Efficient Numerical Approaches for On-Board Rotorcraft Flight Performance Modeling

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Abstract: On-board flight performance models have been installed onto several Army rotorcraft supporting flight performance/mission planning. A flight performance model answers such questions as: what is the available power, what is the power required to hover or cruise, what is the maximum flight/hover weight, what is the maximum flight speed, how much fuel is required or what is the best rate-of-climb that can be achieved.   These models are composed of data and equations that are used to calculate the many performance parameters. Also included in these models are the boundaries and limits for each of these parameters. These models must be efficient in both size and speed while maintaining accuracy for on-board systems, particularly when the flight performance model is used for pilot guidance such as terrain following. A number of numerical approaches have be used to maintain the accuracy of the flight performance model while being efficient for both size and speed.   These approaches include the use of non-dimensional data, higher order interpolation and pre-processing of derivatives and derived results.

Nomenclature
Abbreviations
FPM   Flight Performance Model
PFPS   Portable Flight Planning Software
PPC   Performance Planning Card

Symbols
Area   Rotor Disc Area
C_p Power Coefficient, $= \text{Power}/(\rho\,(V_{tip}^3)\,\text{Area})$
C_t Thrust Coefficient, $= \text{Weight} / (\rho\,(V_{tip}^2)\,\text{Area})$
FAT   Free Air Temperature
$N_{r\text{-ref.}}$ Referred Rotor Speed, $= \text{RPM} / \theta^{0.5}$
RPM   rotor rpm, % of rpm
PA   Pressure Altitude
V_{be} Best Endurance Airspeed
V_{br} Best Range Airspeed
V_{tas} True Airspeed
V_{tip} Tip Speed
μ Advanced Ratio, $= f(V_{tas}, \text{RPM})$
ρ Air density
θ Air Temperature Ratio, $= f(\text{FAT})$

1 INTRODUCTION

Flight systems now allow aviators to complete performance/mission planning on-board the aircraft during flight. Previously, the aviator planned their performance and mission as a pre-mission exercise using either paper charts and pencil or using specialized software on a personal computer. Core to any performance or mission planning exercise is the flight performance model (FPM). Given the ambient conditions and aircraft configuration, a FPM answers such questions as: what is the available power, what is the power required to hover or cruise, what is the maximum flight/hover weight, what is the maximum flight speed and how much fuel is used.

With the introduction of on-board flight performance/mission planning capabilities, comes the requirement to supply the aviator with consistent answers, i.e., the same flight performance results from all applications. These applications include the on-board performance/mission planning, the Army mission planning system - Portable Flight Planning System (PFPS), the performance planning card (PPC) and the performance charts in the operator's manual. Presented in Ref 1 is an architecture that allows for consistent answers to be achieved across all applications. An additional constraint is that the on-board FPM must be efficient for both speed and size. Typically, on-board processors are both limited in processor speed and memory size. Presented here are the methods that were used to achieve the required speed and data set sizes for on-board FPM's.

2 BACKGROUND

2.1 Flight Performance Model Definition

FPM's are software models that provide a digital representation of a specific rotorcraft's performance capabilities and limitations. The development of an FPM is achieved by combining flight test and engineering data with standard mathematical models. FPM's are built for each rotorcraft family and for variations within a rotorcraft family. Each FPM is comprised of two basic components, core capabilities and derived capabilities.

The core capabilities listed in Table 1, are created from installed engine decks, flight test data, and basic rotorcraft physics. Included in the core capabilities are specific limits of the aircraft, such as transmission limits, gross weight limits and engine high temperature limits. Also shown in Table 1 are the typical inputs that are required for each of the core capabilities and the resulting output value. The non-shaded inputs are common for all rotorcraft, while the shaded inputs are rotorcraft specific.

Typical derived capabilities are provided by the functions listed in Table 2. Each derived capability is computed using one or more functions that have implemented the core capabilities. For example the best endurance airspeed capability is computed by finding airspeed at the minimum torque for a given pressure altitude, free air temperature, gross weight and drag. Additional derived capabilities are created as required.

The FPM is constructed by creating specific modes for the flight performance/mission planning applications (see the next section for a description of these applications). The specific application modes (shown in Table 3) are composed of numerous core and derived modes, along with error and warning messages.

2.2 Applications Using the FPM

FPM's are used in three main applications. The first two applications are the mission planning software (PFPS) and the performance planning card (PPC) software directly used by pilots. The third application is the development of performance charts published in the operator's manuals.

PFPS is mission planning software that allows the user to plan their complete aircraft mission through either tabular entry or a graphical map (see Fig. 1). The user specifies each way point of the mission along with the atmospheric conditions, aircraft configuration and aircraft speed at that point. PFPS provides this information to the FPM's to calculate the mission fuel burn and informs the user if the mission is possible, i.e., are any limits exceeded.

The PPC software (see Fig. 2) is used to calculate specific performance parameters at departure, cruise and arrival. Typical performance parameters that are presented by the PPC are torque available, torque required, maximum gross weight for hover, service ceiling, maximum airspeed, velocity never exceed, best endurance airspeed and best range airspeed. The specific values calculated by the PPC software are spelled out in each aircrafts aircrew training manual. These performance parameters are calculated for the specific aircraft configuration and atmospheric conditions, and must be carried with the pilot during the mission. The FPM is used to supply all performance values that are displayed and printed by the PPC.

Army aircraft operator's manuals contain numerous charts (see Fig. 3) that supply the aviator with graphical representations of the specific aircraft's performance and limitations. Typical performance charts that are presented include torque available, hover torque required, cruise torque required, fuel flow, hover ceiling, hover maximum gross weight, and airspeed conversion. FPM's are now being used with graphical generation software to create these charts for the operator's manuals.

2.3 On-Board Flight Performance Models

On-board FPM's are currently used on many Army rotorcraft. The on-board FPM supports performance planning and/or mission planning. The atmospheric conditions are either user entered or are provided by on-board aircraft sensors. On-board FPM must be:
- Fast – minimal computational requirements (e.g. multi-leg missions must be computed in seconds),
- Small – minimal code/data space (on-board hardware typically has very limited space),
- Accurate/consistent – must reflect the same results as desktop or paper charts.

3 METHODS FOR ACHIEVING: REQUIRED ACCURACY, SPEED AND SIZE

A variety of numerical methods were used in concert to achieve the required accuracy, speed and size for an on-board FPM. These methods include the storage and organization of the data, using a fast Akima interpolation method and pre-processing derived results.

3.1 Data

The storage and organization of the data used for interpolation can be an effective way to reduce the time required for interpolation and reduce the data set size. Use of non-dimensional or referred data and rectangular data sets are a number of the ways data can be prepared for numerical efficiency.

Non-dimensional data
Certain rotorcraft data is typically collected in a manner that produces a non-dimensional data. Take for example the power required data set which in dimensional form has the independent values of PA, FAT, gross weight and airspeed and the dependent value of power required. The non-dimensional form (see Table 4 for the relationship of dimensional to non-dimensional values) of power required has C_t, $N_{r\text{-ref}}$, and μ as the independent variables and C_p as the dependent variable. Using the non-dimensional data set, the number of independent variable has decreased by 1, thus decreasing the data set size and reducing the amount of searching required for the interpolation routines. The penalty for using non-dimensional data is the required pre and post calculations of going from dimensional values to non-dimensional values and back to dimensional values. This time penalty is nearly equivalent to time savings for the reduction in the

number of interpolation dimensions. Therefore, a reduction in data set size is realized in using this approach.

Referred Data
Another major reduction in data is also achieved by using referred data versus dimensional. An example of this reduction is the fuel flow data set. Dimensional fuel flow has independent values of PA, FAT and torque and dependent value of fuel flow. The referred fuel flow has a single independent value of Referred Horsepower and dependent value of Referred Fuel Flow. The fuel flow data set is greatly decreased from a 3-dimensional data set to a 1-dimensional data set, a significant reduction in data set size. Similar to power required, the computational time savings for 1-dimensional versus 3-dimensional interpolation is offset by the pre and post calculations for referring and un-referring the data.

Organization of Data
Data set organization also plays a significant role in reducing the computational time.   All multi-dimensional data sets were required to be rectangular, that is, the dependent values were given at the same x, y, and/or z independent values. Abiding by this requirement, reduced the amount of searching that must be performed for multi-dimensional interpolation while performing intermediate calculations.

3.2 Interpolation Methods

Interpolation of data is a typical method used to compute many of the core functions in a FPM. The preferred method of interpolation is linear interpolation due to it speed and ease of implementation. Using linear interpolation is a reasonable approach as long as the data is fairly linear or the data set is large enough to maintain the required accuracy. For example, in Fig. 4, power required is shown for linear interpolation and for Akima interpolation method, respectively. For both plots, the same number of points are used. Obviously, the Akima interpolation represents the nature of the function more accurately. In order to maintain this level of accuracy the data set required for linear interpolation would have to be an order of magnitude or more greater then the data set required for Akima interpolation.   Therefore, in order to maintain a reasonable data set size, the Akima interpolation method is preferred. Although Akima interpolation represents a significant savings in data set size, the computational resources for Akima can be an order of magnitude greater then linear interpolation. In order to reduce the computational requirements for Akima interpolation, an approach that pre-calculates the required derivatives for Akima interpolation is used. This approach greatly reduces the number of calculations for interpolation, while increasing the data set size with only a small increase in the resulting data set.

Akima interpolation for point 0 (see Fig. 5) can be computed as follows:

$$y_0 = p_0 + p_1 (x_0 - x_3) + p_2 (x_0 - x_3)^2 + p_3 (x_0 - x_3)^3 \qquad (1)$$

where
$p_0 = y_3$
$p_1 = t_3$
$p_2 = (3 (y_4 - y_3) / (x_4 - x_3) - 2 t_3 - t_4) / (x_4 - x_3)$
$p_3 = (t_3 + t_4 - 2 (y_4 - y_3) / (x_4 - x_3)) / (x_4 - x_3)^2$

$t_3 = dy_3 / dx_3$
t_3 can be computed as follows:
$$t_3 = (| m_{45} - m_{34} | m_{23} + | m_{23} - m_{12} | m_{34}) / (| m_{45} - m_{34} | + | m_{23} - m_{12} |) \qquad (2)$$

$t_4 = dy_4 / dx_4$

t_4 can be computed as follows:
$$t_4 = (| m_{56} - m_{45} | m_{34} + | m_{34} - m_{23} | m_{45}) / (| m_{56} - m_{45} | + | m_{34} - m_{23} |) \qquad (3)$$

where

4

m_{12} = slope of line segment 12
m_{23} = slope of line segment 23
m_{34} = slope of line segment 34
m_{45} = slope of line segment 45
m_{56} = slope of line segment 56   (Ref. 2)

For one-dimensional Akima interpolation, 6 known points are required (3 points on either side of the point to be interpolated) to construct the derivatives (t_3 and t_4) at the bracketing points (points 3 and 4 in Fig. 5). Whenever the interpolated point is near a boundary (i.e., between points 1 and 3 or points 4 and 6 in Fig. 5), and additional point or points must be constructed in order to compute the derivates at one or both of the bracketing known points. A quadratic extrapolation is used to construct the additional point or points.

For two-dimensional Akima interpolation (see Fig. 6), the required number of points increases to 36. Two-dimensional Akima interpolation proceeds by first interpolating values at points A, B, C, D, E and F. Once these points are interpolated, the value at point 0 can be interpolated. If a completed set of points are available, the interpolation procedure requires the computation of derivates at 14 individual points and 7 one-dimensional Akima interpolations performed. Of course, if the point requiring interpolation is near a boundary (i.e., where data points are not available), extrapolation of the points must occur.

For a 3-dimensional Akima interpolation the complexity increases with a required 216 points, 86 derivative calculations and 43 one-dimensional Akima interpolations. Additionally, when computing the derivatives at each point, logic must be present to check for points that are near the boundary and require extrapolated points to compute the derivatives. As the number of dimensions are increased the amount of work required to perform an Akima interpolation greatly increases. From a speed stand point, this become prohibitive for on-board systems.

To overcome the increases in computational time for multi-dimensional Akima interpolation, the following numerical approach was used. First, derivatives at each point (equations 2 and 3) are pre-calculated and saved as data. For a one-dimensional Akima interpolation, only equation (1) must be calculated. When a two-dimensional Akima interpolation is computed (see Fig. 6), the derivatives at points C and D are obtained by interpolating the derivatives from the nearby points. For point C, derivatives at points 3 and 4 are used. This procedure is repeated for each additional dimension. The reduction in the number of computations that is achieved for 1 thru 4 dimensions is shown in Table 5. Computational times for Akima are now on the same order as linear interpolation, while using data sets that are 1 to 2 orders of magnitude less then data set required for linear interpolation for equivalent accuracy.

3.3 Data Pre-processing

Another approach to reducing the computation time of a flight performance model is to pre-process results that are iterative in nature. The two calculations that are considered here are best endurance airspeed (V_{be}) and best range airspeed (V_{br}). V_{be} requires iterations on the torque required data, while V_{br} requires iterations on the torque required data and fuel flow data. Each require 10 to 15 iterations to obtain the required accuracy.

Best Endurance Airspeed (V_{be})

Best endurance airspeed (V_{be}) is defined as the airspeed at which the torque required is a minimum for given gross weight, PA and FAT. In non-dimensional terms the definition becomes the μ at which the C_p is at a minimum for a given C_t and $N_{r\text{-ref}}$. For a constant C_t and $N_{r\text{-ref}}$, V_{be} is graphically shown in Fig. 7.

The V_{be} data set is created by calculating V_{be} for a range of C_t and $N_{r\text{-ref}}$ values. Using this data set, the computations of V_{be} is reduced from 10 to 15 iterations on a three-dimensional data set ($C_p = f(C_t, N_{r\text{-ref}}, \mu)$) of power required, to a single calculation on a two-dimensional data set ($\mu = f(C_t, N_{r\text{-ref}})$).

Best Range Airspeed (V_{br})

The V_{br} data set has inputs of $N_{r\text{-ref}}$, C_t, flat plate drag change and wind with an output of V_{br} in terms of μ.

The V_{br} was calculated using the following procedure:

1. The core functions, torque required and fuel flow were used to find the true airspeed (V_{tas}) that gives the maximum Specific Range for a given PA, FAT, gross weight, flat plate drag change and wind input combination. Specific Range is defined as follows:

   Specific Range = (V_{tas} – wind) / Fuel Flow   (4)

2. Torque required is computed for the given PA, FAT, gross weight, flat plate drag change and V_{tas}. Using PA, FAT and the Torque Required, the Fuel Flow is computed. Specific Range is computed using the Fuel Flow, V_{tas} and Wind (Equation 4). Shown in Fig. 8 is Specific Range as a function of V_{tas}. This computation is repeated until a maximum value of Specific Range is found within 0.1 knots. The intersection of the "Specific Range" line and the "Maximum Specific Range" line represent the maximum value of Specific Range.

3. The maximum Specific Range found in step 1 was multiplied by 0.99 (See Fig. 8 for 99% of Maximum Specific Range). An iterative procedure is used to compute V_{br}.

4. At 99% Specific Range there are two corresponding values of V_{tas} (at a computational interval of 0.01 knots). The higher value of V_{tas} is used as V_{br} (see Fig. 8 for V_{br}). An iterative procedure is used to compute V_{br}.

It was observed during the creation of this data set that the V_{br} varied for a given value of $N_{r\text{-ref}}$ and C_t computed from different combinations of PA, FAT and gross weight. Therefore, to reduce the error, at each non-dimensional data point, all values except for gross weight and PA were kept constant. Gross Weight was varied from its minimum to its maximum value, with appropriate values of PA used to keep C_t a constant. This resulted in multiple values of V_{br} for the same constant inputs of $N_{r\text{-ref}}$, C_t, flat plate drag change and wind. The final V_{br} value was the average of the minimum and maximum ($V_{br\text{-avg}}$) of the values computed for multiple computed values for V_{br}. Additionally, the computed difference for each data point was computed and is presented in Figure 9. The difference is defined as: $V_{diff} = \max |(V_{br\text{-}i} - V_{br\text{-avg}})|$, where $V_{br\text{-}i}$ is each computed value of V_{br} for a given set of constant inputs. The maximum difference of 0.45 knots, while majority of the differences were less than 0.25 knots. For the single engine, the maximum difference of 0.36 knots, while a majority of the differences were less than 0.2 knots.

The standard method for calculating V_{br} requires two iterative procedures, each requiring up to 15 iteration. This modified approach to computing V_{br} requires a single interpolation of a four-dimensional data set.

4 CONCLUSIONS

On-board flight performance models are an integral part of on-board performance/mission planning. For some on-board systems, the on-board FPM supply the pilot flight guidance, such as the terrain following/terrain avoidance systems. Therefore, it is imperative that the flight performance model maintain a sufficient level of accuracy, while meeting the overall system speed and size requirements. Using a variety of numerical approaches, each of these requirements is met or exceeded. Although each method presented helped to achieve the requirements, it was the combination of all of these methods that were needed to meet the on-board FPM requirements for accuracy, speed and size.

5 REFERENCES

[1] "U.S. Army Approach to On-Board Flight Performance Modeling," Norman Suhs, Charles Williams, Ronald Craft, Proceedings: American Helicopter Society 64th Annual Forum and Technology Display, Montreal, Canada, May 2008.

 [2] "A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures," Hiroshi Akima, Journal of the Association for Computing Machinery, Vol. 17, No. 4, October 1970, pp. 589-602.

Table 1. Core Functions

Core Function /Output	Inputs
Torque Available [%Q]	Pressure Altitude [ft]
	Free Air Temperature [C]
	Air Speed [ktas]
	Anti-Ice [off/on]
	Heater [off/on]
	Engine Torque Factors
	Inlet Barrier Filter[off/on]
	IR Suppressor [off/on]
Cruise Torque Required [%Q]	Pressure Altitude [ft]
	Free Air Temperature [C]
	Air Speed [ktas]
	Gross Weight [lb]
	Delta Drag [sq-ft]
	Rate-of-Climb [ft/min]
	Configuration (Clean/High Drag)
Hover Torque Required [%Q]	Pressure Altitude [ft]
	Free Air Temperature [C]
	Height [ft]]
	Gross Weight [lb]
	Download [T/W]
	Rate-of-Climb [ft/min]
	Configuration (Clean/High Drag)
Fuel Flow [lb/hr]	Pressure Altitude [ft]
	Free Air Temperature [C]
	Torque [%Q]
	Anti-Ice [off/on]
	Heater [off/on]
	Inlet Barrier Filter[off/on]
	IR Suppressor [off/on]
Air Speed [kcas] (Conversion)	Air Speed [kias]
	Rate-of-Climb [ft/min]
Never Exceed Velocity [kts]	Pressure Altitude [ft]
	Free Air Temperature [C]
	Gross Weight [lb]
	Configuration (Clean/High Drag)

Table 2. Derived Functions

Derived Function	Function Of:
Best Endurance Airspeed	Cruise Torque Required
Best Range Airspeed	Cruise Torque Required Fuel Flow Torque Available Never Exceed Velocity
Minimum Airspeed	Cruise Torque Required Torque Available
Maximum Airspeed	Cruise Torque Required Torque Available
Maximum Gross Weight for Cruise	Cruise Torque Required Torque Available
Maximum Gross Weight For Hover	Hover Torque Required Torque Available
Service Ceiling	Cruise Torque Required Torque Available

Table 3. Application Modes

PFPS Modes	PPC Modes
Cruise Standard	Dual Engine Departure
Cruise Best Endurance	Single Engine Departure
Cruise Best Range	Dual Engine Cruise
Climb Standard	Single Engine Cruise
Climb Best Performance	Dual Engine Arrival
Descent Standard	Single Engine Arrival
Hover	

Table 4. Dimensional to Non-Dimensional

	Dimensional	Non-Dimensional
Pressure	PA *(ft)*	δ
Temperature	OAT *(°C)*	θ
Weight	W *(lbs)*	$C_t = W / (\rho_0 \, s \, V_{tip}^2 \, A)$
Airspeed	V *(ktas)*	$\mu = V/V_{tip}$
RPM	N *(RPM)*	$NREF = (N/N_0)/$
Power	P *(SHP)*	$Cp = P / (\rho_0 \, s \, V_{tip}^3 \, A)$

Table 5. Computations Required for Standard Akima Interpolation and Akima Interpolation Using Pre-Calculated Derivatives.

Dimensions	Standard Akima Interpolation			Akima Interpolation Using Pre-Calculated Derivatives	
	# Points	# Derivate Calculations	# 1-D Interpolations	# Derivative Interpolations	# 1-D Interpolations
1	6	2	1	0	1
2	36	14	7	2	3
3	216	86	43	6	7
4	1296	518	259	12	15
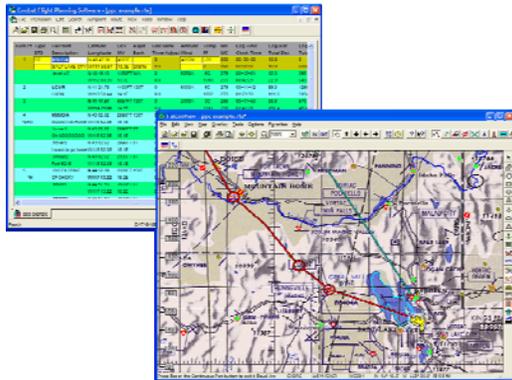
Figure 1: Portable Flight Planning System.
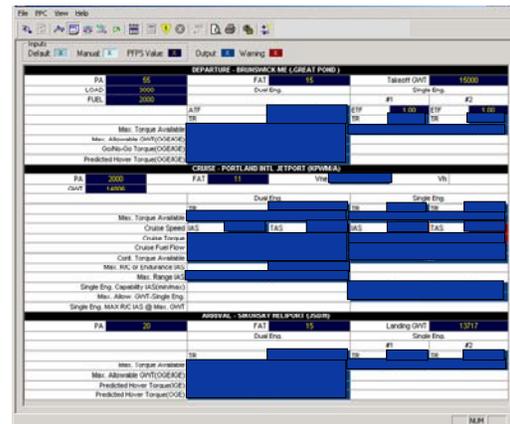
Figure 2: Performance Planning Card.
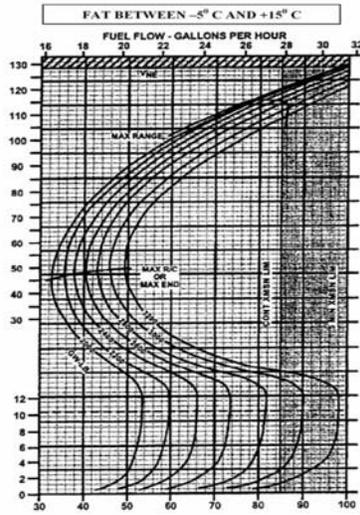
Figure 3. Performance Charts.

| a. Linear Interpolation. | b. Akima Interpolation. |
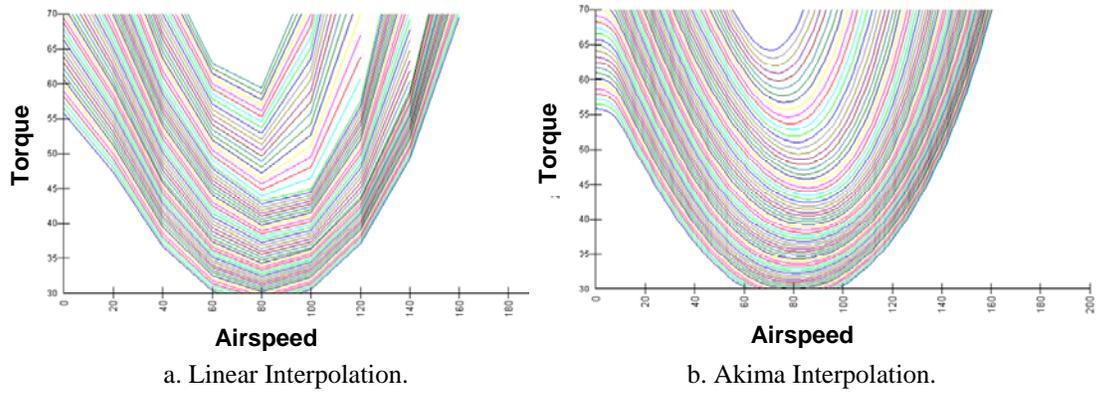
Figure 4. Comparison of Interpolation Methods.
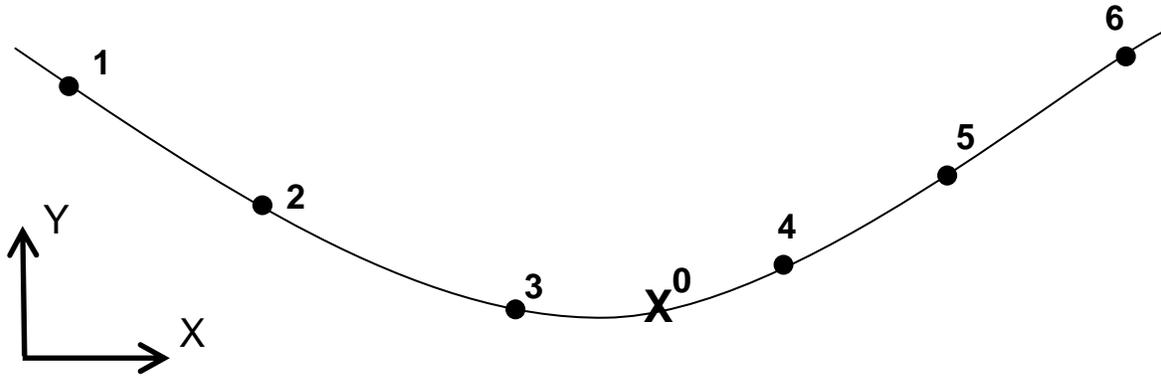
Figure 5. One-Dimensional Akima Interpolation.
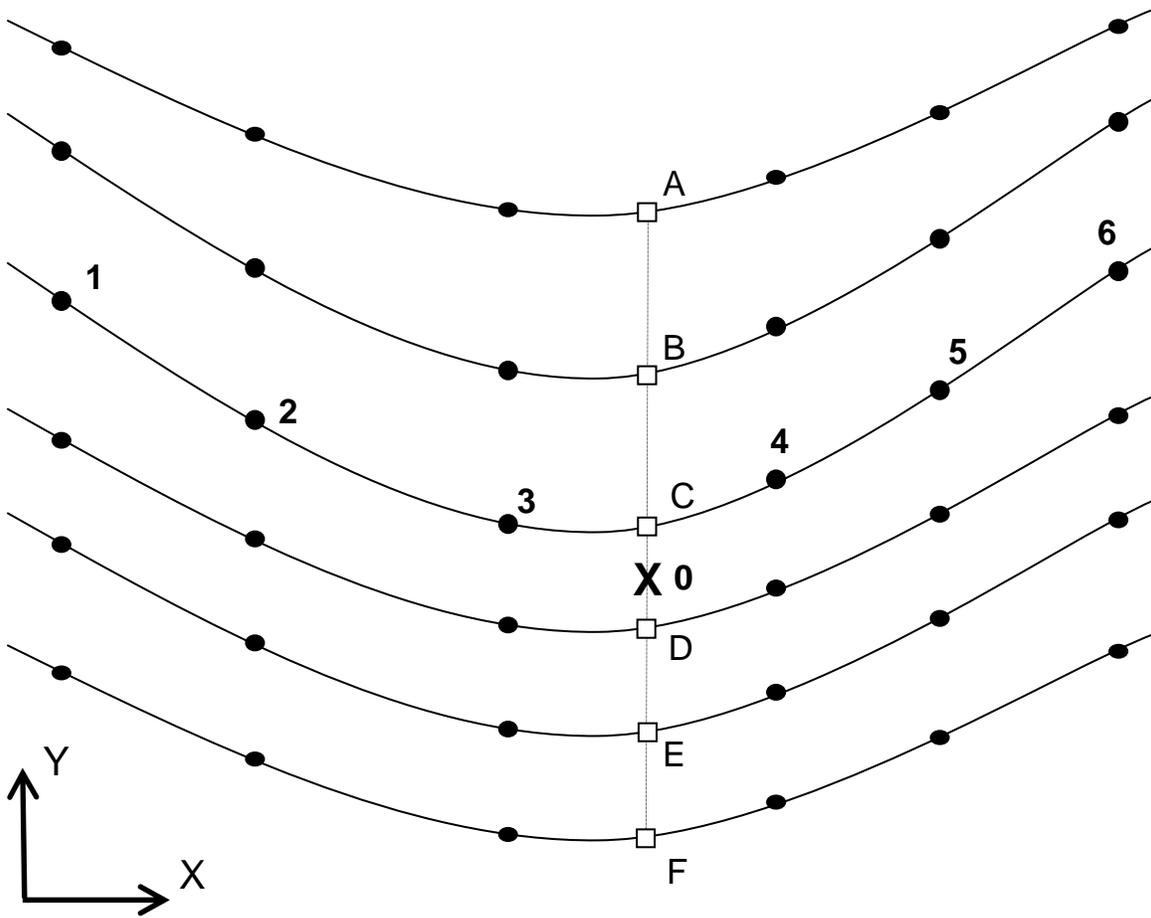
Figure 6. 2-Dimensional Akima Interpolation.
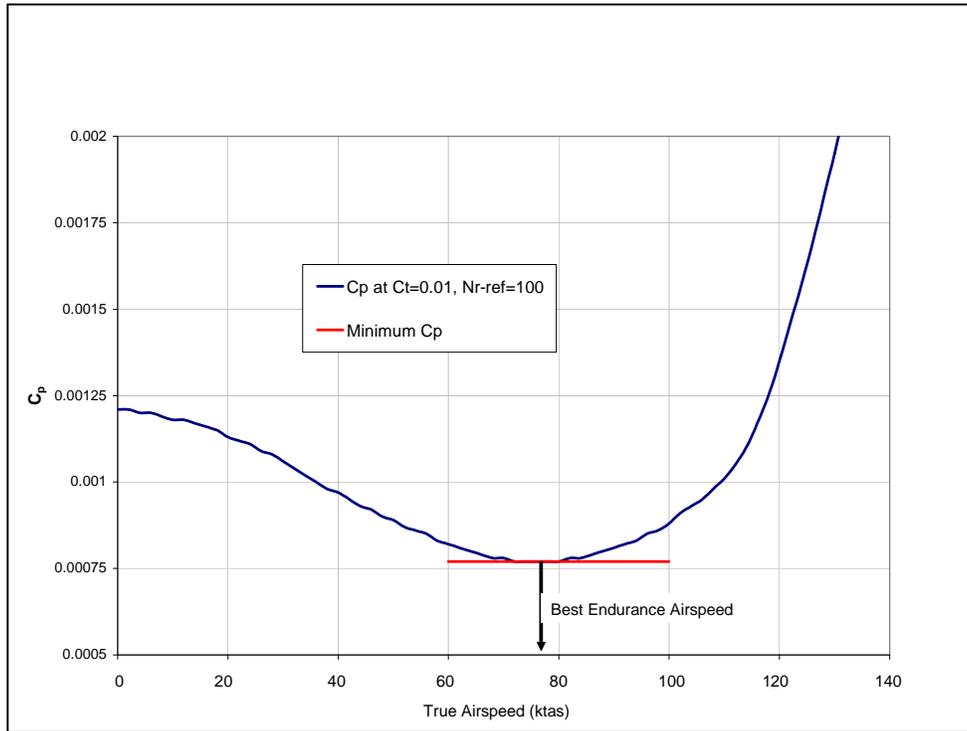
Figure 7. Best Endurance Airspeed
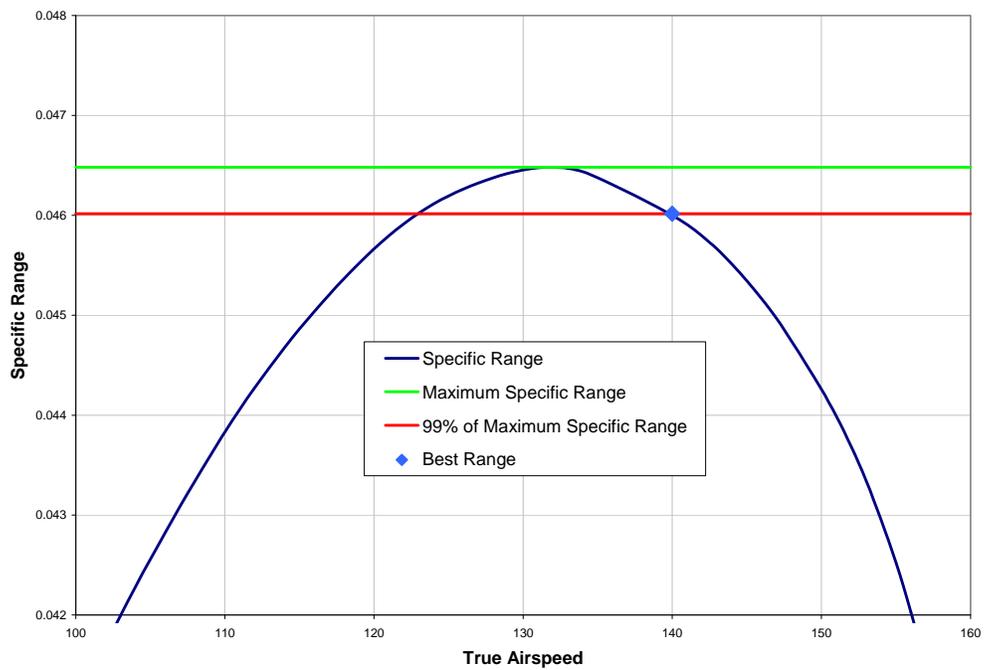
Figure 8. Best Range Airspeed
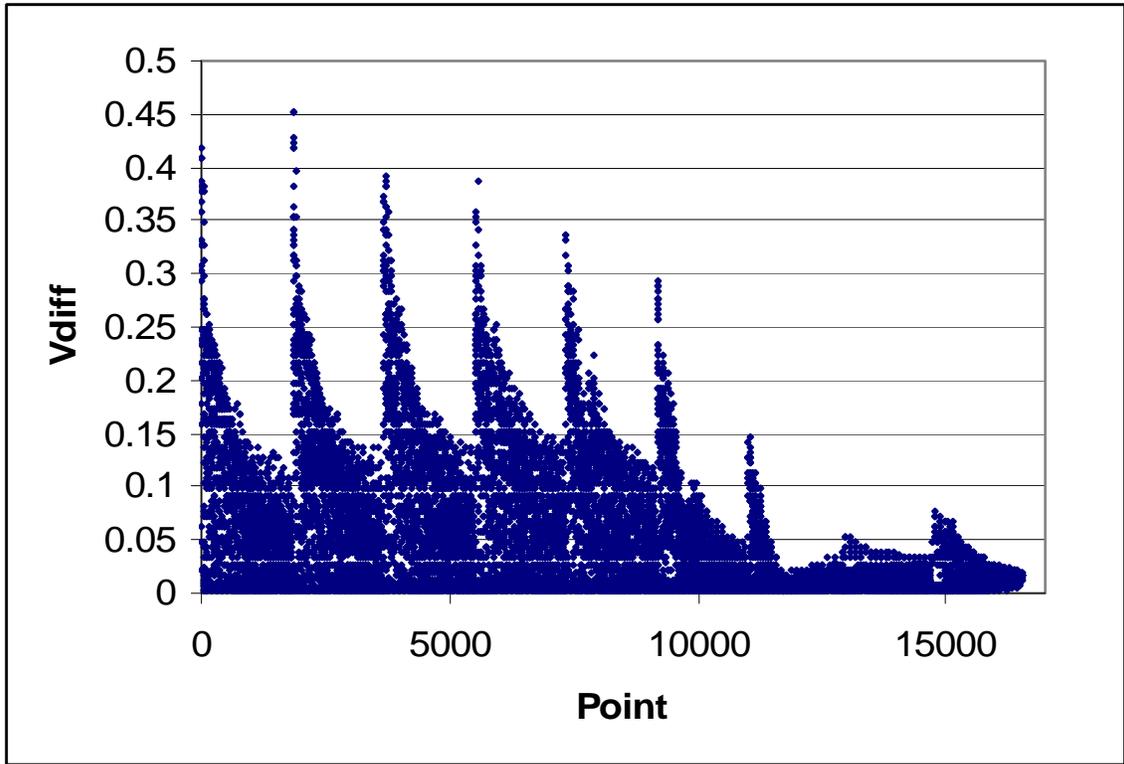
Figure 9. Differences between Computed Vbr and Vbr-avg.