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A 2-D DYNAMIC STALL MODEL BASED ON A HOPF BIFURCATION

by

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A 2-D Dynamic Stall Model Based on a Hopf Bifurcation.

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Abstract: A mathematical model of 2-D dynamic stall is established by identifying stall onset as a Hopf bifurcation. It evolves from a first generation model which has been the object of a recent publication and from a careful analysis of fluid flow mechanisms involved in dynamic stall phenomena. Lift and moment coefficients are shown to be governed respectively by a system of nonlinear ordinary differential equations. The new model gives an insight into the physics of dynamic stall phenomena. The predictions of the model are in good agreement with experimental results in the case of the NACA 0012 airfoil, and are an improvement over those of the ONERA model.

1 Introduction

It is well known that prediction of helicopter rotor loads requires a better understanding of airfoil stall flutter on the retreating blade. Many investigations have been made on dynamic stall phenomena over the past 25 years: extensive wind and water tunnel tests of oscillating airfoils [1, 2], computational fluid dynamics simulation [3]. Reviews of dynamic stall phenomena [4, 5, 6] point out that they remain unsolved, particularly in their 3-D aspects. A mathematical model for airfoil unsteady aerodynamic behavior is needed for engineering rotor airload predictions. Such a model has to fullfil various requirements: to be sufficiently accurate for the prediction of aerodynamic coefficients, to be written in an analytical form compatible for coupling with the structural equations of an airfoil section, to have economical computational demands and to be rationally based. Various mathematical models were proposed and have met some limited success: Boeing Vertol model [7], Lockheed model [8], ONERA model [9], Leishman-Beddoes model [10] and other models reported in a review by McCroskey [6].

The mathematical model of 2-D dynamic stall elaborated at ONERA [9] is referred to as the ONERA model [11]. This model is written in terms of ordinary differential equations with the values of its coefficients deduced from a synthesis of experimental results. It provides predictions of aerodynamic coefficients comparable with those of other models [12]. However, for the next step of modeling 3-D dynamic stall, there is a need for an improved rationally based model.

Our approach relies on an analysis of fluid flow mechanisms involved in dynamic stall phenomena. Two distinct flow phenomena which are stall delay and vortex-shedding are responsible for dynamic stall behavior. The vortex shedding phenomenon is not well modeled by the existing mathematical models. To provide a consistent formulation and theoretical method of modeling this nonlinear aspect of dynamic stall, we have based our approach on the body of theory known as bifurcation theory. Particularly, we have made use of the results of analytical studies by Tobak et al. [13] who have identified stall onset as a Hopf bifurcation. A bifurcation of an aerodynamic system is defined as a replacement of an unstable equilibrium flow by a new stable equilibrium flow when the value of a parameter of the system reaches a critical value. In the case of a Hopf bifurcation, a time-invariant equilibrium flow is replaced by a periodic time-varying equilibrium flow. It is assumed to occur for the flow past an airfoil when the angle of attack exceeds a critical value. A model of 2-D dynamic stall has been established according to this theoretical postulate [14]. It is improved in this study, based on a careful analysis of fluid flow mechanisms which will be exposed in the next paragraph.

2 Modeling dynamic stall on the basis of bifurcation theory

Let us first take up the issue of aerodynamic bifurcation for a static airfoil. The flow past the airfoil remains stationary when the value of the angle of attack α is low. When the value of α is increased incrementally, separation of the flow occurs at a critical value α_{cr} and vortex-shedding begins. Figure 1 shows the unsteady flow structure about a NACA 0012 airfoil visualized in a water tunnel: vortices are shed from both the leading and trailing edges of the airfoil.

The analysis will be based on the behavior of the lift coefficient C_L, it could be done in a similar way by considering the other aerodynamic coefficients C_M and C_D. Typical experimental values of the lift coefficient are reported in figure 2: the values are well determined for $\alpha < \alpha_{cr}$ but are inherently scattered within some finite bounds for $\alpha \geq \alpha_{cr}$. According to Tobak et al. [13], onset of vortex-shedding is associated to a Hopf bifurcation. Experimental results suggest that it is a supercritical bifurcation, as in the case of the flow past a cylinder. No further aerodynamic bifurcation is assumed to occur over the remaining range of angle of attack. Based on this theoretical postulate, the lift coefficient can be decomposed into steady and unsteady components, denoted C_{L_s} and C_{L_u} respectively:

$$C_L = C_{L_s} + C_{L_u} \tag{1}$$

where C_{L_u} is characterized by an amplitude C_{L_0}, a frequency equal to the vortex-shedding frequency called the Strouhal frequency ω_S and a phase ϕ. Typical values of C_{L_s} and C_{L_0} are shown in figure 2.

When the airfoil experiences an unsteady motion, some new features of stall phenomena appear and constitute the so-called " dynamic stall phenomena". Consider for instance an airfoil undergoing a ramp pitch motion $\alpha(\xi)$, depicted in figure 3a/ : the angle of attack α increases linearly with time ξ, reaches the critical value α_{cr} at time $\xi = \tau_1$, attains α_{max} at $\xi = \tau_2$ and is held at this value for $\xi > \tau_2$. Typical experimental values of the lift coefficient recorded during the ramp motion α_ξ are shown in figure 3b/. C_L can be decomposed into the steady component C_{L_s} and the unsteady component C_{L_u}. It appears that during the time interval $[\tau_1, \tau_1 + \tau_d]$, C_{L_s} overshoots its maximum static value and tends toward a value representative of the attached flow condition (cf. Fig. 3b/). The dynamic stall delay phenomenon of airfoils experiencing unsteady motion has challenged aerodynamicists for many years. The fluid flow mechanisms involved in the stall delay phenomenon have been analysed to a great extent by Ericsson and Reding [8]. There are 2 fluid flow effects: "time lag effects" which have a quasi-steady nature and "boundary layer improvement effects". The latter provide the most important contribution to stall delay and are induced by the airfoil motion. In a reference frame attached to the airfoil, the typical instantaneous velocity profile u near the airfoil leading edge is depicted in figure 4. The velocity profile may be viewed, according to Tobak et al. [13], as having 2 components. The first profile u_s corresponds to the time-invariant profile which exists if the airfoil remains static. The second profile u_i represents the velocity induced by the airfoil motion. For a pitch rate $\dot{\alpha} > 0$, the airfoil motion induces a positive contribution and therefore stall is delayed to values of angle of attack beyond the static stall value α_{cr}. For a pitch rate $\dot{\alpha} < 0$, the induced contribution is negative and stall appears at lower values of angle of attack.

After a time delay τ_d (cf. Fig. 3), periodic vortex-shedding begins. The unsteady component C_{L_u} grows until the flow attains a periodic time-varying equilibrium state. Although it is largely recognized that the vortex-shedding phenomenon dominates the behavior of separated flow, few models have incorporated the periodic character of vortex-shedding. Results depicted in Fig.3, representative of experiments by Jumper et al. [15] and Lorber et al. [16], constitute evidence of multiple vortex shedding and its periodic occurrence. Scrutiny of experimental results on oscillating airfoils, published by the group of McCroskey [2], reveals the existence of well defined oscillations on the aerodynamic coefficients loops of $C_L(\alpha), C_M(\alpha$ and $C_D(\alpha)$.

Within our theoretical framework, let us examine an important characteristic of the dynamic stall phenomenon which is the non-repeatability of measurements of the aerodynamic coefficients. This characteristic was revealed by the early investigations of Liiva et al. [1] but wasn't explained. According to Tobak et al. [17], the determination of the periodic time-varying component C_{L_u} requires specification of all the 3 values of amplitude, frequency and phase. The phase value depends on the initial conditions of the flow. Available evidence [16, 18] shows that this dependence is very sensitive. Under these conditions, repeatable measurements could be obtained in low level turbulence tunnels if sufficient time is left between 2 runs: one has to wait for complete decays of the unsteady flow regime and of the flow perturbations generated by the strong dynamic stall vortex. Fig. 5 illustrates the case when insufficient time is left between the 2 consecutive runs: as the initial flow conditions at the second run differ from those of the first run, the lift coefficient C_L has different phase values in the 2 runs and therefore its measurement is non-repeatable. However, measurements of C_L are reproducible within a phase shift, in agreement with experiments [16, 18]. Examination of flow visualizations about oscillating airfoils [19] shows evidence that flow perturbations, generated by the dynamic stall vortex from an oscillatory cycle in pitch motion, subsist at the beginning of the succeeding cycle.

Due to the character of non-repeatablity affecting dynamic stall measurements, it is of standard practice in experimental procedure to do averaging over about 50 cycles. Some experimentalists [16, 20] warned that the averaging procedure smooths out the undulatory structure of aerodynamic coefficient measurements. As far as the undulatory behavior is believed to originate from spurious noise, the averaging procedure does not raise any criticism. However, a recent computational fluid dynamics simulation made by Geissler and Vollmers [3] reveals a pronounced oscillatory structure on the aerodynamic coefficient loops of $C_L(\alpha)$ and $C_M(\alpha)$ which are unmatched with the available experimental results averaged over numerous cycles. Another CFD simulation, done by Isogai [21], also provides evidence of oscillations of non negligible amplitudes. Examination of his computed isovorticity curves reveals that each oscillation on the aerodynamic coefficients loops is associated with a vortex shed from the airfoil leading edge. Recently, Panda and Zaman [22] have found experimentally the existence of oscillations on $C_L(\alpha)$ loops and have shown that it is related to vortices shed from the airfoil.

The Hopf bifurcation based approach appears to be in agreement with experimental results and CFD simulations. The modeling approach furthermore offers the capability of providing predictions about the nature of driven separated flows: these predictions have to be checked. Anticipating the next paragraph, the separated flow may be modeled as a nonlinear oscillator of frequency ω_S. Thus, the aerodynamic system (airfoil - flow) may be modeled as a coupled system of an oscillator of frequency k (the driving reduced frequency) with an oscillator of frequency ω_S. One should expect very diverse features for the flow behavior, as in the case of a cylinder [23] and in particular, a lock-in regime for some range of external frequency and amplitude.

3 Establishment of the mathematical model

3.1 Formulation of the modeling approach

With the knowledge of the fluid flow mechanisms involved in dynamic stall phenomena, various authors have derived semi-empirical mathematical models suited for rotor airload predictions [7, 8, 10, 11] Despite considerable progress made during this last decade, CFD simulation of dynamic stall will not be possible for some time in engineering rotor airloads analyses. Mathematical models are *de facto* practical engineering tools, although they are still imperfect. In particular, they don't incorporate the vortex-shedding phenomenon in a convenient way. To remedy this, a mathematical model has been recently established by the author [14]. The modeling effort has been focussed on the periodic vortex-shedding phenomenon and has ignored the delayed separation phenomenon. The model is updated in this paper to incorporate this aspect of flow phenomena. The key features of the model will be explained below, the details of its development appear in the publication referred to above.

The motion of the airfoil in a reference frame attached to the airfoil is completely determined by two variables: the angle of attack α and the pitch rate $\dot{\Theta} = q$ (Θ: pitch angle), or else the angle of attack and the plunging rate $\dot{h}$. Therefore, the model inherently requires experimental results in pitch and plunge motions as input, unless CFD results become available. The model is established by combining the indicial response approach, the amplitude equations approach and physical reasoning. The indicial response approach has been developed only for steady flow. This approach doesn't take into account the Hopf bifurcation. The amplitude equations approach has been developed in a generic way to apprehend the occurrence of Hopf bifurcation. The equations derived according to the second approach are not specific for the description of the flow about an airfoil. Physical assumptions are necessary to complete the elaboration of the model.

3.2 Modeling steady flow

The indicial response approach was derived rigorously from incompressible Navier-Stokes equations for steady flow which is characterized by a time-invariant equilibrium state [24]. The approach was developed for the modeling of the aerodynamic contribution to the equation governing the motion of an elastically mounted cylinder immersed in a uniform oncoming stream. The result was extended to the case of an airfoil [14]. The lift coefficient is shown to be governed by:

$$\frac{dC_{L_s}}{dt} + b\, C_{L_s} = b\, C_{L_s}^{equil}(\alpha(t), q(t)) + g_1\dot{\alpha}(t) + g_2\ddot{\alpha}(t) + g_3\dot{q}(t) + g_4\ddot{q}(t) + \vartheta(\dddot{\alpha}, \dddot{q}) \qquad (2)$$

where $C_{L_s}^{equil}$ is the equilibrium value of the lift coefficient which coincides with its static value; b, g_1, g_2, g_3 and g_4 are constants.

The same approach applied to the determination of the moment coefficient will give:

$$\frac{dC_{M_s}}{dt} + b' \; C_{M_s} = b' \; C_{M_s}^{equil}(\alpha(t), q(t)) + g_1' \dot{\alpha}(t) + g_2' \ddot{\alpha}(t) + g_3' \dot{q}(t) + g_4' \ddot{q}(t) + \vartheta(\dddot{\alpha}, \dddot{q}) \qquad (3)$$

3.3 Modeling nominally attached flow

Equations (2) and (3) can still be used to describe nominaly attached flow which results from the stall delay phenomenon. "Boundary layer improvement effects" are taken into account by using appropriate values for the equilibrium aerodynamic coefficients $C_{L_s}^{equil}$ and $C_{M_s}^{equil}$. They are chosen according to the model of Leishman-Beddoes. When the airfoil is static, the coefficient $C_{L_s}^{equil}$ is given by:

$$C_{L_s}^{equil} = C_L^{\alpha}(1+\sqrt{f})^2 \, \alpha \qquad (4)$$

where C_L^{α} is a constant and the separation point f is defined as:

$$f = \begin{cases} 1 - 0.3 \exp\left[(\alpha - \alpha_1)/s_1\right] & : \alpha \leq \alpha_1 \\ 0.04 + 0.66 \exp\left[(\alpha - \alpha_1)/s_2\right] & : \alpha > \alpha_1 \end{cases} \qquad (5)$$

where s_1 and s_2 are constants and α_1 is nearly equal to α_{cr}. When the airfoil undergoes an unsteady motion, f is replaced by f' which is governed by:

$$\frac{df'}{dt} + \frac{1}{T_f} f' = \frac{1}{T_f} f \qquad (6)$$

where T_f is a constant. In fact, the authors of the model referred to used 2 different values for T_f according to the values of f'.

To incorporate "time lag effects", instead of solving another ordinary differential equation as in the Leishman-Beddoes model [10], we prefer using the method of Ericsson and Reding [8] which consists in replacing the value of $\alpha(t)$ by a shifted value:

$$\alpha(t) \longrightarrow \alpha(t-\tau) \qquad (7)$$

where τ is a constant. It can be shown that the 2 methods of incorporating time lag effects are equivalent as a first approximation.

The same procedure is adopted for the determination of the moment coefficient.

3.4 Modeling unsteady flow

When the airfoil experiences an unsteady motion, stall occurs when α exceeds a critical value $\alpha_{cr}^{+}(k)$ which can be significantly greater than α_{cr}. The value of $\alpha_{cr}^{+}(k)$ takes into account stall delay phenomena. When α decreases after exceeding α_{cr}^{+}, the flow reattaches to the airfoil at a critical value $\alpha_{cr}^{-}(k)$ which can be significantly lower than α_{cr}. The values of $\alpha_{cr}^{+}(k)$ and $\alpha_{cr}^{-}(k)$ can be deduced from the following relation:

$$f' = 0.7 \qquad (8)$$

according to the Leishman-Beddoes model. In a first generation model [14] it is assumed that $\alpha_{cr}^{+}(k)$ and $\alpha_{cr}^{-}(k)$ are equal to α_{cr} respectively. One has 2 regimes for C_{L_u}, corresponding to growth and decay regimes of the periodic time-varying equilibrium state respectively. It is shown [14] that C_{L_u} obeys to a Van-der-Pol - Duffing type equation during growth regime:

$$\ddot{C}_{L_u} - \omega_S(\beta_L^{+} - \gamma_L^{+} \; C_{L_u}^2)\dot{C}_{L_u} + \omega_S^2(C_{L_u} - \eta_L^{+} C_{L_u}^3) = -E_L^{+}\omega_S\dot{\alpha} - D_L^{+}\omega_S\ddot{\alpha} \qquad (9)$$

where the constants are given a superscript + to characterize growth regime. A Van-der-Pol - Duffing type equation has been the basis of various mathematical models of flow past a cylinder but has never been applied to the case of an airfoil. The simplest way for modeling the decay regime is by a damped oscillator:

$$\ddot{C}_{L_u} - \omega_S \beta_L^- \dot{C}_{L_u} + \omega_S^2 C_{L_u} = 0 \tag{10}$$

where β_L^- is negative. It is possible to model the 2 regimes by the same type of analytical equation with, however, a different set of constants for each regime:

$$\ddot{C}_{L_u} - \omega_S(\beta_L^\pm - \gamma_L^\pm\, C_{L_u}^2)\dot{C}_{L_u} + \omega_S^2(\, C_{L_u} - \eta_L^\pm C_{L_u}^3 - a_{2,L}^\pm C_{L_u}^2) = -E_L^\pm \omega_S \dot{\alpha} - D_L^\pm \omega_S \ddot{\alpha} \tag{11}$$

One notices in Eq. (11) the presence of an additional term in $C_{L_u}^2$: when k increases, it provides a larger shift to C_{L_u} from the equilibrium value $\overline{C}_{L_u} = 0$. Such an analytical term is suggested by studies of Noack et al. [25], related to the description of the Karman vortex street generated past a cylinder. In total, Eq. (11) require 8 parameters.

The moment coefficient C_{M_u} is governed by an equation of the same form:

$$\ddot{C}_{M_u} - \omega_S(\beta_M^\pm - \gamma_M^\pm\, C_{M_u}^2)\dot{C}_{M_u} + \omega_S^2(\, C_{M_u} - \eta_M^\pm C_{M_u}^3 - a_{2,M}^\pm C_{M_u}^2) = -E_M^\pm \omega_S \dot{\alpha} - D_M^\pm \omega_S \ddot{\alpha} \tag{12}$$

It has been observed experimentally that the change in C_M induced by stall occurs at a value of angle of attack greater than that for the rise of C_L [26]. To incorporate this effect, we use different values of $\alpha_{cr}^\pm$ for the coefficients C_L and C_M:

$$\alpha_{cr,M}^\pm = \alpha_{cr,L}^\pm + \Delta\alpha \tag{13}$$

where the values of $\alpha_{cr,L}^\pm$ are deduced from equation (8).

4 Comparison between experimental results and model predictions

The case of the NACA 0012 airfoil in pitch motion is considered. The experimental data come from measurements by McAlister et al. [2] and correspond to the following conditions:

$$Mach = 0.3\ ,\ \Theta = 15^o + 10^o \sin(k\ t) \tag{14}$$

The parameters of equations (2) and (3) governing the steady components C_{L_s} and C_{M_s} can be easily determined according to the procedure of the ONERA model. The parameters involved in calculating the equilibrium values $C_{L_s}^{equil}$ and $C_{M_s}^{equil}$ are obtained according to the procedure of the Leishman - Beddoes model. The 8 parameters characterizing the unsteady components C_{L_u} and C_{M_u} are chosen such that the model reproduces at best the experimental results. The values of the parameters chosen for the lift coefficient are:

$$E_L^+ = 0.186\ ,\ D_L^+ = -0.89\ ,\ \beta_L^+ = 0.015\ ,\ \gamma_L^+ = 0.75\ ,\ \eta_L^+ = -0.6\ ,\ a_{2,L}^+ = 0\ ,\ \beta_L^- = -3.0 \tag{15}$$

The values of the parameters chosen for the moment coefficient are:

$$E_M^+ = -0.62\ ,\ D_M^+ = 0.455\ ,\ \beta_M^+ = 0.015\ ,\ \gamma_M^+ = 7.5\ ,\ \eta_M^+ = 0\ ,\ a_{2,M}^+ = -0.75\ ,\ \beta_M^- = -3.0 \tag{16}$$

The value of ω_S is common to both:

$$\omega_S = 2\pi S\ ,\ S = 0.124 \tag{17}$$

The initial conditions for lift and moment coefficients are fixed to zero.

The predictions of the model are given in Fig. 6 for the lift coefficient and in Fig. 7 for the moment coefficient. Also are represented the experimental results and the predictions of the ONERA model. The predictions of the new model are similar in first approximation to those of the ONERA model for values of reduced frequencies $k < 0.1$. However for $k \geq 0.1$, the predictions of the new model have a better agreement with the experimental results. Furthermore, the model predicts oscillations on the loops of the aerodynamic coefficients. Such oscillatory behavior is not clearly shown in experimental results, as they correspond to the averaging of about 50 cycles. It seems that the oscillatory behavior is more pronounced on other thin airfoils, such as Ames - 01, Wortman FX 69-H-098, Sikorsky SC - 1095, Hughes HH-02, Boeing-Vertol VR-7 and NLR-1 [2].

5 Conclusion

(i) A mathematical model of the aerodynamic contribution to the equations of motion governing an airfoil immersed in an oncoming fluid stream has been elaborated by identifying dynamic stall onset to a Hopf bifurcation. It is found that a set of nonlinear ordinary differential equations (ODE) governs the behavior of the aerodynamic coefficients C_L and C_M. The possibility of describing C_L and C_M beyond the Hopf bifurcation in terms of ODE's originates from the existence of a periodic time-varying equilibrium state of the flow.

(ii) The model provides a global description of dynamic stall phenomena. It gives an explanation of the character of non-repeatability of aerodynamic coefficient measurements based on the sensitivity of the phase of the periodic time-varying equilibrium state upon the initial conditions of the flow. The oscillatory behavior of aerodynamic coefficients during deep stall is associated with the periodic character of the vortex-shedding phenomenon. It is predicted that the flow past an airfoil should have a very varied behavior, and in particular a lock-in regime for an appropriate set of values of external forcing amplitude and frequency.

(ii) The predictions of the new model are in good agreement with experimental results in the case of the NACA 0012 airfoil, and are an improvement over those of the ONERA model.

(iv) The 2-D dynamic stall model requires knowledge not only of static values of aerodynamic coefficients but also of their unsteady behavior. By extrapolation, we expect that modeling of 3-D dynamic stall requires knowledge of 3-D aspects which have to be provided by experiments or by CFD simulation.

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Fig.1.- Flow phenomena of dynamic stall of a NACA 0012 airfoil in the ONERA water tunnel: $Re = 6000$.

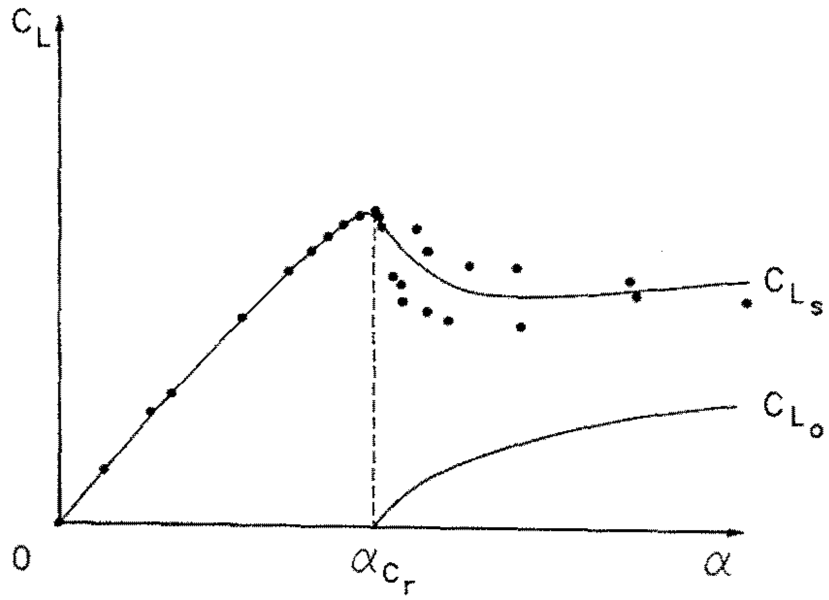


Fig.2.- Typical values of the lift coefficient C_L versus the angle of attack α: the measured static values are denoted by symbols •; beyond the Hopf bifurcation which occurs at α_{cr}, the lift coefficient can be decomposed into steady component C_{L_s} and unsteady component C_{L_u} of amplitude C_{L_o}.

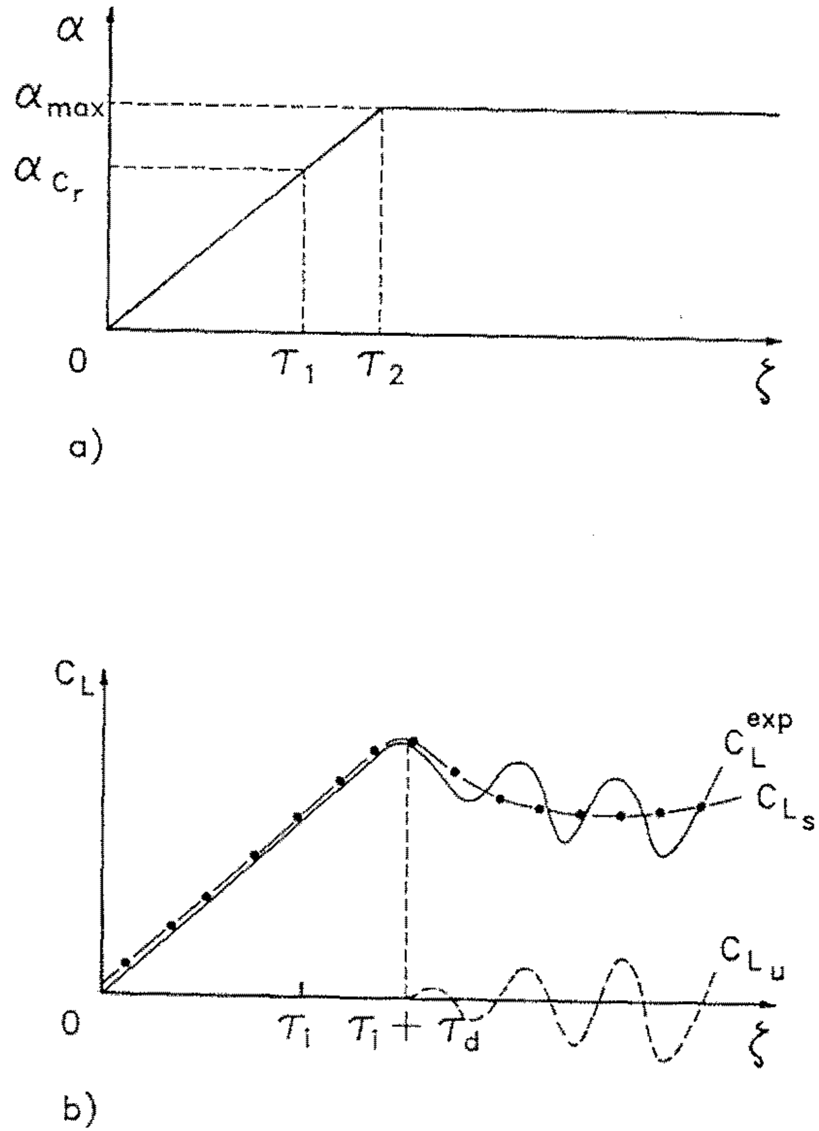


Fig.3.- Typical experimental values of C_L^{exp} (——) recorded during the pitch ramp motion $\alpha(\xi)$: C_L^{exp} is decomposed into steady component C_{L_s} (—·—·—) and unsteady componentC_{L_u} (- - -); C_{L_u} begins to grow at time $\tau_1 + \tau_d$ (τ_d: time delay) to attain its periodic time-varying equilibrium state.

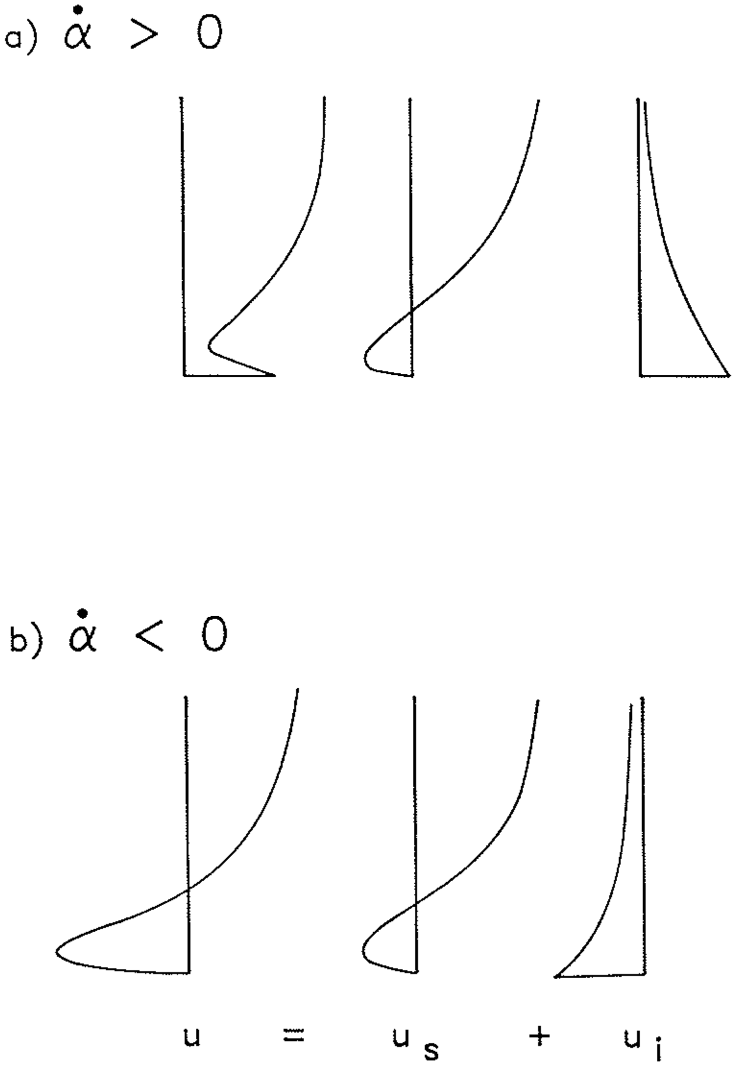


Fig.4.- Velocity profiles near the airfoil leading edge:
a/ pitch rate $\dot{\alpha} > 0$;
b/ pitch rate $\dot{\alpha} < 0$.

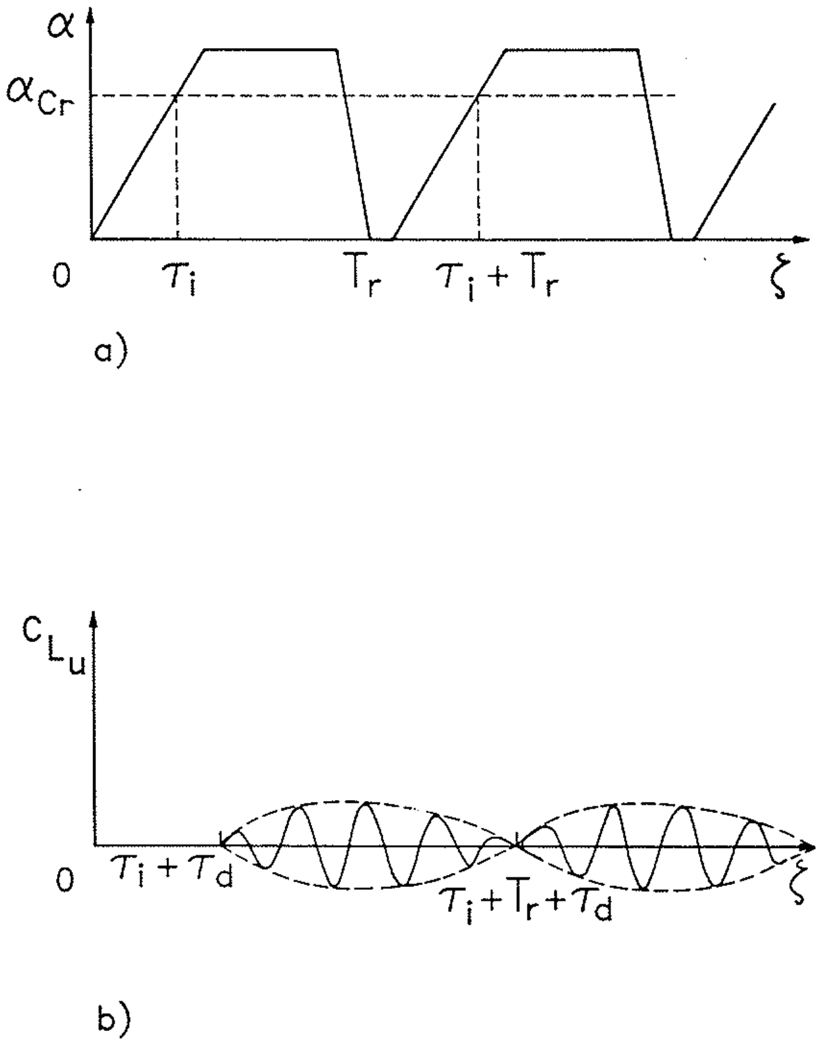
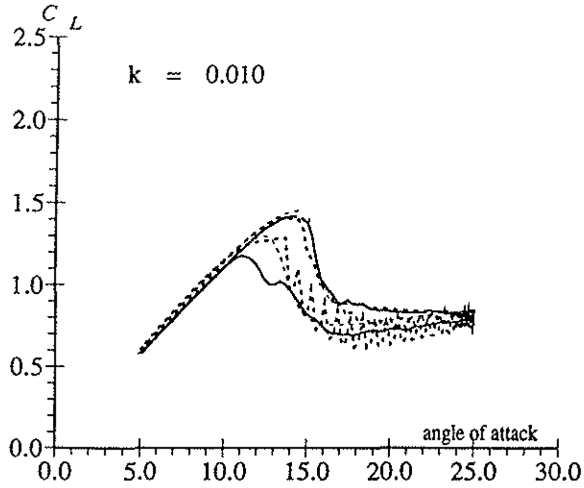
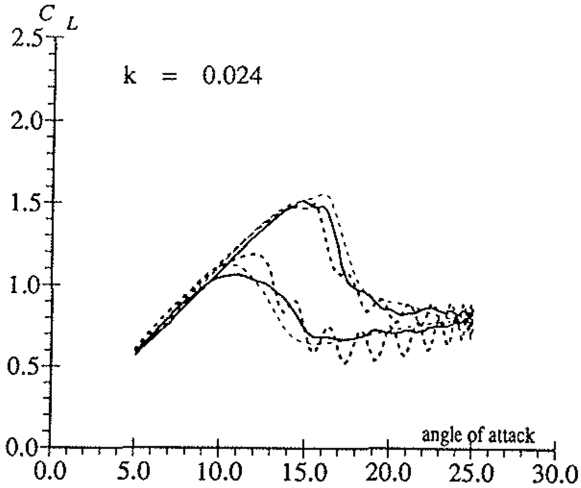
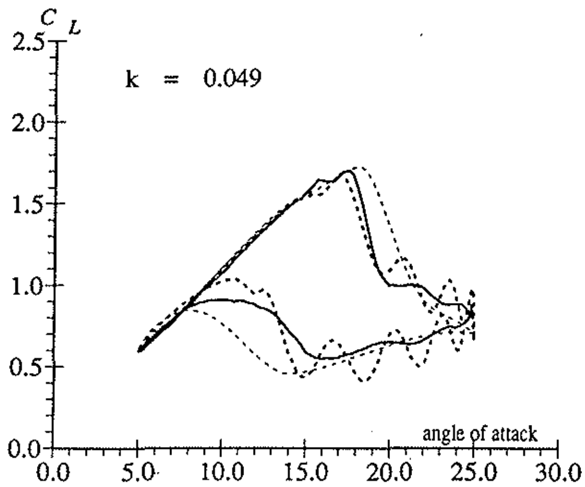
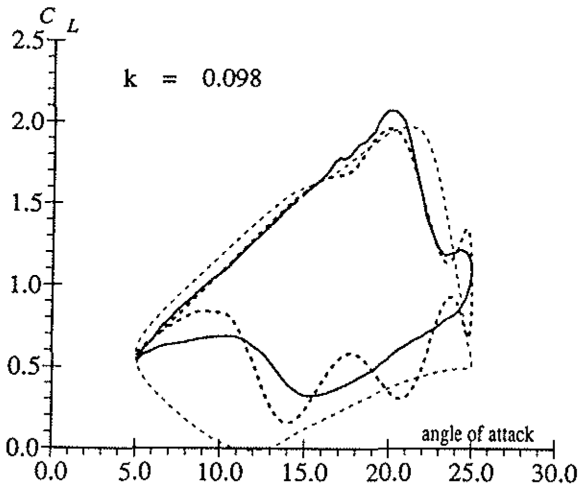


Fig.5.- Typical values of the unsteady component C_{L_u} of the lift coefficient during 2 successive runs of the pitch ramp motion: if insufficient time is left between the 2 runs, the initial conditions of the flow are different in the 2 runs and measurements of C_L are only reproducible within a phase shift.









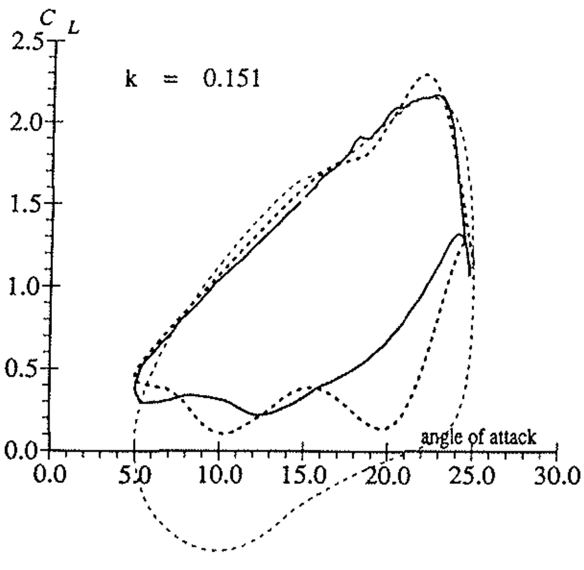
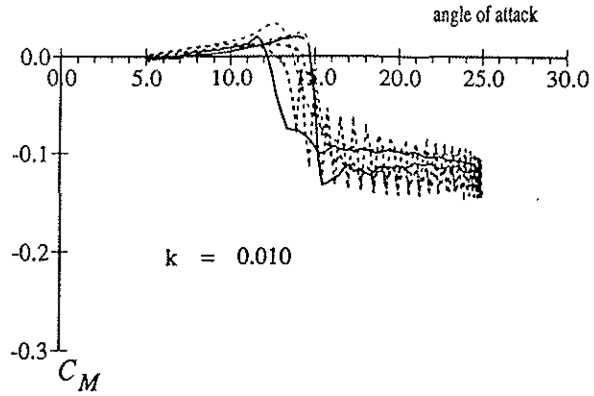
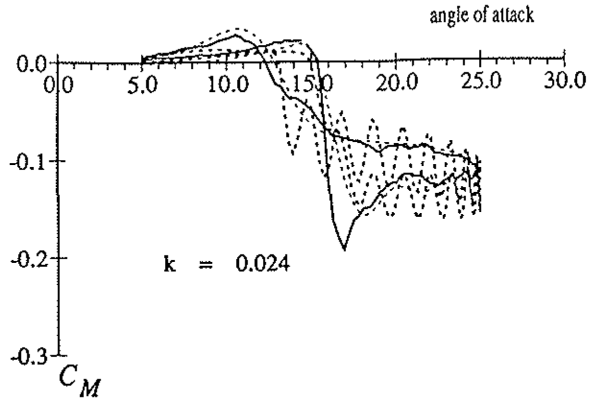
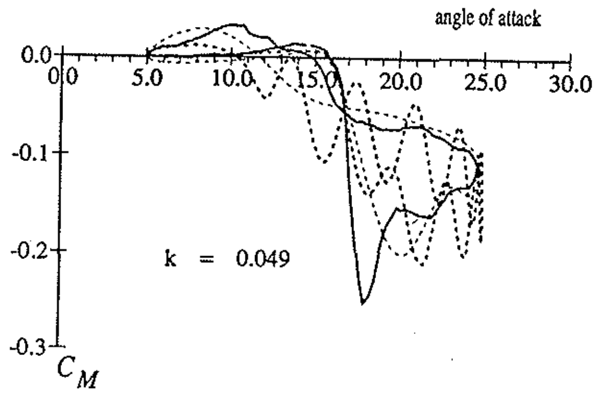
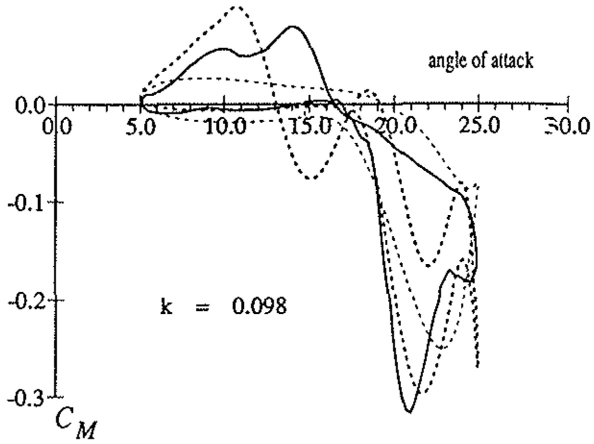


Fig.6. - Comparison of experimental lift coefficient loops $C_L(\alpha)$ for a NACA 0012 airfoil (———) with theoretical values predicted by the ONERA model (- - -) and by the new model (- - -), at various values of reduced frequency k.









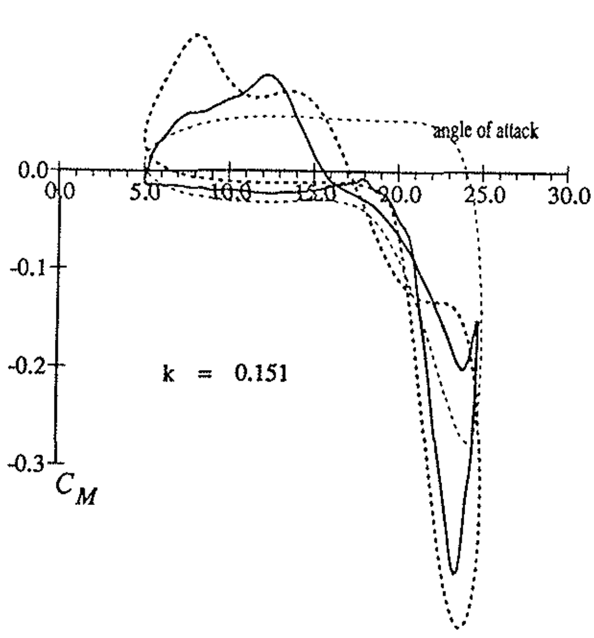


Fig.7. - Comparison of experimental moment coefficient loops $C_M(\alpha)$ for a NACA 0012 airfoil (——) with theoretical values predicted by the ONERA model (- - -) and by the new model (- - -), at various values of reduced frequency k.