ADVANCED COUPLED AEROELASTIC ANALYSIS OF HELICOPTER ROTOR SYSTEM

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Abstract:

Comparing to other flight vehicles, aerodynamics of helicopter blades is time-periodic. For blade flutter analysis this feature requires development of special tools, taking the flow time-dependency into account.

In this study, we develop a framework for the flutter analysis of the full blade model based on Floquet theory. A finite-element beam model governs the blade, whereas the flow is governed by quasi-static aerodynamic functions. An in-house software is developed to integrate the aerodynamics into Nastran finite-element model for solving both static and dynamic aeroelastic problems.

1. Introduction

Flutter of helicopter rotor blades can yield significant blade damage and crash of a helicopter, and is not acceptable at all regimes. To predict flutter conditions, at most flight vehicles the steady flow and equilibrium position of the structure are considered as the base state subject to stability analysis. On the contrary, the base state of helicopter rotor blades is time-dependent, because the blades oscillate during the rotor rotation (blade flapping motion). In Russian helicopter industry, for the flutter prediction two approaches are used. The first is the Floquet applied to a simple two-degree system for the blade section, which can predict flutter in the first bending and first torsional modes [1, 2, 3, 4]. The second approach applied to higher-modes is the averaging of the flow field over the rotation cycle, and flutter analysis of the blade in the time-independent flow [1, 2, 98]. Clearly, both approaches have essential disadvantages: the first can be applied only to the simplest flutter modes, whereas the second does not take the flow periodicity into account.

The history of rotor blades aeroelasticity is shorter than this time for fixed wing. Wright brothers built the first their flying aircraft in 1903, whereas the first flight of Sikorsky R-4 was in 1942. In other words, initial gap of analysis of the fixed and rotary wing is 39 years. Moreover, the aerodynamics and dynamic behavior of the rotors are more complicated. Therefore, nowadays, rotor blades aeroelastic analysis is the wide field of study that needs more and more precise methods and approaches.

One of the first papers was provided by Loewy [13], where the wide spectrum of aeroelastic problems had been consolidated in one article. Books [1, 2, 3] provide information about helicopter design, stress analysis, aerodynamics, structural dynamics and aeroelasticity. The detail review of rotor aeroelasticity was presented by Friedman

[9]. Another detailed analysis is given by Ormiston [14]. The approach of unsteady aerodynamics consideration is shown in [7] by Peters. Some approaches and assumptions based on aerodynamics from forward flight were incorporated into the aeroelastic analysis in [10]. Flutter analysis for helicopter with anisotropic supports is considered by Pavlenko and Zvolanek in [4, 5].The role of unsteady aerodynamics, including dynamic stall, was analyzed, together with the treatment of nonlinear aeroelastic problems in forward flight. Finite element solutions were also considered, together with the treatment of coupled rotor-fuselage problems in [15].

The time-domain fully coupled unsteady aerodynamics and structural elasticity has long history, however, this way is time expensive and difficult for check because of analysis time. One of the examples of this analysis type is shown in [6].

This article describes an advanced coupled aeroelastic analysis of helicopter rotor system that takes the blade flapping motions into account.

In this study, we develop a framework for the flutter analysis of the full blade model based on Floquet theory. A finite-element beam model governs the blade, whereas the flow is governed by quasi-static aerodynamic functions. In-house software is developed to integrate the aerodynamics into Nastran finite-element model for solving both static and dynamic aeroelastic problems.

The paper is organized as follows. The section 2 is the description of structural and aerodynamic models used to design an aeroelastic blade model. Section 3 describes three solution steps used to predict flutter: the first step is the calculation of nonlinear static aeroelastic equilibrium state of the blade under the action of period-averaged flow field; the next step, is the analysis of flapping oscillations by assuming small blade motion; finally, these flapping oscillations are analyzed with respect to aeroelastic stability based on Floquet theory. At each step, coupling of structural and aerodynamic models and the way they are implemented into MSC.Nastran are described. In section 4, we show an example of flutter analysis of a helicopter in design. Finally, section 5 concludes the paper.

2. Computational model

1.1 Structural model

The finite element MSC.Nastran model of the blade is created along the construction axis of the blade (fig. 1). Beam elements (CBEAM) are used for modeling. For each cross section, the following parameters are set: area, moments of inertia, displacement of the shear center, displacement of the neutral axis and non-structural mass. Concentrated masses that model such structural elements as fasteners, are modeled by COMN2 elements.

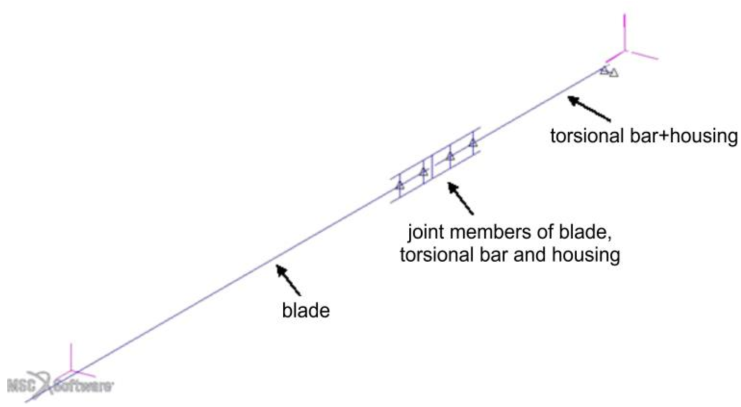

Fig. 1 Coordinate system

2.1 Aerodynamic model

Calculation of blade flutter prediction consists of three steps: 1) Coupled aeroelastic calculation of the stationary (static) state of a rotating blade, taking into account the translational flight of the helicopter; 2) Computation of flapping oscillations of the blade; 3) The calculation of the blade flutter, taking into account the flapping oscillations.

For the first solution step, aerodynamic loads averaged over the period of rotation are computed. To calculate flapping oscillation, variable aerodynamic load, computed without taking oscillations into account, and the change in aerodynamic loads, caused by the motion of the blade section during the oscillation process, are calculated in the linear approximation. Finally, for the last step aerodynamic loads are linearized with respect to flapping oscillation.

Calculation of aerodynamic loads on the blades

The computation of the aerodynamic forces acting on the blades is performed in the following order:

- calculation of the induced speed in the plane of the rotor disk;
- computation of the total velocity vector of the flow around the blade profile section, as well as the angle of attack and the Mach number in the blade section;
- calculation of aerodynamic coefficients in the blade section;
- determination of aerodynamic loads acting on the blade.

Basic assumptions:

- According to the strip hypothesis, the aerodynamic lift and drag coefficients in the section are determined by the experimental aerodynamic characteristics of the blade profile obtained in a plane-parallel flow.
- The influence of the unsteady flow features on the aerodynamic characteristics of the blade cross section is not taken into account.
- Uniform distribution of induced velocities in the plane of the rotor disc in oblique flow regimes around the rotor is accepted.
- The rotations of the flat blade cross-sections caused by deformations of the blade axis are neglected.

Rectangular coordinate systems used

The calculation of the aerodynamic load is conducted in a coordinate system rotating with rotor. This coordinate system is defined as follows: the Y axis is directed along the rotor shaft upwards, and the Z is axis along the axis of the blade. The X axis is perpendicular to the Y and Z axes and complements them to the right-hand triple (fig. 2). This coordinate system will be called the global coordinate system.

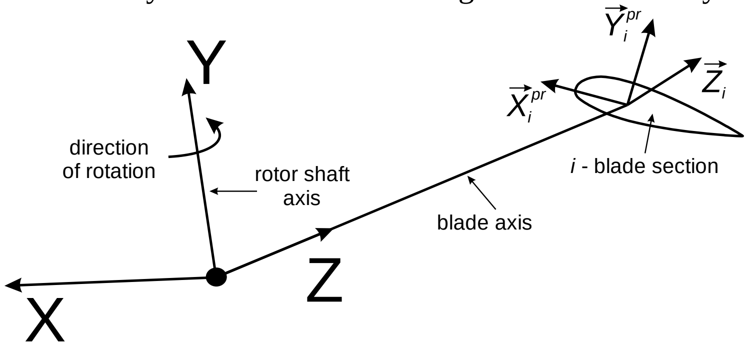

Fig. 2. Coordinate system

The coordinate system associated with the blade section is also used (Fig. 3): $\vec{X}_i^{pr}$ is a unit vector directed along the chord of the i-th section of the blade in the direction of the blade rotation; $\vec{Y}_i^{pr}$ is a unit vector directed upward perpendicular to the chord of the i-th section of the blade; $\vec{Z}_i^{pr}$ is unit vector normal to the i-th blade section (this vector supplements $\vec{X}_i^{pr}$ and $\vec{Y}_i^{pr}$ to the right-hand system).

When calculating aerodynamic loads, a third coordinate system is used in which: $\vec{X}_i^{air}$ is unit vector directed downstream, $\vec{Y}_i^{air}$ is the unit vector directed perpendicular to the flow, $\vec{Z}_i^{air}$ is vector complementary $\vec{X}_i^{air}$ and $\vec{Y}_i^{air}$ to the right-hand system.

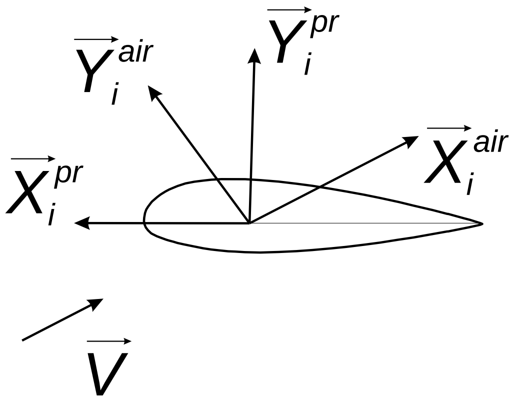

Fig. 3. Coordinate system associated with the flow

Calculation of the induced velocity

Induced velocity ϑ_1, which, according to accepted assumptions, is uniformly distributed over the rotor disk and directed along the axis of the rotor shaft. Induced velocity is opposite to the thrust force. The inducted velocity in the plane of the rotor disk is determined from the expression:

$$(1) \qquad \vartheta_1 = \bar{\vartheta}_1 \omega R$$

where $\omega = 2\pi \cdot \nu$ is the angular velocity of the rotor; ν is the frequency of rotation of the rotor; R is the radius of the rotor; $\bar{\vartheta}_1$ is relative induced velocity, determined from the system of equations (2):

$$(2) \qquad \begin{cases} \lambda = \mu \cdot tg\alpha_{rotor} - \bar{\vartheta}_1 \\ \mathrm{C}_T = a_\infty \cdot \sigma_r \left(\frac{\varphi_{0,7-\alpha}}{3} \cdot \left(1 + \frac{3}{2}\mu^2\right) + \frac{\lambda}{2} \right) \\ \bar{\vartheta}_1 = \frac{0{,}25 \cdot \mathrm{C}_T}{\chi^2 \sqrt{\mu^2 + \lambda^2}} \end{cases}$$

where λ is inflow ratio; $\mu = \frac{V^{rotor} \cdot \cos(\alpha_{rotor})}{\omega R}$ is the coefficient of the rotor operating regime; α_{rotor} is the attack angle of the rotor, i.e., the angle between the design plane of the rotor, perpendicular to the rotor shaft, and the incident flow velocity vector; C_T is rotor thrust coefficient; a_∞ is the derivative of the lift coefficient of the blade profile with respect to the angle of attack; $\sigma_r = \frac{k_b \cdot b}{\pi R}$ is the rotor filling ratio; $\varphi_{0,7-\alpha} = \varphi_{cp} - \alpha_0 + \varphi_d$ is the pitch angle of the undeformed blade at a relative radius $\bar{r} = 0{,}7$ taking into account the angle of zero lifting force of the profile; χ is tip loss factor of the rotor. V^{rotor} is the helicopter velocity relative to the flow, k_b is number of rotor blades; b is length of blade section chord; φ_{cp} is collective pitch set by the rotor control system; α_0

is the angle of zero lifting force of the profile; φ_d is the angle of the geometric blade twist at a relative radius $\bar{r} = 0{,}7$.

One of all the solutions of (2) is accepted as a valid root $\bar{\vartheta}_1$, positive or negative, depending on whether the rotor creates a positive or negative lift:

$$\begin{cases} \bar{\vartheta}_1 > 0\,, & \varphi_{0,7-\alpha} > -\dfrac{3}{2}\dfrac{\mu \cdot tg\alpha_{rotor}}{1+\dfrac{3}{2}\mu^2} \\ \bar{\vartheta}_1 < 0\,, & \varphi_{0,7-\alpha} < -\dfrac{3}{2}\dfrac{\mu \cdot tg\alpha_{rotor}}{1+\dfrac{3}{2}\mu^2} \end{cases}$$

Determination of the velocity vector of flow around blade section

The components of the total velocity vector (in the global coordinate system) of the flow around i-th section of the blade are calculated in the following way:

$$V_{xi}^{tot}(t) = V^{rotor}\cos(\alpha_{rotor})\sin(\omega t) + \omega R_i$$
$$V_{yi}^{tot} = V^{rotor}\sin(\alpha_{rotor}) - \vartheta_1$$

where R_i is the distance from the rotor shaft to the i-th section. The magnitude of the total velocity vector is determined from the expression:

$$\left|\vec{V}_i^{tot}(t)\right| = \sqrt{\left(V_{xi}^{tot}(t)\right)^2 + \left(V_{yi}^{tot}\right)^2}$$

Determination of the angle of attack of the blade section and the Mach number

Mach number M_i of the incident flow in the i-th section of the blade is calculated according to the relation:

$$M_i(t) = \frac{\left|\vec{V}_i^{tot}(t)\right|}{a_s}$$

where a_s is local speed of sound. The attack angle of the blade profile is calculated based on the relationship:

$$\alpha_i(t) = \varphi_{cp} + \varphi_{sec} + R_{3i} + \Phi_{cyclic} + \alpha_{flow}(t)$$

where φ_{cp} is collective pitch set by the rotor control system;

$$\Phi_{cyclic} = \begin{cases} \varphi_{cyclic}\, sin(\omega t) \\ -\varphi_{cyclic}\, sin(\omega t) \\ \varphi_{cyclic}\, cos(\omega t) \\ -\varphi_{cyclic}\, cos(\omega t) \end{cases}$$

where each of the four options corresponds to the phase shift of the cyclic pitch by 0, π/2, π , 3π/2 of azimuth, which corresponds to the creation of a rotor thrust forward or backward, or roll to the right or left (only these four options were considered in analysis); φ_{cyclic} is cyclic pitch amplitude; $\alpha_{flow}(t) = arctg\left(\frac{V_{y_i}^{tot}(t)}{V_{x_i}^{tot}(t)}\right)$ is the angle between the total velocity vector of the flow around of the blade section and the plane of the rotor disk; φ_{sec} is geometric twist of the cross section; R_{3i} is angular displacement (twist) around the Z axis for the i-th section. In the first step of calculation $R_{3i} = 0$.

Determination of aerodynamic coefficients

The lift force coefficient Cy and the drag coefficient Cx are computed on the basis of experimental data, containing the dependencies of Cy and Cx on the angle of attack and the Mach number.

Calculation of the average aerodynamic loads per unit length

The calculation of the load per unit length acting on the blade in the coordinate system associated with the incident flow (coordinate X is directed along the flow) is made on the basis of relations and:

$$\Delta X_{ai}^{air}(t) = C_{xi}(\alpha_i(t), M_i(t)) \frac{\rho \cdot \left|\vec{V}_i^{tot}(t)\right|^2}{2} \cdot b$$

$$\Delta Y_{ai}^{air}(t) = C_{yi}(\alpha_i(t), M_i(t)) \frac{\rho \cdot \left|\vec{V}_i^{tot}(t)\right|^2}{2} \cdot b$$

where ρ is air density. In the global coordinate system, the loads take the form:

$$\Delta \mathrm{X_i(t)} = \mathrm{X_{1i}^{air}} \cdot \Delta \mathrm{X_{ai}^{air}(t)} + \mathrm{Y_{1i}^{air}} \cdot \Delta \mathrm{Y_{ai}^{air}(t)}$$

$$\Delta \mathrm{Y_i(t)} = \mathrm{X_{2i}^{air}} \cdot \Delta \mathrm{X_{ai}^{air}(t)} + \mathrm{Y_{2i}^{air}} \cdot \Delta \mathrm{Y_{ai}^{air}(t)}$$

where $X_{1i}^{\mathrm{air}}, X_{2i}^{\mathrm{air}}, Y_{1i}^{\mathrm{air}}, Y_{2i}^{\mathrm{air}}$ are components of vectors $\vec{X}_i^{\mathrm{air}}$and $\vec{Y}_i^{\mathrm{air}}$, respectively.

In first step of analysis, static aeroelasticity is investigated, for which the loads are averaged over the rotation period of the blade:

$$\overline{\Delta X_i} = \frac{1}{T}\int_0^T \Delta X_i(t)dt$$

$$\overline{\Delta Y_i} = \frac{1}{T}\int_0^T \Delta Y_i(t)dt$$

where $T = \frac{2\pi}{\omega}$ is the period of blade rotation. The integration is performed by the trapezoidal method with 100 steps in the integration period. The components of the aerodynamic forces in a given section of the blade (that is, in a given FEM node) are calculated as follows:

$$F_{aero\ i}^{x} = \overline{\Delta X_\iota} \cdot l_i$$

$$F_{aero\ i}^{y} = \overline{\Delta Y_\iota} \cdot l_i$$

where l_i is the length of the FEM element adjacent to the given node and located closer to the axis of rotor rotation.

The moments of forces acting on the i-th section have the form of a vector product:

$$\vec{M}_i = \left(X_{pc} - X_O\right) \cdot \vec{X}_{pr\ i} \times \vec{F}_{aero\ i}$$

where X_{pc} is the distance along chord from the leading edge of the blade section to the center of pressure, X_O is the distance along the chord from the leading edge of the blade section to the point of intersection of the blade construction axis with the cross section (that is, to the FEM node), $\vec{F}_{\mathrm{aero}\ i} = \{F_{\mathrm{aero}\ i}^{x}, F_{\mathrm{aero}\ i}^{y}, 0\}$ is vector of aerodynamic force.

Linearization of aerodynamic forces

The calculation of linearized aerodynamic forces is carried out in the following order:

- Calculation of the first harmonics of the Fourier series expansion of aerodynamic forces.
- Calculation of disturbances of aerodynamic forces in the first approximation as functions of velocity (translational) and rotation of the cross section.
- The calculation of the Coriolis force in the first approximation as a function of the blade section velocity (translational and angular).

The first harmonics are calculated as follows:

$$F^{x}_{\cos i} = \frac{1}{T}\int_{0}^{T} \Delta X_i(t)\cos(\omega t)dt$$

$$F^{y}_{\sin i} = \frac{1}{T}\int_{0}^{T} \Delta Y_i(t)\sin(\omega t)dt$$

The vector of the total air velocity in case of flapping oscillations is

$$\vec{V}_{total\ i} = \vec{V}_i^{tot} - \vec{v}_i$$

where $\vec{V}_i^{tot}$ is the average value of the velocity obtained in the static aeroelastic calculation, $\vec{v}_i$ is blade section velocity caused by its oscillation. Therefore, the length of the velocity vector is

$$\left|\vec{V}_{total\ i}\right| = \left|\vec{V}_i^{tot} - \vec{v}_i\right| \approx \left|\vec{V}_i^{tot}\right| - \frac{\vec{V}_i^{tot}\cdot\vec{v}_i}{\left|\vec{V}_i^{tot}\right|}$$

The change of the angle of attack caused by the forward movement of the blade section

$$(3) \qquad \alpha'_{vi} \approx \frac{v_{xi}V_{yi}^{tot} - v_{yi}V_{xi}^{tot}}{(V_i^{tot})^2}$$

Loads per unit length without consideration of flapping oscillations are defined as follows:

$$\Delta X_{ai}^{air}(t) = C_{xi}(\alpha_i(t), M_i(t))\frac{\rho\cdot\left|\vec{V}_i^{tot}(t)\right|^2}{2}\cdot b$$

$$\Delta Y_{ai}^{air}(t) = C_{yi}(\alpha_i(t), M_i(t))\frac{\rho\cdot\left|\vec{V}_i^{tot}(t)\right|^2}{2}\cdot b$$

And the expressions for loads, taking into account the flapping oscillations, are of the form:

$$F_i^x = C_{xi}\left(\alpha^0_{pr\ i} + \alpha'_{pr\ i} + \alpha'_{vi}, \frac{\left|\vec{V}_i^{tot} - \vec{v}_i\right|}{a_s}\right)\frac{\rho\left|\vec{V}_i^{tot} - \vec{v}_i\right|^2}{2}b$$

$$F_i^y = C_{yi}\left(\alpha^0_{pr\ i} + \alpha'_{pr\ i} + \alpha'_{vi}, \frac{\left|\vec{V}_i^{tot} - \vec{v}_i\right|}{a_s}\right)\frac{\rho\left|\vec{V}_i^{tot} - \vec{v}_i\right|^2}{2}b$$

where $\alpha_{pr\,i}^{0}$ is the average value of the angle of attack (obtained in a static calculation), $\alpha'_{pr\,i}$ is the additional angle of attack caused by the rotation of the section during vibrations, α'_{vi} is additional angle of attack caused by the translational motion of the section during vibrations (3).

Thus, the expressions for the disturbances of aerodynamic forces take the form:

$$\mathrm{F}_{\mathrm{i}}^{\prime\,\mathrm{x}} = \mathrm{A}_{\mathrm{xi}}^{\mathrm{x}}\mathrm{v}_{\mathrm{xi}}^{\mathrm{glob}} + \mathrm{A}_{\mathrm{yi}}^{\mathrm{x}}\mathrm{v}_{\mathrm{yi}}^{\mathrm{glob}} + \mathrm{A}_{\mathrm{\alpha i}}^{\mathrm{x}}\alpha_{\mathrm{pr\,i}}^{\prime\,\mathrm{glob}}$$
$$\mathrm{F}_{\mathrm{i}}^{\prime\,\mathrm{y}} = \mathrm{A}_{\mathrm{xi}}^{\mathrm{y}}\mathrm{v}_{\mathrm{xi}}^{\mathrm{glob}} + \mathrm{A}_{\mathrm{yi}}^{\mathrm{y}}\mathrm{v}_{\mathrm{yi}}^{\mathrm{glob}} + \mathrm{A}_{\mathrm{\alpha i}}^{\mathrm{y}}\alpha_{\mathrm{pr\,i}}^{\prime\,\mathrm{glob}}$$

where the index «glob» denotes the global coordinate system.

The perturbation of the moment of aerodynamic forces is represented through the vector product:

$$\vec{M}_i = \left(X_{pc} - X_O\right) \cdot \vec{X}_{pr\,i} \times \vec{F}_{total\,i}$$

where $\vec{F}_{total\,i} = \{\bar{F}_i^{\prime\,x} \cdot l_i, \bar{F}_i^{\prime\,y} \cdot l_i, 0\}$

Calculation of Coriolis Force and Moment

The velocity and rotation of the cross section point in case of flapping oscillations is

$$\vec{v}_0 = \begin{pmatrix} V_{0_x} \\ V_{0_y} \\ V_{0_z} \end{pmatrix} e^{i\omega t}$$

$$\vec{\alpha} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} e^{i\omega t}$$

The velocity of rotation of the cross-section point in case of flapping oscillations is

$$\frac{\overrightarrow{d\alpha}}{dt} = \begin{pmatrix} i\omega A_x \\ i\omega A_y \\ i\omega A_z \end{pmatrix} e^{i\omega t} = \begin{pmatrix} \omega_{\alpha_x} \\ \omega_{\alpha_y} \\ \omega_{\alpha_z} \end{pmatrix} e^{i\omega t}$$

The relative velocity of the blade section is:

$$\vec{v}_r = \left(\begin{pmatrix} V_{0_x} \\ V_{0_y} \\ V_{0_z} \end{pmatrix} + \begin{pmatrix} -y\omega_{\alpha_z} \\ x\omega_{\alpha_z} \\ y\omega_{\alpha_x} - x\omega_{\alpha_y} \end{pmatrix} \right) e^{i\omega t}$$

where $\vec{r}$ is radius vector from the FEM node.

Coriolis force at the point of the blade section is

$$\vec{F}_{cor} = f_x\vec{e}_x + f_z\vec{e}_z$$

where $f_x = -2m\delta\tilde{\omega}\left(V_{0_z} + y\omega_{\alpha_x} - x\omega_{\alpha_y}\right)e^{i\omega t}$; $f_z = 2m\delta\tilde{\omega}\left(V_{0_x} - y\omega_{\alpha_z}\right)\vec{e}_z e^{i\omega t}$; $\tilde{\omega}$ is the angular velocity of the blade around the rotor shaft, $\delta = \delta(x - x^*, y - y^*)$ is the delta function (since the mass of the section is concentrated at the point corresponding to the non-structural mass of the FEM). The total force of Coriolis in cross section as follows:

$$\vec{F}_{cor\ tot} = \int_S \vec{F}_{cor} ds = f_{x\ tot}\vec{e}_x + f_{z\ tot}\vec{e}_z$$

$$f_{x\ tot} = \left(-2m\widetilde{\omega}V_{0_z} - 2m\widetilde{\omega}y_0^*\omega_{\alpha_x} + 2m\widetilde{\omega}x_0^*\omega_{\alpha_y}\right) e^{i\omega t}$$

$$f_{z\ tot} = \left(2m\widetilde{\omega}V_{0_x} - 2m\widetilde{\omega}y_0^*\omega_{\alpha_z}\right)e^{i\omega t}$$

where (x_0^*, y_0^*) is the distance between the node FEM and the point of non - structural mass.

The moment of Coriolis force at the point of blade section is

$$\vec{M}_{cor} = \vec{r} \times \vec{F}_{cor} = yf_z\vec{e}_x + (-xf_z)\vec{e}_y + (-yf_x)\vec{e}_z$$

The total moment of Coriolis force in cross section is

$$\vec{M}_{cor\ tot} = \int_S \vec{M}_{cor}\, ds = m_{x\ tot}\vec{e}_x + m_{y\ tot}\vec{e}_y + m_{z\ tot}\vec{e}_z$$

$$m_{x\ tot} = y_0^*\left(2m\widetilde{\omega}V_{0_x} - 2m\widetilde{\omega}y_0^*\omega_{\alpha_z}\right)e^{i\omega t}$$

$$m_{y\ tot} = -x_0^*\left(2m\widetilde{\omega}V_{0_x} - 2m\widetilde{\omega}y_0^*\omega_{\alpha_z}\right)e^{i\omega t}$$

$$m_{z\ tot} = -y_0^*\left(-2m\widetilde{\omega}V_{0_z} - 2m\widetilde{\omega}y_0^*\omega_{\alpha_x} + 2m\widetilde{\omega}x_0^*\omega_{\alpha_y}\right) e^{i\omega t}$$

Linearization of aerodynamic loads relative to flapping oscillations

Expressions for forces linearized with respect to flapping vibrations have the form:

$$\widetilde{F_i'^{x}} = \widetilde{A_{xi}^{x}} v_{xi}^{glob} + \widetilde{A_{yi}^{x}} v_{yi}^{glob} + \widetilde{A_{\alpha i}^{x}} \alpha'^{\,glob}_{\text{пр}\ i}$$

$$\widetilde{F_i'^{y}} = \widetilde{A_{xi}^{y}} v_{xi}^{glob} + \widetilde{A_{yi}^{y}} v_{yi}^{glob} + \widetilde{A_{\alpha i}^{y}} \alpha'^{\,glob}_{\text{пр}\ i}$$

3. Solution sequence

1.1 Calculation of averaged nonlinear aeroelastic state

In this section the deformed state of the blade under the action of centrifugal force and the averaged (over the rotation period) aerodynamic load is calculated. The calculation of the coupled aeroelastic problem is performed in the solver SOL 400 of MSC.Nastran. Aerodynamic loads are calculated as static concentrated forces and moments at the nodes of the FEM.

Since the aerodynamic load depends on the deformed state of the blade, the problem is solved using the method of successive approximations. The calculation algorithm is as follows: in the first step, the aerodynamic forces acting on the undeformed blade are calculated, and the stress-strain state under the action of these forces is calculated; in the next step, the aerodynamic forces acting on the deformed rotor are recalculated and the stress-strain state is recalculated under the effect of recalculated loads; further iterations are repeated until the convergence is achieved.

The resulting stress-strain state is the solution of the two-way coupled aeroelastic problem, since the aerodynamic loads, under the action of which the stress-strain state was obtained, were calculated for the same state.

Calculations at every iteration, are performed taking finite deformations (geometric nonlinearity) and aerodynamic nonlinearity (expressed in a nonlinear dependence of the coefficients of aerodynamic forces on the angle of attack) into account.

Python program was developed to calculate the aerodynamic forces, integrate aerodynamics into the Nastran FEM model and control the calculation process.

2.1 Calculation of forced rotor blade oscillations at horizontal flight regimes

It is assumed that under the action of a periodic aerodynamic load, the blade experiences small oscillations around the static equilibrium position, and the harmonic of rotor rotation frequency gives the dominant contribution to the oscillation process.

In addition to the variable aerodynamic load, calculated without taking oscillations into account, the change of aerodynamic loads caused by the motion of the blade section during the oscillation process is taken into account in the linear approximation. Also, the structural damping of the blade is considered.

Calculation of flapping oscillations is conducted in the frequency domain. The mass matrix is taken from the calculation of static aeroelasticity. The stiffness matrix consists of a structural stiffness matrix obtained from the calculation of static aeroelasticity and a aerodynamic stiffness matrix composed of the corresponding coefficients in the expressions for the linearized aerodynamic forces. The damping matrix consists of structural damping matrix and aerodynamic damping matrix composed of the corresponding coefficients in the expressions for linearized aerodynamic forces.

To export stiffness, mass and structural damping matrices from static nonlinear elastic calculation and use them in the calculation of the flapping oscillations, commands on the DMAP macro language are added to the corresponding calculation files.

3.1 Calculation of blade flutter

At this stage, the flutter is calculated taking into account the flapping oscillations performed by the blade. Since the unperturbed state studied for stability is periodically dependent on time (flapping oscillations), the stability analysis is carried out using the Floquet theory [8].

For computations, solver SOL 109 of MSC.Nastran is used. The following system of equation is solved:

$$M\ddot{u} + B\dot{u} + Ku = B_a\dot{u} + K_a u + B_k\dot{u} + B_{a\,cos}\dot{u}\,cos(\omega t) + B_{a\,sin}\dot{u}\,sin(\omega t) + K_a u\,cos(\omega t) + K_a u\,sin(\omega t)$$

where M, B, K are structural matrices of mass, damping and stiffness, B_a, K_a are aerodynamic matrices of damping and stiffness, B_k is a damping matrix generated by Coriolis force, $B_{a\,cos}\dot{u}\cos(\omega t) + B_{a\,sin}\dot{u}\sin(\omega t) + K_a u\cos(\omega t) + K_a u\sin(\omega t)$ are aerodynamic matrices that define periodically varying coefficients of the system of equations.

Floquet theory

For a linear system:

$$(4) \qquad \frac{dX}{dt} = A(t)X$$

with a piecewise-continuous periodic matrix $A(t + \omega) \equiv A(t)$, the fundamental matrix of solutions normalized at t = 0 has the form:

$$\boldsymbol{X}(t) = \Phi(t)e^{\Lambda t}$$

where Φ(t) is piecewise smooth periodic matrix, and $\Phi(0) = \Phi(\omega) = I$ and Λ is a constant matrix; $\mathbf{X}(t + \omega) = \mathbf{X}(t)\mathbf{X}(\omega)$.

The matrix $\mathbf{X}(\omega) = e^{\Lambda\omega}$ is the monodromy matrix of a system with periodic coefficients, and its eigenvalues are the multipliers of this system. A linear homogeneous periodic system with a continuous matrix is stable if the multipliers lie inside of the unit circle $|\rho| \leq 1$.

In the case of small blade motions around the equilibrium position at flapping vibrations, the linearized system of equations of motion takes form:

$$(5) \quad [\boldsymbol{M}]\ddot{\boldsymbol{u}} + [\boldsymbol{B}(\boldsymbol{t})]\dot{\boldsymbol{u}} + [\boldsymbol{K}(\boldsymbol{t})]\boldsymbol{u} = \boldsymbol{0}$$

where the stiffness and damping matrices have periodic coefficients with a period equal to the rotation period.

The change of variables v = u̇ system (5) is reduced to the form (4):

$$(6) \quad \begin{Bmatrix} \dot{\boldsymbol{v}} \\ \dot{\boldsymbol{u}} \end{Bmatrix} = \begin{bmatrix} -[\boldsymbol{M}]^{-1}[\boldsymbol{B}(\boldsymbol{t})] & -[\boldsymbol{M}]^{-1}[\boldsymbol{K}(\boldsymbol{t})] \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{v} \\ \boldsymbol{u} \end{Bmatrix}$$

To construct the monodromy matrix, it is necessary to solve 2n systems of equations on the same period with the initial conditions

$$\begin{Bmatrix} \boldsymbol{v}(\boldsymbol{0}) \\ \boldsymbol{u}(\boldsymbol{0}) \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$$

where n is the dimension of the system and I is the unit matrix of dimension n.

Unfortunately, this theory can be implemented in practice only for sufficiently low-dimesional systems. Otherwise, it is necessary to integrate over the period too many times and with very small increments. To overcome this problem, we can switch to the modal coordinates of the system [M]ü + [K]u = 0 and restrict ourselves by the first k modes:

$$\begin{Bmatrix} \boldsymbol{v} \\ \boldsymbol{u} \end{Bmatrix} \approx \begin{bmatrix} \boldsymbol{\varphi_k} \\ \boldsymbol{\varphi_k} \end{bmatrix} \begin{Bmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{Bmatrix}$$

The number of solutions per period will be reduced to 2k, and the minimum integration step will be determined by the maximum (k-th) eigenfrequency of the system (convergence study is required for each specific case, but as a rule, 100 steps for the minimum period is sufficient). When normalizing by mass $[\varphi]^T[M][\varphi] = I$, system (6) is converted to the form:

$$(7) \quad \begin{Bmatrix} \dot{\boldsymbol{p}} \\ \dot{\boldsymbol{q}} \end{Bmatrix} = \begin{bmatrix} -[\boldsymbol{\varphi_k}]^{\boldsymbol{T}}[\boldsymbol{B}(\boldsymbol{t})][\boldsymbol{\varphi_k}] & -[\boldsymbol{\varphi_k}]^{\boldsymbol{T}}[\boldsymbol{K}(\boldsymbol{t})][\boldsymbol{\varphi_k}] \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{Bmatrix}, \begin{Bmatrix} p(0) \\ q(0) \end{Bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

And the monodromy matrix of dimension 2k x 2k will be composed of solution vectors (7)

$$\boldsymbol{X}(\boldsymbol{\omega}) = \begin{bmatrix} \boldsymbol{p}(\boldsymbol{\omega}) \\ \boldsymbol{q}(\boldsymbol{\omega}) \end{bmatrix}$$

However, it is not necessary to convert system (5) into (7). Instead, the monodromy matrix in modal coordinates can be obtained as follows: we need to integrate system (5) over the rotation period 2k times with initial conditions (7) or in physical coordinates $\begin{Bmatrix} \dot{u}(0) \\ u(0) \end{Bmatrix} = \begin{bmatrix} \varphi_k & 0 \\ 0 & \varphi_k \end{bmatrix}$. And the matrix composed of solution vectors in physical coordinates of dimension 2n x 2k is converted into modal coordinate.

$$(8) \quad \boldsymbol{X}(\boldsymbol{\omega}) = \begin{bmatrix} [\boldsymbol{\varphi}_{\boldsymbol{k}}]^{\boldsymbol{T}}[\boldsymbol{M}][\dot{\boldsymbol{u}}(\boldsymbol{\omega})] \\ [\boldsymbol{\varphi}_{\boldsymbol{k}}]^{\boldsymbol{T}}[\boldsymbol{M}][\boldsymbol{u}(\boldsymbol{\omega})] \end{bmatrix}$$

If the eigenvalues of the matrix (8) are located in the unit circle, the system is stable.

4. Results of the flutter analysis

The aeroelastic analysis with full blade model of unmanned aircraft (Fig. 1~~Fig. 1~~) was performed. Three parts of analysis were carried out: coupled aeroelastic calculation of the stationary (static) state of a rotating blade, taking into account the translational flight of the helicopter; computation of flapping oscillations of the blade; the calculation of the blade flutter, taking into account the flapping oscillations, using Floquet theory. The calculations were performed at flight 81 regimes. Flutter regimes, as corresponding flutter modes, are found.

Отформатиро

Fig. 1. Heavy unmanned helicopter

In cases of flutter regimes the flutter oscillations were prevented by using additional weights installed on the blade and optimizing their position. Weights with different masses were modeled as concentrated masses at the CEM nodes using the CONM2 elements. The position of the weights was varied both along the axis of the blade and along the chord of the blade section.

5. Conclusions

In this paper, a framework for the flutter analysis of the full blade model based on Floquet theory was developed. Python scripts were written to integrate the aerodynamics into Nastran finite-element model for solving both static and dynamic aeroelastic problems. This framework can be applied to study different flutter modes, and takes the flow periodicity into account.

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