compare the effectiveness of the method a two-dimensional strip type aerodynamic modeling with uniform inflow has been used in present study an in Reference 7.

2. ROTOR BLADE EQUATIONS

Rotor blade equations for flap-lag motions as given in Reference 7 are as follows:

Lag equilibrium,

$$-m V_{,tt} - B_{22} V_{,xxxx} - B_{23} W_{,xxxx} + T V_{,xx} + A_{vt} V_{,t} + A_{wt} + A_{vx} V_{,x}$$
$$A_{wx} W_{,x} + A_v V + A_c = 0 \qquad (1.a)$$

Flap equilibrium,

$$-m W_{,tt} - B_{33} W_{,xxxx} - B_{23} N_{,xxxx} + T W_{,xx} + B_{vt} V_{,t} + B_{wt} W_{,t}$$
$$B_{wx} W_{,x} + B_{vx} V_{,x} + B_v V + B_c = 0 \qquad (1.b)$$

Coefficients are given in Appendix A. V and W are lag and flap displacements nondimensionalized respect to blade length.

The associated boundary conditions are,

at root X = 0

$$V = W = V_{,x} = W_{,x} = 0 \tag{2}$$

at tip X = 1

$$B_{22} V_{,xx} + B_{23} W_{,xx} = 0 \tag{3.a}$$

$$B_{23} V_{,xx} + B_{33} W_{,xx} = 0 \tag{3.b}$$

$$(B_{22} V_{,xx} + B_{23} W_{,xx})_{,x} = 0 \tag{3.c}$$

$$(B_{23} V_{,xx} + B_{33} W_{,xx})_{,x} = 0 \tag{3.d}$$

Similarly, the governing equations for flap-lag-torsion motions of a hingeless rotor blade are written as follows,

Axial equilibrium

$$T_{,x} + m\left[(x_0 + e_1) + 2V_{,t}\right] = 0 \tag{4.a}$$

Lag equilibrium:

$$-(M_{3,x} + GJ\phi_{,x} W_{,xx} - V_{,x} T)_{,x} - q_{3I,x} + P_{YI} + P_{YA} + P_{YD} = 0 \tag{4.b}$$

Flap equilibrium:

$$(M_{2,x} + GJ\phi_{,x} V_{,xx} + W_{,x} T)_{,x} + q_{2I,x} + P_{ZI} + P_{ZA} + P_{ZD} = 0 \tag{4.c}$$

Torsion equilibrium:

$$M_{x,x} + M_1 + q_{1I} + q_{XA} + q_{XD} = 0 \tag{4.d}$$

The associated boundary conditions are:

at root, Xo = 0

$$V = W = \phi = V_{,X} = W_{,X} = 0 \tag{5}$$

at tip, Xo = 1

$$-M_{3,X} - GJ\phi_{,X} W_{,XX} + V_{,X} T - q_{3I} = 0 \tag{6.a}$$

$$M_{2,X} + GJ\phi_{,X} V_{,XX} + W_{,X} T + q_{2I} = 0 \tag{6.b}$$

$$M_3 = -M_2 = M_X = T = 0 \tag{6.c}$$

$$T = 0 \tag{6.d}$$

The coefficients are given in Reference [5]. Indices i, a, d indicates the nature of these quantities are from inertial, aerodynamic and damping terms respectively. In order to apply the proposed numerical scheme to the solution of these equations they can be rewritten as follows:

Lag equilibrium,

$$-V_{,tt} - B_{22} V_{,XXXX} - \tfrac{1}{2} B_{23} W_{,XXXX} - \left[B_{32} W_{,XX} - B_{23} V_{,XX} + GJ\, W_{,XX}\right]\phi_{,XX} + T V_{,XX}$$
$$A_{Vt} V_{,t} + A_{Wt} W_{,t} + A_{Ft}\phi_{,t} + A_{VX} V_{,X} + A_{WX} W_{,X} + A_{FX}\phi_{,X} + A_V V + A_F \phi + A_C = 0 \tag{7.a}$$

Flap equilibrium,

$$-W_{,tt} - \tfrac{1}{2} B_{23} V_{,XXXX} - B_{33} W_{,XXXX} + \left[-B_{23} W_{,XX} - B_{22} V_{,XX} + GJ\, V_{,XX}\right]\phi_{,XX}$$
$$+ T W_{,XX} + B_{Vt} V_{,t} + B_{Wt} W_{,t} + B_{Ft}\phi_{,t} + B_{VX} V_{,X} + B_{WX} W_{,X}$$
$$+ B_{FX}\phi_{,X} + B_V V + B_F \phi + B_C = 0 \tag{7.b}$$

Torsion equilibrium,

$$-\mu_o^2 \phi_{,tt} + (GJ - B_{32}) V_{,XX} W_{,XX} + GJ\phi_{,XX} + \tfrac{1}{2} B_{23}(V_{,XX}^2 - W_{,XX}^2)$$
$$+ C_{FT}\phi_{,t} + C_{VX} V_{,X} + C_{WX} W_{,X} + C_F \phi + C_C = 0 \tag{7.c}$$

where;

$$T = \int_X^1 m(e_1 + x + 2V_{,t})\, dx \tag{7.d}$$

$$T_{,x} = -m(e_1 + x + 2V_{,t}) \tag{7.e}$$

Finite Difference Formulation

The two step finite difference formulation introduces new variables which have been defined as follows,

$$M_V = V_{,xx} \tag{8.a}$$

$$M_W = W_{,xx} \tag{8.b}$$

$$M\phi = \phi_{,xx} \tag{8.c}$$

and

$$VT = V_{,t} \tag{9.a}$$

$$WT = W_{,t} \tag{9.b}$$

$$\phi T = \phi_{,t} \tag{9.c}$$

Besides equations (2.8.a-c) can be rewritten as,

$$VT_{,xx} = Mv_{,t} \tag{10.a}$$

$$WT_{,xx} = Mw_{,t} \tag{10.b}$$

$$\phi T_{,xx} = M\phi_{,t} \tag{10.c}$$

By using time variables the governing set of rotor blade equations for flap-lag motions have been discretized by this two step explicit scheme as,

Lag equation:

$$\begin{aligned} VT_i^{n+1} = VT_i^n + \Delta t \Big\{ & A_{vt}\, VT_i^n + A_{wt}\, WT_i^n - B_{22}\, \delta^2 MV_i^n \\ & - B_{23}\, \delta^2 MW_i^n + T_i^n MV_i^n + A_{vx}\, \delta V_i^n \\ & + A_{wx}\, \delta \;\; V + A_v\, V_i^n + A_c \Big\} \end{aligned} \tag{11.a}$$

$$Mv_i^{n+1} = Mv_i^n + \Delta t\; \delta^2 VT_i^{n+1} \tag{11.b}$$

$$V_i^{n+1} = V_i^n + \Delta t\; VT_i^{n+1} \tag{11.c}$$

Flap Equation

$$WT_i^{n+1} = WT_i^n + \Big\{ B_{vt}\, VT_i^n + B_{wt}\, WT_i^n - B_{23}\, \delta^2 MV_i^n$$
$$- B_{33}\, \delta^2 MW_i^n + T_i^n MW_i^n$$
$$+ B_{wx}\, \delta W_i^n + B_{vx}\, \delta V_i^n + B_v V_i^n$$
$$+ B_c \Big\} \qquad (12.a)$$

and

$$Mw_i^{n+1} = Mw_i^n + \Delta t\ \delta^2 WT_i^{n+1} \qquad (12.b)$$

$$W_i^{n+1} = W_i^n + \Delta t\ WT_i^{n+1} \qquad (12.c)$$

Similar expressions can be written for flap-lag-torsion equations.

The quantities $\delta(\ )_i^n$ and $\delta^2(\ )_i^n$ are first and second central differences of a quantity at i^{th} node at n^{th} time step and are given as,

$$\delta(\ )_i^n = \alpha_{-1}(\ )_{i-1}^n + \alpha_0(\ )_i^n + \alpha_1(\ )_{i+1}^n \qquad (13.a)$$

$$\delta^2(\ )_i^n = \beta_{-1}(\ )_{i-1}^n + \beta_0(\ )_i^n + \beta_1(\ )_{i+1}^n \qquad (13.b)$$

$$\alpha_{-1} = \frac{-\Delta x_{i+1}}{\Delta x_i(\Delta x_{i+1} + \Delta x_i)}\ ,\quad \alpha_0 = \frac{1}{\Delta x_i} - \frac{1}{\Delta x_{i+1}}\ ,\quad \alpha_1 = \frac{\Delta x_i}{\Delta x_{i+1}(\Delta x_{i+1} + \Delta x_i)}$$

$$\beta_{-1} = \frac{2}{\Delta x_i\ (\Delta x_i + \Delta x_{i+1})}\ ,\quad \beta_0 = \frac{-2}{(\Delta x_{i+1})(\Delta x_i)}\ ,\quad \beta_1 = \frac{2}{\Delta x_{i+1}(\Delta x_i + \Delta x_{i+1})}$$

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The two step finite difference formulation introduces new variables which have been defined as follows,

$$M_V = V_{,xx} \tag{8.a}$$

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and

$$VT = V_{,t} \tag{9.a}$$

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Besides equations (2.8.a-c) can be rewritten as,

$$VT_{,xx} = Mv_{,t} \tag{10.a}$$

$$WT_{,xx} = Mw_{,t} \tag{10.b}$$

$$\phi T_{,xx} = M\phi_{,t} \tag{10.c}$$

By using time variables the governing set of rotor blade equations for flap-lag motions have been discretized by this two step explicit scheme as,

Lag equation:

$$\begin{aligned} VT_i^{n+1} = VT_i^n + \Delta t \Big\{ & A_{vt}\, VT_i^n + A_{wt}\, WT_i^n - B_{22}\, \delta^2 MV_i^n \\ & - B_{23}\, \delta^2 MW_i^n + T_i^n MV_i^n + A_{vx}\, \delta V_i^n \\ & + A_{wx}\, \delta \; V + A_v\, V_i^n + A_c \Big\} \end{aligned} \tag{11.a}$$

$$Mv_i^{n+1} = Mv_i^n + \Delta t\ \delta^2 VT_i^{n+1} \tag{11.b}$$

$$V_i^{n+1} = V_i^n + \Delta t\ VT_i^{n+1} \tag{11.c}$$

Flap Equation

$$WT_i^{n+1} = WT_i^n + \Big\{ B_{Vt}\, VT_i^n + B_{Wt}\, WT_i^n - B_{23}\, \delta^2 MV_i^n - B_{33}\, \delta^2 MW_i^n + T_i^n MW_i^n + B_{WX}\, \delta W_i^n + B_{VX}\, \delta V_i^n + B_V V_i^n + B_C \Big\} \qquad (12.a)$$

and

$$MW_i^{n+1} = MW_i^n + \Delta t\ \delta^2 WT_i^{n+1} \qquad (12.b)$$

$$W_i^{n+1} = W_i^n + \Delta t\ WT_i^{n+1} \qquad (12.c)$$

Similar expressions can be written for flap-lag-torsion equations.

The quantities $\delta(\ )_i^n$ and $\delta^2(\ )_i^n$ are first and second central differences of a quantity at i^{th} node at n^{th} time step and are given as,

$$\delta(\ )_i^n = \alpha_{-1}(\ )_{i-1}^n + \alpha_0(\ )_i^n + \alpha_1(\ )_{i+1}^n \qquad (13.a)$$

$$\delta^2(\ )_i^n = \beta_{-1}(\ )_{i-1}^n + \beta_0(\ )_i^n + \beta_1(\ )_{i+1}^n \qquad (13.b)$$

$$\alpha_{-1} = \frac{-\Delta x_{i+1}}{\Delta x_i(\Delta x_{i+1} + \Delta x_i)}, \quad \alpha_0 = \frac{1}{\Delta x_i} - \frac{1}{\Delta x_{i+1}}, \quad \alpha_1 = \frac{\Delta x_i}{\Delta x_{i+1}(\Delta x_{i+1} + \Delta x_i)}$$

$$\beta_{-1} = \frac{2}{\Delta x_i\ (\Delta x_i + \Delta x_{i+1})}, \quad \beta_0 = \frac{-2}{(\Delta x_{i+1})(\Delta x_i)}, \quad \beta_1 = \frac{2}{\Delta x_{i+1}\ (\Delta x_i + \Delta x_{i+1})}$$

For equal meshes Δx, these differences become,

$$\delta(\ )_i^n = \frac{(\ )_{i+1} - (\ )_{i-1}}{2\Delta x} \tag{14.a}$$

$$\delta^2(\ )_i^n = \frac{(\ )_{i+1}^n - 2(\ )_i^n + (\ )_{i-1}^n}{\Delta x^2} \tag{14.b}$$

The associated boundary conditions given by equations (5), (6.a-d) can be written. For example at root Xo = 0.

$$V_1^n = W_1^n = VT_1^n = WT_1^n = \phi_1^n = 0 \ , \quad MV_1^n = 2V_2^n/\Delta x^2 \quad MW_1^n = 2W_2^n/\Delta x^2$$

Similar equations can be written at tip Xo=1.

Any term, $(\ )_i^n$ is the value of the corresponding quantity at the i node at the n^{th} time step, for the nodes i=2,3 ...., m-1. i=1 is the node at the fixed end at Xo=0., i=m is the node at the free end at Xo=1; where $\delta\ (\ )_m^n$ and $\delta^2(\ )_m^n$ $\delta^3(\ )_m^n$ have been approximated by backward differences. It is necessary to consider a fictitious node M+1 .

In spatial directions a second order accuracy is obtained due to the central differencing of the spatial derivatives. Accuracy for displacements, velocities and second derivatives of MV, MW, and Mϕ is still first order in time.

Numerical Stability of Finite Difference Scheme

The numerical stability of the finite difference scheme used for the solution of flap-lag-torsion equations is investigated by Von Neumann stability analysis.[22 23] The mesh values and the errors of any given numerical scheme can be represented by a finite discrete Fourier series at each time level such that each component is multiplied by a scalar amplification factor as the scheme proceeds to the next time level. Thus, in one space dimensions the error vector can be written as,

$$E_k^{(n)} = \sum_{j=1}^{\infty} A_j\, e^{i\sigma_j x_k} e^{c_j \tau_n} \tag{15}$$

It is also possible to consider a single component of error vector given by equation (15) and assume that

$$E_k^{(n)} \approx e^{i\sigma x_k} e^{c\,\tau_n} \tag{16}$$

Then, if the scheme is stable, the amplification factor which is given by

$$e^{c\Delta t} = \frac{E_k^{(n+1)}}{E_k^{(n)}}$$

must satisfy

$$\left| e^{c\Delta t} \right| \leq 1$$

for all c.

By applying the Von Neumann stability analysis to the numerical scheme choosen for the solution of rotor blade equations stability condition is obtained as follows:

$$\left[\left(1 + 8\lambda_A \frac{\Delta t^2}{\Delta x^4}\right)^2 + \lambda_D^2 \frac{\Delta t^4}{4\Delta x^4} \right]^{1/2}$$

$$+ \left[\left(16\lambda_A \frac{\Delta t^2}{\Delta x^4} + 64\lambda_A^2 \frac{\Delta t^4}{\Delta x^8} + \lambda_D^2 \frac{\Delta t^4}{4\Delta x^4}\right)^2 + \frac{\Delta t^4}{\Delta x^4} \lambda_D^2 \right] \leq 1 \quad (17)$$

where Δt and Δx are the time step and the spatial mesh size respectively. λ_A and λ_D are the largest eigenvalues of matrices A and D respectively.

For flap-lag equations, matrix [A] represents the stiffness of the system and it is linear. On the other hand the matrix [D] is nonlinear and is a function of the nodal variables and time in forward flight conditions. The Von Neumann condition which is based on Fourier series analysis applies only if the coefficients of the linear differential equations are constant. In case the differential equation with variable coefficients and nonlinearity this method can still be applied locally.[23] Thus the value of λ_B is a variable in the von Neumann stability condition given by equation (17), the stability condition needs to be checked at every point of the field at each time step.

3. Results and Discussions

In this section numerical results have been presented to illustrate the application of finite difference method to solve rotor blade dynamical equations. Since the main objective of this study is primarly to illustrate the application of finite difference methods to find transient and steady-state responses of the rotor blade motion in hover and forward flight conditions, certain simplifications and assumptions have been,

1. Uniform inflow is used, where λ_0 is given by (18) and the cyclic inflow components, λ_s and λ_c are set to zero.

$$\lambda_0 = \mu \tan\alpha_R + \frac{C_T}{2\sqrt{\mu^2+\lambda_0^2}} \tag{18}$$

2. Hub and tip loses are not included.

3. A two-dimensional, strip type, quasi-steady aerodynamic model is used.

4. Structural and mass properties of blade have been assumed to be uniform along the span. A uniformly equal mesh distribution has been used.

5. Reverse flow effects have not been included.

6. The cyclic pitch variation in forward flight is given as,

$$\theta = \theta_0 + \theta_{1S} \sin\psi + \theta_{1C} \cos\psi \tag{19}$$

where all trim values for a given weight coefficient C_W are obtained from Table 2 of Reference 7.

In all cases, finite difference solutions have been first obtained for hover conditions and the advance ratio μ has been set equal to zero. The cyclic pitch components are not present. A uniform inflow λ has been assumed to be equal to its value at 0.75 span and written as,

$$\lambda = (a\sigma/16)\left[(1+24\,\theta/a\sigma)^{1/2} - 1\right] \tag{20}$$

The pitch setting θ is also set equal to the steady pitch component obtained from trim solutions. After a certain time interval flight condition is switched from hover to forward flight by introducing the cyclic pitch components to the corresponding pitch variation by a linear incremental procedure in n_f time steps. Increments for cyclic pitch components are taken as follows:

$$\theta_{1C}^{n+1} = \theta_{1C}^{n} + \Delta\,\theta_{1C} \qquad \text{for } n \quad n_h < n < (n_h + n_f)$$

$$\theta_{1S}^{n+1} = \theta_{1S}^{n} + \Delta\,\theta_{1S} \qquad (21)$$

where

$$\Delta\theta_{1C} = \frac{\theta_{1C}}{n_f} \quad , \qquad \Delta\theta_{1S} = \frac{\theta_{1S}}{n_f}$$

The advance ratio μ is set to its trimstate value immediately at switching stage to forward flight and solutions have been obtained for different switching steps.

Results have been given in as two groups of figures. As a first case response solutions for flap-Lag motions have been presented. Results, for different advance ratios have been considered as second case. Response results have been presented in a similar manner.

Results for Flap-Lag Motions in Forward Flight

Response solutions for flap-lag motions in forward flight have been obtained by using the formulation given in Section 2. The boundary conditions for this case are linear, and the elastic coupling parameter have been set to R_c = 1.0. The remaining blade parameters are given by Table 1. For the soft inplane blade (W_{L1} = 0.732) these properties are close to those of the Boelkow BO-105 hingeless rotor.

TABLE 1

Configuration Parameters for Flap-Lag-Torsion in Forward Flight

(First rotating lag Fre.)	$\omega_{L1} = 0.732$
(First rotating flap Fre.)	$\omega_{F1} = 1.125$
(First rotating torsion Fre.)	$\omega_{T1} = 3.176$
Semicord	$b/R = 0.0275$
Solidity ratio	$\sigma = 0.07$
Drag coefficient	$C_{D0} = 0.01$
Lock number	$\mu = 5.5$
Slope of lift curve	$a = 2\pi$
Weight coefficient	$C_W = 0.0$

Advance ratio	μ variable
Aerodynamic center offset	$X_4 = 0$
Precone angle	$\beta_p = 0$

Numerical solutions have been obtained by using different finite difference discretization parameter Δx. Smaller number of meshes make the system stiffer but eight meshes have been found to be efficient since a remarkable changes in responses have not been observed for meshes higher than eight. In Figure (1) a typical time history solutions for flap-lag motions have been illustrated. As seen from the figures, the transient of the lag motion is significantly longer than the transient of the flap motion. In Figures 2 and 3 the flap and lag responses for advance ratios $\mu = 0.2$ and $\mu = 0.4$ have been illustrated. Results have been compared with the results of Straub and Freidmann[7] where the same governing equations have been solved by two different numerical techniques. As seen from the figures a good agreement is obtained for both cases.

Results for Flap-Lag-Torsion Motion in Forward Flight

Response solutions for flap-lag-torsion motions have been obtained with the two step explicit finite difference scheme. Boundary conditions for this case are nonlinear due to bending-torsion coupling terms, and solutions have been started with linearized boundary conditions, again for the hover conditions. Forward flight parameters have been introduced by an incremental procedure and the nonlinear terms have been finally included when the transients have attenuated. Typical time history of flap-lag-torsion tip motions have been presented in Figure (4). Two different lag transients for different forward flight switching time intervals have also been presented. As seen from Figure 4 lag transient have attenuated in a shorter interval of time for smoother switching case $\Delta_{tF} = 3.0$ as compared with the case Δ_{tF} 1.2.

Flap-lag-torsion steady tip deflections for advance ratio $\mu = 0.2$ (for blade parameters given in Table 4.1) have been presented in Figure 5. Results are compared with the results of Jonnalagadda.[4] A very good agreement has been obtained for flap and lag responses. Torsion response results of both studies are in the same range of magnitude, but different in shape. Results for advance ratio $\mu = 0.4$ are given in Figure 6 where an agreement is better for flap-lag-torsion responses in shape but different in magnitude. In Figures 7 and 8 similar comparisons have been made with solutions of Freidmann and Kottapalli[14]

CONCLUSIONS

Finite difference methods provide an alternate choice for integrating the rotor blade equations. Because the particle differential equations have been directly discretized in time and space a desired accuracy can be allowed on the basis of the available criteria that have been established are proved. In future it is possible to combine these methods with computational fluid mechanics results to solve aeroelastic problems with more accurate aerodynamics.

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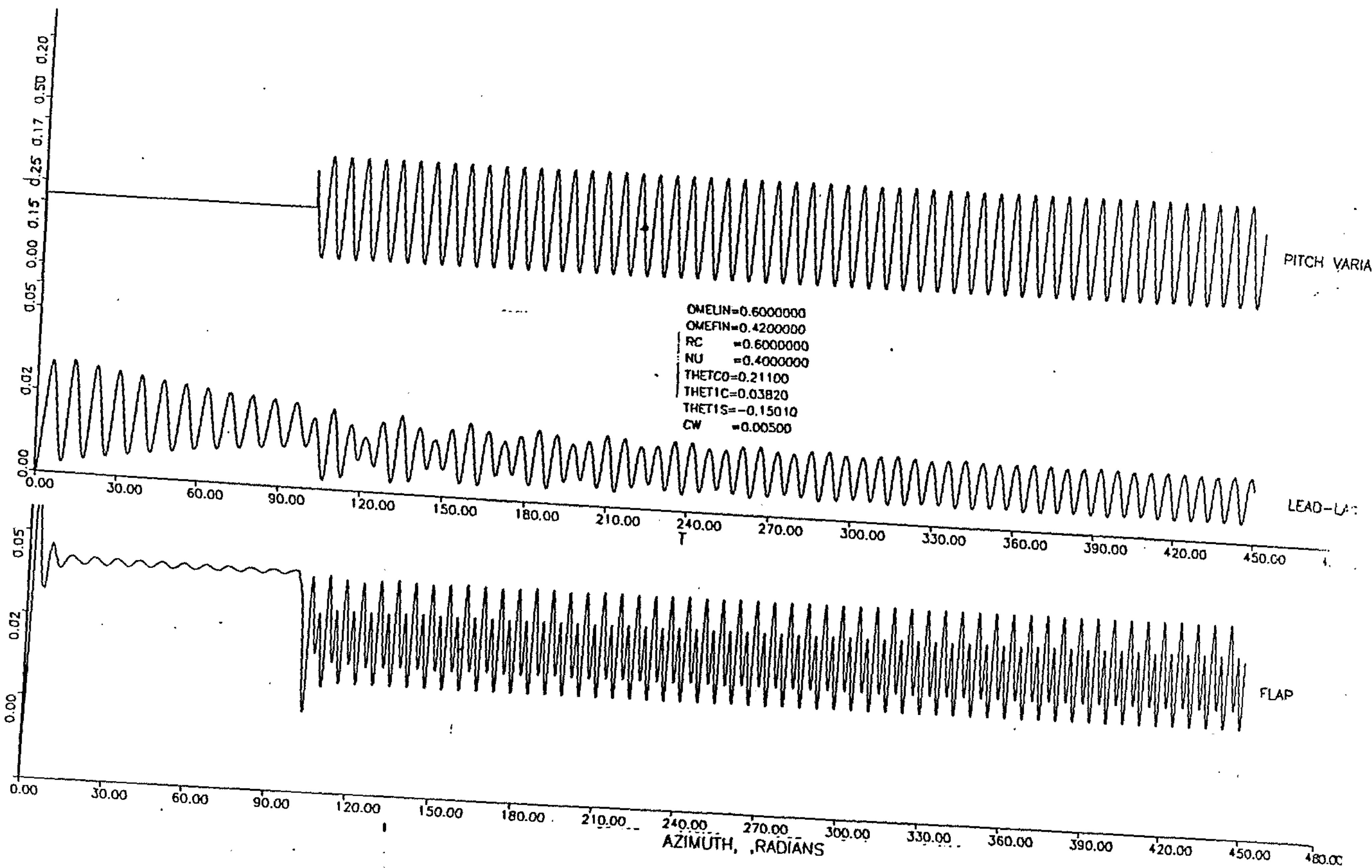

Fig. 1 Response of the soft-in-plane configuration for flap-lag motions, Cw=0.005, μ=0.40

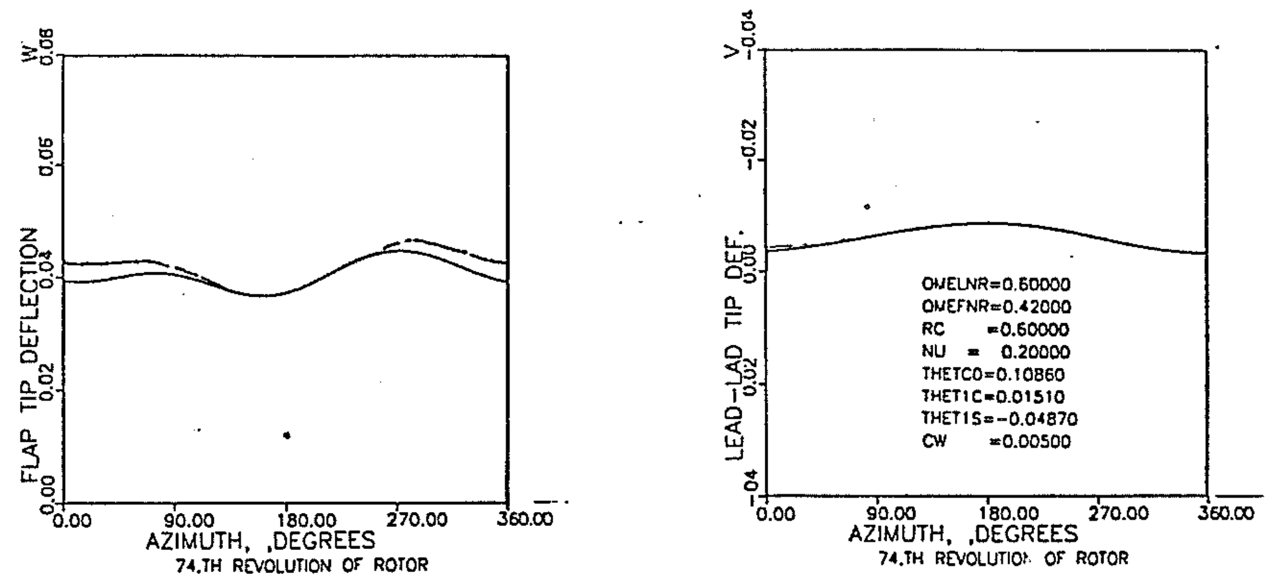

Fig. 2 Response of the soft-in-plane configuration for flap-lag motions, Cw=0.005, μ =0.20 (_____ Present study ,_ . _ . Straub and Freidmann[7])

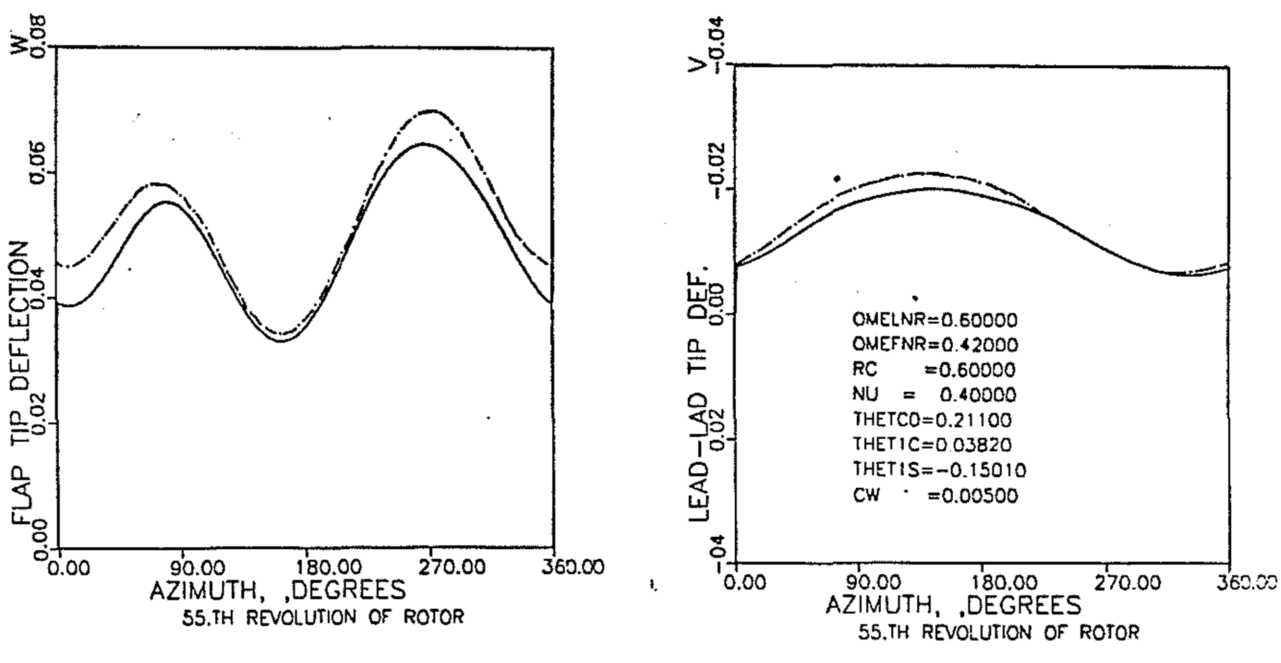

Fig. 3 Response of soft-in-plane configuration for flap-lag motions, Cw=0.005, μ =0.4 (____ Present study, _ . _ . Straub and Freidmann[7])

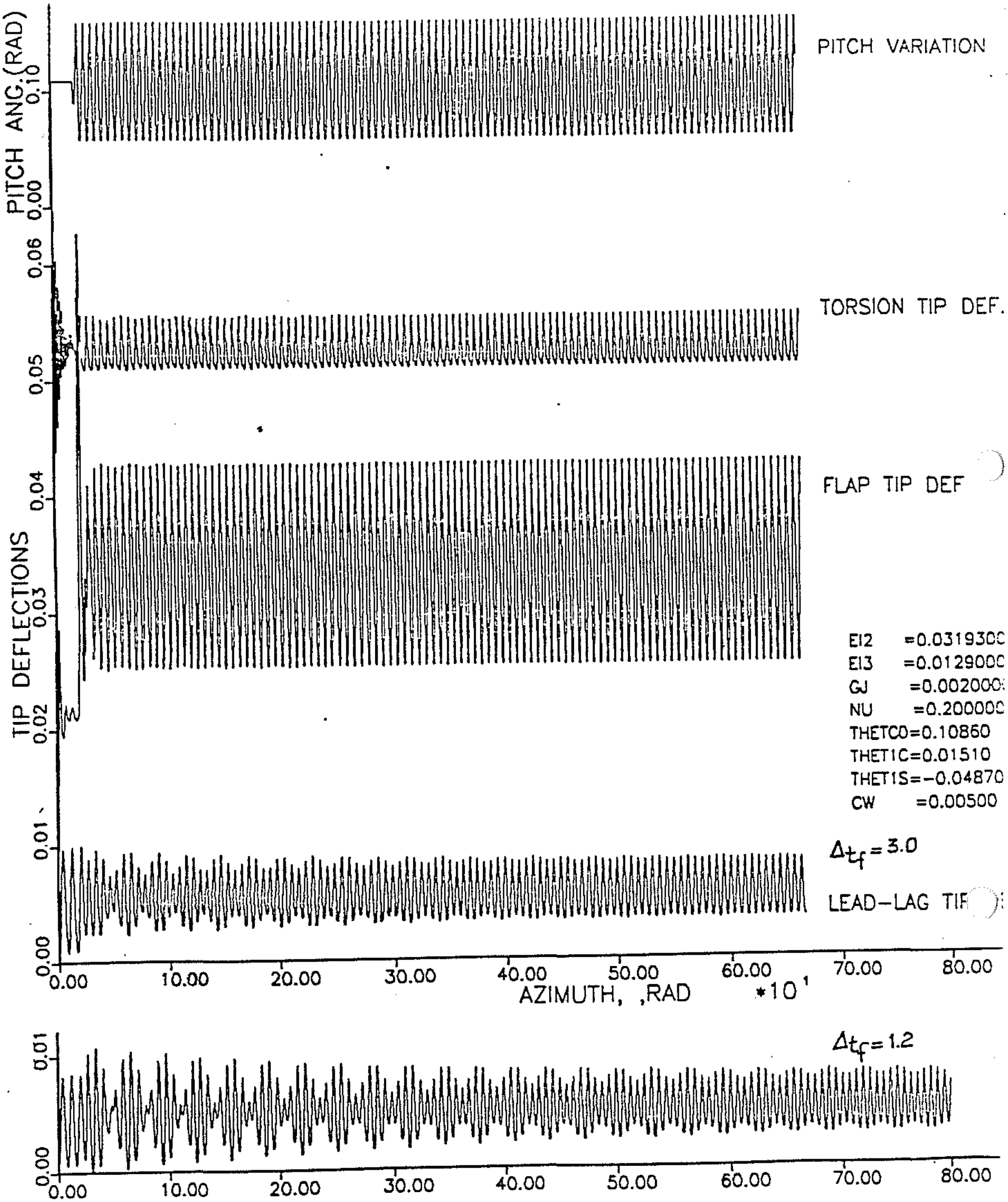

Fig. 4 Response of soft-in-plane configuration for flap-lag- torsion motions, Cw=0.005 μ =0.2

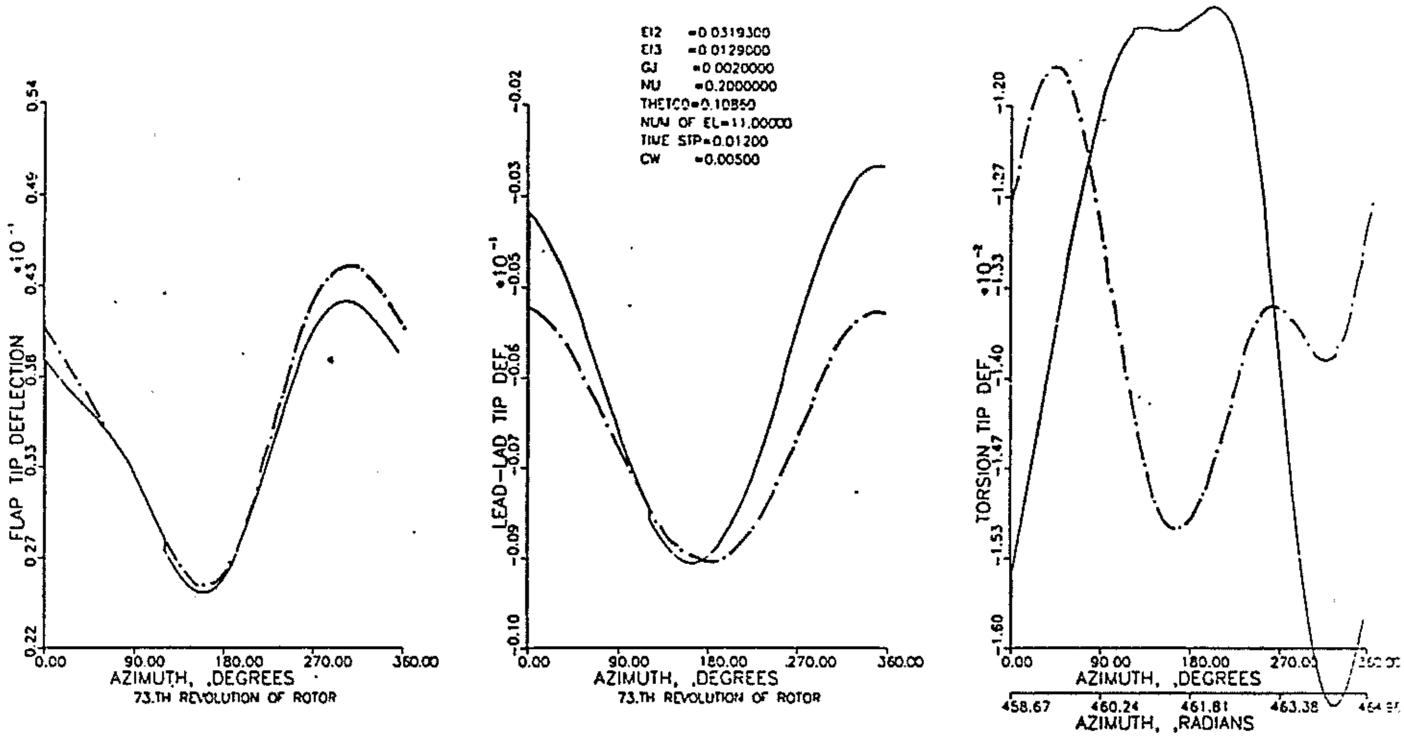

Fig. 5 Response of soft-in-plane configuration for flap-lag-torsion motions, Cw=0.005 μ =0.2 (____ Present study,_ ._ . Jonnalagadda[4])

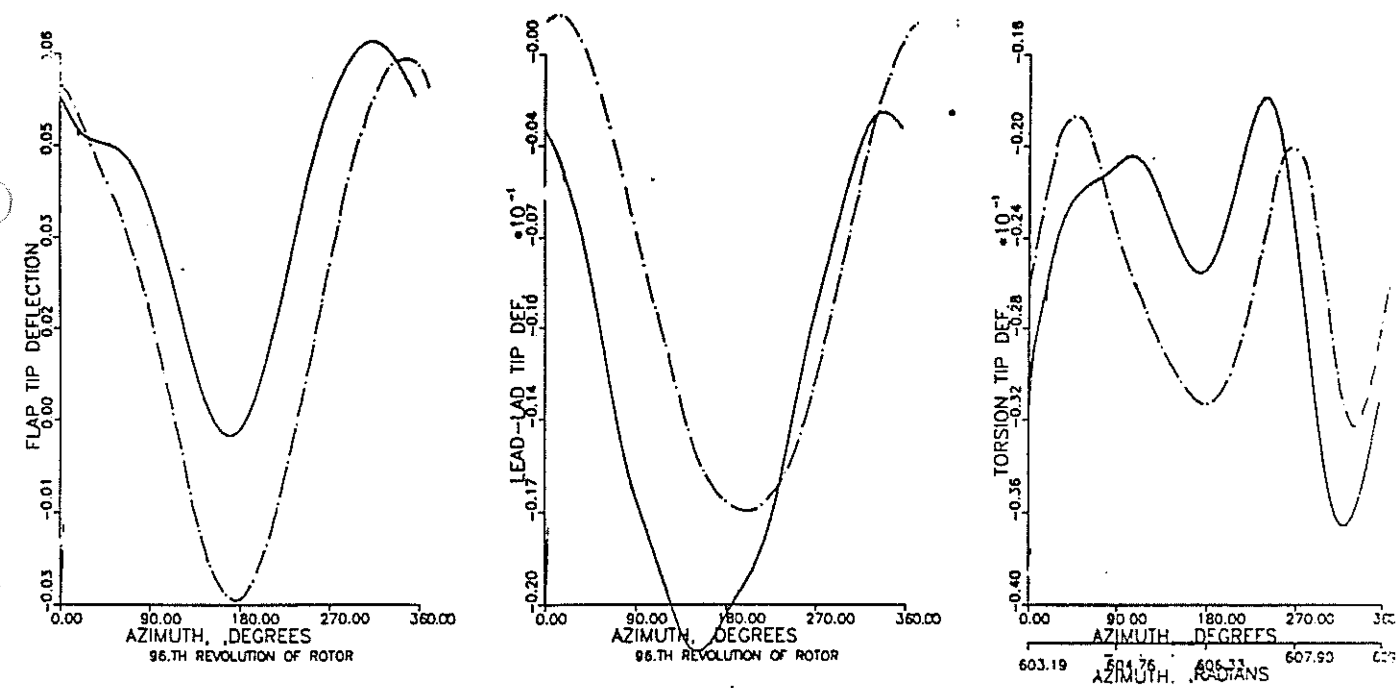

Fig. 6 Response of soft-in-plane configuration flap-lag-torsion motions, Cw=0.005 μ =0.4 Present study,_ . _ . Jonnalagadda[4])

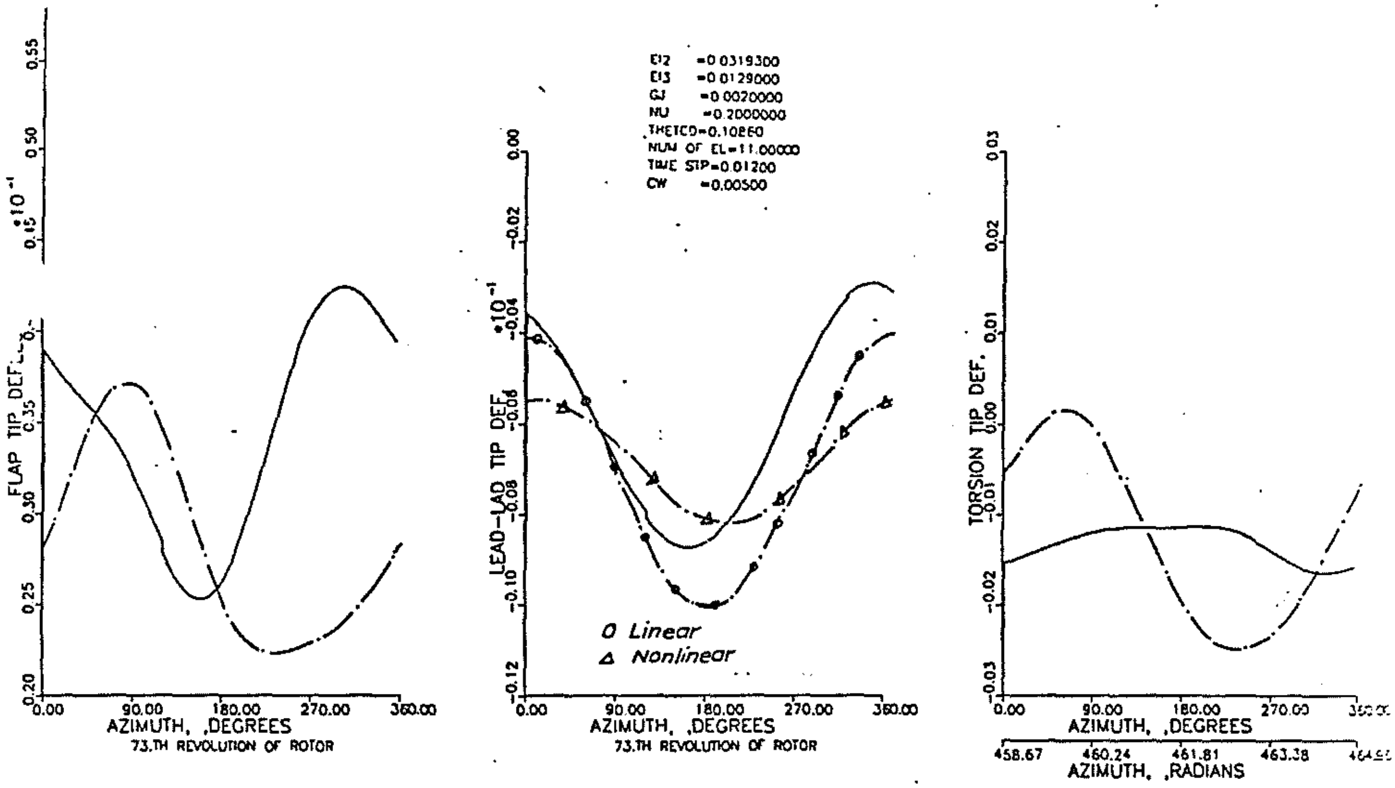

Fig. 7 Response of soft-inplane configuration
for flap-lag-torsion motions, Cw=0.005 μ =0.2
(____ Present study, _ . _ . _ Freidmann and Kottapalli[14])

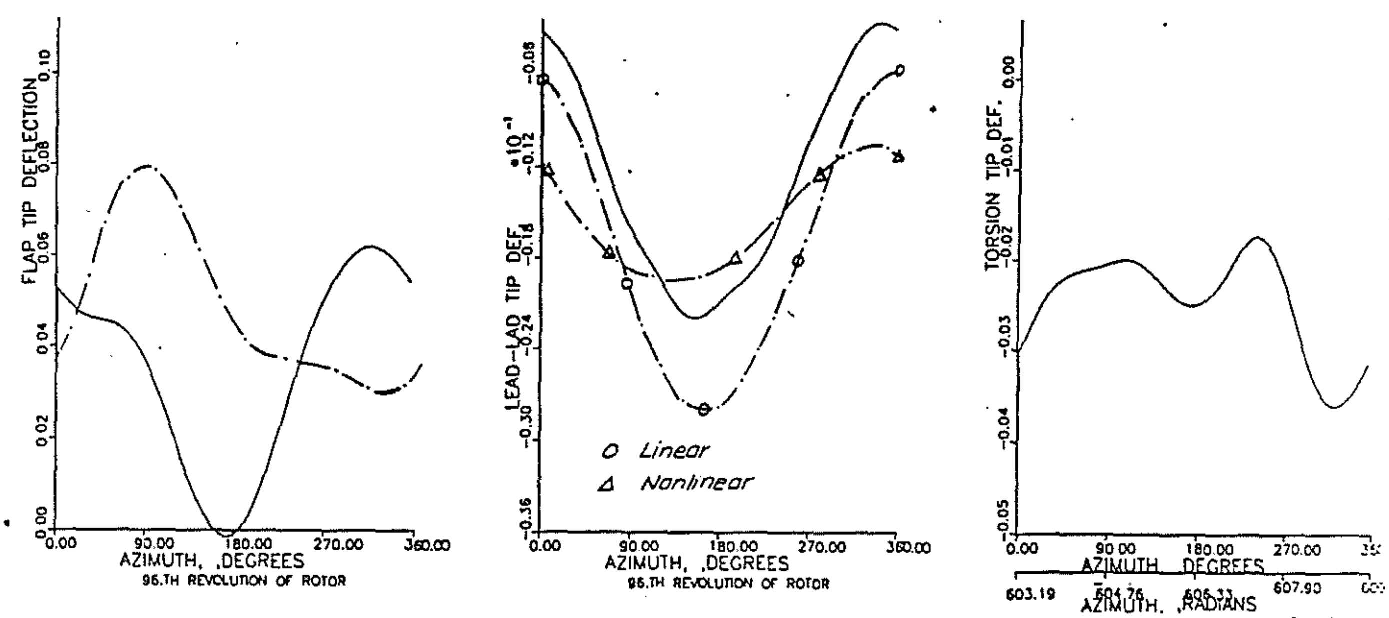

Fig. 8 response of soft-in-plane configuration
for flap-lag-torsion motios, Cw=0.005 μ =0.4
(____ Present study, _ . _ . Freidmann and Kottapalli[14])