THE CHALLENGE OF THE DAMPERLESS ROTOR

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Abstract

Soft-inplane hingeless and bearingless rotor helicopters must be designed to avoid ground and air resonance instability. Conventional approaches rely on auxiliary blade lead-lag dampers incurring weight, cost, and maintenance penalties. Tailoring aeroelastic couplings offers a potential, but not as yet generally accepted solution for eliminating lead-lag dampers. The major evolutionary stages of hingeless and bearingless rotor development are surveyed along with relevant research on aeroelastic and aeromechanical stability applicable to stabilizing ground and air resonance. The basic technical problems of using aeroelastic couplings are reviewed along with some of the practical problems of designing rotor blades to provide such couplings. Possible approaches for realizing a practical damperless rotor are discussed.

Introduction

For many years, the rotorcraft technical community has attempted to devise helicopter rotors that would eliminate the necessity for auxiliary lead-lag dampers to prevent ground and air resonance - customary practice since the development of the conventional articulated rotor helicopter. With the advent of the soft-inplane hingeless rotor in the 1960s, the requirement for auxiliary damping was reduced, but with very few exceptions today's typical helicopter continues to employ elastomeric lead-lag dampers. Advanced rotor development during the last decade has focused considerable attention on the bearingless rotor, but again, configurations currently in production retain auxiliary lead-lag dampers.

During the last two decades, considerable attention has been devoted to better understanding the effects of aeroelastic couplings on ground and air resonance stability to be able to eliminate the need for auxiliary lead-lag damping. However, this effort has been largely unsuccessful. Although significant couplings have been found that will eliminate ground or air resonance instability for some ranges of operating conditions or vehicle configuration variations, these results have not yet been sufficiently successful to eliminate the need for such dampers.

The purpose of this paper is to review the results of research on aeroelastic and aeromechanical stability of hingeless and bearingless rotors and identify the difficulties that must be overcome in applying aeroelastic couplings to achieve a successful damperless rotor. The paper will concentrate on the fundamental aeroelastic issues, but will also relate this knowledge to the evolution of the principal rotor configurations developed over the last three decades. It is hoped that critically examining the currently understood limitations of aeroelastic couplings may provide additional stimulus for researchers and rotor designers to renew their efforts to advance the state-of-the-art in this area. Ultimately such investigation will yield a general solution for damperless bearingless rotors.

The paper will begin with a background discussion of helicopter rotor development, outline alternative approaches to eliminating blade dampers, survey relevant aeroelasticity research, present representative analytical results to illustrate the technical issues in applying aeroelastic couplings to stabilize air and ground resonance, and finally conclude with a few comments on the practical problems of implementing such couplings in rotor design.

Background

Making Rotors Simpler

Throughout the evolution of the helicopter, continual attention has been given to reducing the inherently complex mechanisms of the unique apparatus that provides lift, propulsion and control of the helicopter - the rotor itself. The now classic fully articulated rotor system dates back to the 1920's and 1930's when blade flap and lead-lag hinges were introduced to solve problems of rotor control and inplane blade loads. The presence of lead-lag hinges led to the phenomenon of mechanical instability or ground resonance researched in the early 1940's by Coleman and others and solved with the addition of blade lag hinge and landing gear dampers. A major goal of rotor development ever since has been to eliminate the blade hinges and dampers that encumber the rotor with weight and drag, add to the cost and maintenance burden, and reduce vehicle reliability and safety. Although the development of elastomeric bearings, elastomeric dampers, and composite materials have significantly improved the fully articulated rotor, the impetus continued to further eliminate these parasitic components.

During the 1960's, a surge of development resulted in the first practical hingeless rotors. By exploiting structural properties of advanced metallic and composite materials, the flap and lead-lag hinges were eliminated and necessary blade motions were accommodated by elastic bending in the blade root region. While a major step forward, the preferred soft-inplane hingeless rotor typically required auxiliary blade lead-lag dampers to control ground resonance and the airborne analog, air-resonance, that emerged when the flap hinges were eliminated. The stiff-inplane variant of the hingeless rotor is inherently immune to ground and air resonance, but is less desirable for other reasons.

Continued evolution has produced the bearingless rotor, now finding favor for many helicopter applications, which does away with the conventional pitch bearing of articulated and hingeless rotor blades by introducing a flexbeam element to simultaneously accommodate blade bending and pitch change motions. Early bearingless rotors explored diverse configurations; current versions have converged to similar arrangements for the flexbeam and pitch-change torque tube and include a snubber and elastomeric lead-lag damper to ensure aeromechanical stability. Progress in advanced rotor design has reached a stage where parts count, weight, and reliability have been substantially improved, however, retention of the lead-lag damper in nearly all current helicopters attests to the difficulty of achieving the ideal configuration. The operational cost of lead-lag dampers is a design penalty that extends throughout the life of the vehicle, and a satisfactory solution would significantly improve the helicopter. That challenge - the damperless rotor - is the focus of this paper.

Full-Scale Developments

While the discussion above outlines the principal conceptual framework for the evolution of modern rotors, the actual steps in that evolution were taken by developing real aircraft. A few specifics will add important perspective to the challenge of the damperless rotor. The most significant early examples of hingeless rotor development are represented by the MBB BO-105 (Ref. 1) and Westland WG-13 Lynx (Ref. 2), developed in Europe in the 1960's. These soft-inplane systems (first lead-lag frequency less than rotor frequency) employed different approaches to aeroelastic design of the blades. The WG-13 was designed to minimize blade bending-torsion coupling while the BO-105 was designed to capitalize on such aeroelastic coupling to enhance inherent aeromechanical stability. At the time of development, the aeroelastic phenomena were not well understood and the WG-13 approach ultimately required addition of auxiliary lead-lag dampers while the BO-105 demonstrated adequate stability without dampers. To this day the rotor used by both the Eurocopter BO-105 CBS-5 and BK-117 (Refs. 3, 4) is the only true damperless rotor in production. Other noteworthy hingeless rotor developments of the 1960's included the Lockheed XH-51 and AH-56A Cheyenne stiff-inplane hingeless rotors (first lead-lag frequency greater than rotor frequency) While the stiff-inplane rotor is inherently free of aeromechanical instability, and hence qualifies as damperless, other drawbacks are present. A damperless soft-inplane variant, the XH-51 Matched Stiffness rotor, was flown but exhibited unacceptable air resonance stability due to the lack of lead-lag dampers (Ref. 5). It was also noteworthy as one of the first examples of a bearingless rotor. (Strictly speaking, two-bladed, stiff-inplane teetering rotors, inherently free of aeromechanical instability, represent a special class of damperless rotor.)

The BO-105 approach was successfully adopted for the US Army UTTAS Boeing Vertol YUH-61A (Refs. 6,7). Although the selection of the Sikorsky UH-60 precluded it from entering production, the YUH-61A provided another demonstration of a successful, damperless, soft-inplane hingeless rotor. A conventional wheeled landing gear was used and no oleo dampers were required. Nevertheless, the use of aeroelastic couplings was not as reliable or straight-forward a solution for ground and air resonance as auxiliary lead-lag dampers and during the 1970's and 1980's other manufacturers such as Aerospatiale (AS-350 Starflex, Ref. 8) and Bell (Ref. 9) adapted various configurations of soft-inplane hingeless rotors with elastomeric lead-lag dampers .

Increasing interest in the more advanced bearingless rotor concept led to the US Army sponsored Boeing Vertol Bearingless Main Rotor (BMR) program, and an R&D prototype was flight tested on a BO-105C airframe (Refs. 10, 11). This innovative damperless rotor was successfully operated over the full flight envelope but ground and air resonance damping was not quite as high as desired for some flight conditions. Subsequently, Bell developed the innovative and pioneering Model 680 bearingless rotor based on an flexbeam and torque tube concept with a combination snubber and elastomeric lead-lag damper, that led to a number of prototype variants (Refs. 12, 13). Research under the US Army Integrated Technology Rotor (ITR) Program (Ref. 14) led to further progress but was ended before full-scale development of candidate damperless bearingless rotors. The program did provide an impetus for the successful Hughes Helicopters HARP bearingless rotor (Ref. 15) but this configuration incorporated elastomeric lead-lag dampers similar to the Model 680 rotor. The Model 680 configuration has been followed by roughly similar configurations such as the BO-108 (Ref. 16), MDHS MD Explorer (Ref. 17), Eurocopter EC-135, and the Sikorsky Boeing RAH-66 Comanche (Ref. 18). A refined version of the Model 680 rotor is used for the Bell 430 helicopter. All of these rotor systems employ auxiliary lead-lag dampers without significant reliance on aeroelastic couplings.

To sum up the current state-of-the-art, modern soft-inplane hingeless and bearingless rotors have largely converged to a design philosophy that embraces auxiliary lead-lag dampers to provide freedom from ground and air resonance. By foregoing the development challenge of the inherently stable damperless rotor, the designer has been able to avoid dealing with the limited capabilities of aeroelastic couplings and the attendant subtleties that make more demands on the design synthesis process and analysis tools. Nevertheless, researchers and designers will continue to address alternatives to the dampers embodied in the current rotors.

Alternative Approaches

Before we address the technical details of the damperless rotor, the alternative approaches should be briefly noted. There are at least three possible approaches to the development of a soft-inplane damperless rotor: 1) incorporating high damping material into the blade or flexbeam structure, 2) automatic feedback control systems to actively stabilize the rotor-fuselage dynamic system, and 3) the development of aeroelastic couplings to provide inherent system stability.

High Damping Material The first approach, the use of high-damping material, is desirable but would require the development of new material concepts, and design and manufacturing techniques to incorporate the material into the flexbeam structure itself, as in the Triflex rotor hub (Ref. 19). Thorough testing and qualification would be required to insure the structural integrity of primary load carrying structure of the rotor blade. Alternative approaches could include constrained layer damping methods already studied by some investigators but now receiving increased attention. (Refs. 20, 21).

Active Control The second approach of using automatic control inputs based on vehicle body motions and possibly rotor state feedback, has been shown to be feasible by a number of investigators (e.g. Ref. 22) and offers a number of advantages from the point of view of operational and design flexibility. Furthermore, future rotorcraft will find available ever more capable electronic and computerized control systems. However, despite rapid advancements in these systems, safety of flight issues associated with automatic stabilization of highly unstable modes will be critical and require extensive scrutiny and quality assurance before such approaches will become acceptable and capable of certification.

Aeroelastic Couplings This approach might be termed "natural or inherent stability." For the reasons noted above, the use of aeroelastic couplings is especially attractive. In fact when it is noted that the drawbacks of such an approach are essentially nonexistent, then the merits of this approach are apparent. For the remainder of this paper, it is to be understood that the term damperless rotor will be used for configurations based on the use of aeroelastic couplings. The principal drawback of this approach is simply finding a sufficiently effective design solution. The difficulties include the fact that aeroelastic couplings that may be effective for isolated blade stability may be ineffective in the presence of rotor-body dynamic coupling. Another important factor is the number of different flight conditions and vehicle configurations that must all be stable. Successful aeroelastic designs usually result from carefully tailoring of the system dynamic characteristics, and configuration variations may have an adverse effect on these. Important variations may be vehicle weight, inertia, ground contact conditions, rotor speed, airspeed, descent rate, and load factor.

Analytical and Experimental Research

Before proceeding, it will be useful to briefly survey research on aeroelastic couplings and hingeless and bearingless rotor aeroelastic and aeromechanical stability.

Isolated Blade Stability

Early research on basic flap-lag aeroelastic stability of hingeless rotors (Ref. 23) explored the interaction of aeroelastic couplings with the flap and lead-lag stiffness characteristics of cantilever rotor blades. Pitch-lag coupling, already known to be important for articulated rotors, was found to be very important for hingeless rotors as well. The source of flap-lag structural coupling in the hub and cantilever blades of the hingeless rotor was identified and its influence on hingeless rotor stability was explored including it's interaction with the effects of pitch-lag coupling. Further investigations (Refs. 24, 25) found that certain combinations of aeroelastic couplings could substantially increase lead-lag damping of the isolated rotor blade, particularly at low collective pitch which was anticipated to be a critical operating condition for ground resonance. Experimental investigations confirmed the effectiveness of these aeroelastic couplings, Ref. 26.

Early research on hingeless rotors revealed the fundamental nature of the nonlinear bending-torsion coupling of torsionally flexible cantilever beams. This behavior gives rise to a significant part of the aeroelastic couplings that so strongly influence hingeless and bearingless rotor aeroelastic stability. A detailed investigation of torsionally flexible hingeless rotor blades (Refs. 27, 28, 29) identified design parameters such as blade precone, droop, torque offset, bending and torsion stiffnesses that influenced isolated blade stability. In particular, it was shown how the effective pitch-lag and pitch-flap couplings of torsionally flexible blades could be determined so that these couplings could then be applied to simpler torsionally rigid flap-lag blade models, the approach used for the analytical results presented herein. A description of effective aeroelastic couplings as a function of basic blade design parameters will also be discussed later in the paper.

Coupled Rotor-Body Stability

Considerable research has been devoted to coupled rotor-body air and ground resonance analyses for hingeless rotors, with some of this effort directed toward the effects of aeroelastic couplings (Ref. 30). When the effects of rotor-body dynamic coupling were included, the influence of aeroelastic couplings was found to be significantly altered (Refs. 31, 32). The results of these efforts showed that pitch-lag coupling was generally effective in suppressing air resonance, but the effects of flap-lag structural coupling could strongly destabilize ground resonance for many fuselage and landing gear configurations. For configurations employing lower flap stiffness it was found that the effectiveness of aeroelastic couplings was reduced. Experimental investigations were conducted to confirm the analytical models used to explore the general trends and effectiveness of aeroelastic couplings (Ref. 33). More recently, the effectiveness of pitch-lag coupling for ground resonance has been studied (Ref. 34), and the effects of pitch-flap coupling were found to be helpful in suppressing air resonance for some configurations (Ref. 35). An analysis of blades of composite materials showed the potential for tailoring the ply lay-ups to introduce aeroelastic coupling to enhance aeromechanical stability of a hingeless rotor helicopter in Ref. 36. This work included the effects of aeroelastic coupling of torsion and axial extension.

With increasing interest in advanced bearingless rotors, analytical studies of the effects of aeroelastic couplings have also been conducted for this configuration. In Refs. 37 and 38, several different configurations for pitch control, snubber, and flexbeam arrangements (torque tube, torque rod, snubbed and unsnubbed torque tubes, etc.) were investigated. In addition, such design parameters as precone, droop, pre-sweep, torque offset, flexbeam pre-pitch, and pitch link orientation were explored to determine their influence on air and ground resonance stability. The analytical results of this work identified the difficulty of generating effective aeroelastic couplings for some flexbeam configurations and finding combinations of design variables that would stabilize air and ground resonance for a variety of conditions. Other configurations with relatively low flap stiffness were explored in Ref. 39 and an ITR configuration was studied in Ref. 40.

Ground and Air resonance with Aeroelastic Couplings

The purpose here is to provide a brief overview of ground resonance and hover air resonance characteristics and illustrate how they are influenced by aeroelastic couplings. This will help introduce strategies for optimizing stability and reveal some of the difficulties that are encountered. For a simplified rigid blade model the aeroelastic couplings can be made arbitrarily large, not having to satisfy practical design constraints of torsionally elastic hingeless or bearingless blades and therefore the results to be discussed below may represent, in some sense, an upper bound on potential benefits to be achieved.

Physical System and Mathematical Model

A simplified mathematical model based on the physical system in Fig. 1a is sufficient for the present purposes. It consists of a coupled rotor-body system applicable to the hover or in-ground-contact operating condition. The helicopter is composed of a rotor with hinged, rigid blades and a rigid fuselage having pitch and roll rotation (θ,ϕ) about the body center of mass. The blades rotate against spring restraint about centrally located flap and lead-lag hinges. Body translations and gravitational forces are not included because they are not important for air and ground resonance, Ref. 31.

The blade flap and lead-lag rotations occur about axes parallel and perpendicular to the plane of rotation, Fig 1b. The principal elastic axes of flap and lead-lag springs, K_β and K_ζ, respectively, are not necessarily oriented to coincide with the flap and lead-lag motions shown in Fig. 1b and may be inclined at the angle θ_s to permit arbitrary structural (elastic) coupling of the blade flap and lead-lag motions. When θ_s is zero, the flap and lead-lag deflections are structurally uncoupled. In this paper, two conditions are treated: 1) flap-lag coupling (denoted by R=1) where the spring inclination θ_s is proportional to the blade pitch angle (with an increment for zero blade pitch), $\theta_s = \theta_0 + \theta_{so}$, and, 2) no flap-lag structural coupling, $\theta_s = 0$ (denoted by R=0). These conditions correspond to the cases R=0 and 1 of the model developed in Ref. 23.

The blade aerodynamic forces are derived with quasi-steady theory and no dynamic wake effects are included. The nonlinear equations of motion for this system are linearized for small-perturbation motions and represent the blade motions by rotor flap and lead-lag cyclic multiblade coordinate degrees of freedom. Constant coefficient differential equations, where the coefficients are functions of the equilibrium flap and lead-lag blade deflections β_0 and ζ_0, are solved to yield the eigenvalues and eigenvectors of the system which in turn yield modal frequency and damping of the system. Additional details are presented in Refs. 31 and 32.

Discussion of Results

Frequency and damping results as a function of rotor speed for variations of rotorcraft parameters will be presented to illustrate basic aeromechanical stability characteristics and the influence of various design parameters and aeroelastic couplings. The baseline rotorcraft system properties chosen here represent typical design values. The dimensionless (by rotor radius) body inertia radii of gyration, $k_y{}^2, k_x{}^2 = 0.1, 0.025$ for pitch and roll respectively. The ratio of rotor mass to total system mass, $\mu = 0.1$; the dimensionless rotor mast height $h/R = 0.2$. For ground resonance results, the effective landing gear stiffnesses are represented by pitch and roll springs defined by dimensionless body frequencies, $\omega_\theta, \omega_\phi = 0.2, 0.4$ respectively. A typical soft-inplane lead-lag frequency of $\omega_{\zeta 0} = 0.7$ at normal rotor speed, the flap frequency $p = 1.1$, the Lock number $\gamma = 5$, the blade drag coefficient $c_{d0} = 0.01$, and the rotor solidity $\sigma = 0.05$. The lead-lag structural damping $\eta_\zeta = 0.005$, and the baseline aeroelastic coupling parameters R, θ_{so}, θ_ζ, and θ_β are zero, except when introduced to illustrate their influence on air and ground resonance.

The frequency and damping results (real and imaginary parts of the eigenvalue) are presented in dimensionless form, σ/Ω_0 and ω/Ω_0 for a range of dimensionless rotor speed, Ω/Ω_0, where the normal operating rotor speed $\Omega/\Omega_0 = 1.0$.

Air Resonance

Basic Characteristics The basic coupled rotor-body frequencies and the important frequency coalescences are shown for the baseline configuration in Fig. 2, in vacuo and in air for the $\theta_0 = 0$. A representative of rotor speed operating range of +/- 10 % Ω/Ω_0 is included. At higher rotor speeds, the body pitch and roll frequencies are determined by coupling between the rotor flap regressing mode and the rigid body pitch and roll motions. The coupled frequencies are mainly determined by the blade flap spring stiffness and body inertias (Ref. 8).

The hover air resonance stability of the baseline configuration is illustrated in Fig. 3 by the regressing lead-lag mode damping versus rotor speed for the baseline vehicle. The collective pitch is varied with rotor speed to maintain a constant thrust. Several different values of nominal collective pitch (note that $\theta = \theta_0$ at $\Omega/\Omega_0 = 1.0$) are shown to represent different loading conditions including the zero pitch angle ($\theta_0 = 0$) for reference. The vehicle exhibits typical instability at rotor speeds where the lead-lag regressing mode coalesces with the coupled rotor-body roll and pitch modes. The unstable rotor speed range extends well beyond frequency coalescence region (and the range of in vacuo instability) due to the aerodynamic damping of coupled rotor-body modes. Typically, the air resonance instability intensifies with increasing collective pitch, as the aerodynamic and inertial coupling of rotor blade flap and lead-lag modes increase (from blade steady coning and lead-lag deflections). The effects of additional inherent lead-lag structural damping are also shown in Fig. 3; the baseline η_ζ =0.005 case (0.5% critical) is sufficient for stability at zero collective pitch but nearly 2% is required for higher collective pitch.

Other basic but noteworthy features of air resonance are illustrated by examining parametric variations of several basic design variables. The fundamental effects of blade lead-lag frequency shown in Fig. 4 are well known (Ref. 31). The most unstable case is with $\omega_{\zeta 0} = 0.5$, and as the nominal lead-lag frequency increases, both the air resonance onset rotor speed increases substantially, and the intensity of the instability decreases.

The effects of blade flap bending stiffness, characterized by the blade first flap frequency, p, are interesting. This design variable influences many important helicopter characteristics, from handling qualities to maneuverability, gust response, and blade loads. Lower values (p = 1.05 - 1.08) are desirable but are structurally challenging for the designer. Blade flapping stiffness has a first order influence on the coupled rotor-body mode frequencies, and thus controls the rotor speed for coalescence with the lead-lag regressing mode. This is evident in Fig. 5 where the air resonance onset rotor speed increases directly with flap stiffness (p = 1.02 - 1.4). Since the beneficial effects of rotor aerodynamic damping increase as blade flap stiffness increases, the intensity of air resonance decreases with p, but only until the point where the underlying mechanical instability begins to dominate. The present baseline configuration, p = 1.1, is roughly optimum.

Although not shown here, the effects of body inertia and rotor height are important configuration parameters that strongly influence air resonance stability. Decreasing roll inertia intensifies air resonance, but since it increases the coupled rotor-body roll frequency, it raises the rotor speed for frequency coalescence and therefore increases the rotor speed for air resonance onset. Increasing the rotor height amplifies the destabilizing influence of the regressing lead-lag mode with respect to the body pitch and roll motions and significantly intensifies air resonance.

Aeroelastic Couplings As noted above previous research has explored the effectiveness of aeroelastic couplings for stabilizing air resonance of hingeless rotors as noted above. The results to be presented here will focus specifically on the objective of the damperless rotor. The effects of the two principal aeroelastic couplings on the baseline configuration will be examined in the next three figures. Pitch-lag coupling and flap-lag structural coupling effects are observed separately in Figs. 6 and 7 respectively. Pitch-lag coupling provides significant stabilization for the coupled rotor-body roll mode but produces a small destabilizing effect for the pitch mode. Overall, pitch-lag coupling is generally stabilizing. Flap-lag structural coupling is introduced by inclining the flap-lag principal elastic axes, first equal to the blade pitch angle, and then with an additional increment, ($\theta_s = \theta_{so} + \theta_0$, note here that θ_0 varies with rotor speed as thrust is held constant). Figure 7 shows the typical result that flap-lag structural coupling alone is destabilizing (refer to Ref. 31). When both couplings are included together, a strong stabilizing effect is produced, as shown in Fig 8. In this case, increasing collective pitch increases stability, reversing the trend without aeroelastic couplings, Fig. 3. These air resonance results are typical of a wide variety of configurations, and are the principal basis for optimism regarding damperless rotor feasibility.

Ground Resonance

While the previous examination of air resonance characteristics and the influence of aeroelastic couplings was relatively straightforward, the situation for ground resonance is more complex. The landing gear stiffness characteristics are a major determinant in the fundamental body pitch and roll frequencies. Many factors must be considered in the design of landing gear, and these factors may conflict with damperless rotor design objectives. The body frequencies are also dependent on the ground surface conditions and may vary with the rotor thrust, including possible nonlinear effects. Typically, body frequencies in ground contact are higher than in air, which tends to intensify aeromechanical instability. Finally, the landing gear design and ground contact conditions can influence the relative amount of translation and rotation of the body pitch and roll modes, thus influencing the degree to which rotor aerodynamic damping is available to stabilize ground resonance.

As for the discussion of air resonance, the basic characteristics of ground resonance will be briefly examined, before investigating the potential effectiveness of aeroelastic couplings. Note that for ground resonance results the collective pitch will be held constant with rotor speed.

Basic Characteristics Rotor and body ground resonance frequencies are shown in Fig. 9 for the baseline configuration having relatively soft landing gear and low uncoupled body pitch and roll frequencies, ω_θ, ω_ϕ = 0.2, 0.4 respectively. These low body frequencies will help to clarify the issues involved in stabilizing ground resonance. Results in vacuo and in air (γ = 0, 5) show the effects of aerodynamic damping on the principal coalescences of the regressing lead-lag mode with the body pitch and roll modes; generally the in vacuo coalescences are a better indication of the critical rotor speeds for aeromechanical instability. Since the baseline body stiffnesses are relatively soft, the body pitch mode coalescence occurs within the nominal +/- 10% operating rotor speed range while the body roll mode coalescence occurs at a somewhat higher rotor speed. For a more conventional stiffer landing gear, both coalescences would occur above the normal rotor speed range.

Before investigating the effectiveness of aeroelastic couplings, a few basic results will illustrate the traditional approach of using blade and landing gear dampers to stabilize ground resonance. With the vehicle in a vacuum to remove the effect of aerodynamic damping, and with nominal landing gear damping ($\eta_\theta = \eta_\phi = 0.05$), several variations of blade lead-lag damping are examined as shown in Fig. 10. A case without damping is included to illustrate the classical ground resonance pitch and roll instabilities; the roll mode instability is much more intense than the pitch mode due to the higher frequency and lower body inertia. About 2% blade damping is sufficient to stabilize the rotor-body pitch mode, but 10% or more is required to stabilize the coupled rotor-body roll mode.

The basic effects of rotor aerodynamics are next shown in Fig. 11 (landing gear damping is not included). Aerodynamic forces provide effective rotor-body damping somewhat analogous to landing gear dampers and also introduce the important influence of flap-lag aeroelasticity. At zero collective pitch, aerodynamics provides rotor damping that stabilizes both the pitch and roll modes of ground resonance, but increasing collective pitch is strongly destabilizing. Although the majority of vehicle operation in ground contact occurs with zero or low collective pitch, the vehicle must nevertheless be stable for any collective pitch occurring in ground contact and during the lift-off transition to airborne flight. Figure 11 also shows that the inherent lead-lag structural damping of the baseline rotor ($\eta_\zeta = 0.005$, 0.5% critical) is sufficient to stabilize the pitch mode at zero and higher collective pitch with rotor aerodynamic damping present. The roll mode remains very unstable for all pitch angles.

Aeroelastic Couplings Earlier investigations on the effectiveness of aeroelastic couplings for stabilizing ground resonance have shown that this is generally more difficult to accomplish than for the case of air resonance (Refs. 25, 31). The baseline case is examined for collective pitch of $\theta_0 = 0$ and 0.15 rad with pitch-lag and flap-lag structural coupling introduced separately and in combination. For $\theta_0 = 0$ in Fig. 12, the pitch mode is stable without couplings; adding pitch-lag coupling alone is mildly destabilizing while flap-lag coupling alone is sufficiently destabilizing to produce instability. The combined couplings are the least destabilizing and do not produce a pitch mode instability. For the strong roll mode instability, all couplings are destabilizing but most importantly they reduce the onset rotor speed for instability. These results clearly differ from the case of air resonance and will be examined in more detail below. It could be suggested that aeroelastic couplings are unnecessary at zero collective pitch where ground resonance does not occur in the nominal rotor speed range for the baseline case. In fact this could be accommodated by tailoring the design of real torsionally flexible blades, since aeroelastic couplings are partly generated by blade equilibrium displacements that accompany collective pitch, as will be discussed below

The ground resonance case at non-zero collective pitch, representing a pre-lift-off condition, is shown in Fig. 13. The effects of aeroelastic couplings are more beneficial than for zero collective pitch and the marginally-stable pitch mode of the baseline vehicle is significantly improved. Again, however, the roll mode rotor speed margin is reduced by all combinations of couplings.

In these simplified examples, the ground and air resonance results would be identical except for the landing gear springs and collective pitch variations with rotor speed. Naturally, the body springs produce very significant effects that do in fact make ground resonance far different from air resonance. Nevertheless, it is of interest to trace the evolution of air resonance to ground resonance by simply increasing the body spring stiffnesses continuously from zero to the landing gear values. Such an example is shown in Fig. 14 where the ratio of uncoupled body pitch and roll frequencies is held constant ($\omega_\phi = 2\omega_\theta$) and the nominal in-flight collective pitch (θ_0=0.15) is held constant. As the increasing body spring stiffnesses cause the coalescence rotor speeds to increase, the instability onset rotor speed (for the roll mode) also increases. Note that as the pitch mode instability emerges, the instability onset rotor speed jumps to a lower value before continuing to increase. The basic characteristic that ground resonance intensity increases with body frequencies is investigated in detail in Ref. 31.

The practical importance of the results of Fig. 14 is that while ground resonance intensifies with body frequency, the coalescence rotor speed can increase above the normal operating rotor speed range. In fact, for the highest body frequencies of ω_θ, ω_ϕ = 0.4, 0.8, a reasonably realistic design configuration, the intense roll mode instability becomes inconsequential for this reason and the remaining pitch mode moves just beyond the nominal rotor speed range. Of course, many operational factors can lower the body frequencies, and this is an important practical factor in the possible feasibility of the damperless rotor.

Returning to the relationship of air and ground resonance, we now examine the relative effectiveness of aeroelastic couplings as the uncoupled body frequencies are increased. Three of the configurations from Fig. 14 are individually presented for the θ_0= 0.15 condition, both with and without combined aeroelastic couplings in Figs. 15, 16, and 17. Including results from the air resonance case (Fig. 8) and the baseline ground resonance case (Fig. 13), these results clearly illustrate the progressive decrease in effectiveness of aeroelastic couplings for eliminating ground resonance instability as body frequencies increase. These trends are well summarized in the stability boundary plot of Fig. 18 that shows regions of pitch and roll mode ground resonance mapped as a function of rotor speed and body frequency, clearly indicating that aeroelastic couplings are most beneficial for zero ("air resonance") and low body frequencies. Again, from a practical point of view, Fig. 17 shows that the effects of these couplings are relatively incidental to ground resonance when the body frequencies are moderately high.

One more example of the variability in effectiveness of couplings is given in Figs. 19 and 20. Here a significantly different configuration is chosen with low body frequencies and where the pitch mode frequency is higher than the roll mode (ω_θ, ω_ϕ = 0.2, 0.1). This reversal in frequencies substantially alters the effectiveness of the aeroelastic couplings and they are not at all capable of suppressing ground resonance in this case. Although the roll mode is well stabilized for this case, the pitch mode is strongly destabilized by all combinations of couplings.

Other Considerations

The analytical results presented above only address the broadest issues of the present topic. Many other considerations deserve attention but are beyond the scope of this paper. However, a few will be mentioned. Although the basic characteristics are evident in hover, changes occur in forward flight, sometimes becoming less stable in descent conditions as the blade angle of attack is reduced (Ref. 11). Such characteristics require a more complete analytical approach and the need to examine a wider range of operating conditions. The present results did not address the effects of pitch-flap aeroelastic coupling, primarily because it usually has a smaller influence on air and ground resonance than the other couplings. Nevertheless in some case, the effects may be important and must be included. As noted above landing gear characteristics can vary widely and a more detailed treatment is required. Specifically accounting for the variety of possible ground contact and stiffness conditions must be addressed to insure stability in all cases. A good discussion of these problems, including combined body translation and rotation, is included in Ref. 30.

Practical Design Considerations

Design Strategy

Before analysis of aeroelastic couplings for stabilizing air and ground resonance of a candidate design is undertaken, the basic vehicle dynamic characteristics must be defined. Although other design constraints will necessarily limit the freedom available to tailor dynamic properties, the overall mass and stiffness characteristics of the vehicle will determine the primary rotor and body pitch and roll frequencies and the coalescent rotor speeds. Naturally, to the extent possible, it makes sense to optimize these characteristics to minimize or remove as much of the potential air and ground resonance problem as possible. In this way, the prospects for meeting stability requirements with aeroelastic couplings will be maximized. In some cases other design requirements will take precedence and the aeroelastic problem may be made more difficult.

Implementing Aeroelastic Couplings

So far, the technical results discussed above have represented aeroelastic characteristics of the blade in a relatively abstract fashion serving to illustrate how the principal aeroelastic couplings influence air and ground resonance stability. However, the practical design challenges of obtaining these coupling characteristics in actual blade configurations have not been addressed. While a detailed treatment of this problem is beyond the scope of the present paper, a brief discussion will serve to introduce the principal approaches that are available.

Fundamentally there are two basic approaches to generate effective pitch-lag and flap-lag couplings: 1) favorable arrangement of the geometry or kinematics of the blade and control system components and 2) tailoring the inherent bending-torsion structural coupling of torsionally flexible cantilever blades. The second approach encompasses the broad subclass of coupling characteristics available for blades fabricated from non-metallic composite materials.

A simple way of illustrating the physical origin of a variety of kinematic, geometric, and elastic design parameters that contribute to effective aeroelastic couplings is to introduce a simplified blade and pitch control system having the major features of cantilever hingeless rotor blades, Fig. 21. Simple expressions can then be derived to represent the equivalent couplings manifest in more complete blade structural models. Basic pitch-lag and pitch-flap coupling of torsionally flexible cantilever blades can be directly derived from the nonlinear elastic beam equations. For the simplest basic blade, the important parameters are the flap and lead-lag bending stiffnesses, K_β, K_ζ, torsion rigidity, $K\phi$, and the equilibrium flap and lead-lag bending deflections, β_o, ζ_o, accompanying rotor thrust. Following Refs. 29 and 41 for example, effective pitch-lag and pitch-flap couplings are,

$$\theta_\zeta \sim \beta_o (K_\zeta - K_\beta)/K_\phi, \quad \text{and} \quad \theta_\beta \sim \zeta_o (K_\zeta - K_\beta)/K_\phi$$

These couplings are proportional to the difference in blade bending stiffnesses (vanishing for "matched stiffness" blades) and inversely proportional to blade torsional rigidity. These couplings also vary with rotor thrust; as the blade equilibrium bending occurs for increasing collective pitch, the couplings increase from low to high levels. Such models can be extended (Ref. 29) to represent the effects of blade precone, β_{pc}, droop, β_d, and the ratio, f, of pitch control system stiffness, K_Φ, and blade torsion rigidity K_ϕ. The effective pitch-lag coupling becomes,

$$\theta_\zeta \sim \{\gamma (K_\zeta - K_\beta) (\theta_o - \phi_i) / 8p^2 + (K_\zeta - K_\beta) \beta_{pc}/p^2 - \beta_d [(K_\zeta - K_\beta)/p^2 - K_\zeta /(1 + f)\}/K_\Theta$$

where $f = K_\phi / K_\Phi$, $K_\Theta = K_\phi K_\Phi / (K_\phi + K_\Phi)$, and ϕ_i is the inflow angle.

These simple formulas illustrate the interplay of geometric, elastic, and rotor thrust condition on effective aeroelastic couplings, and the resultant opportunities available to the designer if such factors are fully taken into consideration in design of the rotor blades.

Other more direct kinematic approaches for generating pitch-lag coupling include inclining the blade pitch link connecting the pitch horn to the swashplate away from a vertical orientation. For bearingless rotors, vertically offsetting the inboard torque tube snubber shear pin will produce pitch-lag coupling independent of blade flap deflection.

Flap-lag structural coupling is generated when the flap and lead-lag principal elastic axes are inclined to the rotor plane of rotation. Blade twist generates a small amount of this coupling and pre-pitch of a bearingless rotor flexbeam also produces flap-lag structural coupling. This coupling is discussed in more detail in Refs. 14, 23, 27, and 42.

One point of this discussion is to note that effective aeroelastic couplings can be tailored in a variety of ways. Since air and ground resonance stability varies widely depending on operating conditions, the ability to tailor effective couplings for these different conditions must be exploited. For example, if couplings are destabilizing for ground resonance stability at zero collective pitch, but are stabilizing at higher pitch, it would be appropriate to tailor the couplings to be proportionate to collective pitch. In the case of the bearingless rotor, it may be noted that generating effective aeroelastic couplings may be more difficult than the hingeless rotor due to the constraining influence of the torque tube and inboard snubber, particularly when the flap bending stiffness is low. These issues are considered in Refs. 37, 39, and 40.

Composite Material Considerations

It is well known that non-metallic composite materials offer excellent benefits for rotor blade design, particularly for improving fatigue characteristics of rotor blades subject to continuous rotor vibratory loads. Of particular importance to the damperless rotor is the ability to tailor the stiffness and coupling characteristics of such blades via the detailed structural ply lay-up geometry. Much research has been carried out on development of sophisticated analytical tools to model these complex materials and other work has been done to apply these models to study the benefits of these couplings on aeromechanical stability of hingeless rotor helicopters (Ref. 36).

Landing Gear Considerations

The important influence of landing gear characteristics must also be considered. For ground resonance stability, the body frequencies are the most important first consideration. Although stiffening the landing gear raises the body frequencies and increases the intensity of ground resonance, a point will be reached where the coalescent frequencies fall outside the normal rotor speed operating range. Similarly, softening the landing gear will lower the body frequencies and attenuate or stabilize ground resonance. A combination of aeroelastic couplings and favorable frequency placement will be required to achieve a balanced design for a damperless rotor.

Also important, though not discussed in detail in this paper, are the relative vertical and horizontal stiffnesses, and the height and geometry of the landing gear that govern the mode shape of the body motions as well as the frequencies. Furthermore the choice of wheeled or skid gear will have different consequences. Whatever the design point chosen for the landing gear, all possible off-design conditions and ground contact conditions must be free from instability. Some of these include partial skid gear contact on uneven terrain, ice conditions, low pressure or flat tires for wheeled landing gear, and partial lift-off thrust conditions to mention a few.

Finally, ground resonance dynamics must be balanced against other important vehicle design considerations relating to mission performance, cost, and safety that impact landing gear design. These include strength, crash-worthiness, static stiffness, weight, and aerodynamic drag. All of these are all factors must be taken into account in the total vehicle design. An aeroelastic solution for damperless rotor that adversely affects other important design characteristics will not be acceptable.

Methodology for Design Optimization

The results presented herein illustrate the variations in air and ground resonance characteristics encountered for a range of configuration and operating conditions variables. Some of the key design variables influencing system stability have been identified. It is clear that these variables may in some instances alternately benefit or detract from system stability, depending on the particulars of the vehicles and operating conditions. Clearly finding a practical design solution based on aeroelastic coupling requires balancing these conflicting influences to produce a system stable for a range of operating conditions. This constitutes an optimization problem for the designer - for a given vehicle what are the rotor aeroelastic characteristics that ensure aeromechanical stability over a specified range of operating conditions?

Certainly, this would be a tractable analytical problem for the type of simplified aeromechanical stability analysis employed herein. It is suggested that using such a simplified model would be a logical first step in identifying preliminary optimized configurations. More elaborate analyses, incorporating the details of blade flexibility, flexbeam, torque tube, and pitch control system kinematics as well as composite material characteristics would then be appropriate to confirm the preliminary results and identify further detailed design solutions. Alternative strategies regarding the particulars of such methodologies should be carefully weighed to increase the likelihood of a successful outcome.

Concluding Remarks

1. Experience shown the damperless rotor is truly a challenge.

2. Aeroelastic couplings are generally effective in stabilizing hingeless and bearingless rotor air resonance. The principal couplings of interest are negative pitch-lag coupling and flap-lag structural coupling. A combination of these is most beneficial for increasing air resonance stability.

3. Ground resonance instabilities are usually more intense than air resonance and are also dependent on all of the factors that influence landing gear characteristics. Aeroelastic couplings are not as effective at suppressing these instabilities.

4. There are a variety of approaches for generating effective aeroelastic couplings including rotor hub and blade attachment geometry, pitch control system kinematics, and tailoring of blade bending and torsion coupling characteristics, particularly by using the capabilities of composite materials.

5. Modest refinement of current materials to increase inherent structural damping would significantly help overcome the limitations of aeroelastic couplings. Such a hybrid approach may provide the most practical approach of all.

6. By careful synthesis, tailoring aeroelastic couplings, optimizing landing gear design for nominal and off-design conditions, and maximizing inherent blade structural damping, damperless rotors may well prove feasible for a wide range of helicopter applications.

7. Further research, particularly applying formal optimization techniques to suitable analytical models, should help to meet the challenge of the damperless rotor.

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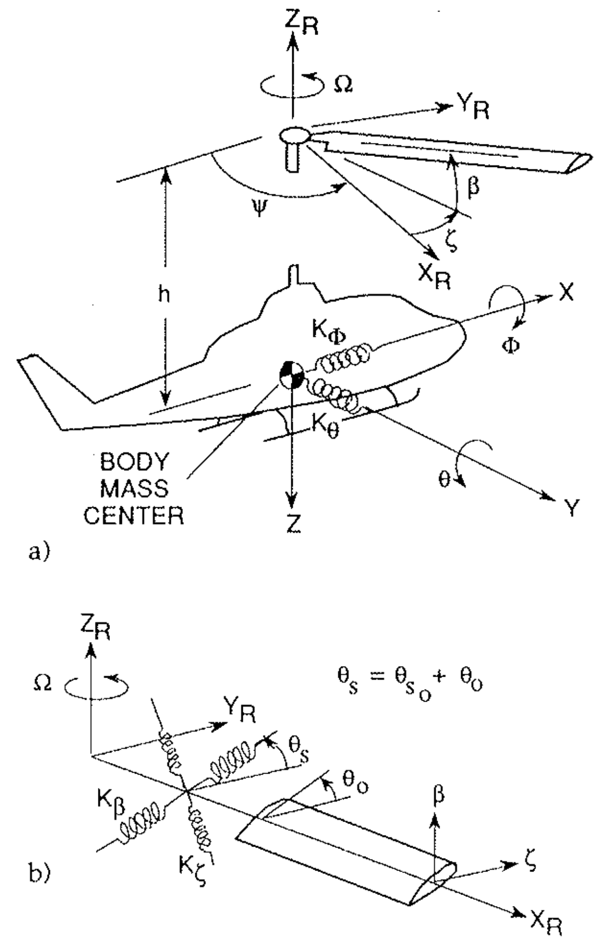

Fig. 1. Analytical model physical system, a) rotor and body with body springs for ground resonance, b) blade collective pitch, flap and lead-lag motions, and spring restrained hinges.

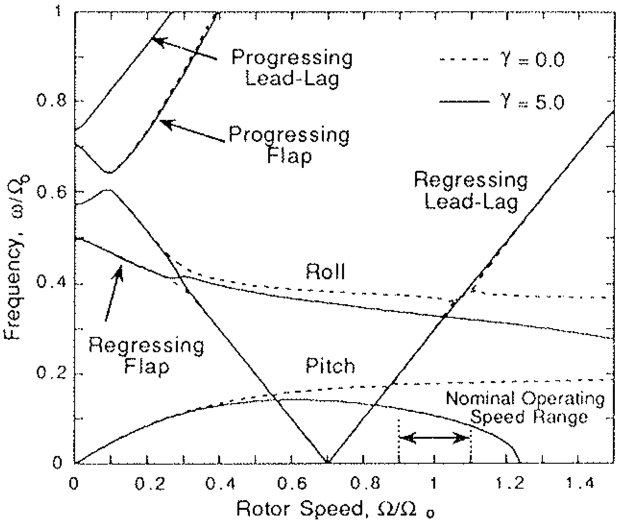

Fig. 2. Air resonance frequencies, in vacuo and in air, $\theta_0 = 0$.

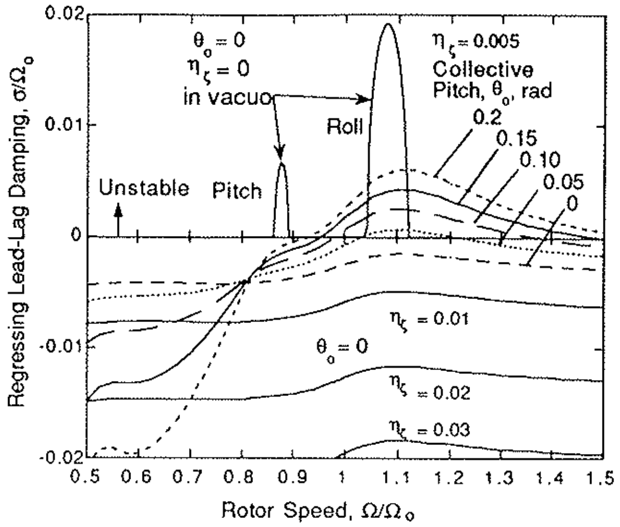

Fig. 3. Effect of collective pitch and lead-lag structural damping on air resonance stability.

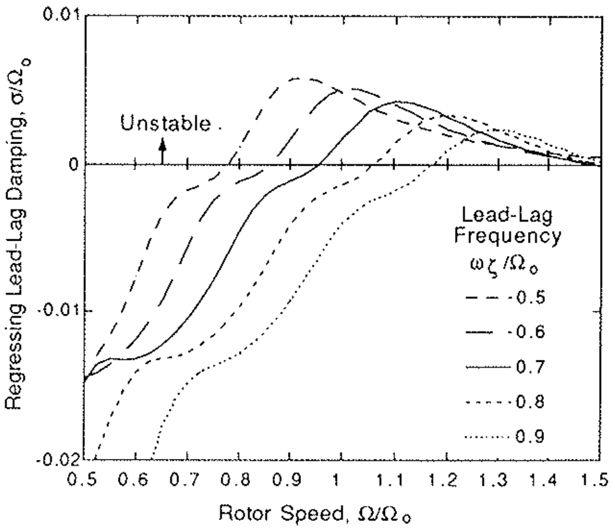

Fig. 4. Effect of lead-lag frequency on air resonance stability, $\theta_0 = 0.15$.

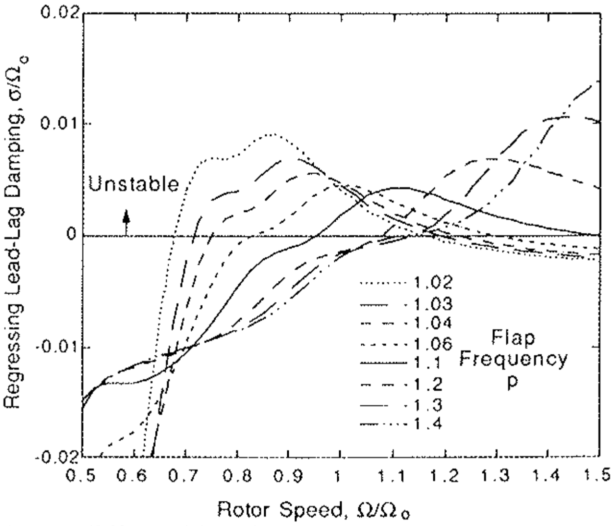

Fig. 5 Effect of flap frequency on air resonance stability, $\theta_0 = 0.15$.

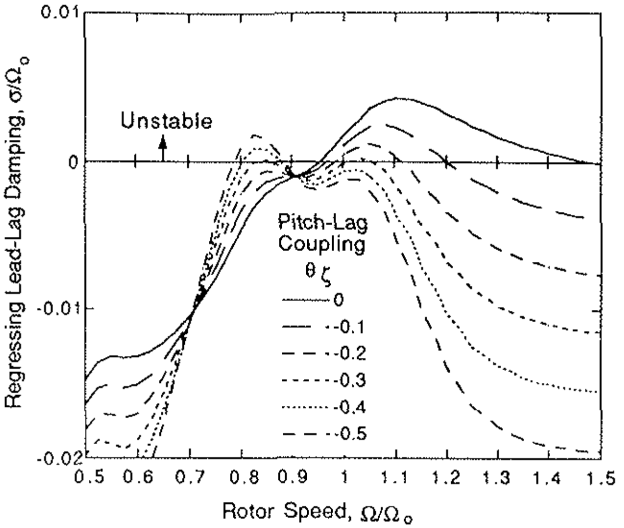

Fig. 6. Effect of pitch-lag coupling on air resonance stability, $\theta_s = 0$ (R = 0), $\theta_o = 0.15$.

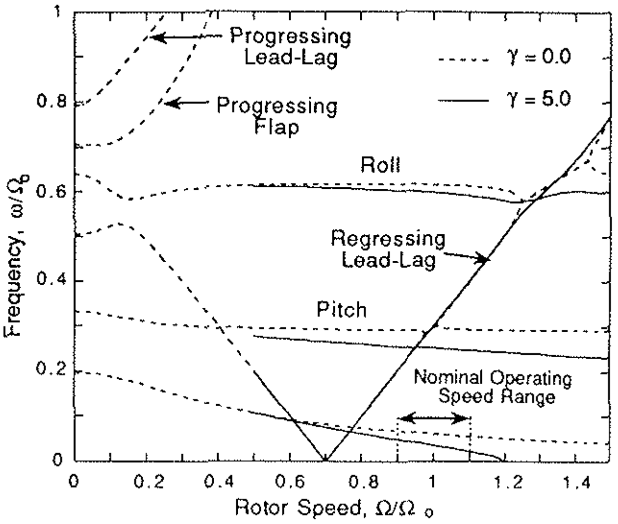

Fig. 9. Ground resonance frequencies, $\theta_o = 0$, $\omega_\theta = 0.2$, $\omega_\phi = 0.4$.

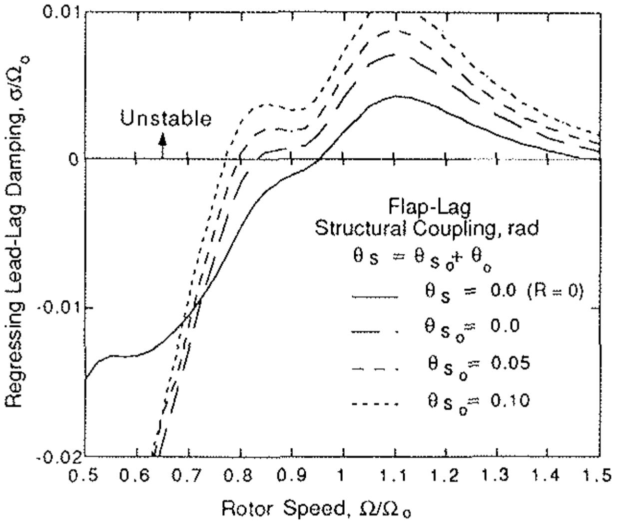

Fig. 7. Effect of flap-lag structural coupling on air resonance stability, R = 1, $\theta_\zeta = 0$, $\theta_o = 0.15$.

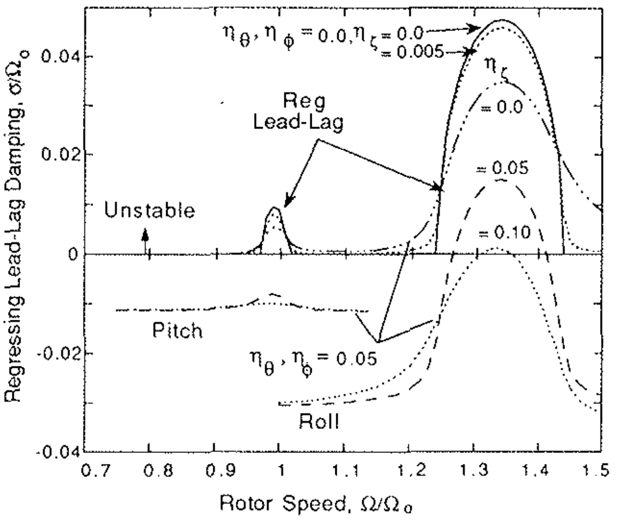

Fig. 10. Ground resonance damping in vacuo with lead lag and body damping , $\theta_o = 0$, $\omega_\theta = 0.2$, $\omega_\phi = 0.4$.

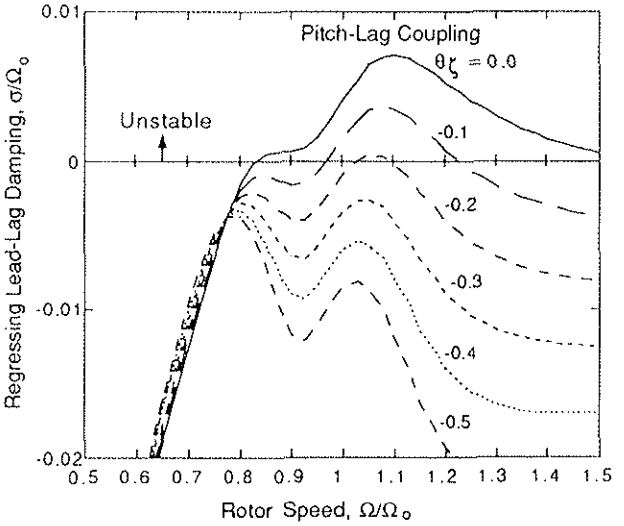

Fig. 8. Effect of combined pitch-lag and flap-lag coupling on air resonance, $\theta_s = \theta_o$ (R = 1), $\theta_o = 0.15$.

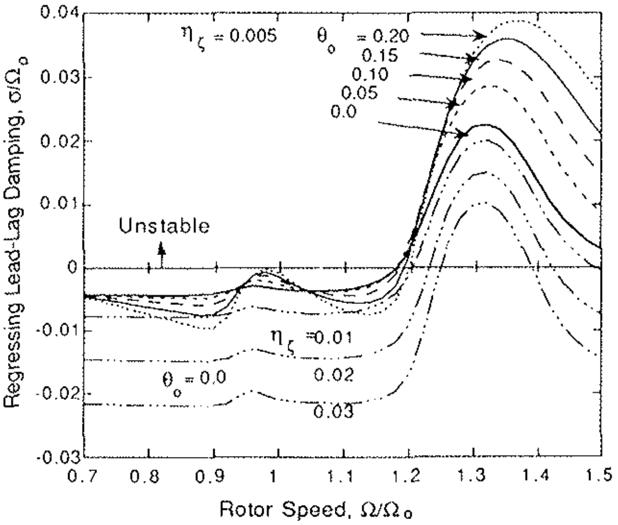

Fig. 11. Effect of collective pitch and lag damping on ground resonance, $\omega_\theta = 0.2$, $\omega_\phi = 0.4$.

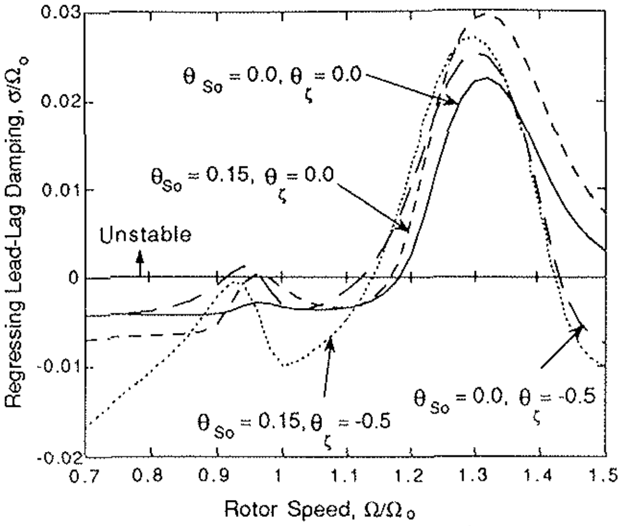

Fig. 12. Ground resonance with aeroelastic couplings, $\omega_\theta = 0.2$, $\omega_\phi = 0.4$, $\theta_o = 0$.

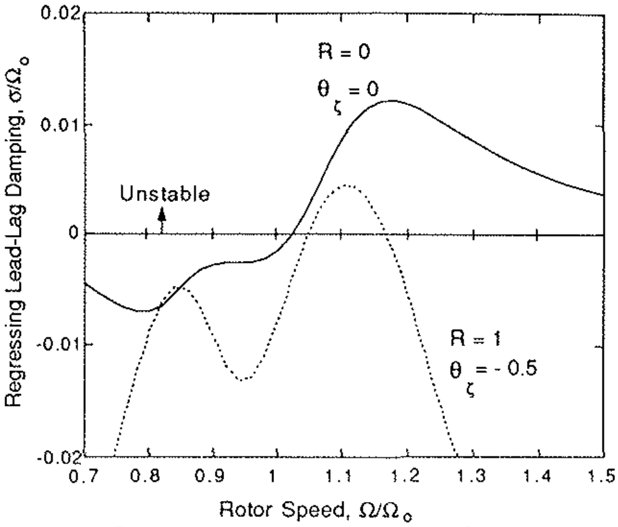

Fig. 15. Ground resonance with aeroelastic couplings, $\omega_\theta = 0.1$, $\omega_\phi = 0.2$, $\theta_o = 0.15$.

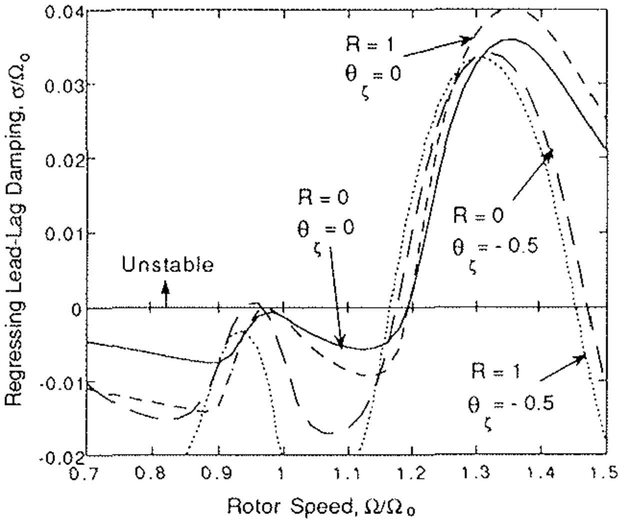

Fig. 13. Ground resonance with aeroelastic couplings, $\omega_\theta = 0.2$, $\omega_\phi = 0.4$, $\theta_o = 0.15$.

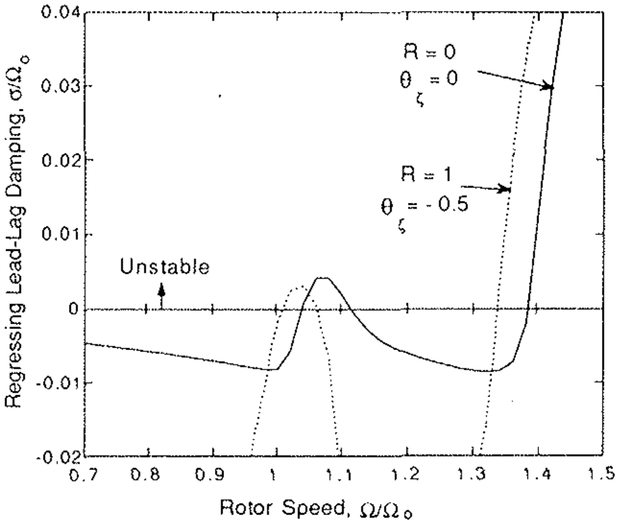

Fig. 16. Ground resonance with aeroelastic couplings, $\omega_\theta = 0.3$, $\omega_\phi = 0.6$, $\theta_o = 0.15$.

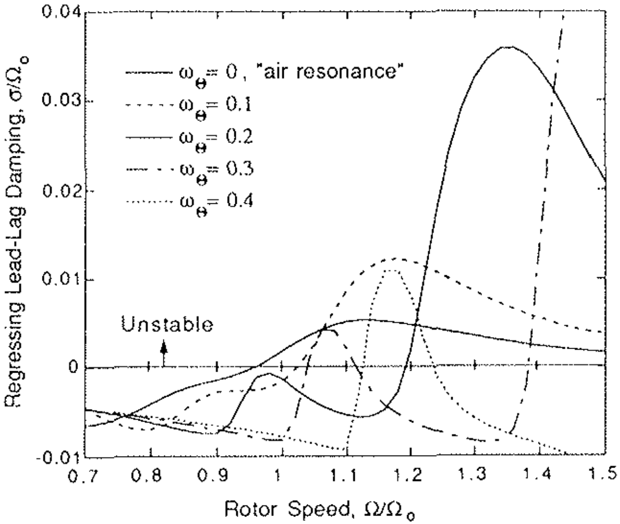

Fig. 14. Effect of landing gear frequency on ground resonance stability, $\theta_o = 0.15$, $\omega_\phi = 2\omega_\theta$.

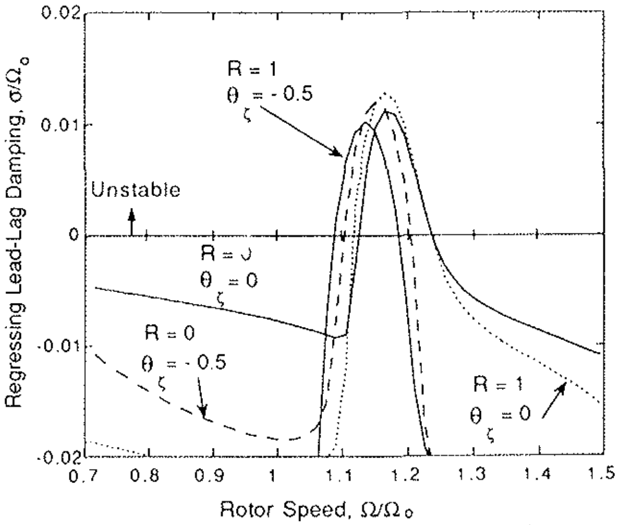

Fig. 17. Ground resonance with aeroel couplings, $\omega_\theta = 0.4$, $\omega_\phi = 0.8$, $\theta_o = 0.15$.

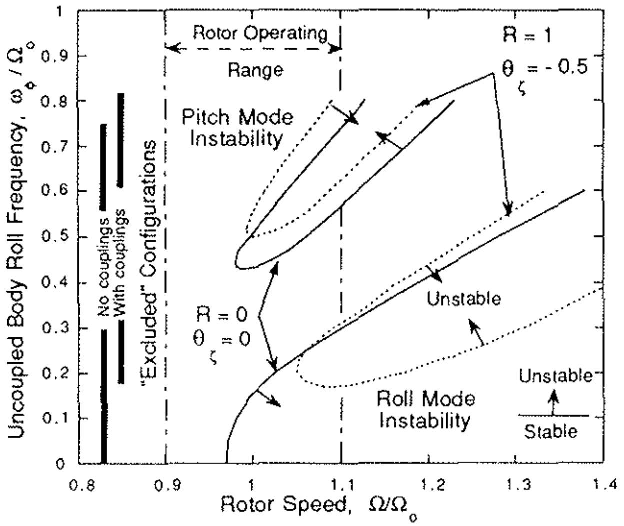

Fig. 18. Ground resonance stability boundaries with and without aeroelastic couplings, tho = 0.15.

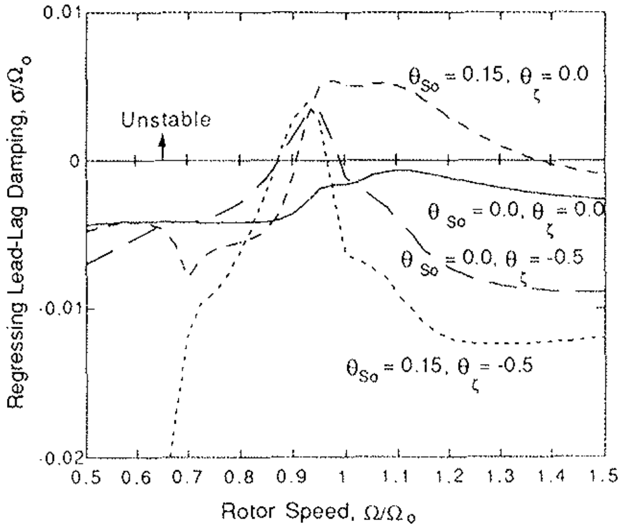

Fig 19. Ground resonance, alternate configuration, $\omega_\theta = 0.2$, $\omega_\phi = 0.1$, $\theta_o = 0$.

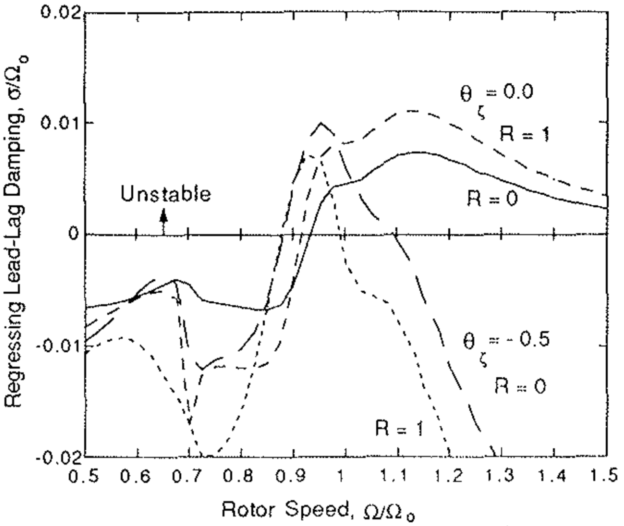

Fig 20. Ground resonance, alternate configuration, $\omega_\theta = 0.2$, $\omega_\phi = 0.1$, $\theta_o = 0.15$.

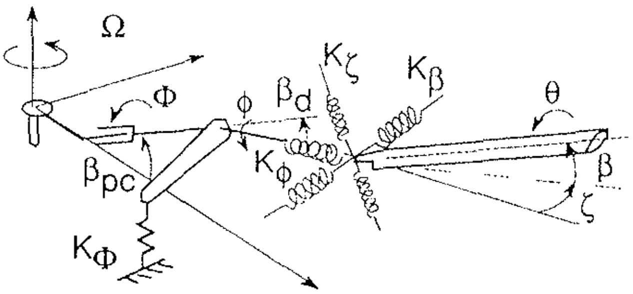

Fig 21. Simplified blade representation showing configuration geomety parameters.