A TIME-DEPENDENT TIP LOSS FORMULA FOR ROTOR BLADE DYNAMIC ANALYSIS

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Abstract

Although, a constant tip loss factor is at best a crude representation of three-dimensional flow effects, it has found widespread use because of its simplicity and accuracy in both hover and forward flight performance and dynamic analysis calculations. This paper will show that a time-dependent formulation for tip loss factor, instead of a constant, is preferable for rotor blade dynamic analysis in forward flight. Substantiation for this new formulation is based upon analysis and simple reasonings that relate to H-34 flight test data, with respect to time histories of the radial distributions of blade airloads and flapwise bending moment.

Notation

B	=	tip loss factor
b	=	number of blades
c	=	blade chord, inches
ΔB	=	ratio of tip loss length to blade radius, 1-B
C_T	=	thrust coefficient
e	=	flapping hinge offset, inches
R	=	blade radius, inches
r	=	distance of blade element from axis of rotation, inches
s	=	wake spacing, inches
t	=	time, seconds
$T(t)$	=	tip loss time dependent function
$T.M._\Psi$	=	thrust moment (moment of lift on blade about flapping hinge), in-lb
U	=	free stream velocity, in/sec
Ω	=	rotor blade angular velocity, rad/sec
ω_{1F}	=	frequency of blade first flapwise bending mode
Ψ	=	rotor blade azimuth angle, degs.

Introduction

In design and analysis of rotor blades, a general practice is to use two-dimensional aerodynamic theory along the entire length of the blade with exception of the blade tip region, where three-dimensional flow effects are more pronounced. For the blade tip, these effects are approximated by the use of a tip loss factor, $B < 1$. In this manner the assumption is made that the lift acts out to the radial station, BR, and is, zero in the region, $1 - BR$. Two-dimensional profile drag is applied throughout the length of the blade and assumed unaffected by the tip loss factor. The tip loss factor, B, is selected so as to best approximate the lift distribution of the true three-dimensional blade. A thorough discussion of the theoretical consideration of tip losses in helicopter analysis is given by Johnson (Ref. 1). With respect to tip loss factor, a recent review is reported in the literature by Peters et al. (Ref. 2). The classical tip loss formulae due to Wheatley and Sissingh, which are a function of blade aspect ratio are given respectively by

$$\Delta B = \frac{c}{2R} \quad (1)$$

and

$$\Delta B = \frac{2c}{3R} \quad (2)$$

Other forms of the factor, derived from Prandtl's two-dimensional theory, are a function of the wake spacing (Refs. 2, 3, and 4) and given by

$$\Delta B = \frac{\sqrt{2C_T}}{b} \quad (3)$$

or

$$\Delta B = \frac{\sqrt{C_T}}{b} \quad (4)$$

where b is the number of blades, and C_T is the thrust coefficient.

Peters et al. (Ref. 2) introduce a new tip loss formula which includes both wake spacing and aspect ratio:

$$\Delta B = 1.283\frac{c}{R}\left[\frac{1 + 6.394(\frac{c}{s})}{1 + 16.06(\frac{c}{s}) + (3.856\frac{c}{s})^2}\right] \quad (5)$$

Tip loss factor as set forth in equations (1) through (5), can be extremely useful in hover analysis and performance calculations in forward flight. However,

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this paper will show that a preferred formulation for rotor dynamic analysis in forward flight should be time dependent. The time-dependent formulation is a simple extension of any of the tip loss factors given in equations (1) through (5) as follows:

$$\Delta B(t) = \Delta B(\psi) = \Delta B\, T(t) \qquad (6)$$

where

$$T(t) = \frac{1 + \cos 2\Psi}{2} = \frac{1 + \cos 2\Omega t}{2} \qquad (7)$$

Substantiation for this proposed formulation will now be discussed.

Rationale for Time-Dependency

In forward flight the radial distribution of airloads on the blade varies with azimuthal angle, Ψ , and hence with time. It follows that the gradient of lift at the tip and hence the gradient of bound circulation at the tip, $\frac{d\Gamma}{dr}$, also vary with time. Consequently, the three-dimensional flow at the blade tip and the associated tip loss factor also vary with time.

Figure 1 compares the radial distribution along the blade as determined by two-dimensional blade element theory, with the actual loading. In contrast to fixed wing lift distribution, the dynamic pressure on a rotor blade is proportional to r^2. This concentrates the lift more toward the tip such that the gradient or loss of lift, shown by $\frac{d\Gamma}{dr}$, exceeds that for fixed wings.

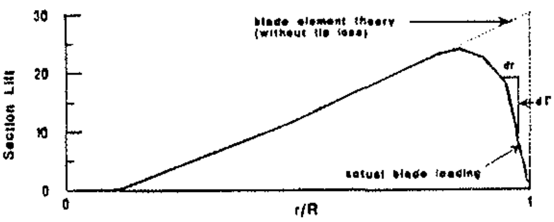


Figure 1. Actual vs. two-dimensional blade loading.

Shown in Figure 2 are measured airload distributions for an H-34 helicopter in level flight at 112 knots. The data is from Reference 6. Plotted are measured airload distributions for $\Psi = 6°, 36°, 66°$ and $96°$.

Clearly, the gradient of blade airloads and hence tip loss factor are a function of Ψ, blade azimuth position.

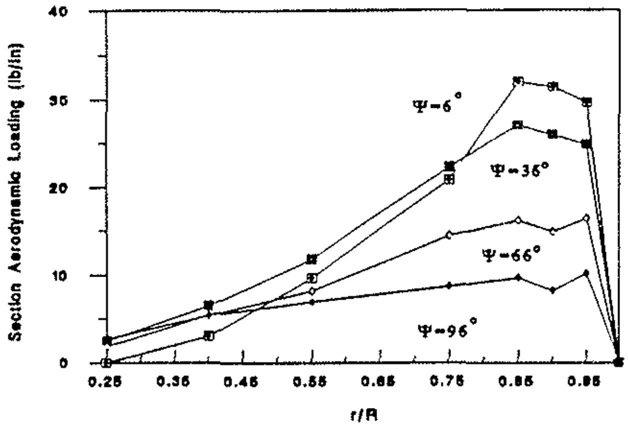


Figure 2. Change in blade airload distribution with azimuth. H-34 at 112 kts. (Ref. 6).

Figure 3 illustrates the importance of properly accounting for tip loss to accurately predict blade flapwise bending moment. In Fig. 3.*a* tip loss is neglected and lift is assumed fully effective to the blade tip. Here, aerodynamic lift is balanced by centrifugal force and the resulting flapwise moment in the tip region is small. In contrast, Fig. 3.*b* shows the blade with tip loss factor introduced.

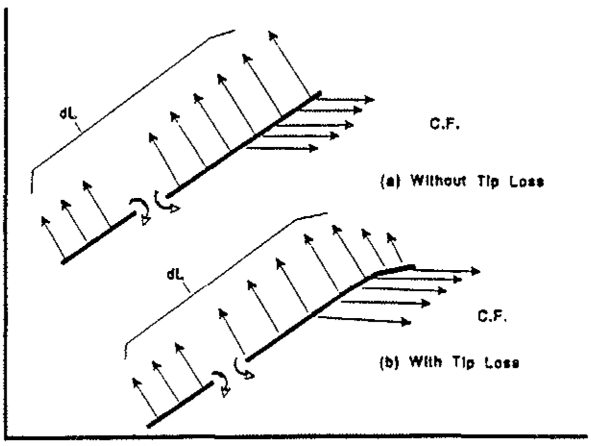
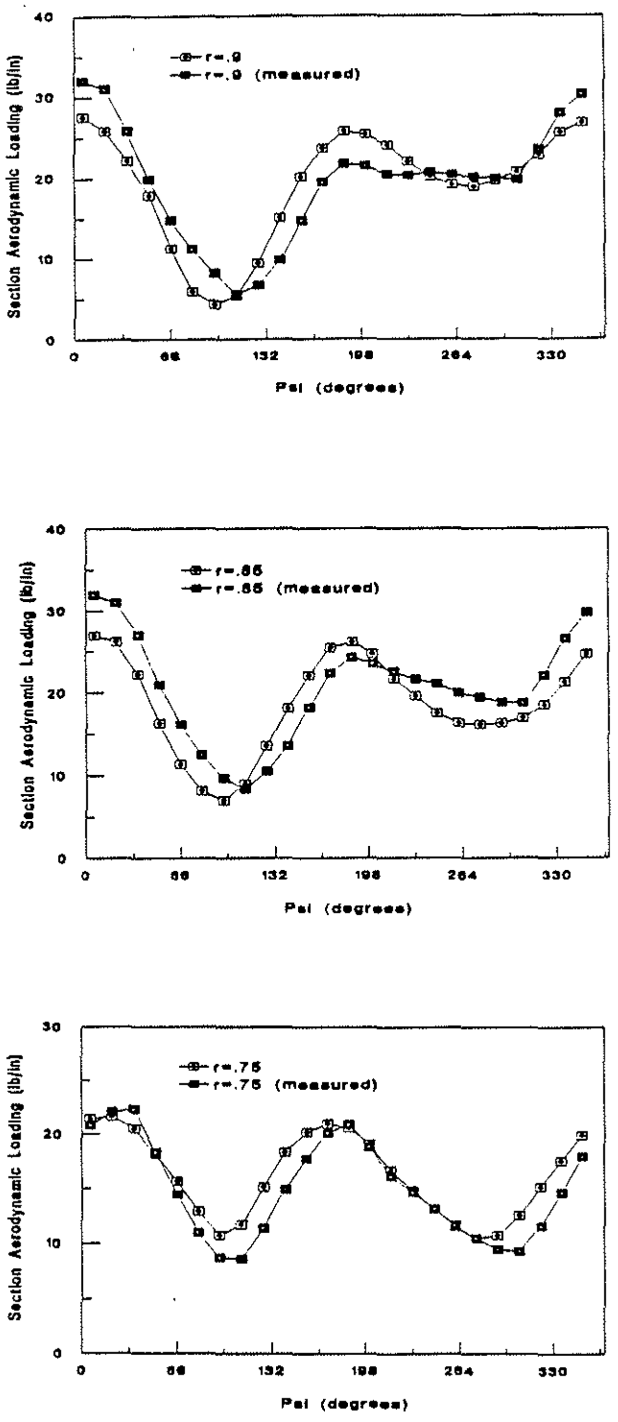


Figure 3. Effect of tip loss on blade flapwise bending.

We observe that as the tip is unloaded, a substantial down bending of the tip is introduced resulting in a significant negative flapwise bending moment being introduced. We conclude that variation of tip loss with azimuth would result in significant flapwise bending motion of the blade tip. Anyone



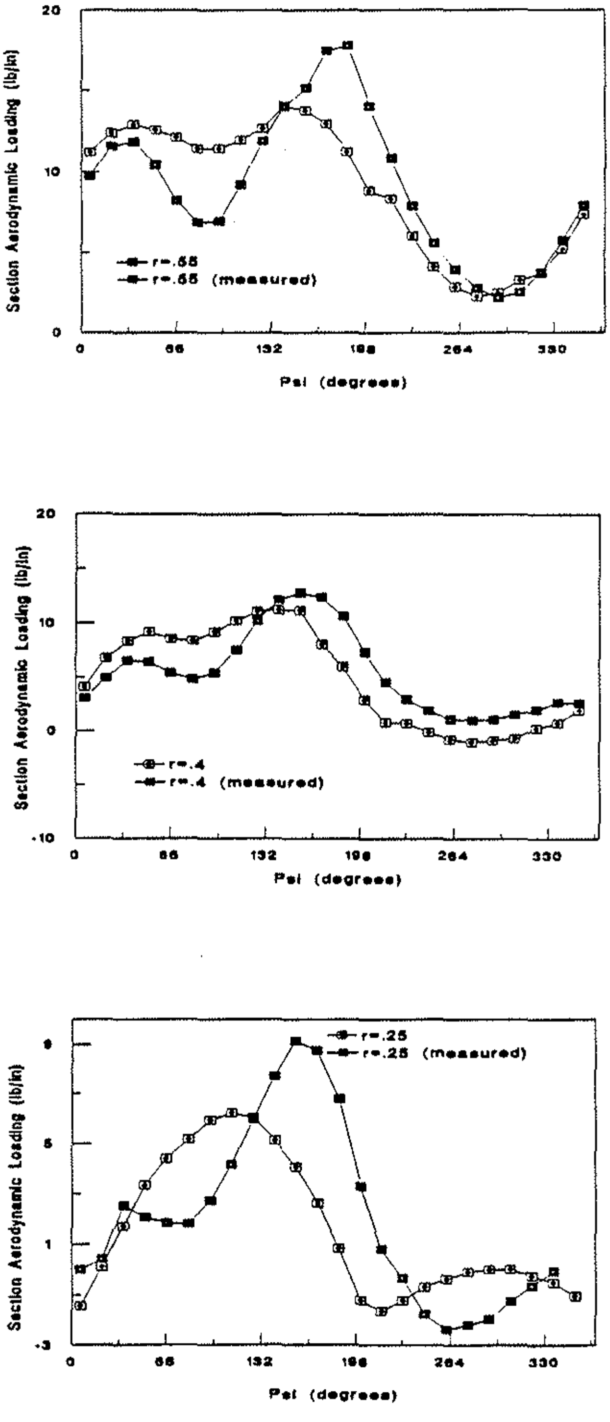


Figure 4. Comparison of calculated and measured blade airload time histories (outboard blade stations).

Figure 5. Comparison of calculated and measured blade airload time histories (inboard blade station).

who has viewed motion pictures of rotor blades in forward flight as recorded by a hub-mounted camera that turns with the rotor, will observe noticeable flexing of the blade tip per rotor revolution. This can be attributed to azimuthal or time-dependent tip loss variation.

Comparison with Test Data

To this date, Scheiman's (Ref. 6) H-34 flight test data represents the most complete set of measured airloads and corresponding flapwise, chordwise, and torsional moments as well as blade motions, ever published. In using this data to compare with analysis, a number of researchers have found the same paraodx that we will report in this paper. The paradox is that at high speed ($\mu > 0.20$) one can obtain reasonably good correlation with the measured airloads using 2-D airfoil data in a forward flight blade airloads analysis that assumes uniform inflow. Such a comparison is shown in Figures 4 and 5 for the 112–knot case. Figure 4 compares calculated and measured blade airload time histories for the outboard sections of the blade ($r/R = .9, .8,$ and.75), while Figure 5 gives the corresponding comparison for those sections of the blade further in ($r/R = .55, .40,$ and 0.25). Calculations are done using a fully-coupled blade dynamic analysis (Myklestad), as described in References (5) and (7), while flight test data is taken from a NASA report, Ref. (6). With this kind of agreement on blade airloads, the researcher is somewhat taken back when the corresponding blade response results in the relatively poor agreement in blade flapwise bending moment time history as shown for $r/R = 0.80$ in Figure 6.

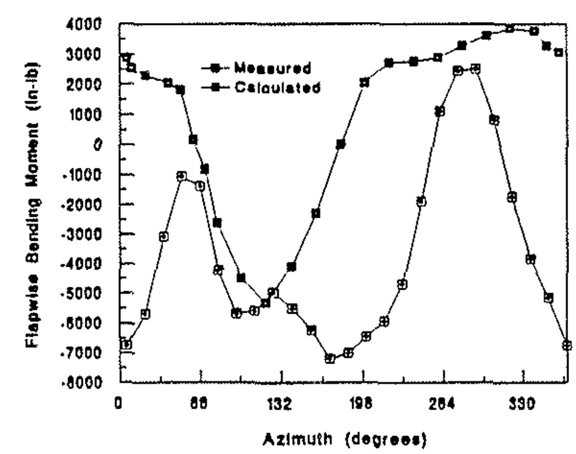


Figure 6. Comparison of measured and calculated flapwise bending at $r/R = 0.80$ (constant tip loss).

Observe that maximum discrepancy between test and analysis occurs in the regions $\Psi = 0^o$ and $\Psi = 180^o$.That is, even neglecting rational considerations, a totally emperical approach to resolving the difference between test and analysis might consider a second harmonic cosine function with zero phase. Also, recall that the first flap bending mode for a uniform articulated blade occurs about

$$\omega_{1F} \approx 2.5\Omega \quad (8)$$

This means we could expect good blade response to $2P$ excitation.

In proposing a time-dependent tip loss factor $\Delta B(t)$, it would be desireable to develop the function based upon previous constant tip loss functions, ΔB. Since constant functions were developed for both hover and forward flight, it seems reasonable to assume for the general case, ΔB was derived based upon a lift distribution corresponding to the hover case. In forward flight the blades experience a lift distribution at $\Psi = 0^o$ and at $\Psi = 180^o$, that is similar to the lift distribution in hover. Recall the moment of the thrust about the flapping hinge is essentially a second harmonic function with its maximum values at these positions. For this reason, a well known expression relating to trim of a rotor in forward flight is,"The helicopter flies on the rotor blades in the fore ($\Psi = 180^o$) and aft ($\Psi = 0^o$) positions."

Derivation of Function, T(t)

With these general concepts understood, now consider the lift distribution on the blade at a representative azimuth angle Ψ. This is shown in Figure 7.

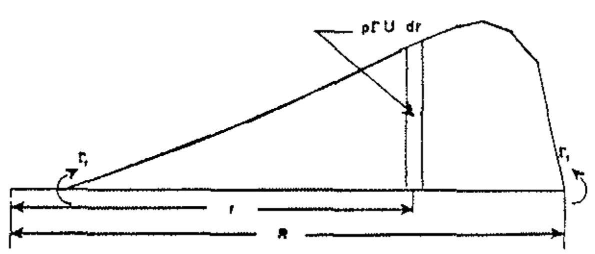


Figure 7. Lift distribution on blade at representative azimuth angle, Ψ.

In the figure, Γ_r and Γ_t represent the strength of the blade root and tip vortices respectively, where the blade's total bound circulation is given by

$$\Gamma_0 = \Gamma_r + \Gamma_t \quad (9)$$

and from Figure 7 we write

$$d\Gamma_t = \Gamma(r)\,\frac{r}{R} dr \quad (10)$$

Integrating, the total tip vortex can be written

$$\Gamma_t = \frac{1}{R}\int_e^R \Gamma(r)\, r\, dr \quad (11)$$

But thrust moment or the moment of the lift about the flapping hinge is given by

$$T.M._{\Psi} = \int_{e}^{R} \rho \, U \, \Gamma(r) \, r \, dr \qquad (12)$$

Comparing the integrands of Equations (11) and (12) we conclude that the strength of the tip vortex will vary with azimuth, Ψ, similar to the azimuth variation of thrust moment. We would expect that the tip loss would vary in the same manner.

Thrust Moment Time Histories

Plotted in Figure 8 are the measured and calculated time histories of thrust moment from the 112-knot case of Ref. 6, where the H-34 is in high-speed level flight. Note the good agreement between calculated and flight-measured values.

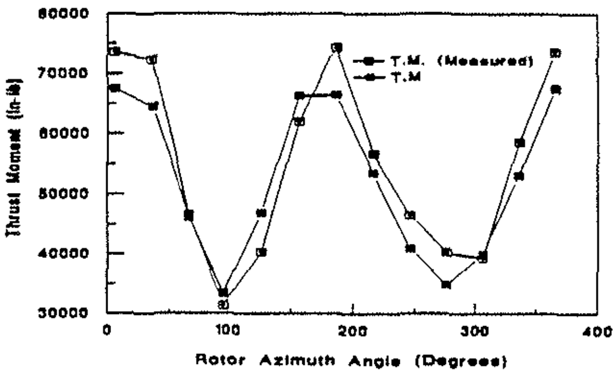


Figure 8. Comparison of calculated and measured time histories of thrust moment.

Observe that the function is primarily a second harmonic function of the form:

$$T(t) = \frac{1+\cos 2\Omega t}{2} = \frac{1+\cos 2\Psi}{2} \qquad (14)$$

From the previous discussion we conclude for forward flight that tip loss is a function of time (azimuth, Ψ). In addition the strength of the tip vortex can be related to the integrated thrust moment along the blade. For a trimmed rotor in forward flight the azimuthal history of thrust moment closely follows

$$f(\Psi) = \cos 2\Psi. \qquad (15)$$

This leads to the following tip loss factor for forward flight:

$$\Delta B(t) = \Delta B T(t) = \Delta B \left(\frac{1+\cos 2\Omega t}{2} \right). \qquad (16)$$

Substantiation of Results

Consider once more the 112-knot case for the H-34 in level flight for which both time histories of blade airload (at 6 radial stations) and blade bending moment (at 5 radial stations) are available. Recall that Figures 4 and 5 show reasonably good correlation between measured and calculated airloads using a simple uniform inflow model. In contrast, if we now assume constant tip loss, ΔB, and obtain the blade response at $r/R = 0.80$, we obtain the relatively poor agreement shown by Figure 6.

Let's repeat the calculation, except now let the tip loss factor vary with time such that

$$\Delta B(t) = \Delta B \, T(t) = \Delta B \left(\frac{1+\cos 2\Omega t}{2} \right)$$

The function $T(t)$ is shown plotted in Figure 9.

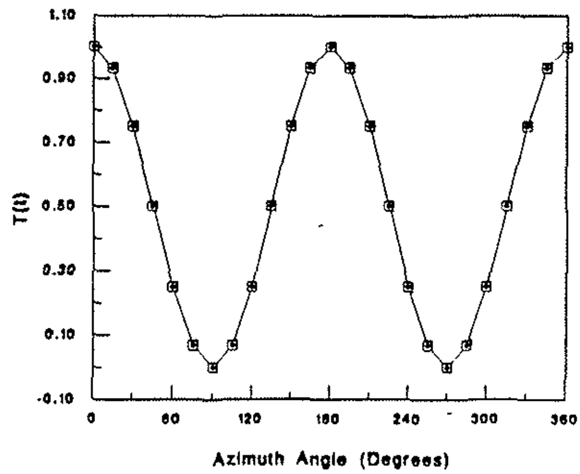


Figure 9. Time-dependent tip loss function, $T(t)$.

Physically, the function $\Delta B \, T(t)$, represents a varying of the rotor tip effective area, $\Psi = 0^{o}$ to $\Psi = 360^{o}$.

Figure 10 compares calculated and measured bending moment time histories with the time dependent tip loss factor introduced into the 112–knot H-34 calculations. Comparison of these results shows the dramatic improvement in correlation that can be achieved by more accurate accountability for tip loss. Study of similar calculations done by other researchers shows in general (with constant tip loss) that there is failure to duplicate the relatively sharp increase and decrease in the bending moment distribution that occurs on the retreating side of the rotor.

Conclusions

This paper has shown that a time-dependent formulation for tip loss factor, instead of a constant, is preferable for rotor dynamic analysis in forward flight. For application, tip loss, ΔB, is obtained from the equation of the analyst's choice (see Eqtns. (1) through (5)). This term is then multiplied by the function, $T(t)$, given by Eqtn. (7) and used in the blade analysis in this form.

Rationale for substitution of this function is based upon the following:

- Tip loss is a function of time (azimuth, Ψ).
- Tip loss factor at $\Psi = 0^o$ and $\Psi = 180^o$ can be taken as that for hover, ΔB.
- The amount of tip loss is related to the strength of the tip vortex.
- The strength of the tip vortex is related to the integrated thrust moment along the blade.
- For a trimmed rotor in forward flight the azimuth history of thrust moment is primarily a second harmonic function (Eqtn. (15)).

From the above we can postulate a tip loss factor of the form

$$\Delta B(t) = \Delta B\, T(t) = \Delta B \left(\frac{1 + \cos 2\Omega t}{2} \right).$$

Introducing this factor into the airloads as applied to a blade dynamic analysis in forward flight yields excellent correlation between measured and calculated blade bending moments as shown in Figure 10.

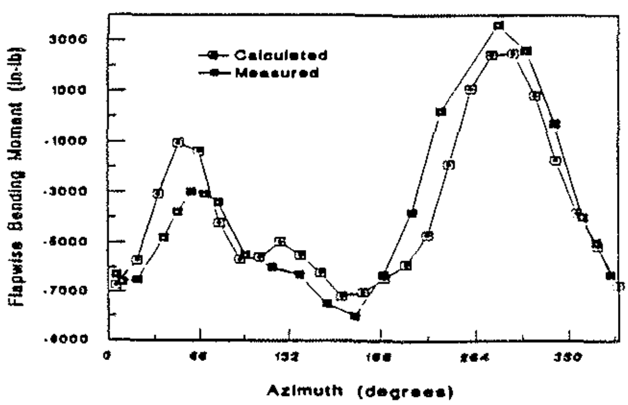


Figure 10. Comparison of measured and calculated flapwise bending at $r/R = 0.80$ (time-dependent tip loss).

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