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AN APPROACH FOR DETERMINATING ROTOR BLADE LOADS

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ABSTRACT

An approach of predicting the rotor blad loads is presented in this paper. This approach, based on load identifition and blade dynamic response analysis, utilizes the measured response of geometric scale model rotor blade or the "old" full scale blade to obtain the "new" full scale rotor blade loads. In investigation, the aerodynamic loads of blade were considered as combination of two parts , one is the function of the blade deflections and another ,which is equal for two blade considering the geometric scale of two blade, isn't. This approach combinating the test data and theory analysis predictes the rotor blade loads in terms of geometric scale between rotor blades. For steady forward flight, the equation of determinating the blade loads were drived. Finally, an example is given to illustrate this approach.

NOTATION

T	rotor blade thrust
r. R	rotor radius
ρ	air density
b	blade chord
Ω	rotor rotational speed
ψ	blade azimuth
v	induced velocities
μ	advance ratio
C_y^α	lift curve slope
$(\bar{\ })$	nondimensional
α	blade attack angle
Y(r)	out of plane blade deflection
q	generalized coordinates
φ	modal shape

INTRODUCTION

Predicting rotor blade loads have been one of the important problem that rotor designer faces. The methods of predicting rotor blade loads have been studied by many researchers in the past time. Now, the development in structural dynamics , aerodynamics and aerolastics of blade have made the methods predicting rotor blade loads improved continuously. Even so, these mothods are only partly successful in predicting rotor blade loads considering that the available results can only be obtained in respective definite conditions. As one way, the rotor blade loads identification mothod based on modal analysis and test data has also been developed by Ref. 1—2 which compute rotor blade loads from the response of rotor blade and for steady forward flight, this mothod can give a satisfactory results.

The rotor blade loads identification mothod requires the blade response data that usually can not be measured for helicopter blade, partically, for a new designed rotor blade. In researching for a solusion, an approach is described in this paper which combines the blade loads identification and blade response analysis and predicte the rotor blade loads from scale model or full scal rotor blade loads. This approach have only need of the geometric scale between the "new" rotor blade and the model or full scale rotor blade, i. e. no need for dynamic scale. So, the scope of applicability of this approach is more expansive than rotor blade loads identification method described in Ref. 1—2.

In the investigation, the rotor blade responsee equations are set up based on the modal analysis. The aerodynamic forces in the equations derived from liner left theory are considered as combination of the two parts, one is expressed as the function of the blade deflections , another is not, so that the part as the function of blade deflection can be further expressed as the function of generalized coordinates, and then regarded as the additional item of left side of blade vibration equations, while the part that is not the function of blade deflection is laid the right side of equation as the vibration forces of blade vibration equation.

For two scale rotor blades, the relations that non — dimensional right side items of equations are equal made the response of one rotor blade transformed to predicte the loads on another scale rotor blade.

AERODYNAMIC LOAD ANALYSIS

For steady forward flight, using the liner lift theory, the aerodynamic force on the airfoil section at location r is written as:

$$\frac{dT}{dr} = \frac{1}{2}\rho b W_z^2 C_y^a \cdot \alpha \qquad (1)$$

Considering the geometric twist along the blade, cyclic pitch and flap adjestment, the equation (1) can be written as:

$$\frac{dT}{dr} = \frac{1}{2}\rho b C_y^a \Omega^2 R^2 [\overline{W}_z^2 (\varphi_0 + \varphi_\delta + \varphi_s \sin\psi + \varphi_c \cos\psi - K \frac{\partial y(0.\psi)}{\partial r}) + \overline{W}_z (\mu tg\, \alpha_s + \bar{v} - \mu\cos\psi \frac{\partial Y}{\partial r} - \frac{1}{\Omega R}\frac{\partial y}{\partial t})] \qquad (2)$$

Furthermore, the aerodynamic force can be expressed as the combination of two parts $P_y(r,\psi)$, $P_0(r,\psi)$. The part $P_y(r,\psi)$ is the function of the blade deflection $y(r,\psi)$.

$$P_y(r,\psi) = \frac{1}{2}\rho \bar{b} \Omega^2 R^3 C_y^a [(\bar{r} + \mu\sin\psi)^2 (-K \frac{\partial y(r,\psi)}{\partial r}) + (\bar{r} + \mu\sin\psi)(-\mu\cos\psi \frac{\partial y(r,\psi)}{\partial r} - \frac{1}{\Omega R}\frac{\partial y(r,\psi)}{\partial t})] \qquad (3)$$

The blade deflection at location r can be represented in terms of a Fourier series.

$$y(r,t) = y_0(r) + \sum_k (Y_{kc}(r)\cos k\psi + Y_{ks}(r)\sin k\psi) \qquad (4)$$

Where, $Y_0(r)$, $Y_{kc}(r)$, $Y_{ks}(r)$ are the modal summation at location r of blade

$$Y_0(r) = \sum_j \varphi_j(r) q_{j,0}$$
$$Y_{kc}(r) = \sum_j \varphi_j(r) q_{j,kc} \qquad (5)$$
$$Y_{ks}(r) = \sum_j \varphi_j(r) q_{j,ks}$$

where, $q_{j,0}$, $q_{j,kc}$, $q_{j,ks}$ are the jth genralized coordination. then,

$$P_y(r,\psi) = C_0\{-K(\bar{r}^2 + 2\bar{r}\mu\sin\psi + \mu^2\sin^2\psi) \cdot (\frac{dY_0(0)}{dr} + \sum_k (\frac{dY_{kc}(0)}{dr}\cos k\psi + \frac{dY_{ks}(0)}{dr}\sin k\psi)] + (-\mu^2\sin\psi\cos\psi - \mu\bar{r}\cos\psi)[\frac{dY_0}{dr} + \sum_k (\frac{dY_{kc}(r)}{dr}\cos k\psi + \frac{dY_{ks}(r)}{dr}\sin k\psi)] + (-\bar{r} - \mu\sin\psi)\frac{1}{R}[-\sum_k (Y_{kc}(r)k \cdot \sin\kappa\psi - Y_{ks}(r)k\cos k\psi)]\} \qquad (6)$$

Using the formula of trigonometric function, the loads which is not the function of blade deflection can be written as:

$$P_y(r,\psi) = C_0\{P_{y0}(r) + \sum_k (P_{ykc}\cos k\psi + P_{yks}\sin k\psi)\} \quad (7)$$

where,

$$P_{y0}(r) = -C_0\left\{\frac{\mu^2}{4}[\varphi']\{q_{j,2s}\} - \frac{\mu}{2R}[\varphi]\{q_{j,1c}\} + [\frac{\bar{r}\mu}{2}][\varphi']\{q_{j,1c}\}\right\}$$

$$P_{ykc}(r) = -C_0\{\frac{\mu^2}{4}[\varphi']\{q_{j,k+2s}\} - \frac{\mu(k+1)}{2R}[\varphi]\{q_{j,k+1c}\} + [\frac{\bar{r}\mu}{2}][\varphi']\{q_{j,k+1c}\} + [\frac{\bar{r}k}{R}][\varphi]\{q_{j,ks}\} + \frac{\mu(k-1)}{2R}[\varphi]\{q_{j,k-1c}\} + \frac{\bar{r}\mu}{2}[\varphi']\{q_{j,k-1c}\} - \frac{\mu^2}{4}[\varphi']\{q_{j,k-2s}\}\}$$

$$\{P_{yks}\} = -C_0\{-\frac{\mu^2}{4}[\varphi']\{q_{j,k+2c}\} - \frac{\mu(k+1)}{2R}[\varphi]\{q_{j,k+1s}\} + [\frac{\bar{r}\mu}{2}][\varphi']\{q_j, k+1s\} - [\frac{\bar{r}k}{R}][\varphi]\{q_{j,kc}\} + \frac{\mu(k-1)}{2R}[\varphi]\{q_{j,k+s}\} + [\frac{\bar{r}\mu}{2}][\varphi']\{q_{j,k-1s}\} + \frac{\mu^2}{4}[\varphi']\{q_{j,k-2c}\}\}$$

(8)

like the $P_y(r,\psi)$, the aerodynamic force $P_0(r,\psi)$ which is not the function of blade deflection can be written as:

$$P_0(r,\psi) = P_{0,0}(r) + \sum_k (P_{0,kc}(r)\cos k\psi + P_{0,ks}(r)\sin k\psi) \quad (9)$$

TRANSFORMING EQUATIONS

BLADE DYNAMIC ANLYSIS

Using the generalized coordination, the vibrations equations of the blade can be written as:

$$[(\gamma_j^2 - (k\Omega)^2)]\{q_{j,k}\} = [\varphi]^T\{F\} \quad (10)$$

where the structural damp is ignored. Substitute the aerodynamic force for the right side of equations, then

$$[\gamma_j^2 - (k\Omega)^2)]\{q_{j,k}\} = [\varphi]^T(\{P_{0,k}\} + \{P_{y,k}\}) \quad (11)$$

Assume

$$\{\rho_{0k}\} = [m][\varphi]\{f_{j,k}\} \quad (12)$$

then, the equation is further written as:

$$[(\gamma_j^2 - (k\Omega)^2][q_{j,k}\} - [\varphi]^T\{P_{y,k}\} = \{f_{j,k}\} \quad (13)$$

where, the subscript j, expresses jth mode, the subcript k, expresses kth harmonic, γ_j, is the jth natural frequencies of rotating blade, $q_{j,k}$, is the generalized coordination, $f_{j,k}$, is the generalized forces of aerodynamic force part which is not the function of blade deflections. Considering the $\{P_{y,k}\}$ is not only the function of $\{q_{j,k}\}$, but also the function of the $\{q_{j,k\pm1}\}$ and $\{q_{j,k\pm2}\}$. The equation is now written as:

$$[G][q] = [f] \quad (14)$$

where, $[q]$, the generalized coordination matrix, consist of $\{q_{j,k}\}$, $(K=0,1,2,\cdots)$.

LOADS ANALYSIS

The generalized coordination $\{q\}$ is determined from the measured moments $\{M\}$ along the rotor blade,

$$\{q\} = ([M_R]^T[M_R])^{-1}[M_R]^T\{M\} \quad (15)$$

where $[M_R]$ is the modal moments of rotor blade. Using the equation (14), the aerodynamic loads $\{f_{j,k}\}$ which is not the function of blade deflection, can be obtained. Consequently, the aerodynamic loads $\{f_{j,k}\}$ which is not the function of scale blade deflection can be written in terms geometric scale as

$$\{f_{j,k}\}_{scale} = [\varphi]^T_{scale}([m][\varphi]\{f_{j,k}\}) \quad (16)$$

Also, using the equation (14) and equation (8) for scale rotor blade, the genrealized coordination of scale blade can be determined, then, the scale rotor blade aerodynamic loads which is the function of scale blade function can be calculated, then the aerodynamic forces of scale rotor blade are calculated as:

$$[f_{j,k}] + [\varphi]^T_{scale}[P_{yk}]_{scale} \qquad (17)$$

EXAMPLE

A computer program has been developed to calculate the aerodynamic forces from the scale rotor blade load analysis. For illustrating the analysis and computer program, the two cantilevels with respective mode shape $[\varphi]$, $[\varphi]_{scale}$, and vibration force, $([r][\varphi]\{q_j\} + \{P_i\}) sin\omega t$, $([r]_{scale}[\varphi]_{scale}\{q_j\}_{scale} + \{P_i\}_{scale}) sin\omega t$, are assumed. In vibration forces, the part $[r][\varphi]\{q_j\} sin\omega t$ is the force relating the blade deflection, and the part $\{P_i\} sin\omega t$ is the force no relation with blade deflection. The aerodynamic loads parts of both cantilevels which are not function of blade deflection are assumed equally, so as to stimulate the force of two scale rotor blade. To investigate the effect of mode numbers, the first three, five, and ten modes were involved.

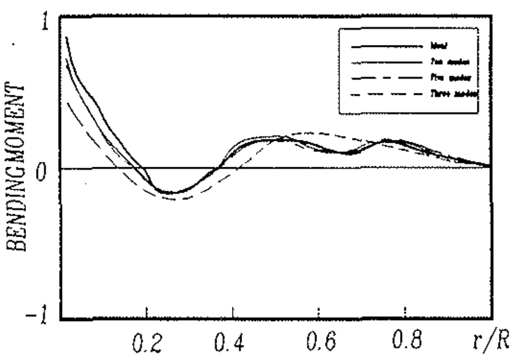

Fig. 1. Comparison of Computed Bending Moments

The results of cantilevel loads from this analysis showed that for the high enough harmonic mode involved, here, the five modes, this analysis can get a good coorelation between the assumed loads and predicted loads. So, the present analysis is a effective way of predicting the blade loads.

REMARK

The rotor blade loads identification and loads transforming between geometric scale rotor blades applied to predicte aerodynamic loads in steady forward flight is an available way. Even though there are no experimental data available for comprision with the predictions, only computed results are presented. An important feature of present method is its ability to predicte the rotor blade loads from only geometric scale rotor blade response. This method has an obvious effect in using the model rotor test results. But, it is necessary to note that the aim of this method is only to investigate a new way to predicting the rotor blade loads combinating the experiment and theory analysis and need further investigated.

REFERENCE

1) William. G. B., Estimation of Blade Airloads From Rotor Blade Bending Moments. paper presented at 13th European Rotorcraft Forum, 1987.

2) S. S. Liu and G. A. O. Davies, The Determination of Rotor Blade Loading From Measured Strains. paper presented at 13th European Rotorcraft Forum, 1987.