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A GENERAL PURPOSE PROGRAM FOR ROTOR BLADE DYNAMICS

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Abstract

The work is being developed within the frame of a cooperation plan between Costruzioni Aeronautiche Giovanni Agusta and Aerospace Department of Politecnico of Milan, and covered by research contract n° 782.

The purpose of such a program was to produce and certificate an unique computer program for rotor blade dynamics, capable of dealing both with hinged and hingeless rotors, suitable for steady state flight analysis and stability evaluation.

Blade motion is represented by finite elements in the space-time domain, covering the span of one or more rotor revolutions, and allowing for different nonlinearities.

This method gives a flexible unitary approach for the very different model complexities required by the different design phases, such as the simplest rigid blade schemes, and the more sophisticated ones including nonlinearities arising from large blade flexibilities and large movements in control links. The aerodynamics, at the present development state, considers prescribed wake geometry and blade element theory, including stall and compressibility effects.

The paper presents a short discussion of the models and of the procedures employed and shows the first computed results.

1. - Introduction

The analysis of an helicopter rotor requires the determination of a periodic trim condition and the study of its stability; due to the high degree of nonlinearities involved in such an aeroelastic system, these studies are in general numerically performed with the help of a digital computer.

Different methods are available in the literature to solve each of the previously cited computational steps, but none of them provides an unified approach to both trimming and stability analysis. An extensive review of all these methods can be found in [1],[2] and [3] .

The helicopter rotor is generally modeled with a finite number of generalized coordinates by variable separation in the space and time domains.

Tipically the space shape of the motion is approximated by the natural modes of the blade rotating in vacuum, while the time dependency is provided by the time variation of their amplitudes. In this way we are led to the modeling of the rotor by a set of second order nonlinear ordinary differential equations in the time domain.

When a complete linearization of the problem can be accepted, a solution is assumed in a Fourier series form, and the unknown coefficients are determined by solving the resulting linear set of equations. With this procedure a periodic solution is implied and, although the method seems quite general and theoretically able to deal with nonlinear problems, the analytical burden posed by the coefficient matrix computation becomes practically unacceptable in a realistic analysis.

A second way makes use of step by step explicit numerical integration method, which seems to be free from any limitation on the form of the differential equations, and thus capable of handling more general systems, provided they can be reduced to the first order.

Halfway through the collocation and the integration method there is the procedure of [4] , which seems to be suitable only for linear equations, at least in the form there presented.

Hence a step by step method of integration seems to be the most general one in solving the set of nonlinear differential equations describing the motion of the rotor, but even such a method does not seems to be free from drawbacks.

It must be noted that, whilst the investigation of a periodic solution corresponds to the imposition of boundary conditions, the step by step procedure requires the knowledge of the initial conditions, and in general, for nonlinear equations, it may be difficult to found those initial conditions related to periodic solutions. Then the procedure is arbitrarily started and the integration performed until the steady state periodic solution is reached. It is obvious that the computer effort, and related cost, is stricly dependent on the stability of the set of equations, and that the lack of stability makes the method fail to converge. Such an event may be an indication of instability, but no systematic information is provided for a stability analysis, which has to be dealt with by a different approach.

The models used for rotor dynamics range from simplified schemes, for preliminary design, to general purpose detailed formulations, for stress analysis and whole aircraft response simulation.

The simple models are generally used in order to represent a particular design facet, and many variations are developed even by different groups of people involved in the same project.

On the other hand the overall models are often of general use only within a predetermined set of configurations, and in some cases, new design solutions cannot be evaluated. Thus both the simple and the general purpose models have to be rewritten and heavily modified in such a case.

There is then the need for an unified way to approach the modeling of the rotor so that either the preliminary design, detailed analyses and flight test validation of the design process can be undertaken within a single program. This unique tool should clearly be able to model the widest possible spectrum of rotor configurations, with the capability of dealing both with simple and sophisticated models; moreover the basic formulation should be " open-ended ", in the sense of being able to fulfill new needs by simply adding new options to the existing ones.

A joint effort has been established between Costruzioni Aeronautiche Agusta and the Politecnico of Milan in order to develop a computer program satisfying these requirements.

This paper outlines the basic concepts on which this program is being built. It is worth to note that the method here presented can be viewed as a nice application of the finite element method to the Hamilton's variational principle since, as it will be shown, periodic solutions can be easily imposed and both linearized stability and time response to an assigned control law can be studied, within an unified approach.

Another relevant feature of the finite element application to the Hamiltonian formulation is the capability to provide an automatic way to derive the system equations, with a minimum of analytical effort and with the consequent reduction in man hours spent to develop and check the needed formulas.

2. - Generalized Hamilton's principle

We assume the configuration of an arbitrary mechanical system be represented by a set of generalized coordinates {q} which can be submitted to a set of holonomic constraints given by :

$$\{ \Phi (q,t) \} = 0 \qquad (2.1)$$

and to a set of nonholonomic constraints in the form :

$$\{ \Psi (q,\dot{q},t) \} = | A | \{\dot{q}\} + \{a\} = 0 \tag{2.2}$$

It can then be shown [5] , [6] that a generalized form of the Hamilton's principle can be given by :

$$\int_{t_i}^{t_f} (\delta\pounds + \delta\pounds_e + \delta\pounds_c) \, dt + \int_{t_i}^{t_f} \{\delta\lambda\}^T \{\Phi\} \, dt + \int_{t_i}^{t_f} \{\delta\mu\}^T \{\Psi\} - \{\delta q\}^T \{p\} \Big|_{t_1}^{t_f} = 0 \tag{2.3}$$

where $\delta\pounds$ is the virtual change of the Lagrangian function, $\delta\pounds_e$ the external virtual work for those forces which cannot or are not included in the Lagrangian function and $\delta\pounds_c$ the virtual work of the forces of reaction which maintain the kinematical constraints. The virtual work $\delta\pounds_c$ has an expression of the type :

$$\delta\pounds_c = \{\delta q\}^T \{C\} \tag{2.4a}$$

where :

$$\{C\} = | B |^T \{\lambda\} + | A |^T \{\dot{\mu}\} \tag{2.4b}$$

and

$$|B| = |\frac{\partial \Phi}{\partial q}| \tag{2.4c}$$

$\{\lambda\}$ and $\{\mu\}$ are the vectors of the Lagrangian multipliers associated with the holonomic and nonholonomic constraints respectively.

It is important to note that the Hamilton's principle formulation, entailed by Eq.(2.3), allows to use an arbitrary virtual change $\{\delta q\}$ which does not vanish at the end points, because of the appearance of the generalized momentum :

$$\{p\} = \{ \frac{\partial \pounds}{\partial q} \}$$

in the left hand side of Eq.(2.3) [7] , [8]. Moreover the time derivatives $\{\dot{\mu}\}$ of the Lagrangian multipliers of the nonholonomic constraints in relationship (2.4b) leads to constraint reactions that have congruent virtual work related to the $\{\mu\}$ vector; at last the constraint relationship (2.1) and (2.2) are taken into account in the variational formulation itself by the second and third terms in the left hand side of Eq.(2.3).

Thus by this approach the system response equations, the boundary conditions and the constraints can be all obtained directly from the same Eq.(2.3), provided that the initial or boundary conditions satisfy Eq.(2.1) and (2.2) .

3. - Discretization and incremental formulation of Hamilton's principle

The system response is determined by a direct numerical discretization of Eq.(2.3). Clearly this discretization has to be performed simultaneously in space and time for the configuration unknowns and for the Lagrangian multipliers too; nevertheless, for sake of simplicity in notation and in hope to make the basic features of this approach more clear, the space dimensions and the constraints are not taken into account in this presentation.

On the basis of a finite element discretization in time domain, let $\{q\}_k$ be the generalized coordinate at any of the given N instants of time and :

$$\{qN\}^T = | \{q\}_1^T \{q\}_2^T \quad \cdots \quad \{q\}_N^T | \tag{3.1}$$

the vector describing the overall configuration of the system between ini-

tial and final time; the generalized coordinates are then determined at any time instant by means of appropriate shape functions $|N|$:

$$\{q(t)\} = | \; N(t) | \{qN\} \tag{3.2}$$

Recalling that :

$$\{\dot{q}\} = | \; \dot{N} \; | \{qN\} \tag{3.3a}$$

$$\{\delta q\} = | \; N \; | \{\delta qN\} \tag{3.3b}$$

$$\{\delta\dot{q}\} = | \; \dot{N} \; | \{\delta qN\} \tag{3.3c}$$

and by substitution of Eq.(3.2) and (3.3) into Eq.(2.3), we have :

$$\{\delta qN\}^T (\{L\} - \{BT\}) = 0 \tag{3.4}$$

with :

$$\{L\} = \{L_{\ell}\} + \{L_Q\} \tag{3.5}$$

$$\{L_{\ell}\} = \int_{t_i}^{t_f} (\; | \; \dot{N} \; | \{\ell_{\dot{q}}\} + | \; N \; | \; \{\ell_q\}) \; dt \tag{3.6a}$$

$$\{L_Q\} = \int_{t_i}^{t_f} | \; N \; |^T \{Q\} \; dt \tag{3.6b}$$

where $\{Q\}$ is the vector of the generalized external forces, $\{BT\}$ is related to the boundary terms, and $\{\ell_q\}$ and $\{\ell_{\dot{q}}\}$ are the derivatives of the Lagrangian function with respect to $\{q\}$ and $\{\dot{q}\}$, respectively.

If $\{Q\}$ does not contain impulsive forces, we have :

$$\{BT\}^T = | \; -\{p\}_1^T,0, \quad \ldots\ldots \quad ,0,\{p\}_N^T | \tag{3.7}$$

Eq.(3.4) can be used either for the solution of problems with given initial conditions or to obtain periodic solutions, if present.
In the first case we can write :

$$\{L\} = \{BT\} \tag{3.8}$$

and, as $\{q\}_1$ and $\{p\}_1$ are assigned, Eq.(3.8) becomes a set of $n \times N$, generally nonlinear, equations with the N-1 unknowns $\{q\}_k$,(k=2, ... ,N) and $\{p\}_N$.
In the second case, because of the periodicity, we have :

$$\{q\}_1 = \{q\}_N \tag{3.9a}$$

$$\{p\}_1 = \{p\}_N \tag{3.9b}$$

and

$$\{\delta q\}_1 = \{\delta q\}_N \tag{3.9c}$$

then from Eq.(3.4) we can write :

$$\{L\} = 0 \tag{3.10}$$

which, because of relationship (3.9a), constitutes a set of n x(N-1) equations with the N-1 unknowns $\{q\}_i$,(i=1, ... ,N-1) .

Clearly the functional of Eq.(2.3) can be discretized with many finite time elements by use of the well known assembly technique peculiar of the finite element method, and, in this case, Eq.(3.9a) is satisfied by trivially summing the contributions of the elements of closure on the same unknowns and equations.

It is now important to note that the discretization of Eq.(2.3) requires the continuity of only the shape functions and not that of their derivatives, in order to insure the convergence of the discretized solution. This is due to the fact that the Hamilton's principle is a weak integral formulation of the equations of dynamic equilibrium. This circumstance greatly enhances the efficiency of the approach, as compared to previous applications [9],[10] of the same idea, since no nodal derivatives explicitly appear as unknown, and also impulsive forces can be correctly taken into account.

The numerical solution of Eq.(3.8) and (3.10) requires an incremental formulation of the discretization, as given by :

$$\{L\}_0 + |K|_0 \{\Delta qN\} - \{BT\} = 0 \tag{3.11}$$

where

$$|K|_0 = |K_\pounds|_0 + |K_Q|_0 \tag{3.12a}$$

and

$$|K_\pounds|_0 = \int_{t_i}^{t_f} (|\dot{N}|^T |\pounds_{\dot{q}\dot{q}}|_0 |\dot{N}| + |\dot{N}|^T |\pounds_{\dot{q}q}|_0 |N|) \, dt +$$

$$+ \int_{t_i}^{t_f} (|N|^T |\pounds_{q\dot{q}}|_0 |\dot{N}| + |N|^T |\pounds_{qq}|_0 |N|) \, dt \tag{3.12b}$$

$$|K_Q|_0 = \int_{t_i}^{t_f} (|N|^T |Q_{\dot{q}}|_0 |\dot{N}| + |N|^T |Q_q|_0 |N|) \, dt \tag{3.12c}$$

where $|\pounds_{\dot{q}\dot{q}}|_0 |\pounds_{\dot{q}q}|_0$ and $|\pounds_{qq}|_0$ are the second derivatives of the Lagrangian function, and $|Q_{\dot{q}}|_0$ and $|Q_q|_0$ the derivatives of the generalized external forces . At a solution point we clearly have $\{L\} = \{BT\}$ and thus Eq.(3.11) can afford a linearized response for small disturbances, i.e.

$$| K |_0 \{\Delta qN\} = \{\Delta BT\} \tag{3.13}$$

With no impulsive forces we can reduce Eq.(3.13) to :

$$\begin{bmatrix} |K_{II}| & |K_{IF}| \\ |K_{FI}| & |K_{FF}| \end{bmatrix} \begin{Bmatrix} \{\Delta q\} \\ \{\Delta q\} \end{Bmatrix} = \begin{Bmatrix} -\{\Delta p\} \\ \{\dot{\Delta} p\} \end{Bmatrix} \tag{3.14}$$

which, having defined the incremental state vector as :

$$\{\Delta X\}_i = \begin{Bmatrix} \{\Delta q\}_i \\ \{\Delta p\}_i \end{Bmatrix} \qquad \{\Delta X\}_f = \begin{Bmatrix} \{\Delta q\}_f \\ \{\Delta p\}_f \end{Bmatrix}$$

can be rearranged to :

$$|TM| \{\Delta X\}_i = \{\Delta X\}_f \tag{3.15}$$

where :

$$|TM| = \begin{bmatrix} -|K_{FF}||K_{IF}|^{-1} & |K_{FI}| - |K_{FF}||K_{IF}|^{-1}|K_{II}| \\ \hline -|K_{IF}|^{-1} & |K_{IF}|^{-1}|K_{II}| \end{bmatrix} \tag{3.16}$$

is clearly the transition matrix, since it relates the final to the initial state. Eq.(3.15) is a linear matrix difference equation, a solution of which can be written in the form :

$$\{\Delta X\}_f = \tau \, \{\Delta X\}_i \tag{3.17}$$

which after substitution in Eq.(3.15) gives rise to the following eigenproblem :

$$|TM| \{\Delta X\}_i = \tau \, \{\Delta X\}_i \tag{3.18}$$

The eigenvalue of maximum modulus τ_{max} is related to the stability of the given solution, since, depending if the modulus of maximum eigenvalue is larger, equal or lower than 1, the solution of Eq.(3.17) is diverging, neutral or stable.

If the previous formulation is applied to a periodic solution of Eq.(3.10), we are led directly to a Floquet type study of the stability of linearized periodic set of equations [5],[11].

We note now that in order to take into account the constraints, we have just to make appropriate discretizations for the Lagrangian multipliers as :

$$\{\lambda\} = | N(t) | \{\lambda N\} \tag{3.19a}$$

$$\{\mu\} = | N(t) | \{\mu N\} \tag{3.19b}$$

where the shape functions need neither to be the same as the previously assigned nor to be equivalent for holonomic and nonholonomic constraints.

We can then arrive [6] to the same set of Eq.(3.4) with an augmented vector of generalized coordinates :

$$\{q^*\}^T = | \{qN\}^T, \{\lambda N\}^T, \{\mu N\}^T | \tag{3.20a}$$

and boundary terms of the type :

$$\{BT^*\}^T = | \{BT\}^T, \ 0 \ , \ 0 \ | \tag{3.20b}$$

4. - Rotor modeling and implementation problems

The helicopter rotor is modeled by beam, hub, scalar and kinematical constraint elements, whose degrees of freedom are referred to the rotating hub reference frame. The hub frame motion can either be assigned, or its translational and angular speeds can be assumed as unknowns along with overall associated loads and other global quantities, such as the mean induced velocity and swash plate position and orientation.

The beam,hub and kinematical constraint element are developed as space-time finite elements and their configuration is related both to their states at different time instants or azimuthal positions and to the global independent coordinates[5].

An isoparametric beam element has newly been developed for special purpose of curved and twisted rotor blade modelization; it is capable to describe large displacements and rotations with respect to a prescribed, generally moving, reference frame, and it can also include the possibility of rigid behaviour to be used in simplified analyses[5].

The hub can be modeled by the beam element itself or by a given flexibility and inertia matrix, which is extended as a time finite element.

Kinematical constraint elements are used to represent, if they are present, flap, lead-lag and feathering hinges. These elements are developed by use of Lagrangian multipliers, and they impose the congruence between blade and hinges degrees of freedom. The pitch control is determined from the swash plate position and orientation, considering the pitch control rod as inextensible.

The scalar elements are used to model dampers, concentrated stiffnesses and masses and, since they have no space dimensions, only a time development is required.

By means of these elements the analyst can develop the most suitable model to carry out different types of analyses, within an unified approach.

The development of the above elements, within the framework of the finite element discretization outlined in the previous paragraph, limits the hand work to write only the basic formulas, leaving all the integration problems to the computer; thus all tedious algebraic manipulations, which are prone to errors and difficult to check, are avoided.

The basic analyses that can be performed are the obtainment of a trimmed periodic solution for the rotor and the study of its stability. These problems require the solution of the nonlinear set of algebraic Eq.(3.8), which, for detailed elastic models, implies a large number of unknowns, and thus an heavy utilization of the computer resources is required. This circumstance is stressed on by the fact that a rather wide spectrum of trimmed conditions in hovering and forward flight are generally required.

The problem to handle a large number of unknowns is not related to this formulation only, and it is one which generally suggests the use of the natural modes of some typical configurations, in order to reduce the elastic degrees of freedom, while insuring a good convergence.

Nevertheless the use of displacements and rotations of the nodal points as unknowns simplifies the development of a general finite element: hence this choice of degrees of freedom has been preferred, making use of all the appropriate techniques developed to cope with the problems with a large number of unknowns. Thus, for instance, the need to obtain many trimmed solutions in different flight conditions is properly fulfilled by a solution in a continuation form, and a modal condensation technique can be viewed as a way to improve the speed of each iteration, without using the modes in the basic formulation; moreover full advantage is drawn from the sparsity of the coefficient matrix [12],[13],[14], [15],[16] .

Many other techniques are available to improve the efficiency of the nonlinear solver and they are intended to curtail the number of incremental steps, subsequently reducing the coefficient matrix computations and factorizations [17].

The stability analysis of the trimmed motion is just a by-product of the previous phases, since the obtainment of Eq.(3.13) and of the transition matrix can be embedded in the factorization of the $|K|$ matrices at convergence; then

the solution of the eigenproblem is performed by generally available library routines.

It is important to note that the explicit evaluation of the transition matrix, representing the perturbed motion near a trim condition, is a great undertaking, especially if flexible blades are taken into account.

The difficulty to automatically write the governing differential equations and to evaluate the transition matrix by numerical integration of the differential equations, for a whole set of independent initial conditions, has in practice limited the use of the Floquet method to rather simple models of rotors [18], [19],[20] .

An area of great concern to this approach to rotor dynamics analysis comes from the aerodynamic modeling, especially if the aerodynamic is taken within the |K| matrix. The problem of interfacing the structural to the aerodynamic model is common to all the aeroelastic calculations and generally different rappresentation are taken for each model, and an appropriate interface is established by the use of matrices of generalized aerodynamic forces, which are the aerodynamic part of the |K| matrix. Generally these matrices are full and this is a further reason for the use of some kind of modal coordinates in place of displacements of the nodal points.

It must be noted that a correct formulation of the motion dependent incremental aerodynamic matrix is more important in a correct stability analysis than in the trim condition search, in which it is only important to correctly determine the force unbalance at the nodes, in order to assure a correct solution, while the contribution to the |K| matrix can be approximated in any way that does not damage the convergence speed.

At the present time the program adopts a rather crude strip theory with prescribed induced inflow and some basic correction for unsteady effects [5].

The improvement of the aerodynamic model is the real crux of the formulation, since a correct and complete account of the historical effects related to unsteadyness, wake and dynamic stall completely couples the set of equations; this fact, when it is added to the cost of the aerodynamic computations, can destroy the efficiency of the method.

Therefore the strip theory will be maintained, but it will be joined iteratively to a separate aerodynamic module, which should give improved inflow and unsteady corrections, on the basis of an assumed trim motion.

The use of modal reduction techniques [14],[15],[16] can be of some help in improving the economy of the coupling procedure.

5. - Basic validation and concluding remarks

The development of a computer program based on the formulation outlined in this paper requires substantial computer and manpower resources, both because the fairly new formulation implies the need of careful and extensive basic tests and because it is necessary to implement state of the art numerical techniques in order to obtain an acceptable efficiency of the program being produced.

A basic complete version has been by now completely programmed and it is undergoing final tests. Here we will now show some of the basic examples used to validate the formulation.

The first one is concerned with the trimming and stability analysis of periodic solutions of Duffing's equation :

$$p^2\eta'' + 2p\xi\eta' + \eta + \delta\eta^3 = \cos\psi$$

which has been used to check the basic concepts of this approach [21] . Fig. 1 shows the trimmed solution in terms of the dynamic amplification factor versus the frequency parameter, for different dampings factor and a rather high nonlinear term δ.   This results are obtained by using six three-node elements covering one period and they have been checked by an accurate explicit integration method, iterated till a periodic motion is reached. Figures 2 and 3 show the behaviour of the eigenvalue of largest absolute module, as obtained by following
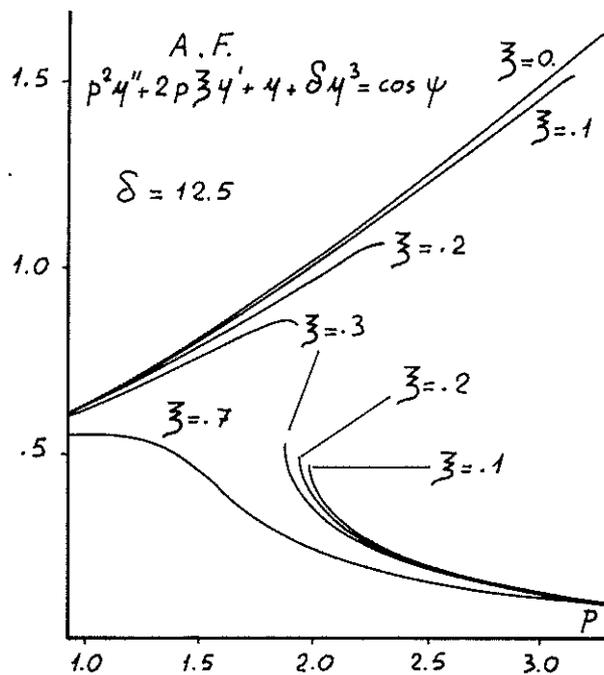
Fig. 1 - Amplification factor vs. frequency parameter
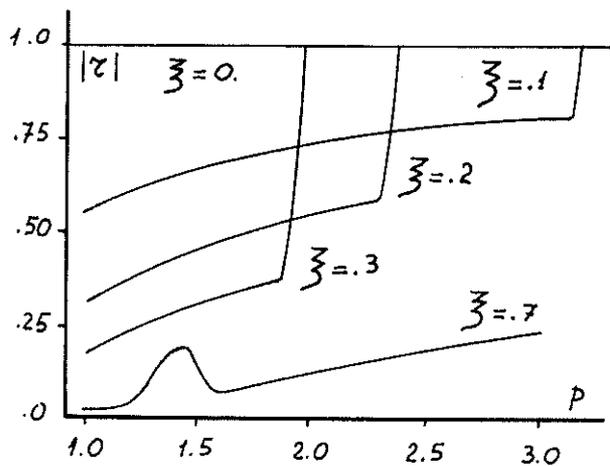
Fig. 2 - Stability behaviour
increasing p
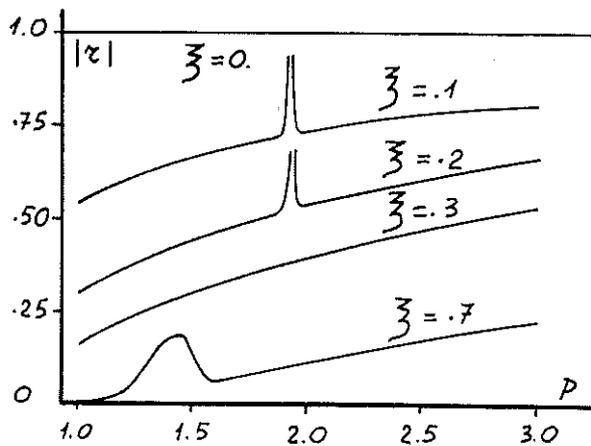
Fig. 3 - Stability behaviour
decreasing p

the stable branches of the response curves at increasing and decreasing frequency parameters.

It can be noted that the unstable branch of the response curve is well marked by these diagrams, while it is sometimes difficult to ascertain the stability by means of the explicit integration.
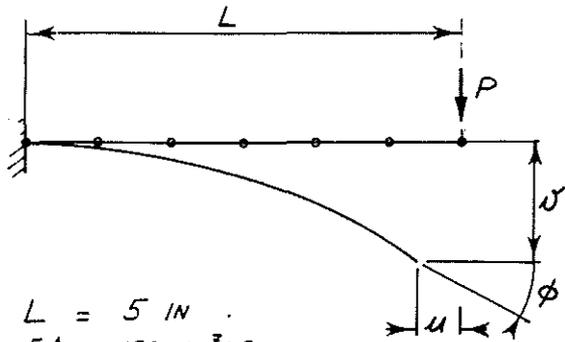
L = 5 IN .
EA = .150·10⁷LB
GA = .573·10⁷LB
EJ = .312·10⁵LB·IN²

20 LOADING STEPS
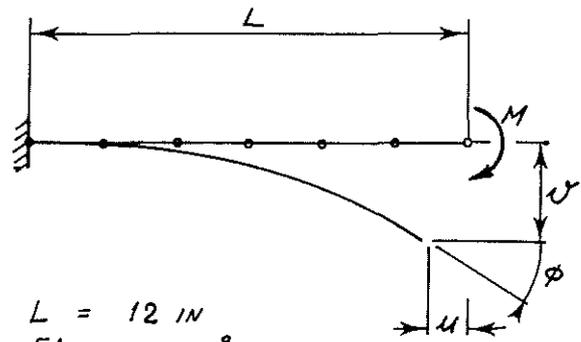
L = 12 IN
EA = .300·10⁸ LB
GA = .114·10⁸ LB
EJ = .250·10⁷ LB·IN²

6 LOADING STEPS

TWO FOUR-NODES BEAM ELEMENT
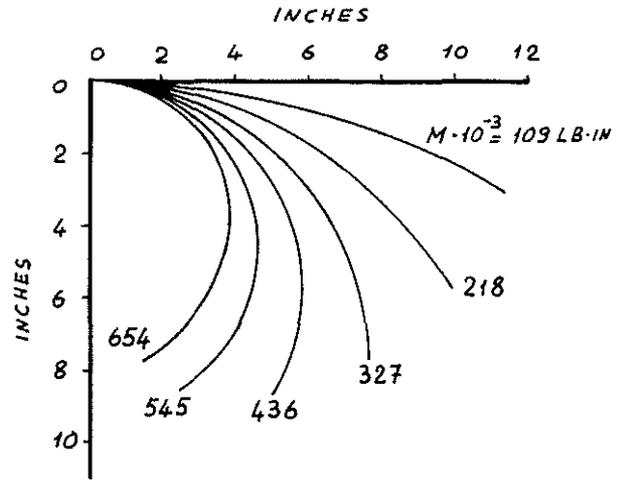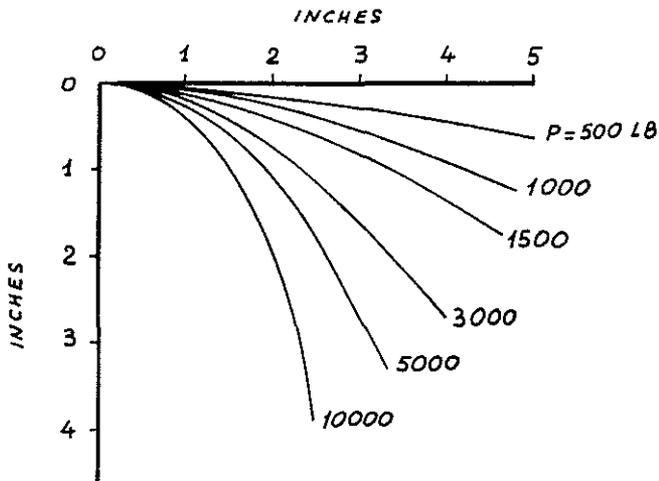
DEFLECTION SHAPES

$$\zeta = \frac{PL^3}{EJ}$$

$$\eta = \frac{ML}{2\pi EJ}$$
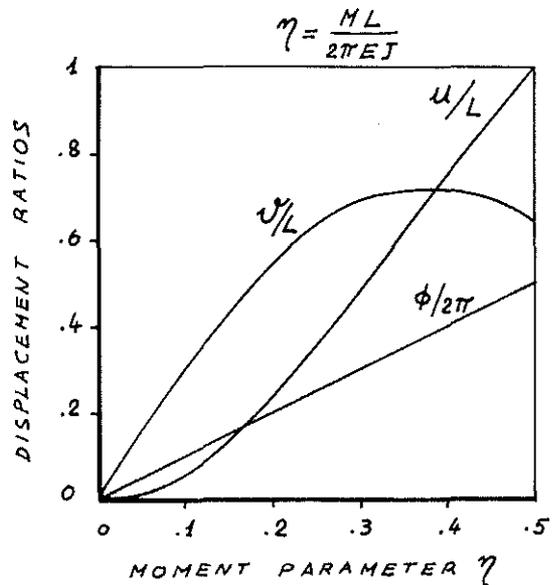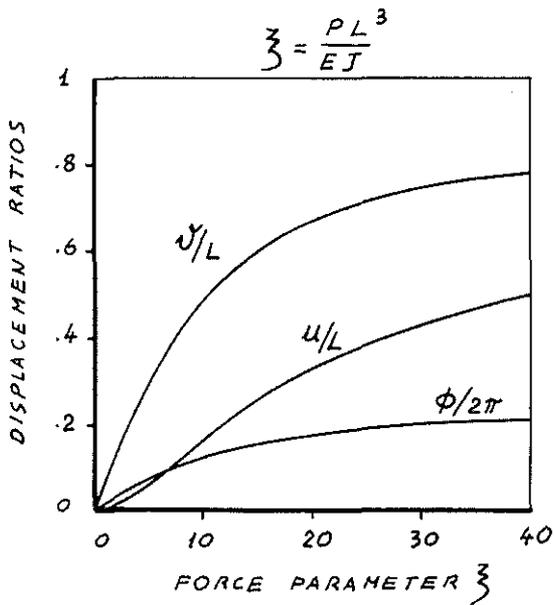
DISPLACEMENT RATIOS

Fig. 4 - Nonlinear static tests of the beam element

36-10

The capability of the blade beam element to correctly describe large displacements is shown by Fig. 4, which favorably compares with the results of Ref.[22] and [23]. This element is capable to represent arbitrarily curved and pretwisted blades with non coincident deformation and stress resultant center points. The previous results are obtained   within the general program by a constant forcing over an assigned periodic motion, with aerodynamic and inertia forces suppressed.

ɣ	FH	LH	CO	R	C	RS	L	E
36°	150	85	1630	5265	335	180	240	125
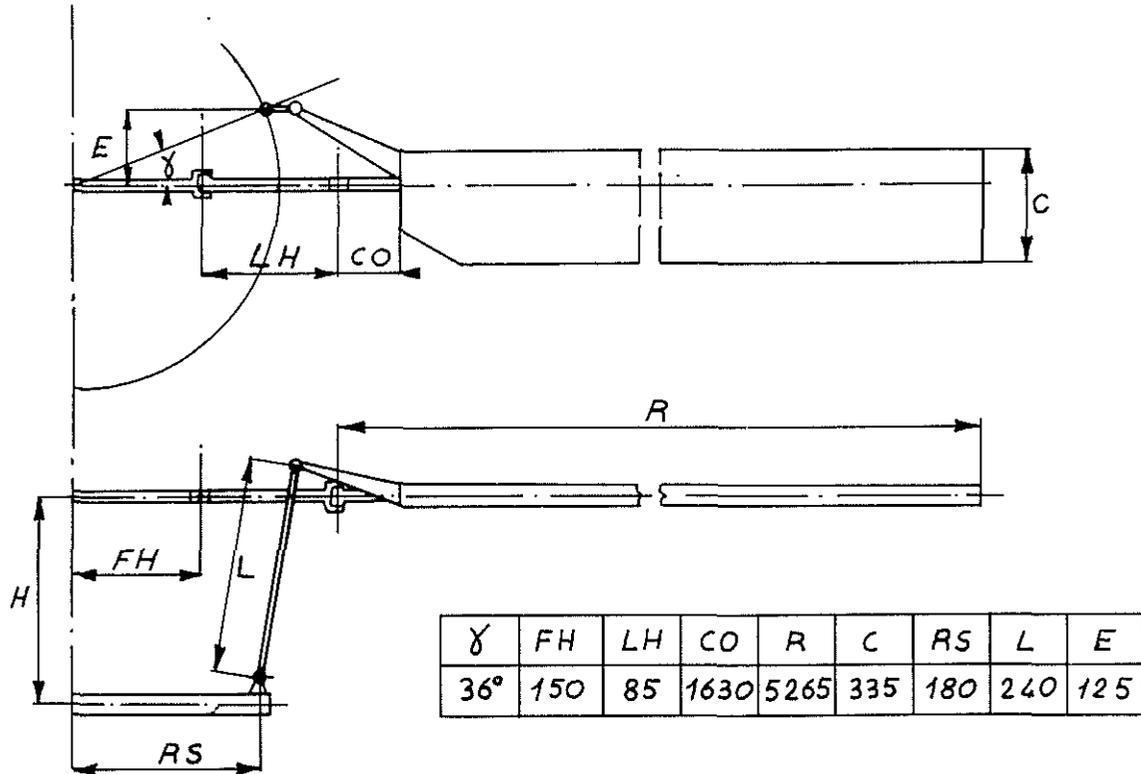
Fig. 5 - Rotor geometry

The remaining figures show some trim and stability results related to a realistic rigid blade of the articulated rotor, sketched in Fig. 5.

In particular Fig. 6 demonstrates fast convergence to the hovering trim, which is important in establishing a base solution for a continuation approach to other conditions, such as the one of Fig. 7, which is related to an high advance ratio and presents a rather severe stall on the retreating blade.

Fig. 8 and 9 demonstrate the stability margins for some flight conditions and at different rotor speeds.

More substantial results of calculations relating to a rotor with an elastic blade cannot be presented yet, but nevertheless we think the results shown substantiate the main features of this approach, i.e. :

- minimum analytical development effort and almost automatic formulation of the response equation;
- capability of directly trim the rotor to a periodic motion;
- sound approach to the stability check of the periodic solution, with the transition matrix afforded as a by-product of the procedure;
- open ended formulation usable for either simple or sophisticated analyses.

A point that remain to be solved is the development of a procedure that can efficiently interface this method with more sophisticated aerodynamic formulations than simple strip theory. This problem is more crucial in this formulation than in other ones, because it can substantially effect the sparsity of the incremen-

tal matrix, the speed of convergence to the trimmed state and the correct deter-
mination of the transition matrix.
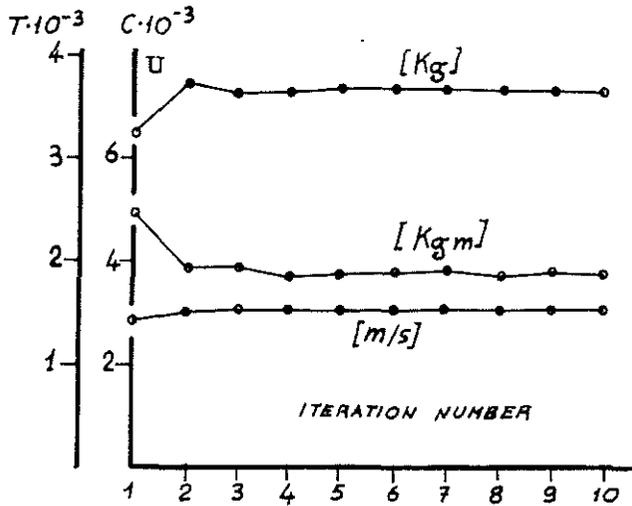
Fig. 6 - Convergence behaviour
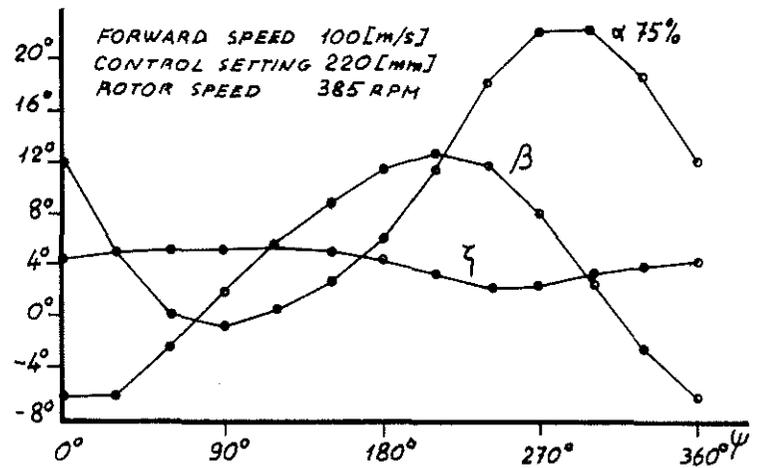
Fig.7 - Flap β, lead-lag ζ and blade angle
angle of attack α against azimuthal
position
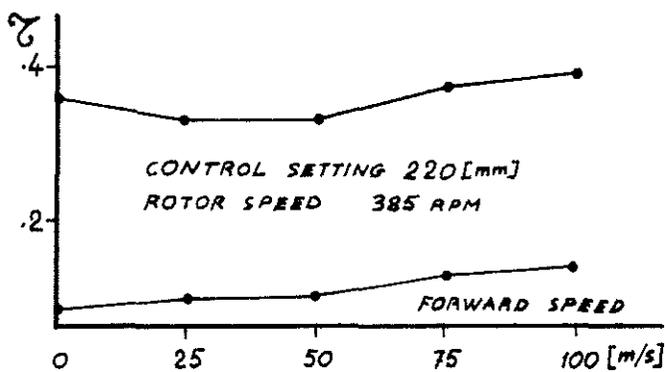
Fig. 8 - Stability behaviour against
forward speed
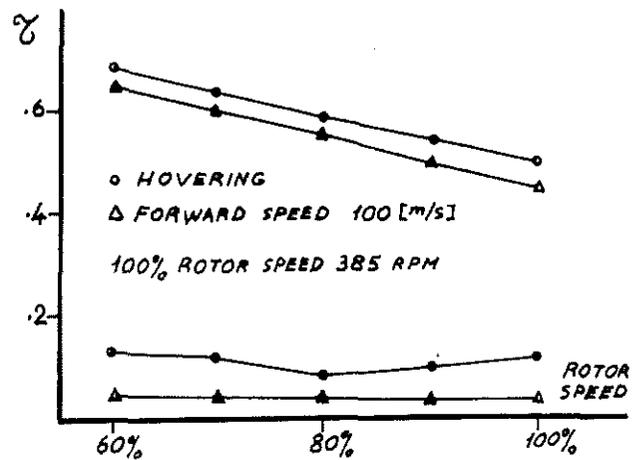
Fig.9 - Stability behaviour against
rotor speed

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References

1. Many authors   Specialist Meeting on Helicopter Rotor Loads Prediction Methods.
AGARD CP 122 (1973)

2. K.H. Hohenemser   Hingless Rotorcraft Flight Dynamics.
AGARD AG 197 (1974)

3. Many authors   Rotorcraft Dynamics.
NASA SP-352 (1974)

4. A. Berman   A New Approach to Rotor Blade Dynamic Analysis.
Journal of the American Helicopter Society, July 1965

5. M. Borri, M. Lanz, P. Mantegazza   Studio dinamico delle condizioni di regime e delle instabilità dei rotori di elicottero : formulazione dei modelli e del metodo di integrazione.
Rapporto Tecnico AG03/01, October 1980

6. M. Borri, P. Mantegazza   Studio dinamico delle condizioni di regime e delle instabilità dei rotori di elicottero : descrizione dei vincoli articolazione mozzo e vincoli di comando.
Rapporto Tecnico AG 03/03, December 1980

7. C. Lanczos   The Variational Principle of Mechanics.
2nd ed., The University of Toronto Press, (1964)

8. T.H.H. Pian, T.F. O'Brien   Transient Response of Continuous Structures Using Assumed Time Functions.
9th Int. Cong. of Appl. Mech., Brussels (1956)

9. I. Fried   Finite Element Analysis of Time Dependent Phenomena.
AIAA Journal, (1969) 7 (6) 1170-1173

10. M. Geradin   A Classification and Discussion of Integrator Operators for Transient Structural Response.
AIAA Paper 74-105 (1974)

11. G. Sansone   Equazioni differenziali nel campo reale.
2nd ed., Zanichelli, Bologna (1975)

12. P. Hood   Frontal Solution for Unsymmetric Matrices.
Int. Journ. of Num. Meth. in Eng. (1976) 10 379-399

13. J.M. Ortega, W.C. Rheinboldt   Iterative Solution of Nonlinear Equations in Several Variables.
Academic Press, New York (1970)

14. R.W. Clark, R.M. James   New Approach to the Solution of Large, Full Matrix Equations.
AIAA Journal (1981) 19 (2) 184-190

15. F.K.Straub, P. Friedmann   A Galerkin Type Finite Element Method for Rotary-wing Aeroelasticity in Hover and Forward Flight.
VI European Rotorcraft and Powered Lift Aircraft Forum, paper n° 15, Bristol 1980

16. A.K. Noor   Recent Advances in Reduction Method for Non-linear Problems.
Computer & Structures (1981) 13 31-44

17. M. Geradin, S. Idelsohn, M. Hogge   Computational Strategies for the Solution of Large Nonlinear Problems via Quasi-Newton Methods.
Computer & Structures (1981) 13 73-81

18. K.G.H. Gaonkar, D.S. Simha Prasad, D. Sastry   On Computing Floquet Transition Matrix of Rotorcraft.
V European Rotorcraft and Powered Lift Aircraft Forum, paper n° 45, Amsterdam 1979

19. P.P. Schrage, D.A. Peters   Effects of Structural Coupling Parameters on the Flap-Lag Forced Response of a Rotor Blade in Forward Flight Using Floquet Theory.
IV European Rotorcraft and Powered Lift Aircraft Forum, paper n° 23, Stresa 1978

20. M. Borri, P. Mantegazza   Su alcune forme di instabilità dei rotori semirigidi bipala.
l'Aerotecnica Missili e Spazio (1974) 5 313-322

21. M. Borri, G.L. Ghiringhelli, P. Mantegazza, D. Orlandi   Studio dinamico delle condizioni di regime e delle instabilità dei rotori di elicottero : casi prova metodo di integrazione. Applicazione all'equazione di Duffing.
Rapporto Tecnico AG03/02, November 1980

22. S.C. Tang, K.S. Yeung   On the Tangent Stiffness Matrix in a Convected Coordinate System.
Computer & Structures (1980) 12 849-856

23. K.J. Bathe, S. Bolourchi   A Geometric and Material Nonlinear Plate and Shell Element.
Computer & Structures (1980) 11 23-48