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SOME CALCULATIONS OF TIP VORTEX - BLADE LOADINGS

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Summary.

Transient tip vortex-blade loadings are calculated from an inviscid steady model in which the tip vortex is idealised as a discrete line vortex while the blade loading is represented by a vortex lattice arrangement. Mach number effects are neglected.

1. INTRODUCTION

Tip vortex-blade interaction is an important ingredient in rotor aerodynamics. Locations of interactions are summarised in Fig. 1, taken from reference 1, for a four bladed rotor at two different advance ratios. It is seen that there is a wide range of conditions of vortex inclinations relative to a blade and transverse velocities across a blade.

An experimental and theoretical programme is being undertaken at Queen Mary College into this area. This note describes some of the preliminary theoretical developments.

2. MATHEMATICAL PROBLEM

The idealised problem described in this note is shown in Fig. 2. Consider a rectangular wing of high aspect ratio swept at an angle Λ to a uniform low speed stream of velocity U, neglecting Mach number effects; note that the sweep angle can be either positive or negative. A line vortex of strength Γ, height h above the wing, and parallel to the plane of the wing is inclined at an angle θ to the free stream. Since the line vortex is free each element of the vortex convects in the direction of the free stream at the free stream velocity U. The successive locations of the free vortex at successive times t_1, t_2 and t_3 are shown in Fig. 2. Thus there is a transient loading across the span of the wing with time as the line vortex traverses across the span.

This unsteady problem can be reduced to a steady problem by taking moving axes Ox_1y_1 travelling along the span of the wing with the line vortex, as shown in Fig. 3. To determine the velocity of translation of the axes along the span consider the velocity vector diagram in Fig. 3; if AC is equal to the free stream velocity U thus the velocity OC is required.

Now

$$OB = AB\tan\theta = (U-BC)\tan\theta = BC\cot\Lambda$$

hence

$$BC = \frac{U\tan\theta\tan\Lambda}{1+\tan\theta\tan\Lambda} .$$

Then

$$OC = BC\text{cosec}\Lambda = \frac{U\tan\theta\sec\Lambda}{1+\tan\theta\tan\Lambda}. \qquad (1)$$

Eqn. (1) gives the translational velocity of the moving axes along the span. Note that the axes do not move when $\theta=0$, which is consistent with the problem formulated in Fig. 2, that the translational velocity remains finite as $\theta\to\Lambda$, and that the translational velocity approaches infinity as

$$\theta \to \frac{\pi}{2} + \Lambda,$$

i.e. as the line vortex becomes parallel with wing leading edge.

Relative to the moving axes the problem reduces to a steady state problem as shown in Fig. 4. The relative free stream becomes aligned with the line vortex, the relative free stream velocity becomes $(U\cos\Lambda\sec(\theta-\Lambda))$, and the line vortex and free stream become inclined at an angle $(\theta-\Lambda)$ to the normal to the wing leading edge.

The spanwise load distribution will be of the form

$$L(y_1) = \tfrac{1}{2}\rho U^2\cos^2\Lambda\left(\frac{\Gamma}{Uc}\right)\sec^2(\theta-\Lambda)F((\theta-\Lambda),\ y_1/c,\ h/c) \qquad (2)$$

Thus the steady problem need only be solved for variables $(\theta-\Lambda)$ and h/c. Since Γ can be positive or negative (i.e. clockwise or anticlockwise), the range of variables can be either

$$\Gamma>0,\ \frac{-\pi}{2} \leqslant (\theta-\Lambda) \leqslant \frac{+\pi}{2}\ ;$$

or

$$\Gamma>0,\ \Gamma<0,\ 0\leqslant\theta\leqslant\frac{\pi}{2}.$$

The transient loading relative to fixed axes (x,y), with the y axis along the wing span, is obtained from eqn. (2) by substituting

$$y_1 = y + \left[\frac{U\tan\theta\sec\Lambda}{1+\tan\theta\tan\Lambda}\right]t, \qquad (3)$$

where t is a time from a datum.

The reduction of the transient problem to a steady problem breaks down as $(\theta-\Lambda)$ approaches $\pi/2$ because the translational velocity of the axes along the span (eqn. 1) approaches infinity and the spanwise loading (eqn. (2)) also apparently tends to infinity. In the physical problem, as seen from Fig. 2, as θ approaches $(\pi/2+\Lambda)$ the line vortex becomes parallel to the wing leading edge and passes in a two dimensional manner across the chord of the wing. It is impossible to reduce this transient flow to a steady problem. Nevertheless this type of situation is a possible practical one. It is shown in this note that the results from the steady three dimensional problem do indeed tend to the correct two dimensional transient results and $(\theta-\Lambda)$ tends to $\pi/2$.

3. MATHEMATICAL MODEL FOR THE THREE DIMENSIONAL STEADY PROBLEM

The steady mathematical problem posed in Section 2, and shown in Fig. 4, is solved by separating the effects of wing profile, camber and vortex induced loads on the overall loads. For the vortex induced loads, the wing surface is replaced by a planar system of vorticity whose strength is such that it counteracts the flow field across the wing surface induced by the line vortex. The planar wing vorticity is represented by a discrete vortex lattice arrangement, as shown in Fig. 5. The vortices on the wing are arranged parallel and normal to the wing leading edge while the shed vortices into the wake aft of the trailing edge are assumed to be straight lying in the direction of the resultant spanwise flow, due to the combined effects of the free stream and the induced field of the line vortex, in the region of the wing trailing edge.

It is seen from Fig. 5 that the vortex lattice mesh on the wing is more dense spanwise the region of the line vortex interference but then spreads out spanwise at distances removed from the line vortex. In these calculations there are 8 chordwise vortices.

The unknown strengths are those of the wing vortices parallel to the leading edge, (the so-called bound vortices); the strengths of the vortices normal to the leading edge (the so-called trailing vortices) and the strengths of the shed vortices into the wake (also known as trailing vortices) are known in terms of the unknown bound vortices because of the Helmholtz condition of continuity of vorticity. The downwash field at the collocation points, taken at the mid points of the vortex lattice mesh, due to the wing vortex lattice can be expressed in terms of a linear expression involving an influence coefficient matrix and the unknown bound vortex strengths. These wing downwash velocities cancel the induced downwash velocities due to the line vortex at the collocation points so the unknown bound vortex strengths can be calculated.

Once the strengths of all of the wing vortices are known, the loads on each wing vortex, for both bound and trailing vortices, can be estimated. Loads arise from the interaction of inclined free stream with both bound and trailing wing vortices; there are additional non-linear loads associated with the induced velocities from the line vortex acting on lthe wing bound and trailing vortices.

4. MATHEMATICAL MODEL FOR TWO DIMENSIONAL TRANSIENT PROBLEM

The mathematical model for the two dimensional transient problem shown in Fig. 6 has been developed by adaptation of a general unsteady program developed at Queen Mary College by Cheung. Planar aerofoil and wake vorticity is treated as piecewise linear between nodes. The strengths of the vorticity at the nodes on the aerofoil are unknown, the strength of the wake vorticity is related to the aerofoil vorticity by the rate at which circulation is shed into the wake and convected downstream. The problem is solved at successive intervals of time as the line vortex approaches and passes over the aerofoil chord.

5. RESULTS AND DISCUSSION

Fig. 7 shows the spanwise lift distribution for the case when $(\theta-\Lambda)$ is zero (i.e. the line vortex lies normal to the wing leading edge) for various values of the height h/c of the line vortex above the wing and a fixed value of $\Gamma/2\pi Vh$ in a stream with a reference velocity V. The constant value of $\Gamma/2\pi Vh$ for varying h/c implies that the induced velocities underneath the vortex remain constant; the transverse induced velocity at the wing surface is $0.1V$ in Fig. 7.

The variation of spanwise lift follows the well known character. These numerical curves agree closely with some analytic results obtained by Hancock[2]. One feature is the non-linear effect which gives a larger negative peak C_L than the positive peak C_L. It has been shown that this effect increases as the effective vortex strength (Γ/V) increases.

Fig. 8 shows the spanwise lift distributions for various values of $(\theta-\Lambda)$ with a line vortex of fixed strength and fixed height above the wing. As the effective sweep angle $(\theta-\Lambda)$ increases the peak values of C_L increase and the load 'spreads' itself across the span; the difference in the positive and negative peak values of C_L now becomes more pronounced. The spanwise distributions for Γ negative differ only slightly from those shown in Fig. 8 for the particular value of (Γ/Vh); however as (Γ/Vh) increases there are substantial non-linear differences between the results for Γ positive and negative. There can be mathematical difficulties at large values of (Γ/Vh) for then the shed vortices from the trailing edge can be 'blown' back over the wing.

Finally Fig. 9 shows a comparison of transient loading from two calculations, in the first calculation results are obtained from the above steady model with the line vortex nearly parallel to the leading edge, while in the second calculation results are obtained from the two dimensional unsteady model. The agreement is gratifying.

6. CONCLUSIONS

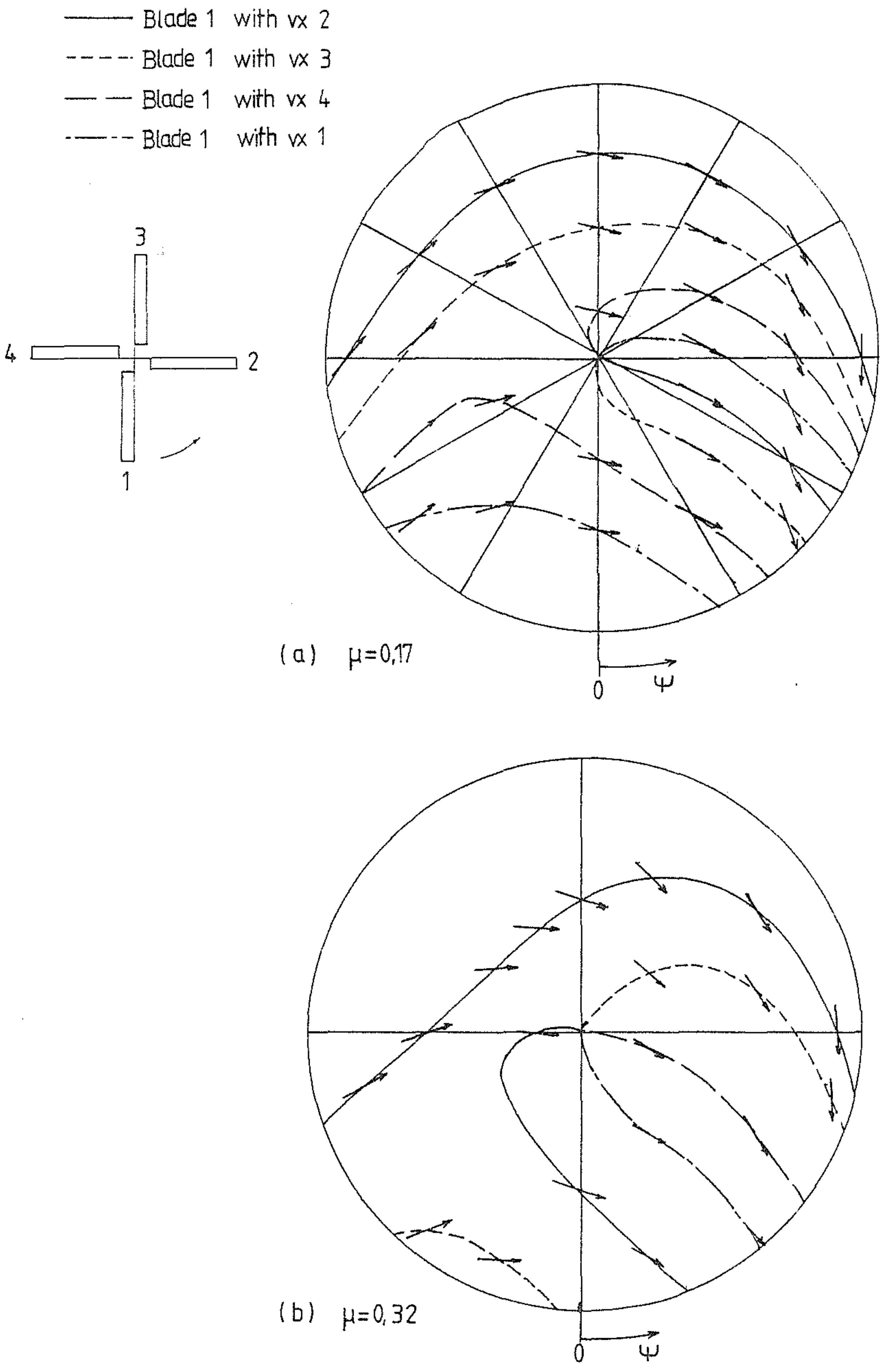
A vortex lattice model has been formulated to estimate transient loads associated with tip vortex-blade interference, neglecting Mach number effects. A range of results have been obtained.

This model is probably too complicated to apply in the context of an overall rotor aerodynamic prediction method but it is hoped to reduce the results obtained in this note to an empirical form which can be applied more widely.

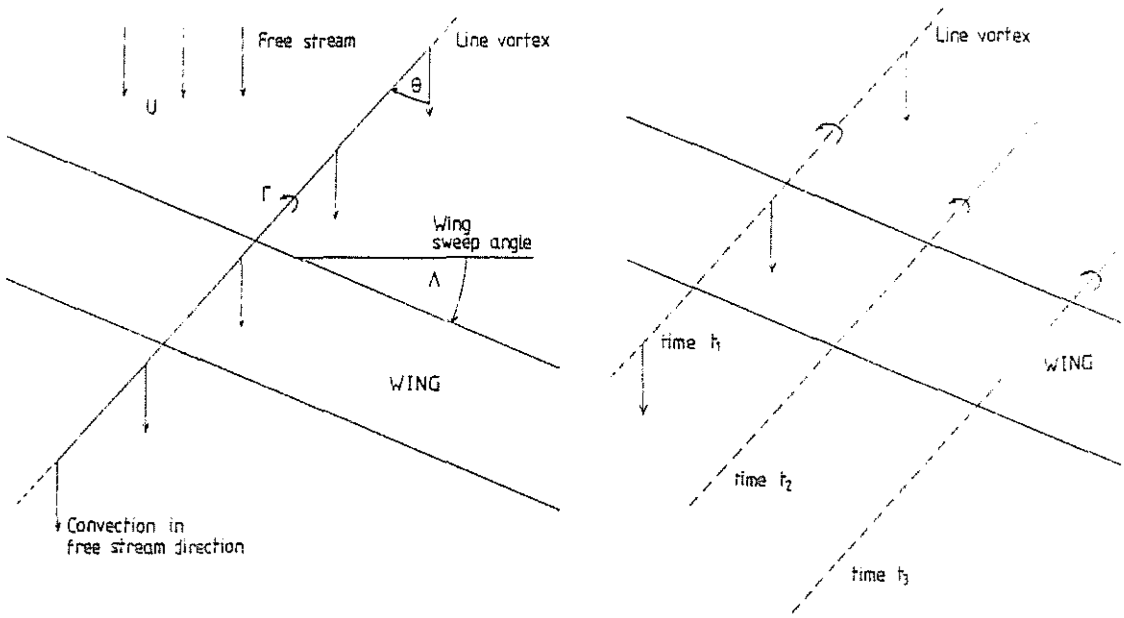
7. REFERENCES

1. P. Brotherhood and C. Young. "The measurement and interpretation of rotor blade pressures and loads on a PUMA helicopter in flight". 5th European Rotorcraft and Powered Lift Aircraft Forum, 1979.

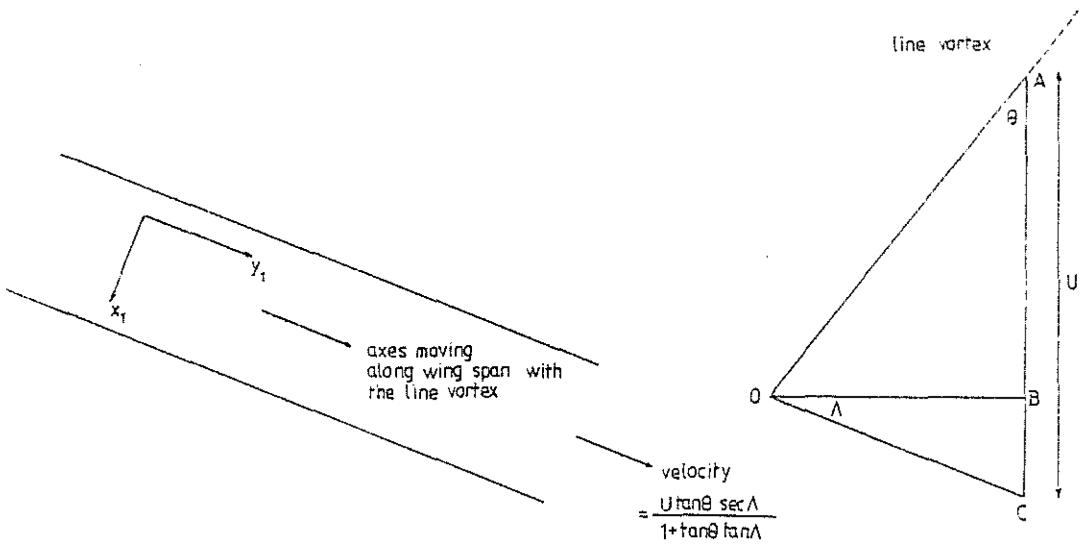
2. G.J. Hancock. "Aerodynamic loading induced on a two-dimensional wing by a free vortex in incompressible flow". The Aeronautical Journal of the Royal Aeronautical Society, Vol. 75. June 1971.



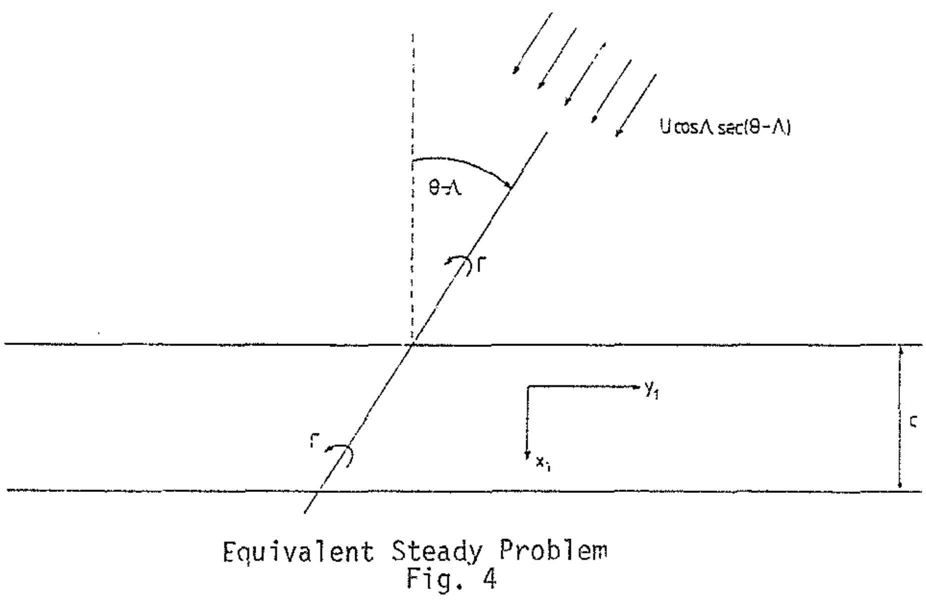
Some Typical Tin Vortex Blade Geometries
Fig. 1



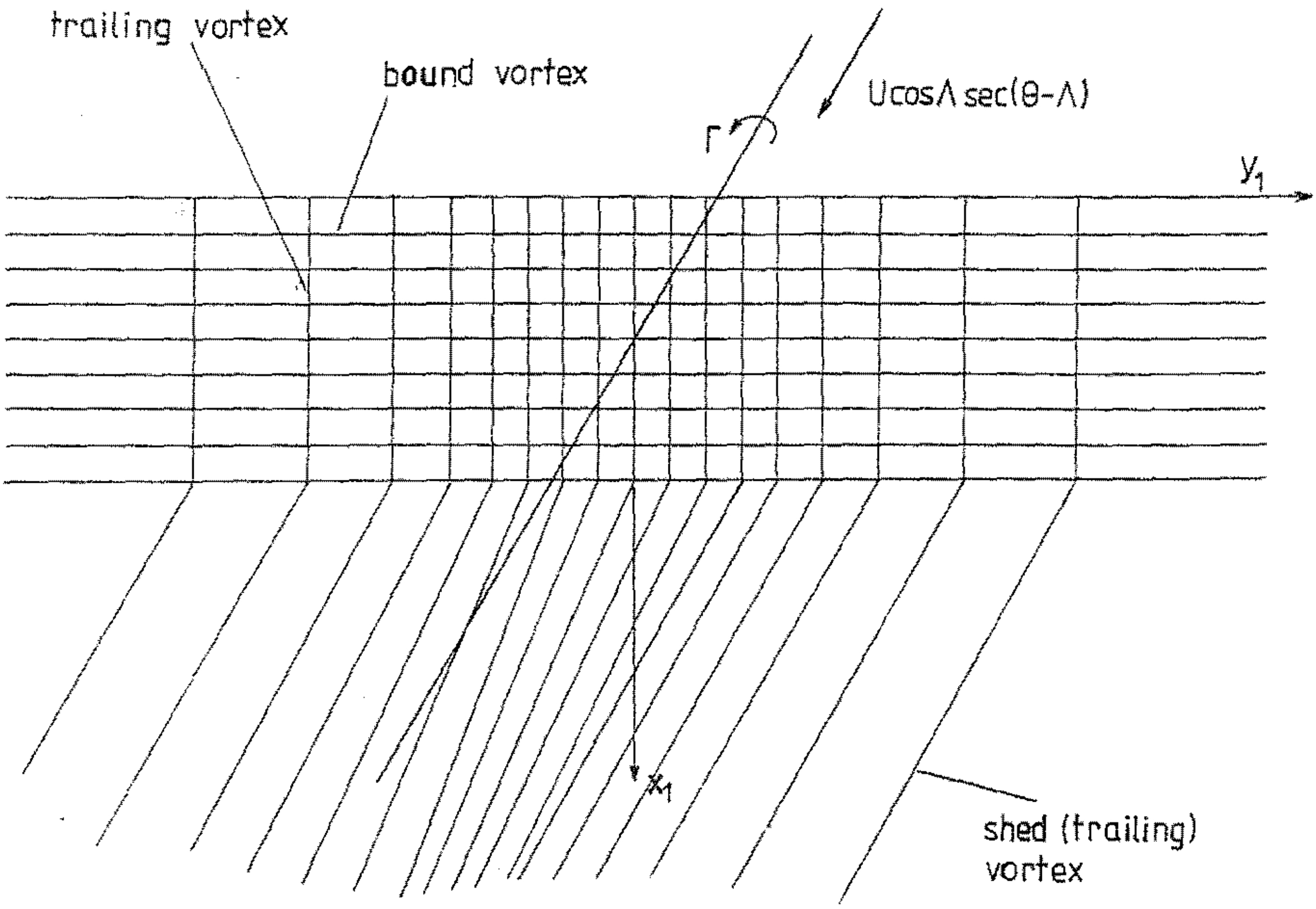
Travelling Line Vortex
Fig. 2



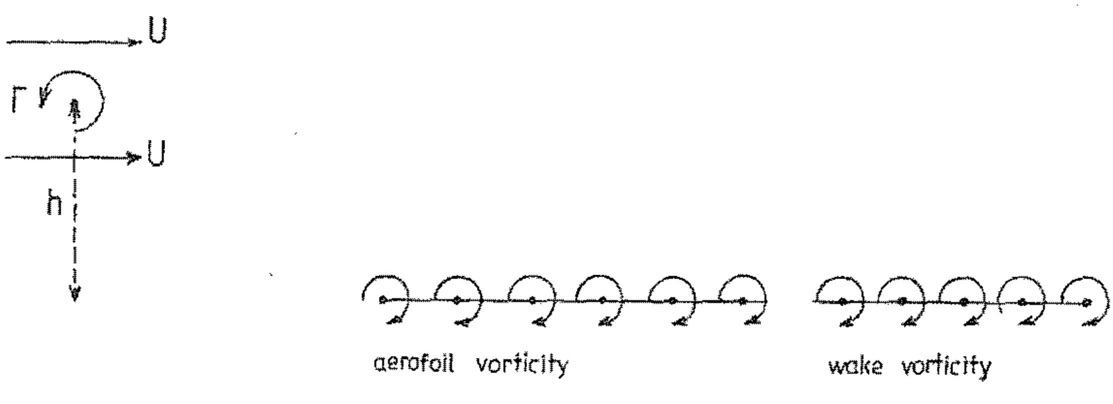
Moving Axes
Fig. 3



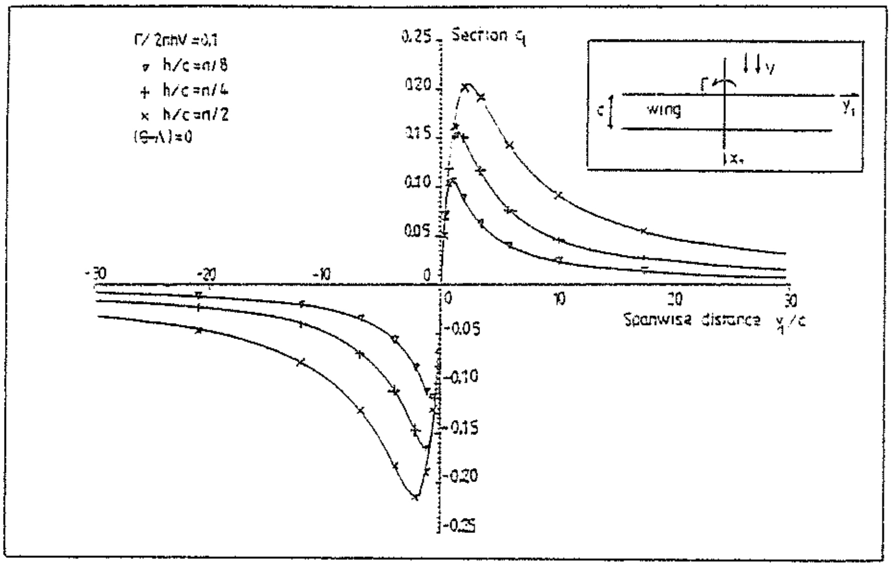
Equivalent Steady Problem
Fig. 4



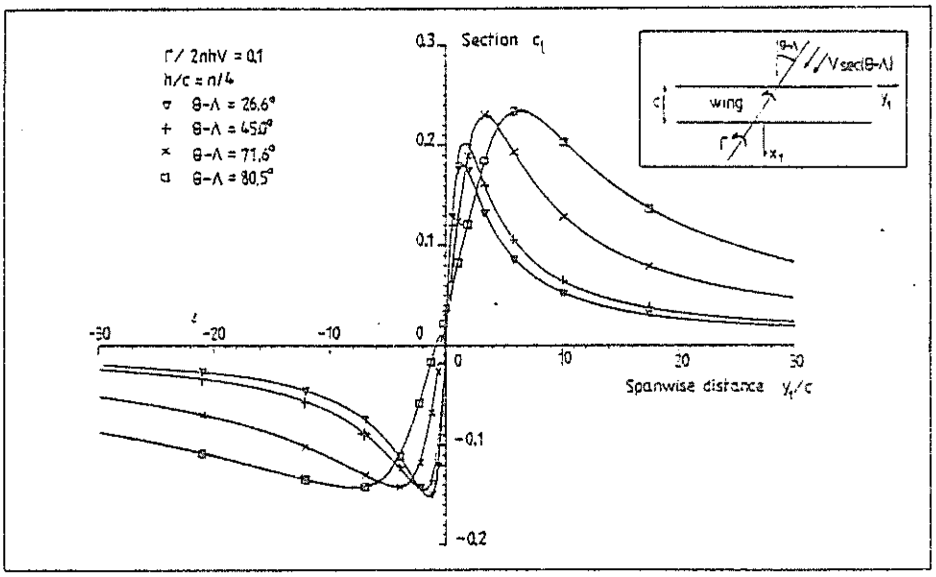
Three Dimensional Steady Vortex Model
Fig. 5



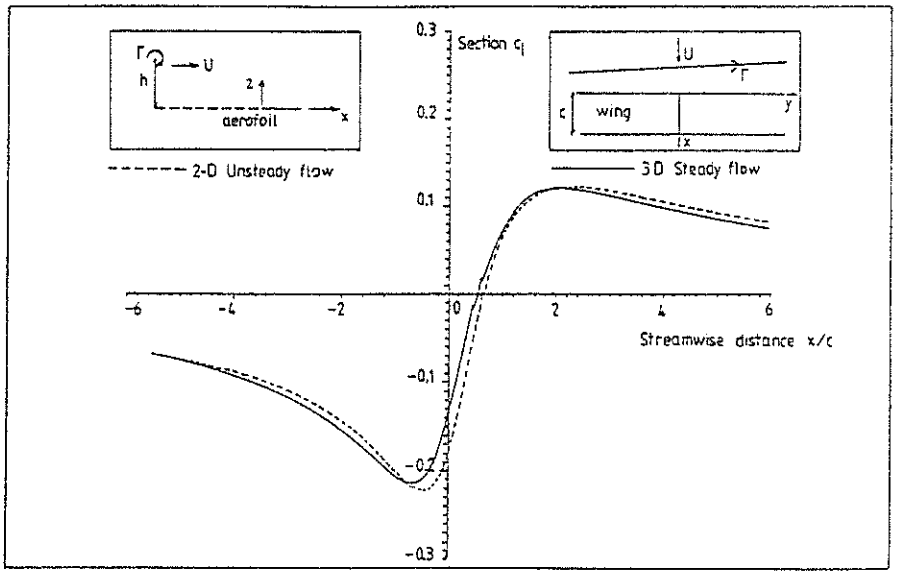
Two Dimensional Unsteady Model
Fig. 6



Spanwise Load Distribution for Varying Line Vortex Strength
Fig. 7



Spanwise Load Distribution for Varying Angles of Sweep
Fig. 8



Equivalence of Steady and Unsteady Models.
Fig. 9