CONTINUOUS-TIME PREDICTOR-BASED SUBSPACE IDENTIFICATION FOR HELICOPTER DYNAMICS

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In this paper the current state-of-the-art in subspace model identification is reviewed, specific issues such as the estimation of continuous-time models are discussed and a set of methods suitable for time-domain, continuous-time identification of rotorcraft dynamics is presented. The proposed technique, which can deal with data generated under feedback, is illustrated by means of simulation results.

INTRODUCTION

The derivation of accurate dynamic models for helicopter aeromechanics is becoming more and more important, as progressively stringent requirements are being imposed on rotorcraft control systems: as the required control bandwidth increases, accurate models become a vital part of the design problem. In this respect, system identification has been known for a long time as a viable approach to the derivation of control-oriented dynamic models in the rotorcraft field (see for example the recent books [28, 13] and the references therein).

In the system identification literature, on the other hand, one of the main novelties of the last two decades has been the development of the so-called Subspace Model Identification (SMI) methods (see for example the books [31, 36]), which have proven extremely successful in dealing with the estimation of state space models for Multiple-Inputs Multiple-Outputs (MIMO) systems. Surprisingly enough, in spite of the ease with which SMI can be exploited in dealing with MIMO modelling problems, until recently these methods have received limited attention from the rotorcraft community, with the partial exception of some contributions such as [35, 5, 19]). SMI methods are particularly well suited for rotorcraft problems, for a number of reasons. First of all, the subspace approach can deal in a very natural way with MIMO problems; in addition, all the operations performed by subspace algorithms can be implemented with numerically stable and efficient tools from numerical linear algebra. Finally, information from separate data sets (such as generated during different experiments on the system) can be merged in a very simple way into a single state space model. Recently, see [17], the interest in SMI for helicopter model identification has been somewhat revived and the performance of subspace methods has been demonstrated on flight test data. However, so far only methods and tools which go back 10 to 15 years in the SMI literature (such as the MOESP algorithm of [32] and the bootstrap-based method for uncertainty analysis of [6]) have been considered. Therefore, the further potential benefits offered by the latest developments in the field have not been fully exploited. Among other things, present-day approaches can provide:

- unbiased model estimates from data generated during closed-loop operation, as is frequently the case in experiments for rotorcraft identification (see, e.g., [7, 12]);
- the possibility to quantify model uncertainty using analytical expressions for the variance of the estimates instead of relying on computational statistics (see [7]);
- the direct estimation of continuous-time models from (possibly non-uniformly) sampled input-output data (see [2, 3, 4] and the references therein).

In view of the above discussion, this paper has the following objectives. First, the current state-of-the-art in SMI is reviewed and specific issues such as the estimation of continuous-time models are discussed. Second, a set of methods suitable for time-domain, continuous-time identification of rotorcraft dynamics is presented. The proposed technique can deal with data generated in closed-loop operation as it does not require restrictive assumptions in this sense. Finally, the achievable model accuracy is illustrated by means of simulation results for a full-scale helicopter.

SUBSPACE MODEL IDENTIFICATION: A SHORT OVERVIEW

As recounted in [9], by the late 70s the theory of MIMO linear systems had been completely understood, and yet from a practical point of view black-box identification of MIMO systems remained an issue until the late 80s. The cause for this was the estimation of the structural indices that characterize the parameterizations of MIMO systems, which is tricky and often leads to ill-conditioned numerical problems (see e.g., [10]). Therefore, there was a strongly felt need for simple, possibly suboptimal, procedures bypassing the need for estimating structural indices. SMI methods offered exactly the potential to overcome this difficulty. In the last twenty years or so, SMI algorithms have been developed, which have proven extremely successful in dealing with the estimation of discrete-time state space models for MIMO systems. Classical SMI methods, developed in the early 90s for the estimation of discrete-time models, are the MIMO Output-Error State sPace (MOESP, see [32] and the references therein) class of algorithms, based on the idea of estimating a basis of the observability subspace directly from data, and the N4SID algorithm (see [29]), which relies on the estimation of the state sequence for the system as an intermediate step for the estimation of the state space model. A tutorial, detailed account of such methods can be found in the textbooks [31, 36]. Besides the possibility of dealing with MIMO problems in a simple and natural way, one of the keys to the success of SMI methods in applications is that all the operations performed by subspace algorithms can be implemented with numerically stable and efficient tools from numerical linear algebra, based on the numerically robust QR factorisation and on the singular value decomposition (see, e.g., [26]).

Not surprisingly, the problem of extending SMI methods to the identification of continuous-time systems has been studied in a number of contributions. In [30] a frequency-domain approach to subspace identification of continuous-time models was proposed, while a time-domain SMI algorithm able to identify a continuous-time model from sampled input-output data was first proposed in [11], building on the framework introduced in [14]. More precisely, in the cited thesis the class of SMI algorithms was extended to the identification of continuous-time models through the use of Laguerre filters: this allowed the development of a method that deals with noise in a similar way as its discrete-time counterparts. More recently, in [22] the version of the MOESP algorithm presented in [33, 34] was adopted and a discrete-time algebraic equation was derived starting from sampled input-output data by describing derivatives of stochastic processes in the distribution sense, while in [1, 20] the combination of the MOESP algorithm with filtering methods to avoid the need to compute numerical derivatives of input-output signals was proposed. In [25] a novel approach to the problem of continuous-time SMI has been presented, based on the adoption of orthonormal basis functions to arrive, again, at a MOESP-like data equation for a continuous-time system.

All the above mentioned contributions, however, assume that the system under study is operating in open-loop. This assumption is frequently restrictive in practice and is typically violated in aerospace applications, in which partial loop closures must be retained during identification experiments, primarily for safety issues (see, e.g., [13, 16, 28]). The problem of closed-loop SMI has been studied extensively in recent years due to its high relevance for practical applications (see, e.g., [18, 8, 7, 12] and the references therein). The present state-of-the-art is represented by the so-called Predictor-Based Subspace IDentification (PBSID) algorithm (see, again, [7]) which, under suitable assumptions, can provide consistent estimates of the state space matrices for a discrete-time, linear time-invariant system operating under feedback. The problem of closed-loop subspace identification in continuous-time has been first considered in the literature in [21], where the application of the errors-in-variables approach of [8] is proposed to deal with correlation in a continuous-time setting. More recently, see [2, 3, 4], novel continuous-time SMI schemes, based on the derivation of PBSID-like algorithms within the all-pass domains proposed in [11] and [25] and relying, respectively, on Laguerre filtering and Laguerre projections of the sampled input-output data have been proposed.

PROBLEM STATEMENT AND PRELIMINARIES

Consider the linear, time-invariant continuous-time system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t),\ x(0) = x_0 \\ y(t) &= Cx(t) + Du(t) + v(t) \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are, respectively, the state, input and output vectors and $w \in \mathbb{R}^n$ and $v \in \mathbb{R}^p$ are the process and the measurement noise, respectively, with covariance given by

$$E\left\{\begin{bmatrix} w(t_1) \\ v(t_1) \end{bmatrix} \begin{bmatrix} w(t_2) \\ v(t_2) \end{bmatrix}^T\right\} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta(t_2 - t_1).$$

The system matrices A, B, C and D are such that (A, C) is observable and $(A, [B, Q^{1/2}])$ is controllable. Assume that a dataset $\{u(t_i), y(t_i)\}$, $i \in [1, N]$ of sampled input/output data (possibly associated with a non equidistant sequence of sampling instants) obtained from system (1) is available. Then, the problem is to provide estimates of the state space matrices A, B, C and D (up to a similarity transformation) on the basis of the available data. Note that unlike most identification techniques, in this setting incorrelation between u and w,v is not required, so that this approach is viable also for systems operating under feedback.

In the following Sections a number of definitions will be used, which are summarised hereafter for the sake of clarity (see, e.g., [37, 14, 23] for further details).

Let $\mathcal{L}_2(0, \infty)$ denote the space of square integrable and Lebesgue measurable functions of time $0 < t < \infty$. Consider the first order all-pass (inner) transfer function

$$w(s) = \frac{s - a}{s + a}, \quad (2)$$

$a > 0$, together with the associated realisation

$$w(s) = \frac{c_w b_w}{s - a_w} + d_w, \quad (3)$$

where $a_w = -a$, $b_w = -\sqrt{2a}$, $c_w = \sqrt{2a}$, $d_w = 1$. $w(s)$ generates the family of Laguerre filters, defined as

$$\mathcal{L}_i(s) = w^i(s)\mathcal{L}_0(s) = \sqrt{2a}\frac{(s - a)^i}{(s + a)^{i+1}}. \quad (4)$$

Denote with $\ell_i(t)$ the impulse response of the i-th Laguerre filter. Then, it can be shown that the set

$$\{\ell_0, \ell_1, \ldots, \ell_i, \ldots\} \quad (5)$$

is an orthonormal basis of $\mathcal{L}_2(0, \infty)$, i.e., all signals in $\mathcal{L}_2(0, \infty)$ can be represented by means of the set of their projections on the Laguerre basis.

FROM CONTINUOUS-TIME TO DISCRETE-TIME USING LAGUERRE PROJECTIONS

The continuous-time algorithms discussed in this paper are based on the results first presented in [25, 23], and further expanded in [15, 24], which allow to obtain a discrete-time equivalent model starting from the continuous-time system (1), along the following lines.

First note that under the assumptions stated in the previous section, (1) can be written in innovation form as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ke(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \quad (6)$$

and it is possible to apply the results of [25] to derive a discrete-time equivalent model, as follows. Consider the first order inner function $w(s)$ defined in (2) and apply to the input u, the output y and the innovation e of (6) the transformations

$$\begin{aligned} \tilde{u}(k) &= \int_0^\infty \ell_k(t)u(t)dt \\ \tilde{y}(k) &= \int_0^\infty \ell_k(t)y(t)dt \\ \tilde{e}(k) &= \int_0^\infty \ell_k(t)e(t)dt, \end{aligned} \quad (7)$$

where $\tilde{u}(k) \in \mathbb{R}^m$, $\tilde{e}(k) \in \mathbb{R}^p$ and $\tilde{y}(k) \in \mathbb{R}^p$. Then (see [25] for details) the transformed system has the state space representation

$$\begin{aligned} \xi(k+1) &= A_o\xi(k) + B_o\tilde{u}(k) + K_o\tilde{e}(k),\ \xi(0) = 0 \\ \tilde{y}(k) &= C_o\xi(k) + D_o\tilde{u}(k) + \tilde{e}(k) \end{aligned} \tag{8}$$

where the state space matrices are given by

$$\begin{aligned} A_o &= (A - aI)^{-1}(A + aI) \\ B_o &= \sqrt{2a}(A - aI)^{-1}B \\ K_o &= \sqrt{2a}(A - aI)^{-1}K \\ C_o &= -\sqrt{2a}C(A - aI)^{-1} \\ D_o &= D - C(A - aI)^{-1}B. \end{aligned} \tag{9}$$

CONTINUOUS-TIME PREDICTOR-BASED SUBSPACE MODEL IDENTIFICATION

In this Section a summary of the batch continuous-time PBSID algorithm proposed in [2, 4] is provided, and its implemention is discussed.

Starting from system (6), in this Section a sketch of the derivation of a PBSID-like approach to the estimation of the state space matrices A_o, B_o, C_o, D_o, K_o is presented. Considering the sequence of sampling instants t_i, $i = 1, \dots, N$, the input u, the output y and the innovation e of (6) are subjected to the transformations

$$\begin{aligned} \tilde{u}_i(k) &= \int_0^\infty \ell_k(\tau)u(t_i+\tau)d\tau \\ \tilde{y}_i(k) &= \int_0^\infty \ell_k(\tau)y(t_i+\tau)d\tau \\ \tilde{e}_i(k) &= \int_0^\infty \ell_k(\tau)e(t_i+\tau)d\tau \end{aligned} \tag{10}$$

(or to the equivalent ones derived from (7)), where $\tilde{u}_i(k) \in \mathbb{R}^m$, $\tilde{e}_i(k) \in \mathbb{R}^p$ and $\tilde{y}_i(k) \in \mathbb{R}^p$. Then (see [25] for details) the transformed system has the state space representation

$$\begin{aligned} \xi_i(k+1) &= A_o\xi_i(k) + B_o\tilde{u}_i(k) + K_o\tilde{e}_i(k),\ \xi_i(0) = x(t_i) \\ \tilde{y}_i(k) &= C_o\xi_i(k) + D_o\tilde{u}_i(k) + \tilde{e}_i(k) \end{aligned} \tag{11}$$

where the state space matrices are given by (9).

Letting now

$$\tilde{z}_i(k) = \begin{bmatrix}\tilde{u}_i^T(k) & \tilde{y}_i^T(k)\end{bmatrix}^T$$

and

$$\begin{aligned} \bar{A}_o &= A_o - K_oC_o \\ \bar{B}_o &= B_o - K_oD_o \\ \widetilde{B}_o &= \begin{bmatrix}\bar{B}_o & K_o\end{bmatrix}, \end{aligned}$$

system (11) can be written in predictor form as

$$\begin{aligned} \xi_i(k+1) &= \bar{A}_o\xi_i(k) + \widetilde{B}_o\tilde{z}_i(k),\ \xi_i(0) = x(t_i) \\ \tilde{y}_i(k) &= C_o\xi_i(k) + D_o\tilde{u}_i(k) + \tilde{e}_i(k), \end{aligned} \tag{12}$$

to which the PBSID$_{opt}$ algorithm, summarised hereafter, can be applied to compute estimates of the state space matrices A_o, B_o, C_o, D_o, K_o. To this purpose note that iterating $p-1$ times the projection operation (i.e., propagating $p-1$ forward in the index k the first of equations (12), where p is the so-called past window length) one gets

$$\begin{aligned} \xi_i(k+2) &= \bar{A}_o^2\xi_i(k) + \begin{bmatrix}\bar{A}_o\widetilde{B}_o & \widetilde{B}_o\end{bmatrix}\begin{bmatrix}\tilde{z}_i(k) \\ \tilde{z}_i(k+1)\end{bmatrix} \\ &\vdots \\ \xi_i(k+p) &= \bar{A}_o^p\xi_i(k) + \mathcal{K}^pZ_i^{0,p-1} \end{aligned} \tag{13}$$

where

$$\mathcal{K}^p = \begin{bmatrix}\bar{A}_o^{p-1}\widetilde{B}_0 & \dots & \widetilde{B}_o\end{bmatrix} \tag{14}$$

is the extended controllability matrix of the system in the transformed domain and

$$Z_i^{0,p-1} = \begin{bmatrix}\tilde{z}_i(k) \\ \vdots \\ \tilde{z}_i(k+p-1)\end{bmatrix}.$$

Under the considered assumptions, $\bar{A}_o$ has all the eigenvalues inside the open unit circle, so the term $\bar{A}_o^p\xi_i(k)$ is negligible for sufficiently large values of p and we have that

$$\xi_i(k+p) \simeq \mathcal{K}^pZ_i^{0,p-1}.$$

As a consequence, the input-output behaviour of the system is approximately given by

$$\begin{aligned} \tilde{y}_i(k+p) &\simeq C_o\mathcal{K}^pZ_i^{0,p-1} + D_o\tilde{u}_i(k+p) + \tilde{e}_i(k+p) \\ &\vdots \\ \tilde{y}_i(k+p+f) &\simeq C_o\mathcal{K}^pZ_i^{f,p+f-1} + D_o\tilde{u}_i(k+p+f) + \\ &\quad + \tilde{e}_i(k+p+f), \end{aligned} \tag{15}$$

so that introducing the vector notation

$$\begin{aligned} Y_i^{p,f} &= \begin{bmatrix}\tilde{y}_i(k+p) & \tilde{y}_i(k+p+1) & \dots & \tilde{y}_i(k+p+f)\end{bmatrix} \\ U_i^{p,f} &= \begin{bmatrix}\tilde{u}_i(k+p) & \tilde{u}_i(k+p+1) & \dots & \tilde{u}_i(k+p+f)\end{bmatrix} \\ E_i^{p,f} &= \begin{bmatrix}\tilde{e}_i(k+p) & \tilde{e}_i(k+p+1) & \dots & \tilde{e}_i(k+p+f)\end{bmatrix} \\ \Xi_i^{p,f} &= \begin{bmatrix}\xi_i(k+p) & \xi_i(k+p+1) & \dots & \xi_i(k+p+f)\end{bmatrix} \\ \bar{Z}_i^{p,f} &= \begin{bmatrix}Z_i^{0,p-1} & Z_i^{1,p} & \dots & Z_i^{f,p+f-1}\end{bmatrix} \end{aligned} \tag{16}$$

equations (13) and (15) can be rewritten as

$$\begin{aligned} \Xi_i^{p,f} &\simeq \mathcal{K}^p\bar{Z}_i^{p,f} \\ Y_i^{p,f} &\simeq C_o\mathcal{K}^p\bar{Z}_i^{p,f} + D_oU_i^{p,f} + E_i^{p,f}. \end{aligned} \tag{17}$$

Considering now the entire dataset for $i = 1, \dots, N$, the data matrices become

$$\begin{aligned} Y^{p,f} = [&\tilde{y}_1(k+p)\dots\tilde{y}_N(k+p)\dots \\ &\tilde{y}_1(k+p+f)\dots\tilde{y}_N(k+p+f)], \end{aligned} \tag{18}$$

and similarly for $U_i^{p,f}$, $E_i^{p,f}$, $\Xi_i^{p,f}$ and $\bar{Z}_i^{p,f}$. The data equations (17), in turn, are given by

$$\begin{aligned} \Xi^{p,f} &\simeq \mathcal{K}^p\bar{Z}^{p,f} \\ Y^{p,f} &\simeq C_o\mathcal{K}^p\bar{Z}^{p,f} + D_oU^{p,f} + E^{p,f}. \end{aligned} \tag{19}$$

From this point on, the algorithm can be developed along the lines of the discrete-time PBSID$_{opt}$ method, i.e., by carrying out the following steps. Considering $p = f$, estimates for the matrices $C_o\mathcal{K}^p$ and D_o are first computed by solving the least-squares problem

$$\min_{C_o\mathcal{K}^p, D_o} \|Y^{p,p} - C_o\mathcal{K}^p \bar{Z}^{p,p} - D_o U^{p,p}\|_F, \qquad (20)$$

where by $\|\cdot\|_F$ we denote the Frobenius norm of a matrix. Defining now the extended observability matrix Γ^p as

$$\Gamma^p = \begin{bmatrix} C_o \\ C_o\bar{A}_o \\ \vdots \\ C_o\bar{A}_o^{p-1} \end{bmatrix} \qquad (21)$$

and noting that the product of Γ^p and $\mathcal{K}^p$ can be written as

$$\Gamma^p\mathcal{K}^p \simeq \begin{bmatrix} C_o\bar{A}^{p-1}\widetilde{B}_o & \dots & C_o\widetilde{B}_o \\ 0 & \dots & C_o\bar{A}\widetilde{B}_o \\ \vdots & & \\ 0 & \dots & C_o\bar{A}^{p-1}\widetilde{B}_o \end{bmatrix}, \qquad (22)$$

such product can be computed using the estimate $\widehat{C_o\mathcal{K}^p}$ of $C_o\mathcal{K}^p$ obtained by solving the least squares problem (20).

Recalling now that

$$\Xi^{p,p} \simeq \mathcal{K}^p\bar{Z}^{p,p} \qquad (23)$$

it also holds that

$$\Gamma^p\Xi^{p,p} \simeq \Gamma^p\mathcal{K}^p\bar{Z}^{p,p}. \qquad (24)$$

Therefore, computing the singular value decomposition

$$\Gamma^p\mathcal{K}^p\bar{Z}^{p,p} = U\Sigma V^T \qquad (25)$$

an estimate of the state sequence can be obtained as

$$\widehat{\Xi}^{p,p} = \Sigma_n^{1/2}V_n^T = \Sigma_n^{-1/2}U_n^T\Gamma^p\mathcal{K}^p\bar{Z}^{p,p}, \qquad (26)$$

from which, in turn, an estimate of C_o can be computed by solving the least squares problem

$$\min_{C_o} \|Y^{p,p} - \widehat{D}_o U^{p,p} - C_o\widehat{\Xi}^{p,p}\|_F. \qquad (27)$$

The final steps consist of the estimation of the innovation data matrix $E^{p,p}$

$$E^{p,p} = Y^{p,p} - \widehat{C}_o\widehat{\Xi}^{p,p} - \widehat{D}_o U^{p,p} \qquad (28)$$

and of the entire set of the state space matrices for the system in the transformed domain, which can be obtained by solving the least squares problem

$$\min_{A_o,B_o,K_o} \|\widehat{\Xi}^{p+1,p} - A_o\widehat{\Xi}^{p,p-1} - B_o U^{p,p-1} - K_o E^{p,p-1}\|_F. \qquad (29)$$

The state space matrices of the original continuous-time system can then be retrieved by inverting the (bilinear) transformations (9).

SIMULATION STUDY: MODEL IDENTIFICATION FOR THE BO-105 HELICOPTER

We consider the BO-105, possibly the most studied helicopter in the rotorcraft system identification literature. The BO-105 is a light, twin-engine, multi-purpose utility helicopter. In this example it is considered in forward flight at 80 knots, a flight condition which corresponds to unstable dynamics, with the aim of demonstrating the identification of a six-DOF state-space model with test data extracted from a simulator based on the nine-DOF model from [27]. As described in the cited reference, the model includes the classical six DOF and some additional states to account for some additional effects, namely:

- The BO-105 exhibits highly coupled body-roll and rotor-flapping responses; their interaction is represented in the model with a dynamic equation that describes the flapping dynamics using the cyclic controls.
- A second order dipole is appended to the model of roll rate response to lateral stick in order to account for the effect of lead-lag rotor dynamics.

Therefore, the simulator includes a nine-DOF model including the six-DOF quasi steady dynamics, the flapping equations and the lead-lag dynamics modelled with a complex dipole. Delays at the input of the model are also taken into account in the simulation, though they are not estimated. The helicopter is considered in forward flight at 80 knots. The state vector and the trim values are

$$x = \begin{bmatrix} u & v & w & p & q & r & \phi & \theta & a_{1s} & b_{1s} & x_1 & x_2 \end{bmatrix}$$

and, respectively,

$$u_0 = 40 \text{ m/s}, \quad v_0 = 3 \text{ m/s}, \quad w_0 = -5 \text{ m/s}, \quad \theta_0 = 0. \qquad (30)$$

The state vector includes the longitudinal flapping a_{1s}, the lateral flapping b_{1s} and two state variables x_1 and x_2, coming from the lead-lag dynamics complex dipole. Finally, the output vector is

$$y = \begin{bmatrix} u & v & w & p & q & r & \phi & \theta \end{bmatrix}, \qquad (31)$$

i.e., the state variables related to quasi steady dynamics are measured. Note that in this example it has been chosen to identify only the six-DOF quasi steady dynamics of the helicopter. The identification experiment is performed in closed-loop because of the instability of the model. The input variables (δ_{lat},δ_{lon},δ_{ped},δ_{col}) have been excited in the same experiment with pseudorandom binary signals with a duration of 60 s. The perturbation of the control inputs has a 1% amplitude and the sampling time is 0.008s. The parameters of the algorithm presented in the previous Section have been chosen as $p = 40$ and $a = 45$. The obtained results are illustrated in Figures 1-12, which provide a comparison between the frequency response of the nine-DOF model (solid lines in the figures) and the frequency response of the identified model (dashed lines in the figures). As can be seen from the figures, the agreement is quite satisfactory. In particular, some discrepancies between the nine-DOF model and the reduced order identified one appear only at frequencies where either the neglected modes start playing a significant role in the dynamics of the system and/or the excitation level provided by the perturbation inputs starts to prove insufficient.

Finally, a time-domain validation of the identified model has been also carried out, by measuring the accuracy of the model in response to a doublet input signal on each input channel. The input sequence used in the validation experiment is illustrated in Figure 13, while the time history for two of the outputs (u and w) is presented in Figure 14. Again, even though the open-loop system is unstable, the simulated

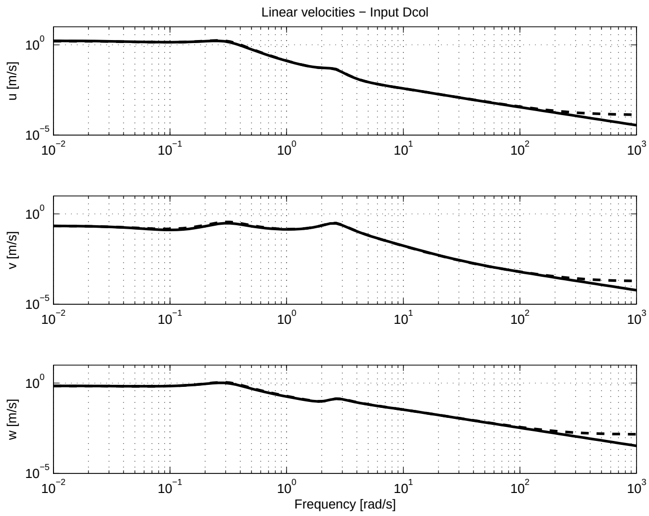

Fig. 1. Frequency response from collective input to linear velocities. (real: solid line; estimated: dashed line)

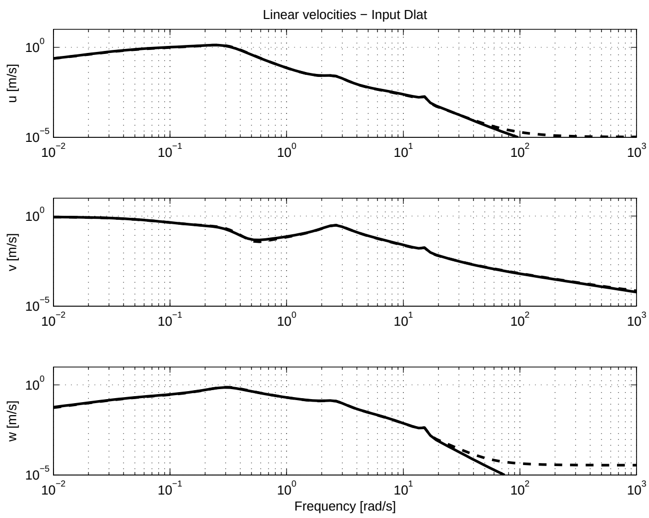

Fig. 2. Frequency response from lateral cyclic input to linear velocities. (real: solid line; estimated: dashed line)

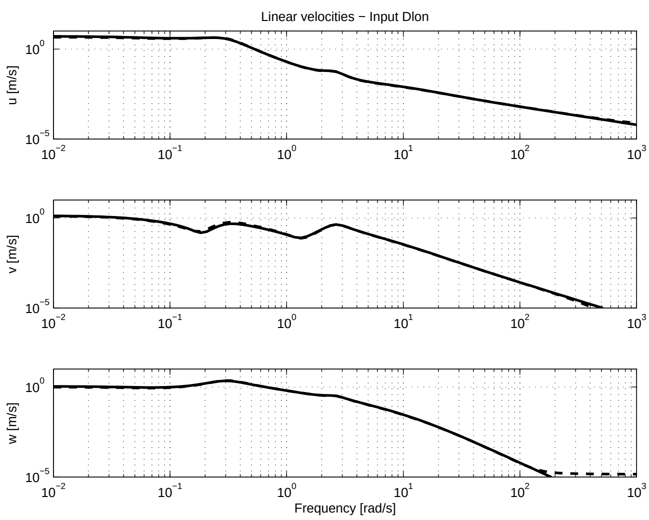

Fig. 3. Frequency response from longitudinal cyclic input to linear velocities. (real: solid line; estimated: dashed line)

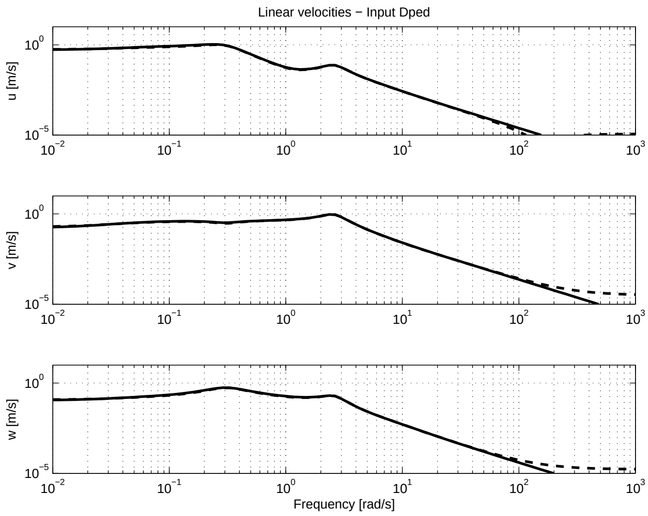

Fig. 4. Frequency response from pedal input to linear velocities. (real: solid line; estimated: dashed line)

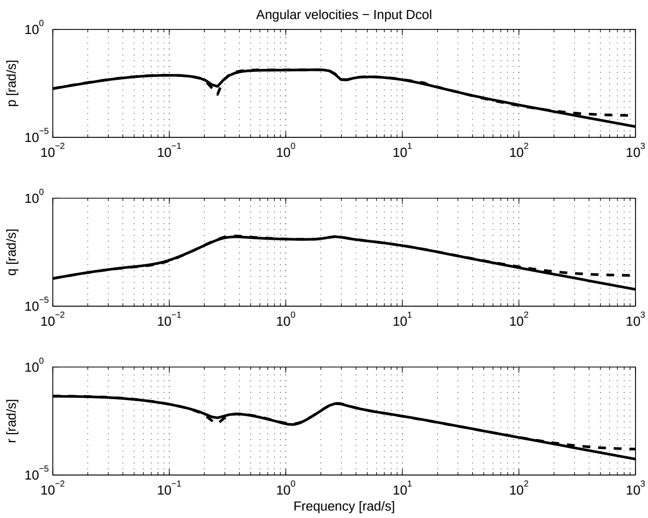

Fig. 5. Frequency response from collective input to angular velocities. (real: solid line; estimated: dashed line)

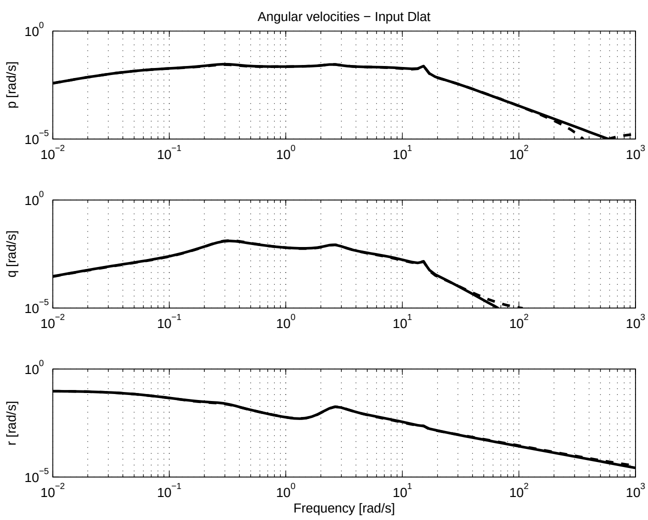

Fig. 6. Frequency response from lateral cyclic input to angular velocities. (real: solid line; estimated: dashed line)

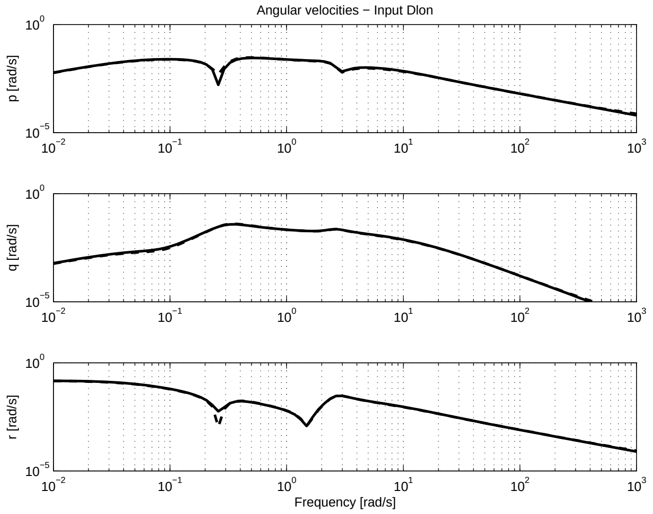

Fig. 7. Frequency response from longitudinal cyclic input to angular velocities. (real: solid line; estimated: dashed line)

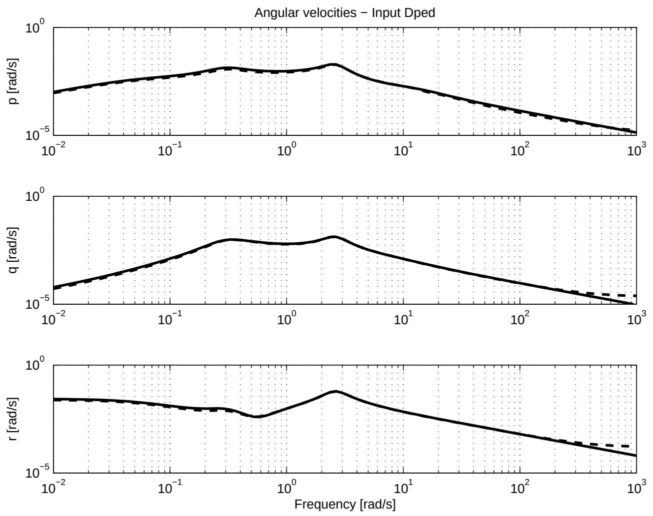

Fig. 8. Frequency response from pedal input to angular velocities. (real: solid line; estimated: dashed line)

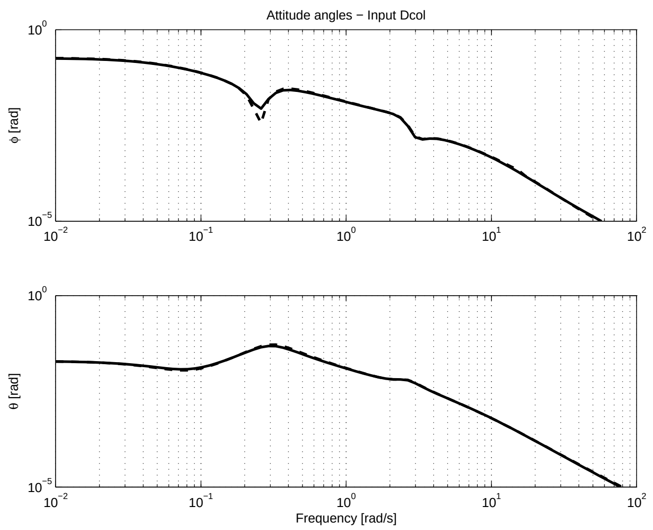

Fig. 9. Frequency response from collective input to attitude angles. (real: solid line; estimated: dashed line)

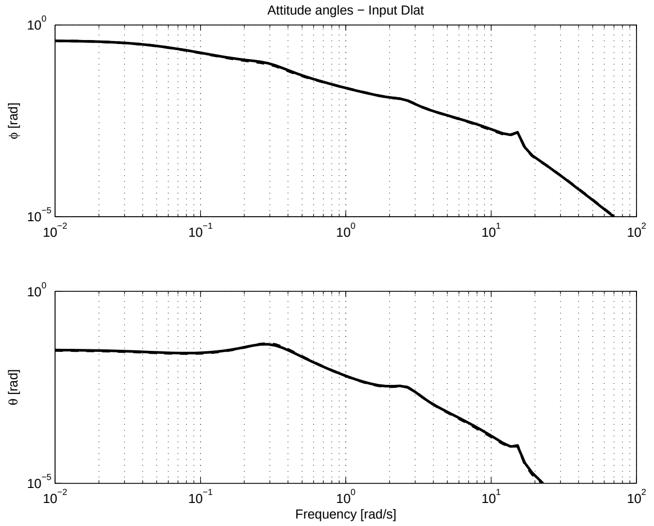

Fig. 10. Frequency response from lateral cyclic input to attitude angles. (real: solid line; estimated: dashed line)

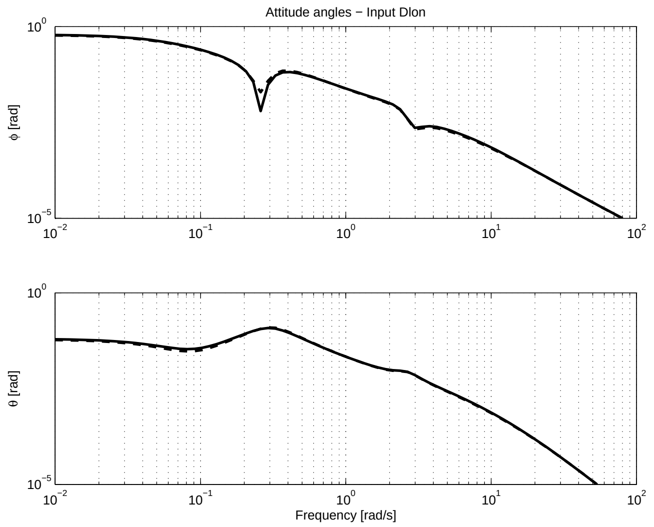

Fig. 11. Frequency response from longitudinal cyclic input to attitude angles. (real: solid line; estimated: dashed line)

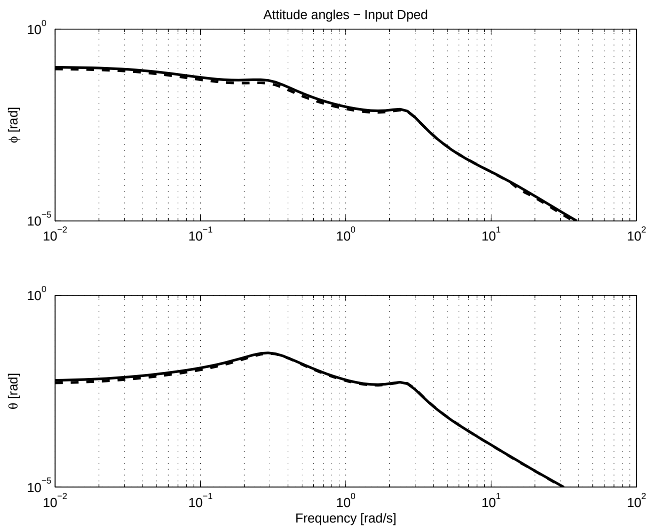

Fig. 12. Frequency response from pedal input to attitude angles. (real: solid line; estimated: dashed line)

outputs obtained from the identified model (dashed lines) match very well the ones computed from the nine-DOF model (solid lines). In quantitative terms, considering the relative error norm, defined as $\frac{\|e\|_2}{\|y\|_2}$, its value is below 5% on all the considered output variables.

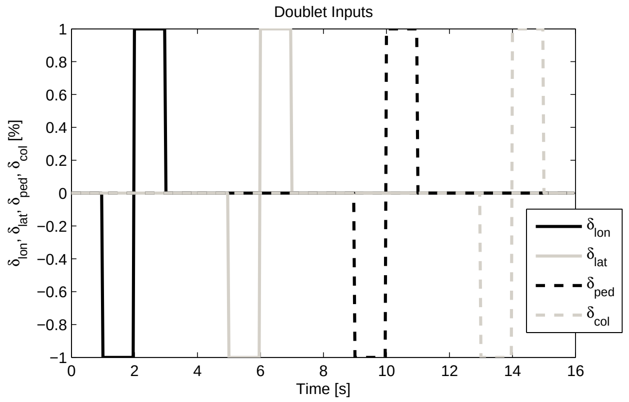

Fig. 13. Doublet input signal used for model validation.

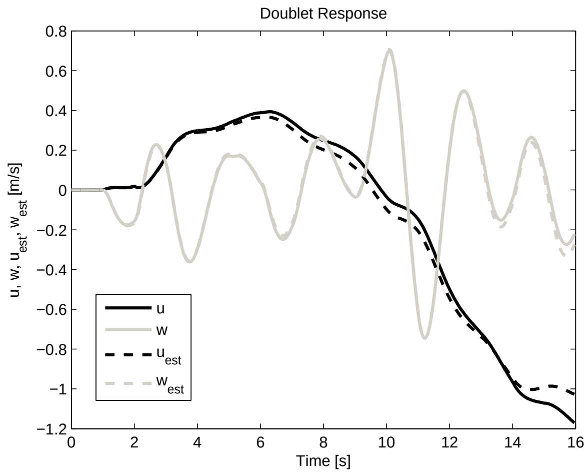

Fig. 14. Doublet output signals (real: solid line; estimated: dashed line).

CONCLUDING REMARKS

The problem of continuous-time subspace model identification has been considered and a batch algorithm based on Laguerre projections of the input-output variables followed by a PBSID identification step has been proposed. Simulation results show that the proposed schemes are viable for rotorcraft applications and can deal successfully with data generated during closed-loop experiments.

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