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MATHEMATICAL MODELING OF LOADING OF BEARINGLESS MAIN ROTOR WITH ELASTIC ELEMENTS OF TORSIONAL TYPE

BY

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Summery

Some questions of creation the model of bearingless main rotor are under discussion in offered report. This model can be used in deciding serial tasks for specification of aircraft performance and calculation of load in rotorcraft flight structure of light helicopter ANSAT. In this modeling the laminated and multichannel elastic elements' behavior is described by parametric matrix of compliance (flexibility). The parametric matrix of compliance (flexibility) is defined by calculation and is revised by experimental researches. The developing model let us be more accurately in definition of load on elements of main rotor construction and to refine parameters of control actions on helicopter and supply (maintain) tuning of helicopter's wire guidance flight control.

Modern science and technical achievements ensure stability and a resource of units which are designed with use of composite. These achievements allow to create the main rotor with elastic holding of blades to the rotor bushing. In contemporary research papers this rotor is called as fillowsbearingless, rigid, semirigid, elastic, etc. A rotor with elastic attaching of blades is a special case of bearingless main rotors with absent axial, vertical and horizontal bearings (joints). Functions of all these bearings are executed thanks to the elastic element - an integral bearing. Using such elastic elements significantly simplifies the design, increases maneuverability and controllability of helicopter. In addition it ensures a service in the system "on the condition". The elastic elements are supposed to have definite softness in planes of flap, rotation and torsion. These are the conditions ensured the stiffness and air-elastic stability of main rotor constructive elements. The softness of such elements should be optimized in number of parameters. Constructive elastic elements can be composed of anisotropic composites with different elasticity modulus. In this case the modeling of their mode of deformation can be done thanking to account the relations between bending and torsion. The main gold of these work is to develop the strategy of numerical simulation of the main rotor elasticity when the rotor is composed of elastic elements of torsion types made of rubber-fabric composites.

The modern tendency is to replace the metallic elastic elements of spring type (Bo-105, BK-117, Ka-62, Mi-34) by elastic elements made of composites (MD-900, EC-135) [1]. A modeling of rotor' aeroelasticity for helicopters having the take-off weight equal to 3000 kg has several characteristics which are analyzed in the given report. The object of the research is the torsion made of the rubber-fabric composites with the design cone angle and the displacement of axis in the plane of rotation. Constructive power scheme of such an element is a flat system of laminated space deformed beams of variable stiffness. This system is supposed to have the multichannel transfer of loads.

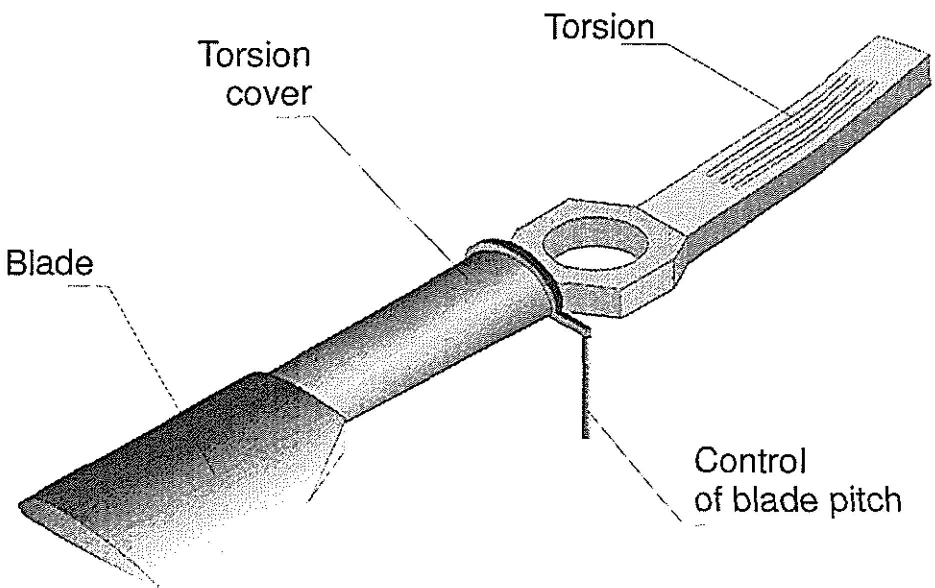


Fig. 1

The basic design power scheme of blades attaching to the bearing of rotor is shown in the Fig. 1. A blade is attached to the bushing by the spring torsion which allows the flapping motion and curving in the planes of the blade rotation. It is possible to transfer the controlling moment on the blade by means of the cover rotation. The torsion twisting is as effective as the rotation of the axial joint. The load transfer from blades onto the bushing can be done by several independent beams. That's why a rotor of this type sometimes is called a rotor with redundant, or multichannel ways of load transfer to the bearing [2]. It differs from a hinge and hingeless main rotor, which has only one way of load transfer onto the blade.

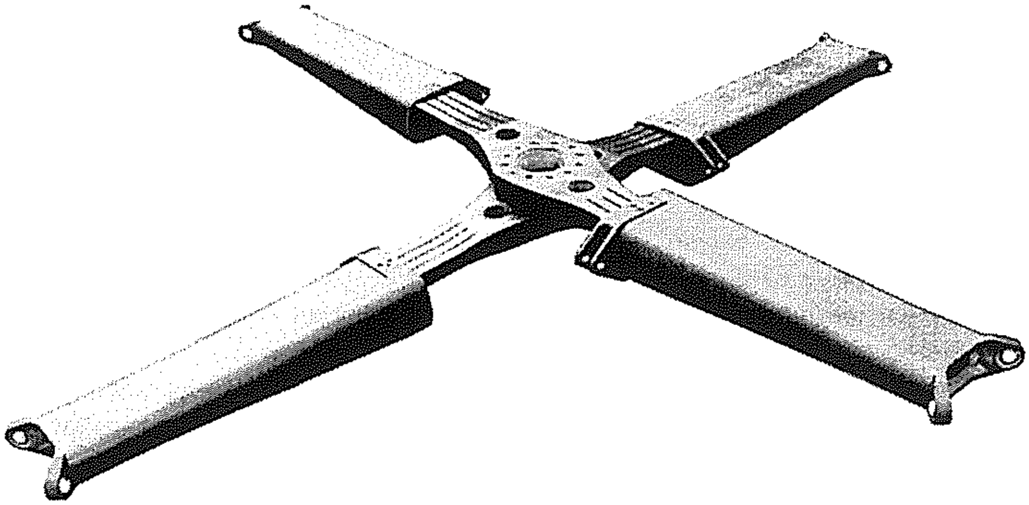


Fig. 2

When the load transfer to the bushing from the blade butt is done in the many-server way while the torsion is deformed the constructive nonlinear bending-torsion relations come to exist. This bending-torsion relations are caused by linear extention of the twisted elastic element. In this connection the analysis of bearingless rotor aeroelasticity is more complex then the analysis of the hingeless and hinge rotors. It is a very important task in developing of methods of aeroelastic analysis of the elastic main rotor to create the adequate concept of nonlinear behavior of elastic element under the controlling moment and environmental stress of blade. This analysis which includes the elastic element behavior can be divided into three main stages:

a) preliminary calculation of deforming of the elastic element in the prescribed limits of deformation and loading; determination of flexibility matrixes on the boundary of elastic blade element;

b) determination of blade dynamic behavior under the influence of aerodynamic and inertial loads taking into accounte the nonlinear behavior of elastic element;

c) validaty test of the level of flexibility matrixes elements for a set of operational conditions of the elastic element operating.

The process of separate beam deforming is described in the terms of shear models of Timoshenko type. The complete definition of kinematic variables and internal power factors for every beam of torsion can be given by using the expression of the internal power factors in kinematic parameters and corresponding rigidities. The analytical formulas for calculation of generalized displacement and forces in free section of beam number "k" can be written as [3]:

$$V_{k,x}(z^k) = V_{k,x}(0) + \omega_{k,y}(0)z^k + \frac{1}{EJ_y^k}[\frac{Q_x^k(0)}{k_x^3}(k_x z^k -$$

$$- sh(k_x z^k)) + \frac{M_y^k(0)}{k_x^2}(ch(k_x z^k) - 1)] + \frac{1}{GF_x^k}[\frac{Q_x^k(0)}{k_x} sh(k_x z^k) -- M_y^k(0)(ch(k_x z^k) - 1)],$$

$$V_{k,y}(z^k)=V_{k,y}(0)-\omega_{k,x}(0)z^k+\frac{1}{EJ_x^k}[\frac{Q_y^k(0)}{k_y^3}(k_y z^k-$$

$$-sh(k_y z^k))-\frac{M_x^k(0)}{k_y^2}(ch(k_y z^k)-1)]+\frac{1}{GF_y^k}[\frac{Q_y^k(0)}{k_y}sh(k_y z^k)+M_x^k(0)(ch(k_y z^k)-1)],$$

$$\omega_{k,x}(z^k)=\omega_{k,x}(0)+\frac{1}{EJ_x^k}[-\frac{Q_y^k(0)}{k_y^2}(1-ch(k_y z^k))+\frac{M_x^k(0)}{k_y}sh(k_y z^k)], \quad (1)$$

$$\omega_{k,y}(z^k)=\omega_{k,y}(0)+\frac{1}{EJ_y^k}[\frac{Q_x^k(0)}{k_x^2}(1-ch(k_x z^k))+\frac{M_y^k(0)}{k_x}sh(k_x z^k)],$$

$$V_{k,z}(z^k)=V_{k,z}(0)+\frac{N_z^k z^k}{EF_k}-\frac{1}{2}\int_0^{z^k}[(V_{x,z}^k)^2+(V_{y,z}^k)^2]dz,$$

$$V_{x,z}^k=\omega_{k,y}(0)+\frac{Q_x^k(0)}{k_x^2 EJ_y^k}-\frac{Q_x^k(0)ch(k_x z^k)}{N_z^k}+\frac{M_y^k(0)sh(k_x z^k)}{N_z^k}k_x,$$

$$V_{y,z}^k=-\omega_{k,x}(0)+\frac{Q_y^k(0)}{k_y^2 EJ_x^k}-\frac{Q_y^k(0)ch(k_y z^k)}{N_z^k}-\frac{M_x^k(0)sh(k_y z^k)}{N_z^k}k_y,$$

$$M_x^k(z^k)=\frac{Q_y^k(0)}{k_y}sh(k_y z^k)+M_x^k(0)ch(k_y z^k),$$

$$M_y^k(z^k)=-\frac{Q_x^k(0)}{k_x}sh(k_x z^k)+M_y^k(0)ch(k_x z^k),$$

$$Q_x^k(z^k)=Q_x^k(0)\tilde{n}h(k_x z^k)-M_y^k(0)k_x sh(k_x z^k)\ ,$$

$$Q_y^k(z^k)=Q_y^k(0)\text{–}h(k_y z^k)+M_x^k(0)k_y sh(k_y z^k).$$

In the formulas (1) z^k is a local coordinate of beam number “k”. The values (.)(0) correspond the coordinate $z^k=0$, and k_x, k_y are defined by expressions

$$k_y=\frac{N_z^k}{EJ_x^k(1+\frac{N_z^k}{GF_y^k})},$$

$$k_x=\frac{N_z^k}{EJ_y^k(1+\frac{N_z^k}{GF_x^k})}. \quad (2)$$

The correlations given above allow us to form a system of nonlinear algebraic equations which describe the process of torsion deformation. This equations system can be solved by methods, accepted in [4]. The mode of deformation and flexibility matrixes of elastic element with predetermined charac-

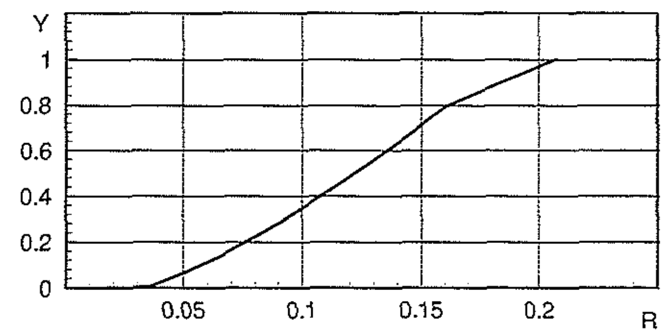


Fig. 3.

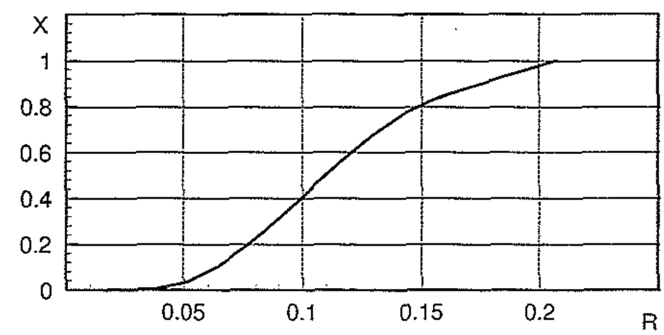


Fig. 4.

teristics of material and configuration are determined under the using of fine-element method.

The characteristic feature of torsion deforming is S-type form of the elastic line, which differs in quality from the beam line (Fig. 3, 4).

The parametric calculations allowing to find a relationship between the external load and deformation of torsion were conducted for various designs which are shown in Fig. 5. This relation can be presented using matrix mode of the following form:

$$\nu = \nu_0 + A(N_z, \varphi_{\ddot{o}\tilde{\imath}\eth}) \cdot q = \begin{bmatrix} \nu_{x0} \\ \nu_{y0} \\ \omega_{x0} \\ \omega_{y0} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} Q_x \\ Q_y \\ M_x \\ M_y \end{bmatrix} \tag{3}$$

where ν - generalized displacement vector of torsion end section, ν_0 - initial displacement vector of torsion end section depended from the initial configuration; q - generalized loading vector, corrected the blade butt; A - flexibility matrix which elements depend of centrifugal load and twisting angle of torsion end section (angle of incidence). Elements of flexibility matrix are represented in the calculation as Lagrange polynomial of fifth degree. The approximation error cannot be over 2% (1).

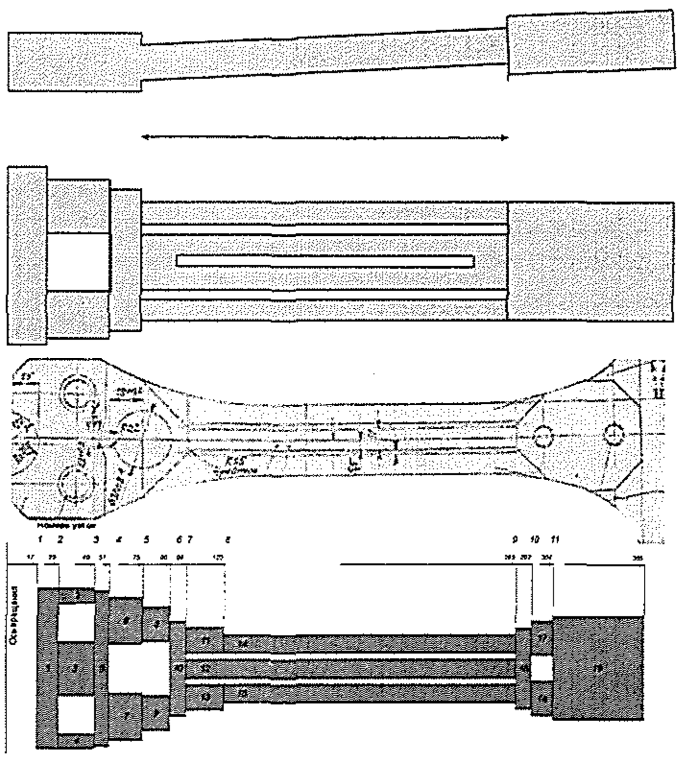


Fig 5

The series of parametrical flexibility matrix for elastic elements of the main and tail rotor was obtained using the correlations given above.

The blade loads are obtained by solving nonlinear equations of wing airfoil's beam motion. The system of these equations takes the following shape:

$$\begin{gathered} M\ddot{u}_i = Q_i' + q_i \\ \mathbf{J}\ddot{\vartheta}_i = M_i' + e_3 \times Q_i + m_i \\ \dot{u}_i' = \tilde{\dot{u}}_i' + \varepsilon_{kij}\chi_i \dot{u}_j = \varepsilon_{ij1}\omega_j \\ M_{\vartheta_i} = \mathbf{G}_i(\chi_i - \chi_{io}^{(0)}) \\ \chi_i = \mathbf{A}^*\tilde{\vartheta}_i' + \mathbf{A}\chi_{io}^{(0)} \\ \vartheta_i' - \chi_i = \varepsilon_{kij}\vartheta_i\chi_j . \end{gathered} \tag{4}$$

Q_i - internal force vector with components composed cutting and axial force; M_i - internal moment vector with components composed bending and rotational moment; q_i,m_i - distributed load and moment per-unite length in projection on coupled axes; u_i, ϑ_i - linear and angular replacement of beam' element; $\dot{u}_i$, $\dot{\vartheta}_i$, $\ddot{u}_i$, $\ddot{\vartheta}_i$ - first and second time derivative for linear and angular replacement.

Environmental stress on the blade is non-conservative and is defined in each blade section as distribution of cutting force and moment per-unit length. Three components of each vector are determined in coordinate system connected with the blade section. Centrifugal force and force per-unit length are reduced to center of rigidity. The moments are defined in correlation with the main axes of blade section. Using a nonlinear system of integral-differential equations of motion it is possible to determine the elastic line position on each stage of calculation and in given moment of time.

The solution of the equation lets us to determine the loads on rotor system's elements in oder to define the stability and endurance (life time) of main rotor system and also to standardize external load for static and dynamic tests. The position of elastic axis of blade and torsion in one the given stage of calculation and load on the blade in flapping plane corresponding to this position is a good illustration of the concept

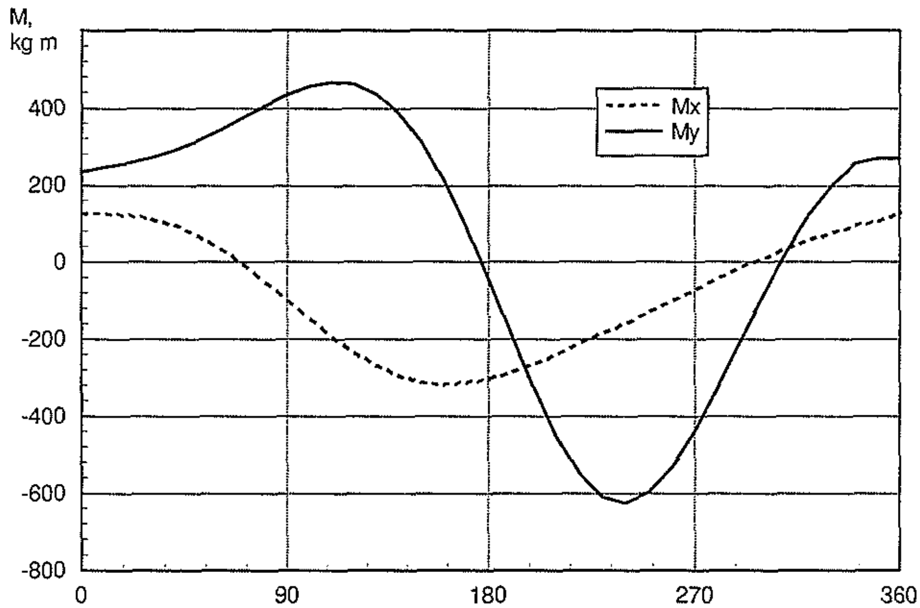


Fig. 6

Another way of studies of the bearingless main rotor with elastic torsional suspension of blades is to ensure helicopter balancing, stability and controllability for all modes of flight.

As a rule a number of parameters, which are found in the certain single aerodynamic calculation of main rotor are also used in the calculation of some features of helicopter balancing. They are corrected using the approximated formulas when the control parameters are changed. It is a traditional way to use a hinge main rotor model with the absolute rigid structure of blades and minor offset of axis of horizontal flight (<5%) in the calculation of aerodynamic features. Sometimes the spatial motion of helicopter is divided into two separate motions: longitudinal and transverse one.

This approach results in hard restrictions on control parameters and design features. The bearingless rotor with the elastic element of torsional type corresponds to these requirements only partly. Concerns to the studying rotor, it is necessary to choose the equivalent joint of such a rotor taking into consideration the coincidence of the first fluctuation of its own frequency in planes of flap, or the equality of controlling moments on the bushing. But these approach give different results.

In the given report there is a mathematical model of calculation of helicopter balancing features which is done using the spatial scheme. Rectilinear steady flight of single-rotor helicopter can be operated either without the slide but with roll angle, or without roll but with slide angle. This division of problem into two parts (longitudinal and transverse motion) is not quite correct.

Let's use the following equation of motion of a helicopter for solving a problem of helicopter balancing:

$$\begin{aligned}
&m(\dot{V}_x+\omega_y V_z-\omega_z V_y)=R_x-G_x;\\
&m(\dot{V}_y+\omega_z V_y-\omega_x V_z)=R_y-G_y;\\
&m(\dot{V}_z+\omega_x V_y-\omega_y V_x)=R_z+G_z;\\
&J_x\cdot\dot{\omega}_x+(J_z-J_y)\cdot\omega_z\omega_y=M_x;\\
&J_y\cdot\dot{\omega}_y+(J_z-J_x)\cdot\omega_x\omega_z=M_y;\\
&J_z\cdot\dot{\omega}_z+(J_y-J_x)\cdot\omega_x\omega_y=M_z.
\end{aligned} \tag{5}$$

where $\overline{G}=\{G_x,G_y,G_z\}$ — gravity force (attached in center of mass of helicopter);; $\overline{\dot{V}}=\{\dot{V}_x,\dot{V}_y,\dot{V}_z\}$ —acceleration (speedup) of helicopter's center of mass in the coupled coordinate system; $\overline{\dot{\omega}}=\{\dot{\omega}_x,\dot{\omega}_y,\dot{\omega}_z\}$, —angular acceleration in relation of helicopter' center of masses, $\overline{\omega}=\{\omega_x,\omega_y,\omega_z\}$ — rate of helicopter' angular motion, $\overline{J}=\{J_x,J_y,J_z\}$— tensor of helicopter' inertia, $\overline{R}=\{R_x,R_y,R_z\}$ $\overline{M}=\{M_x,M_y,M_z\}$ — main vector and main moment of aerodynamic force

The parameters of control (operation) which are necessary for helicopter balancing are obtained using an equations system of type (10). The main unknowns of the system are six controlling actions. A number of these actions are equal to the number of equations in the system: φ_0 - pitch blades main rotor; χ, η - deflection angles of wobble plate; φ_{tr} - pitch blades of tail rotor; γ - roll angle; ϑ - helicopter pitch angle.

In the calculation of spatial motion of the main rotor blades the elastic blade model is developed on the basis of theory of fine beam of wing airfoil when greate displacement takes place[5]. Besides the angular spatial turning of accounting sections in the local coordinate system is used.

Equilibrium equations of deformed blade in local coordinate system take the shape:

$$\begin{aligned}
&\frac{\partial\varphi_1^*}{\partial s}=\frac{1}{\cos\varphi_1^*}\left[\left(\frac{M_\xi}{EI_\xi}+\omega_{01}\right)\cdot\cos\varphi_3^*-\left(\omega_{02}+\frac{M_\eta}{EI_\eta}\right)\cdot\cos\varphi_3^*\right];\\
&\frac{\partial\varphi_2^*}{\partial s}=\left(\frac{M_\xi}{EI_\xi}+\omega_{01}\right)\cdot\sin\varphi_3^*+\left(\omega_{02}+\frac{M_\eta}{EI_\eta}\right)\cdot\cos\varphi_3^*;\\
&\frac{\partial\varphi_3^*}{\partial s}=\left(\omega_{03}+\frac{M_\zeta}{GI_\delta}\right)-tg\varphi_2^*\cdot\left[\left(\frac{M_\xi}{EI_\xi}+\omega_{01}\right)\cdot\cos\varphi_3^*-\left(\omega_{02}+\frac{M_\eta}{EI_\eta}\right)\cdot\sin\varphi_3^*\right],
\end{aligned} \tag{6}$$

where $\varphi_1,\varphi_2,\varphi_3$ — an angle of rotation of section in the (flight) reference point, M_ξ,M_η,M_ς — moments of external (applied) forces, $\omega_{01},\omega_{02},\omega_{03}$ — initials of the curvature, EI_ξ,EI_η,GI_p — blade's rigidity in relation to the main axis of design section.

Numerical decision of equations of the blades motion in the frame of helicopter balancing equations system is fined by development of desired functions of a blade moving in harmonic series:

$$\Delta\beta_i=a_o^i+\sum_{j=1}^{\infty}\left(a_j^i\cdot\cos j\psi+b_j^i\cdot\sin j\psi\right),. \tag{7}$$

were a_o^i,a_j^i,b_j^i — factor of (resolution) development; $\psi=\omega t$ - blade's azimuth; ω — angular velocity of rotation; t — time. When magnitudes a_o^i,a_j^i,b_j^i are known, it's possible to calculate the moving in every point of blade. A nodal point of blade's elastic axis can be described in following expression:

$$y_n = \sum_{i=1}^{n}\left[a_o^i + \sum_{j=1}^{\infty}\left(a_j^i \cdot \cos j\omega t + b_j^i \cdot \sin j\omega t\right)\right] \cdot \sum_{j=i}^{n} \Delta r_j \quad (8)$$

Under the integrating matrixes [6] the obtained integral-differential equations can be reduced to the nonlinear algebraic equations using one of the methods of optimization search [7]. The flexibility matrix of torsion of type (3) is used as boundary conditions in the blade butt.

Thereby, when solving the balance equations it is possible to find not only parameters of helicopter control, but also dynamic loads and moving of the blades of main and tail rotor. The task of a helicopter balancing, calculation of loads on the blades of main and tail rotor of definition of required horsepower, which is necessary to calculate the aircraft performance characteristics can be correctly solved as a whole.

The strategy described above allows to receive the light helicopter balancing parameters. The influence of particularities of torsion elastic models on the light helicopter trim characteristics can be evaluate by means of comparative calculations for the follows three models of blade attaching to the rotor bushing:

- ·a classical rotor with hinge blades attaching; its shoulder of horizontal joint can be chosen according to the fundamental tone of fluctuations;
- ·a rotor which has rigidity in the horizontal joint. It linearly depends on rotation angle of blade;
- ·a rotor based on the elastic element of torsion type with parameters, defined by correlations of (3) shape.

Comparative analysis of the results which based on the models given above shows that a number of parameters have a different sensitivity degree connected with the chosen calculation models. Among the parameters which have low sensitivity there are hauling capacity, rotational moments and angle of incidence on main and tail rotors. The sensitivity of the operational parameters of longitudinal motion are less than sensitivity of transverse control parameters (Fig. 7 -19)

Thereby, characteristics of helicopter longitudinal moving with bearingless rotor can be correctly calculated using the rotor with the equivalent offset of horizontal joint. The reliable results connected with the parameters of transverse moving are obtained using the models with "real" elastic torsion. In other cases the divergence of results is over then 50%. It depends on the model chosen. The corner of transverse deflection of wobble plate changes even the sign depending on velocity of horizontal flight (Fig. 7).

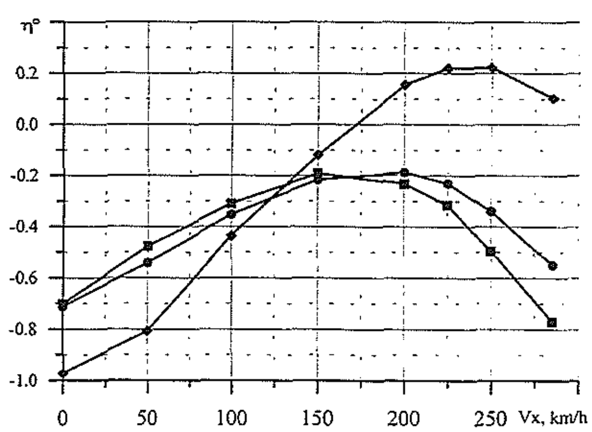
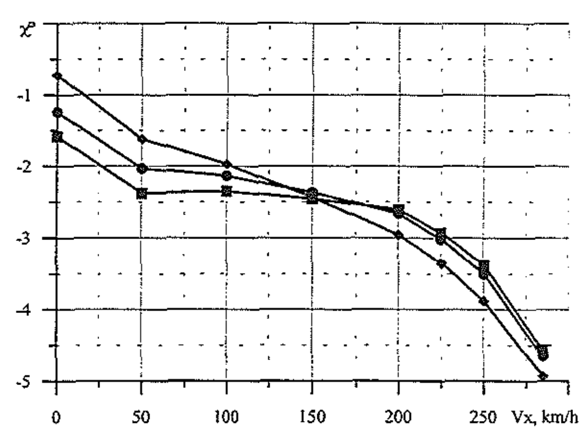


Fig. 7.



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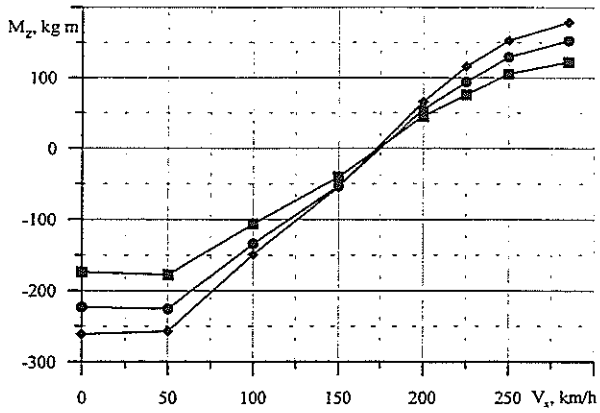

Fig. 9

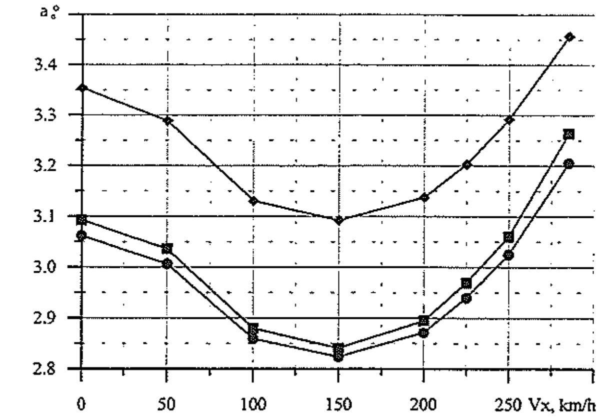

Fig. 13

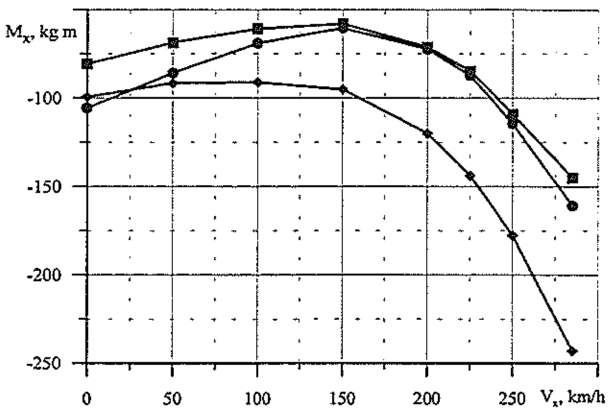

Fig. 10

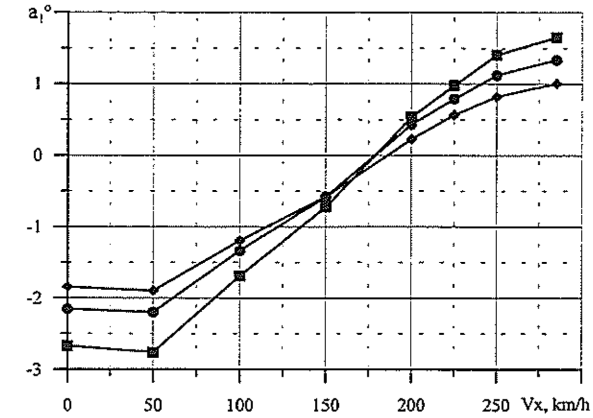

Fig. 14

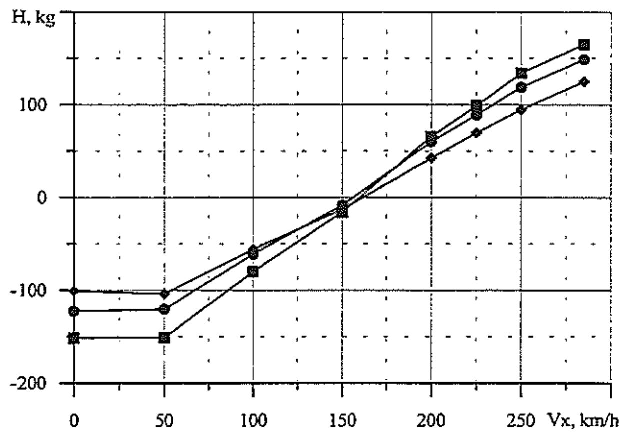

Fig. 11

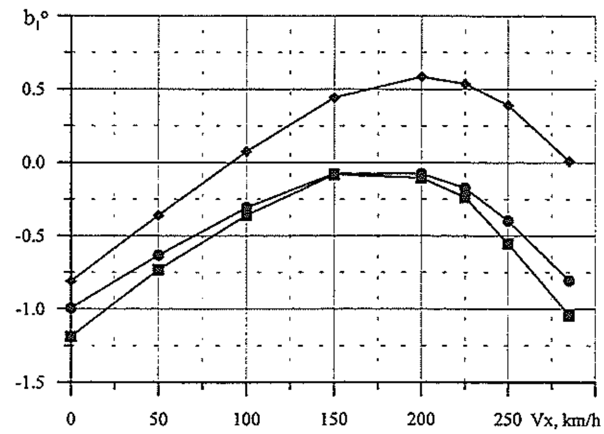

Fig. 15

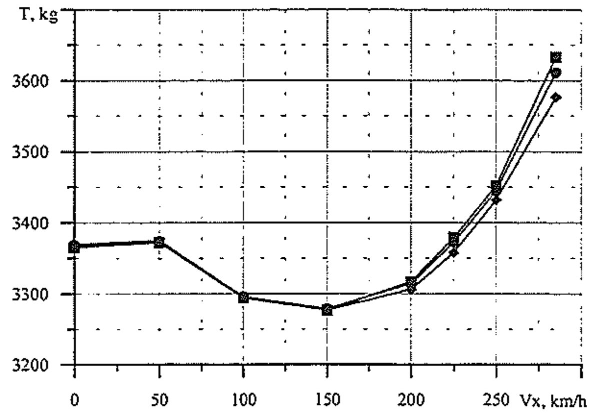

Fig. 12

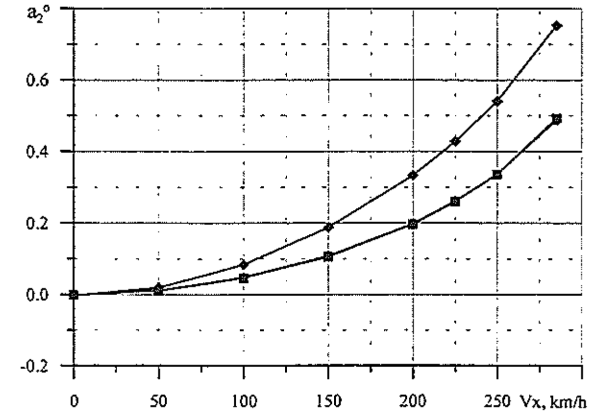

Fig. 16

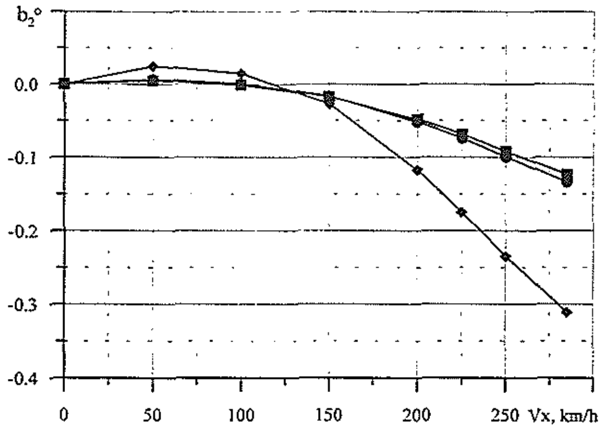

Fig. 17

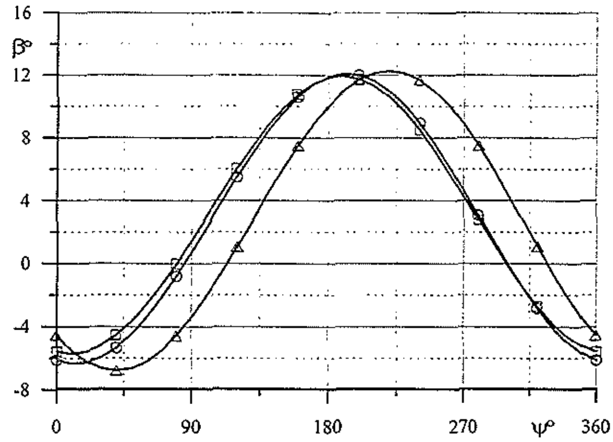

Fig. 19

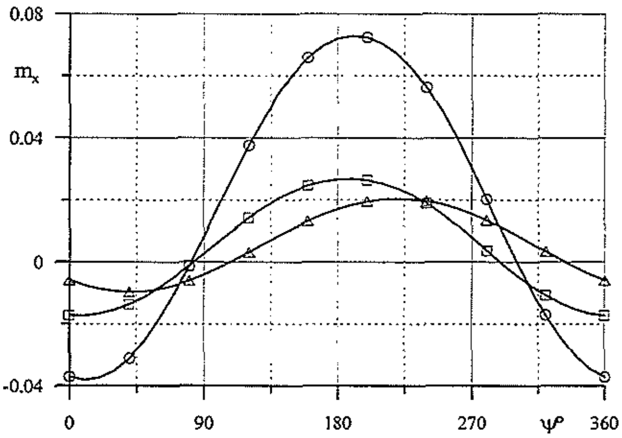

Fig. 18

Introduction of the elastic torsion models into numerical simulation has greatly influenced on the harmonic composition of the blades flapping motion. The latter determines the variable loads in sections, and enlarges bending moments on the rotor axis (Fig. 13-17).

The experimental studies of the choice and regulation of advanced angle of operation have shown the advantages of numerical evaluations of advanced angle on the stage of the overland regulation of helicopter rotor system (Fig. 8-10).

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