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PROGRESS IN THE SEMI-EMPIRICAL PREDICTION OF THE AERODYNAMIC FORCES DUE TO LARGE AMPLITUDE OSCILLATIONS OF AN AIRFOIL IN ATTACHED OR SEPARATED FLOW

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ABSTRACT

In this paper, we study the possibility to modelize the unsteady behaviour of a profile, stall being taken into account, with a set of differential equations, starting from small amplitude oscillation tests in a wind-tunnel.

The application on several profiles have shown that the same set of equations could be used. Furthermore, we realised that the identified parameters behaved very similarly.

Our conclusion is that unsteady stalled behaviour is not very hard to predict, and can be introduced in rotor mechanical equations.

I. INTRODUCTION

The aerodynamic conditions on a rotor blade of an helicopter at cruising speed are such that we always have stall on some part of the retreating blade. Unsteady stall cannot yet be taken into account by pure theory.

ONERA proposes this effect to be modelised by an empirical set of differential equations, the coefficients of which are deduced from a series of wind tunnel tests. The feasibility of such a method h already been shown on a OA 12% profile (reference 1 and 2).

This method has now been extended to several profiles. This has led to a new well established set of equations. Morever, the needed coefficients show a remarkably common behaviour. This is the object of this paper.

II. PRESENTATION

II.1 - General principles of modelisation by differential equations

The input and output parameters of a system usually can be related through a set of differential equations. The classical methods of physics often display them readily.

Even in the cases where theory is too heavy to lead to them, differential equations can nevertheless be written, by searching for a set of equations with similar transfer functions than that of our physical system, at least on a certain plage of frequency.

In case our system has a non-linearity in X for example, the notion of transfer function has no meaning any more. We can use the general property that for a sufficiently small variation of X around an average value Xo, the system can be considered as linear and so admits a transfer function. A differential equation is then choosen, that fit this transfer function for each value of Xo ; Xo becomes a parameter in the equations.

This differential equation is valid for a small vibration around Xo. It can then be applied if Xo itself varies versus time, with the condition that this variation is sufficiently small. In fact, the constraint can be loosened : if the transfer function versus Xo does not change, Xo can vary freely. Our real constraint is that our transfer functions varies slowly enough as a function of time.

Such a modelisation has been used by E. SZECHENYI to predict the lift created on a cylinder by the vortex shedding, as a function of its movement (reference 3). The results have been excellent. To complete them, the author has introduced a supplementary term in X^3, that is negligeable at a low amplitude or at a small frequency, but which have allowed to correctly predict the limit of oscillation of the cylinder in case of diverging motion.

II.2 - Constraints due to the helicopter application

The application of modelisation by differential equations to the case of a helicopter profile will be done assuming a two-dimensional aerodynamics in each section. The input will describe the airfoil position, the output the aerodynamic forces.

At first, a set of differential equations has to be found, that reproduces the behaviour of the aerodynamic forces. Then the transfer function is curve fitted. The variables that create non-linearities have to be introduced as parameters in the equation coefficients.

They are incidence and Mach number at the moment, but Reynolds number, and with much ambition, parameters that describe the profile shape could be taken into account later.

Two problems arise :

- Stall

The transfer functions at small amplitude of vibration usually show a strong discontinuity when stall occurs. This discontinuity is contrary to the hypothesis of a slow variation of the transfer functions introduced at the preceding paragraph.

The steady flow on an airfoil can be of two kinds : attached or detached. The passage from one to the other corresponds to a strong variation of the studied mechanism for which our equations cannot be applied without hesitation. So far, the experience has shown us that the introduction of a stall delay is usually needed.

- Pitching and heaving

(heaving = translation perpendicular to the wind speed)

The problem of modelising by differential equations is theoretically solved. It suffices to curve fit the transfer functions aerodynamic forces/airfoil position.

In practice, the transfer functions are numerous enough, and the fittings complex enough since pitching and heaving are related (heaving creates an aerodynamic incidence), for us to try to simplif; the problem.

- we shall consider that the effect of fore-aft motion is quasi steady and so, it will not be modelised here. A study of the influence of that degree of freedom is scheduled.

- simple aerodynamic models ($C_z = \frac{\partial C_z}{\partial \theta}\theta$) are able to predict the behaviour of a helicopter in non externe flight conditions. We shall take pitching and heaving into account in the non-stall domain, and we shall see at the next chapter that information on heaving can be deduced from pitching.

- in the stall domain, a pitching/heaving differenciation could be useless, because the differences between the two tests can be lost in measurement incertainties. These two movements differ only by pitching speed which is null in case of heaving. Morever, tests realised by the institut de Mecanique des Fluides de Marseille, have shown that for high amplitude vibrations, heaving at amplitude H creates the same forces that pitching at amplitude $\gamma H/V$, for the same mean incidence (reference 6).

II.3 Modelisation for non-stalled flow

Instead of considering pitching and heaving, we shall separate the effects of two types of induced velocities on profiles.

a) 

constant distribution

θ or $\dot{h}/V$

b) 

linear distribution

$\dot{\theta}$

The distribution b) can hardly be isolated experimentally.

For the unstalled flow, we shall use these distributions of induced velocity as inputs. We have been led to write our equations as follow :

$$\left\{\begin{array}{l} \dot{C}_z + d\,C_z = (a_1 \;\; a_2)\begin{pmatrix}\theta + \dot{h}/V \\ \dot{\theta}\end{pmatrix} + (b_1 \;\; b_2)\,\dot{\overline{\begin{pmatrix}\theta + \dot{h}/V \\ \dot{\theta}\end{pmatrix}}} \\ C_m = (c_1 \;\; c_2)\begin{pmatrix}\theta + \dot{h}/V \\ \dot{\theta}\end{pmatrix} + (d_1 \;\; d_2)\,\dot{\overline{\begin{pmatrix}\theta + \dot{h}/V \\ \dot{\theta}\end{pmatrix}}} \end{array}\right.$$

A term $\dot{C}_m$ seems useless. The unknown coefficients are easily identified on experimental or theoretical (Van de Vooren, reference 4 for example) transfer functions.

We shall rather write these expressions as :

$$\begin{cases} \dot{C_z} + \lambda C_z = \lambda C_{z_s}\big|_{\theta + \dot{h}/V} + \lambda s \dot{\theta} + \sigma(\dot{\theta} + \ddot{h}/V) + \delta \ddot{\theta} \\ C_m = C_{m_s}\big|_{\theta + \dot{h}/V} + s_m \dot{\theta} + \sigma_m(\dot{\theta} + \ddot{h}/V) + \delta_m \ddot{\theta} \end{cases}$$

for which $C_z = C_{z_s}$ and $C_m = C_{m_s}$ in steady conditions.

The fact that the imaginary part of the heaving transfer function equals ν times the real part of the pitching transfer function at small reduced frequencies leads to :

$$\delta \simeq s \qquad \delta m \simeq s m$$

When this property is used, heaving is deduced completely from tests in pitch. Figure 1 shows a fitting in pitch for a flat blade at M = 0.5, and the forces in heaving that are deduced from it though the use of δ = s, compared with the theorical values. The accuracy is sufficient, knowing that heaving usually is less important.

Modelisations of the theoretical flat blade at a Mach number of 0.5 gave the following equations :

$$\begin{cases} \dot{F} + 0.12\, F = -\pi \rho V^2 b^2 \ell \left\{ 0.276\,(\theta + \dot{h}/V) + 0.228\,\dot{\theta} + 1.38\,(\dot{\theta} + \ddot{h}/V) + 1.9\,\ddot{\theta} \right\} \\ M^t = -\pi \rho V^2 b^3 \ell \left\{ 0.6\,\dot{\theta} + 0.65(\dot{\theta} + \ddot{h}/V) + 0.22\,\ddot{\theta} \right\} \end{cases}$$

We shall finally remark that in case of an incompressible flow on the flat blade, the equation for moment is exact and is written :

$$M^t = -\pi \rho V^2 b^3 \ell \left\{ 0.5\,\dot{\theta} + 0.5(\dot{\theta} + \ddot{h}/V) + 0.375\,\ddot{\theta} \right\}$$

II.4 Modelisation in the stalled domain

Transfer functions at small amplitude in the stalled domain clearly show the following properties :

- a high frequency behaviour much like the unstalled flow.
- a medium frequency behaviour of the resonance type.

These properties, the need of a certain continuity of the model with the no-stall equations, the respect of some limit conditions, have led to the following set of equations :

$$\begin{cases} \dot{C}_{z_1} + \lambda C_{z_1} = \lambda C_{z_L}\big|_{\theta + \dot{h}/V} + \lambda s \dot{\theta} + \sigma(\dot{\theta} + \ddot{h}/V) + \delta \ddot{\theta} \\ \ddot{C}_{z_2} + a \dot{C}_{z_2} + r C_{z_2} = -(r \Delta C_z + E \dot{\theta}) \\ C_z = C_{z_1} + C_{z_2} \end{cases}$$

$$\begin{cases} C_{m_1} = C_{m_L}\big|_{\theta + \dot{h}/V} + s_m \dot{\theta} + \sigma_m(\dot{\theta} + \ddot{h}/V) + \delta_m \ddot{\theta} \\ \ddot{C}_{m_2} + a_m \dot{C}_{m_2} + r_m C_{m_2} = -(r_m \Delta C_m + E_m \dot{\theta}) \\ C_m = C_{m_1} + C_{m_2} \end{cases}$$

C_{zl}, ΔC_z, C_{ml}, ΔC_m are defined on figure 2. C_{zl} is C_{zs} when no stall, but has to be extrapoled in the stall domain. It is the value C_{zs} of static C_z that would be obtained if boundary layer were sucked for example.

We see that the first equation is kept the same, but to the forces it predicts, a correction given by the second equation has to be added.

In every case, as well for lift, moment or drag, ΔC_z will play the role of the parameter measuring stall. $\Delta C_z = 0$ means no stall, ΔC_z large means large stall. The parameter $\ddot{\theta}$ would not define stall quantity as well.

The form of equation we have written is now well established. It is the investigation on several profiles that made us choose these equations among other possibilities. It does not seem possible to choose another formulation. On the one hand, we cannot diminish the number of parameters in the model for the sake of simplification, and on the other hand, the curve fits are good enough, and taking another parameter into account (term $\ddot{\theta}$ in the second side of the second equation for example) would be useless.

The only simplification we can think of would be to derive moment equations from lift through a simplified formula, instead of treating it independantly. This possibility is soon going to be tested

II.5 - Transition between unstalled and stalled regions

Care has to be taken for that transition, because it corresponds to a rapid variation of the flow, which is contrary to our hypothesis.

Three cases have been met :

. when stall is mild, the direct use of the equations we have derived from the transfer functions, yields good results_see reference 1.This has been the case with OA 12 profile for which some lift big loops are displayed figure 5 (a stall delay improves the loops but can be neglected). For this airfoil, we had no sudden loss of lift at stall, on the static curves.

. for other profiles with a more classical behaviour, the natural delay brought by the differential equations is quite insufficient to represent the actual delay. This delay is a non-linear phenomenon invisible on small amplitude tests in which we never jump from one type of flow to the other.

The experiment shows that stall happens at a higher incidence than the static stall angle θ_s , and as long as stall has not occured we must not use the equation of the stall regime. So, our model needs at least a dynamic criteria of stall. We shall rely upon BEDDOES result (reference 5) and say that stall effectively happens with a delay $\Delta\tau$ in reduced time ($\Delta t = b\Delta\tau/V$ in real time) after meeting the static stall angle(BEDDOES used a delay of 10.8 for lift and 5 only for moment).

This hypothesis is taken into account by constraining the second member of the second equation to zero till stall actually occurs. This is done by constraining $\Delta\zeta$ to 0 till stall occurs, furthermore, we need the property E =($\Delta\zeta$ = 0)=0, which is well verified.

★ It can happen that the static stall is so severe that quasi steady stall and reattachment occur at two different angles ($\theta_{reatt} < \theta_{stall}$)
In that case we have a zone $\theta_{reatt} \leq \theta \leq \theta_{stall}$ for which both types of flow are stable. We had then to introduce a second delay for reattachment too.

In conclusion of this paragraph, we see that we usually have to use a more complex model than the one which consists to say that every flow change occurs at the static stall angle. Till now, small amplitude tests were sufficient to define our models. But in fact :

- It is more prudent to complete the small amplitude data by some large amplitude loops in the domain we are interested in (helicopter cyclic pitch frequency, for example) for checking the model.

- Some special tests at high amplitude could give interesting information on the value of the parameters a and r when the stalled quantity ΔC_z becomes null (big loops, or better : step of incidence between say $\theta = \theta_{stall}+3°$ and $\theta = \theta_{stall}-5°$)

- If we admit the scheme of a constant delay $\Delta\zeta$, we only add one parameter to the model. Moreover, if we impose $\Delta\zeta$ = 10 systematically, no degree of freedom is added. This value seems convenient at least in the cases we have treated so far.

III. APPLICATIONS

III.1 - Performed modelisations

Modelisation has been performed on several profiles of the OA family (9%, 12%, 13%). The behaviour of the VR7 airfoil has been studied too. This work made us converge toward the set of equations described before, which at the present seems the most adapted.

The models have been identified from pitch tests. The hypothesis of paragraph I.4 enables us to apply them to heaving, by introducing the variable h' and h" as described, but no test has been done to check that.

The experimental transfer function were well fitted, the only problem having been met on the real part of lift, at zero degree of incidence : the model expects an asymptotic value, that measurements do not always display. When this experimental tendency happened, it has been neglected (but the low frequencies were always well fitted).

In every case, the parameters has been easily found and show remarkable behaviours.

III.2 Mean model

Our model has two sets of parameters :

- λ, s, σ which characterize the unstalled flow. We know that every airfoil behaves almost the same in that regime.

- r, a, e which characterize the stalled flow. In fact, we had to introduce a correction in the parameter T of the first equation, a correction due to stall.

These coefficients are expressed as fucntion of ΔC_z , which we consider as a measure of stall. The results obtained sofar have given about the same value of (r, a, e, σ) for each airfoil, and most remarkably, the same dependence versus ΔC_z , as figure 3 shows.

The dots come from an automatic curve fitting using a least square method, and the curve is the retained function for the parameter. The parameter b replaces ($\lambda s+\sigma$) and is related to the asymptotic value of the real part of the transfer function. This value is not well defined by experiment, which explains dot dispersions.

We see that, as well for lift or for moment :

$\sqrt{r} \simeq r_o + r_2 \Delta C_z^2$ ($\sqrt{r}$ is a circular frequency)

$a \simeq a_o + a_2 \Delta C_z^2$

$E \simeq e_o + e_2 \Delta C_z^2$ $e_o < 0$ for lift, $e_o > 0$ for moment

$\sigma \simeq \sigma_o + \sigma_1 \Delta C_z$ $\sigma_1 < 0$ " $\sigma_1 > 0$ "

Futhermore, the parameter r happens to be the same for lift and for moment. This means that this "resonance circular frequency" is in fact a property of the flow which manifests itself in every type of force.

As a conclusion of this paragraph, it seems that the dynamical stalled behaviour of an airfoil at small smplitudes of vibration is quite easy to modelize, and that it depends little on its geometry (at least for the classical airfoils that have been tested).

In fact, the airfoil geometry has an important influence on static characteristics only.

III.3 - Application to large amplitude motion

Our model is now able to reproduce the behaviour of our airfoil for every low amplitude vibration, even non sinusoïdal ones. The requirement for this is linearity at low amplitude, which have been well verified (in fact, we noticed a loss of linearity at high Mach number, when shock waves are present).

Our equations can be used with high amplitude motion, provided the transfer functions evolve slowly with our parameters (incidence, Mach number or any other).

The slower, the evolution, the better the results. Obviously this hypothesis fails at stall, and experience effectively revealed problems here. In fact, these problems happen to be stronger with more sudden stalls. Here are the three cases we have dealt with :

OA 12 profile

This is the first airfoil we have tested. A model, that was quite heavy was derived manually, by trial ans error from small amplitude vibrations. The direct application to large loops gave results that were published (reference 1 or 2) and that we judged very satisfactory. In fact it seems that comparaison model-experiment could have been better if we had taken a delay into account.

Modelisation has been done again, using our automatic code and the equations toward which we have converged. The new big loops are drawn figure 5, taking the BEDDOES delay into account. They are not so good at very high incidence. This is due to the fact that we used fast automatic procedures that can ignore certains tendencies. Modelisation is easily performed, but is slightly less accurate (for example, we write certain coefficients as parabola versus ΔC_z whereas the parabolic behaviour could come from different kinds of formulae ; a possibility would be to replace these formulae simply by adjusted tables of data in which we would interpolate).

OA 9 profile

Here, we have met the problem of the fidelity of wind tunnel tests. The low and high amplitude tests were conducted on two different models, several years apart. Our equations based on the low-amplitude tests were unable to reproduce the big loops, till we realised that different static curves were measured during the two tests.

The loops drawn in figure 4 use the coefficient of the low amplitude tests (we had no choice) and the static curves of the high amplitude ones. The model restitutes at last the general appearance of the loops.

Tjis shows something we have met several times : the static curves are very sensitive, as well to the profile shape as to experimental conditions, but non the unsteady characteristics.

VR 7 profile

The experimental data come from a water tunnel test run by Ken Mc ALISTER of the US Army. This profile has the peculiarity to show different stall and reattachment angles (θ s and θr), even at a quasi-null pitch rate. In fact the zone between θs and θr is stable for the unstalled flow as well as for the stalled regime. This makes stall a very sudden event.

This airfoil shows a static lift curve with three separate zones (see fig. 2), which made us quite uncertain for the choice of C_{zl} and ΔC_z . Our choice has worked for lift ; on moment, results were very interesting but the introduction of the necessary delay broke this good behaviour. Some choices we have done here are probably inappropriate. This problem cannot arise on more classical profiles. Because of the very sudden stall and reattachment, we had to introduce a delay in each case, still BEDDOES value of $\Delta\tau = 10$.

III.4 - Modelisation on the whole helicopter flight domain

The OA 9 profile is going to be used on a helicopter rotor. In order to estimate the complete behaviour of that rotor, a model was to be established that covers the whole flight domain of a blade.

The small amplitude tests had to be done in two different wind tunnels, the low Mach number at Toulouse (CEAT) and the High Mach number at Modane (ONERA). Mach 0.4 was common to the two series of tests.

It happened that the two series of tests show little continuity. We already had this kind of problem with small amplitude and high amplitude tests (see OA 9, prece ding chapter) : wind tunnel tests on the same profile differ from each other and so cannot be really quantitative : so how can we restitute the blade in-flight behaviour ?

Nevertheless, we have established a general model for OA 9, that respects the general tendencies we have met. Such a model should restitute the behaviour of a helicopter profile, qualitatively. It will be used in that goal, coupled with the mechanical equations of the rotor (reference 7). For more quantitative evaluations, more confident wind tunnel tests are needed. In any case the use of this model should improve calculations that would use a quasi-steady aerodynamic theory.

Figure 7 shows the model behaviour. At first the lift and moment static curves and the transfer functions when no stall, at several Mach numbers, then the transfer functions versus incidence at a Mach number of 0.40.

IV. CONCLUSION

An investigation on several profiles has allowed us to propose the simplest form of equation that can modelise their aerodynamic characteristics, even in the stalled region. This study has clearly revealed a common behaviour of the model coefficients. This fact simplifies their research, which is now quite automatic.

The necessary wind tunnel tests are quickly done if one uses the method of random low amplitude excitations to get the needed transfer functions. However, it is preferable to complete them by some high amplitude tests.

The application of these equations to the high amplitude movements that are met on a helicopter blade gives accurate enough results, excepts for the onset of stall. The stall delay that is usually met can be introduced artificially into the equation under the form of the constant in reduced time stall delay of BEDDOES (reference 5).

It finally looks as if the static behaviour of a profile at high incidence were more difficult to predict than the dynamic behaviour. It is more sensitive too, to the external conditions (wind tunnel for example).

Further investigation will study the properties of thinner airfoils. Drag behaviour will have to be checked too. There exists a possibility to make moment derive directly from lift, a procedure that would save one degree of freedom for resolution.

Modelisation of stall through differential equations is now established upon a solid base. It is currently used at ONERA and Aerospatiale to check the effects of forced vibrations of rotors and is in the process of being integrated in a computer code which uses the linear 3D lifting surface theory to determine the induced velocity on each blade section.

V. REFERENCES

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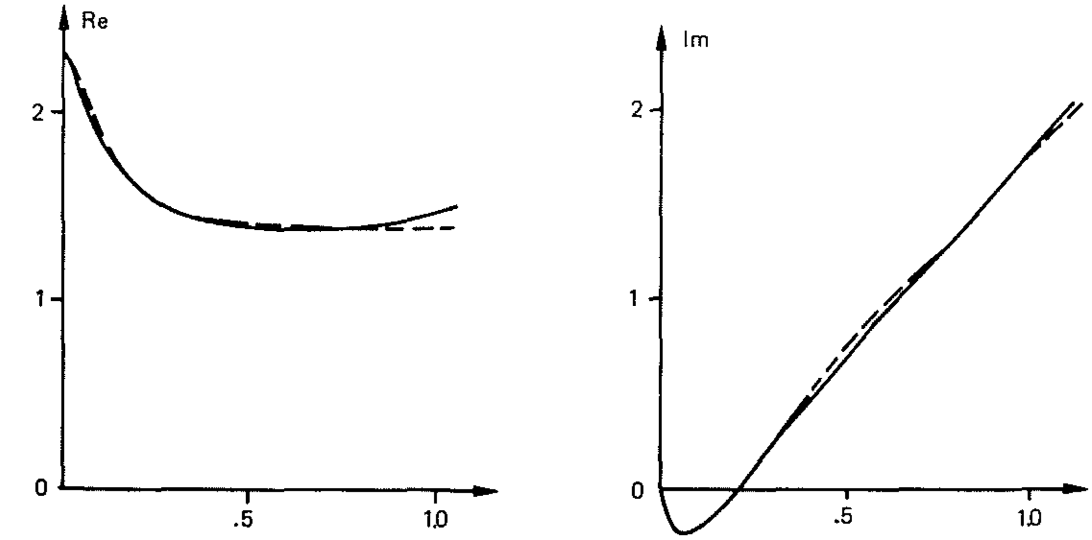
FIGURE 1



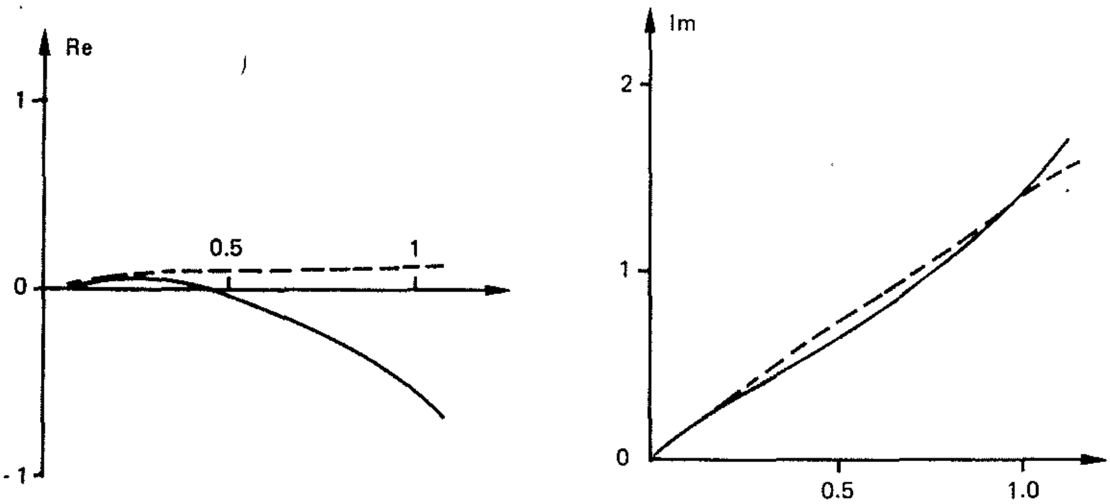
Pitch around quarter chord

_________ Van de Vooren lift coefficients (Mach = 0.5)

------- Model obtained after fitting by

$$\dot{F} + \lambda F = \lambda\alpha\,(\theta + \dot{h}/V) + \lambda s\,\dot{\theta} + \sigma\,(\dot{\theta} + \ddot{h}/V) + s\,\ddot{\theta}$$

with $\alpha = 2.3$ $\quad\sigma = 2.38$ $\quad s = 1.9$



Heaving _________ Van de Vooren lift coefficient

---------- Model obtained by fitting pitch

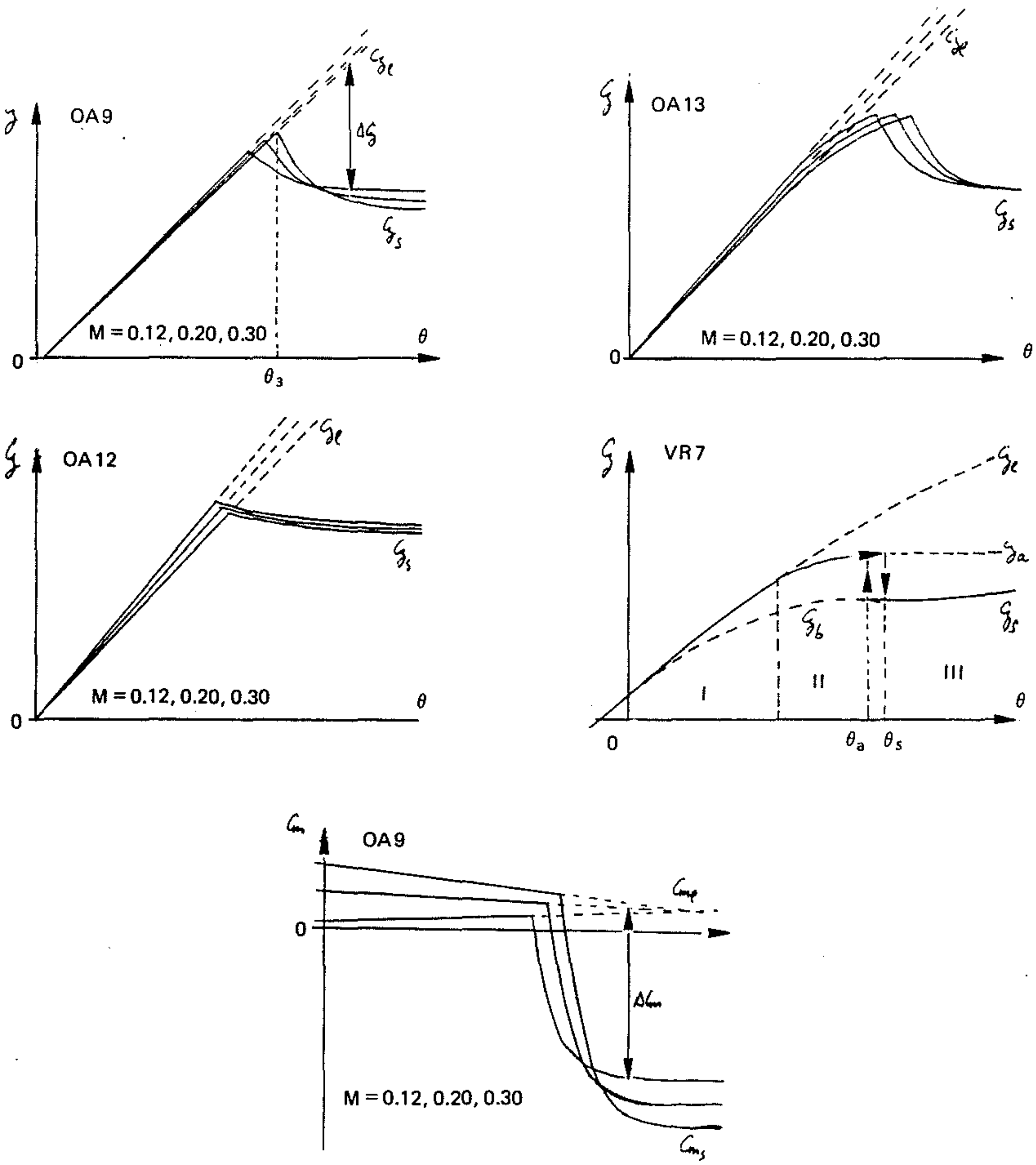


FIGURE 2 : STATIC CURVES USED FOR THE DIFFERENT AIRFOILS

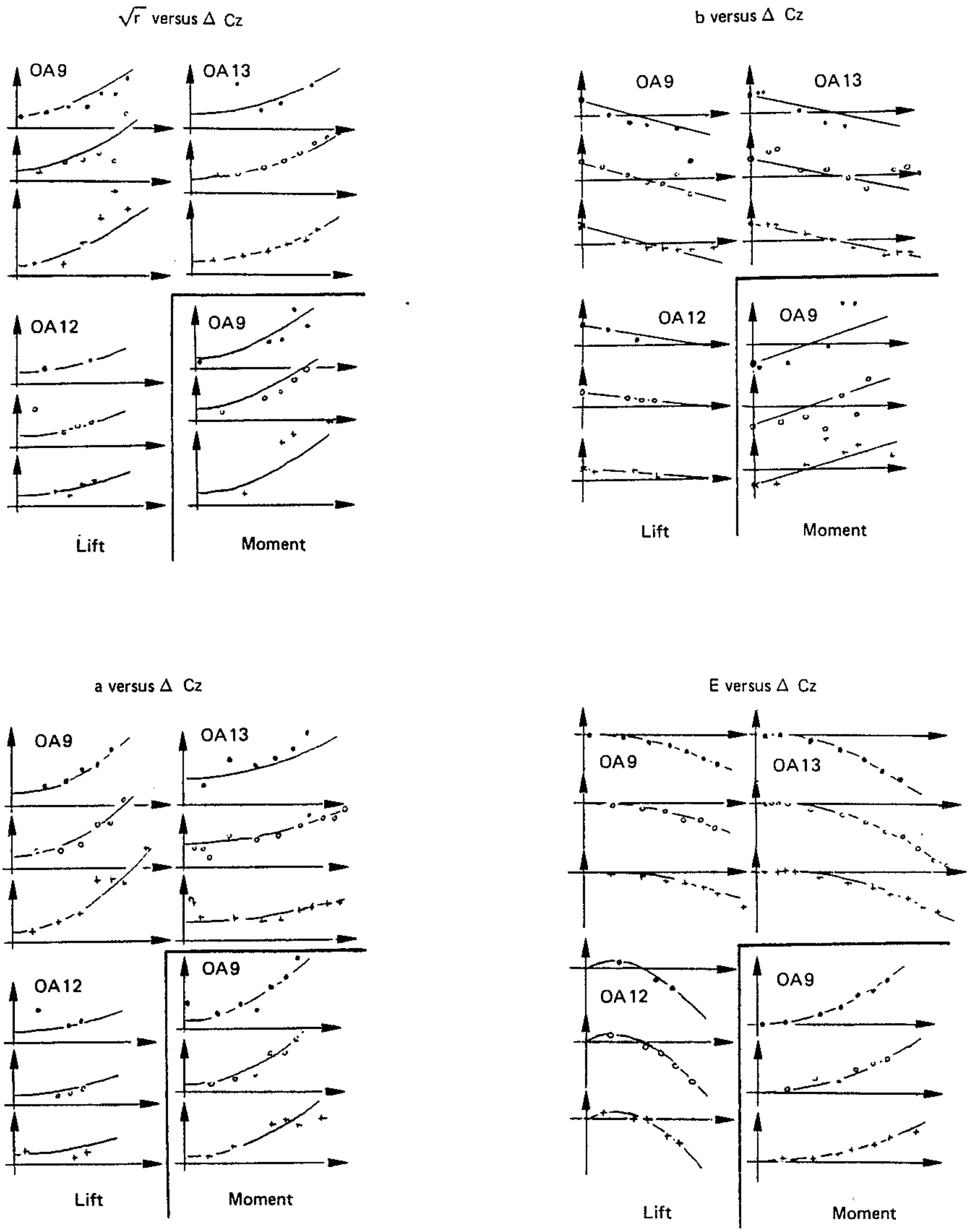
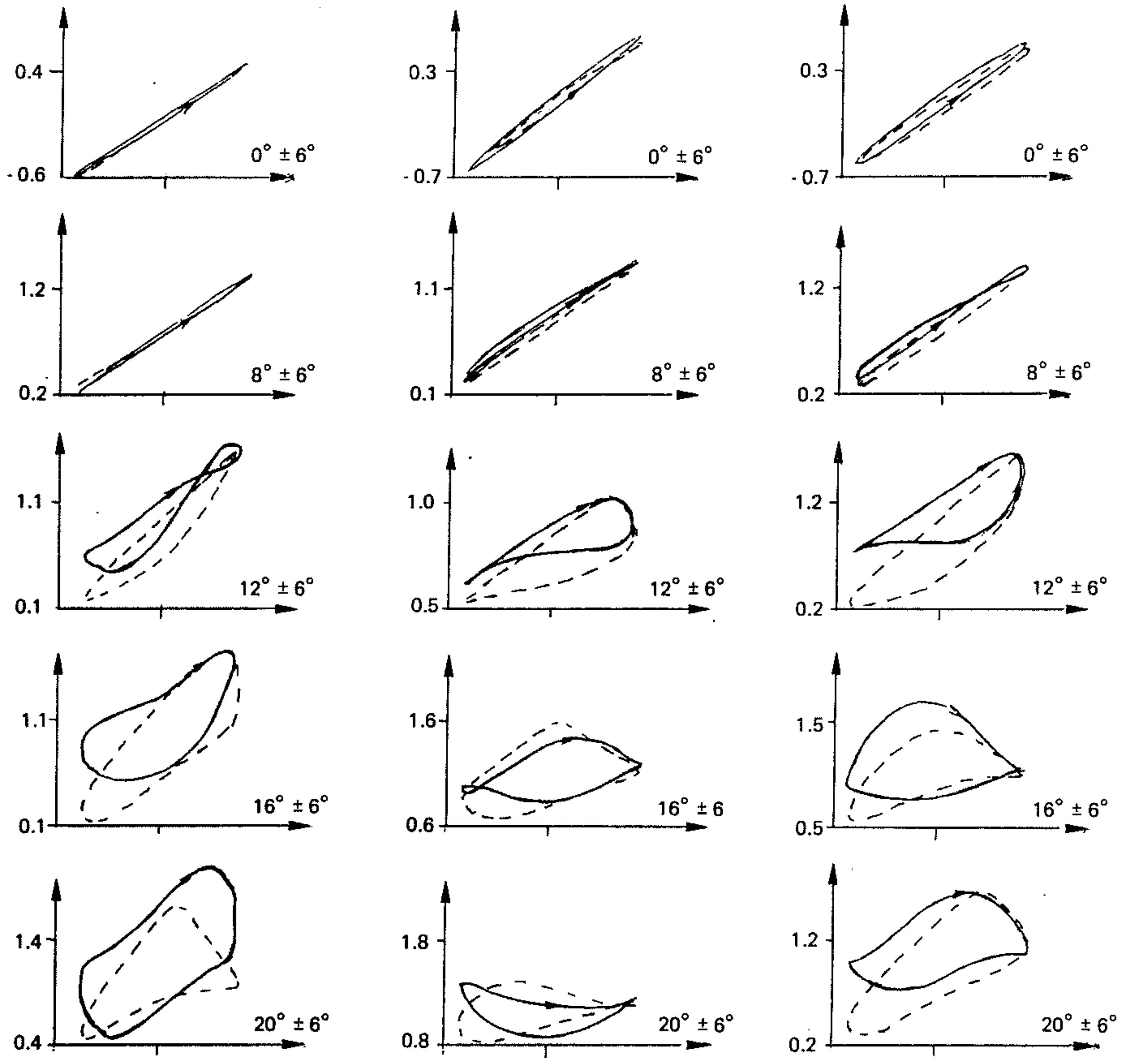


FIGURE 3 : EVOLUTION OF THE MODEL PARAMETERS VERSUS STALLED QUANTITY ΔC_z



M = 0.12 ν = 0.12
f = 4 Hz

M = 0.30 ν = 0.05
f = 4 Hz

M = 0.30 ν = 0.10
f = 8 Hz

_____ experiment

_ _ _ model

FIGURE 4 : OA 9 PROFILE - LARGE AMPLITUDE LIFT LOOPS

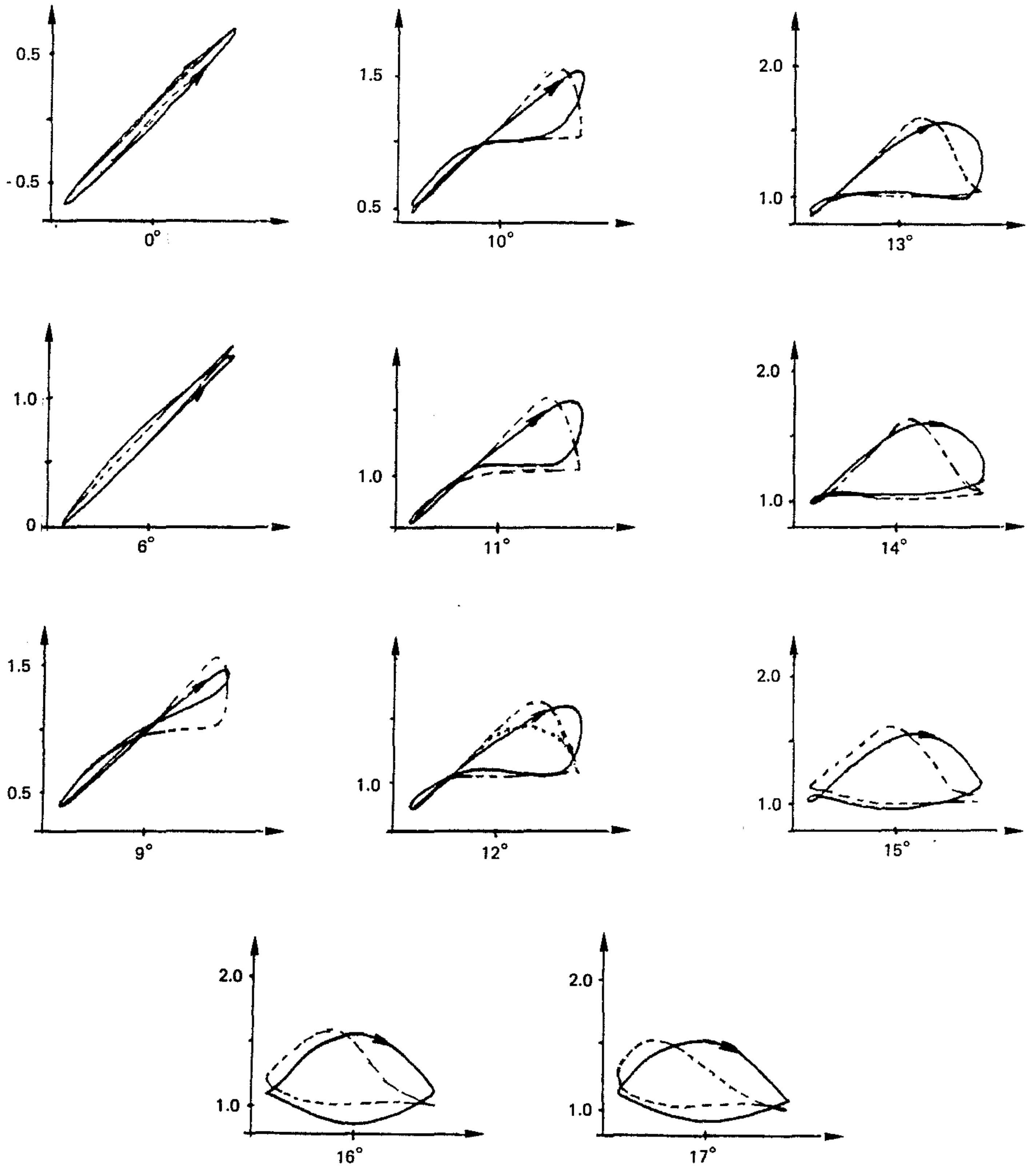


FIGURE 5 : OA 12 PROFILE : LIFT LOOPS, AMPLITUDE OF OSCILLATION : ± 6°

M = 0.30 ν = 0.05 (freq = 4 Hz)

——— Experiment
— — — Model
.... Model without stall delay

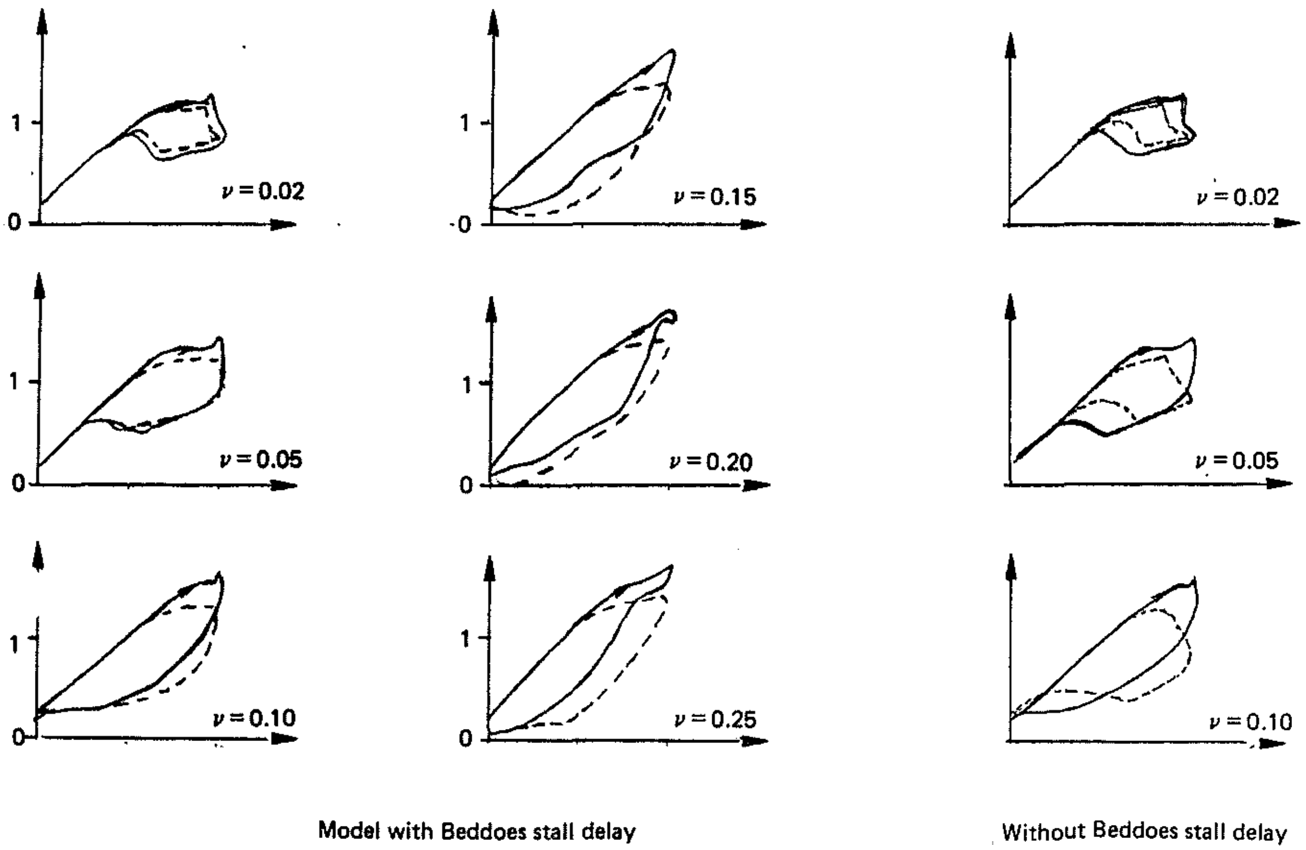


FIGURE 6 : VR 7 PROFILE - LARGE AMPLITUDE LIFT LOOPS

______ Experiment

_ _ _ Model

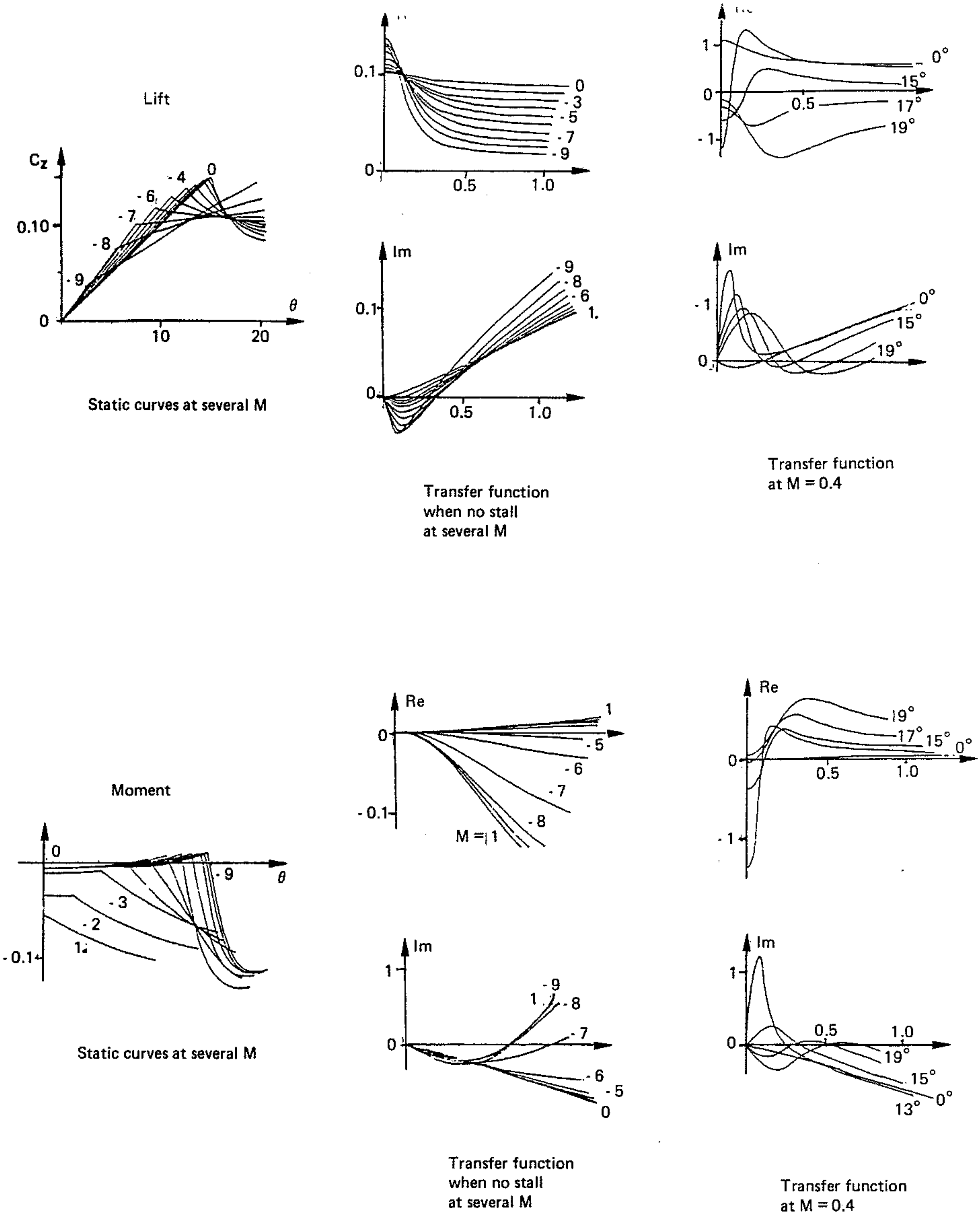


FIGURE 7 : MODELISATION ON THE WHOLE MACH DOMAIN, BASED ON OA 9 TESTS

VI . APPENDICES

VI.1. OA 9 Lift ; $\mathcal{M} \in [0, 0.35]$

$$\begin{cases} \dot{C}_{z_1} + d\, C_{z_1} = d\, C_{z_e} + (d s + \sigma)\,\dot{\theta} + s\,\ddot{\theta} \\ \ddot{C}_{z_2} + a\, \dot{C}_{z_2} + r\, C_{z_2} = -(r\, \Delta C_z + E\, \dot{\theta}) \\ C_z = C_{z_1} + C_{z_2} \end{cases}$$

$C_{z_e} = C_{z_0} + p_0\, \theta$

$C_{z_s} = C_{z_e}$ $\qquad \theta \leq \theta_s$

$C_{z_s} = C_{z_0} + p_0\, \theta_s + c\left[e^{h(\theta - \theta_s)} - 1\right]$ $\qquad \theta_s \leq \theta$

$\Delta C_z = C_{z_e} - C_{z_s}$

Stall delay. $\Delta \tau_s = 10$

$\theta_s = 16 - 10\,\mathcal{M}$

$C_{z_0} = -0.05$

$p_0 = 0.1 / \sqrt{1 - \mathcal{M}^2}$

$c = -4/3\,\mathcal{M} + 0.7$

$h = -0.5$

$d = 0.20$

$\sigma = 0.0775 - 0.08\,\mathcal{M} - 0.13\,\Delta C_z$

$s = 0.087$

$\sqrt{r} = 0.1 + 0.80\,\Delta C_z - 1 + 1/(0.80\,\Delta C_z + 1)$

$a = 0.20 + 0.30\,\Delta C_z^2$

$E = (0.3\,\mathcal{M} + 0.10)\,\Delta C_z^2$

VI.2 – OA 12 Lift ; $\mathcal{M} \in [0, 0.35]$

$$\begin{cases} \dot{C}_{z_1} + d\, C_{z_1} = d\, C_{z_e} + (d s + \sigma)\,\dot{\theta} + s\,\ddot{\theta} \\ \ddot{C}_{z_2} + a\, \dot{C}_{z_2} + r\, C_{z_2} = -(r\, \Delta C_z + E\, \dot{\theta}) \\ C_z = C_{z_1} + C_{z_2} \end{cases}$$

$C_{z_e} = (0.1 + 0.08\,\mathcal{M})\,\theta$

$C_{z_s} = C_{z_e}$ $\qquad \theta \leq \theta_s$

$C_{z_s} = (0.1 + 0.08\,\mathcal{M})\,\theta_s + 0.35\left[e^{(-0.09 - 0.15\,\mathcal{M})(\theta - \theta_s)} - 1\right] + 0.01\,(\theta - \theta_s)$ $\qquad \theta_s \leq \theta$

$\Delta C_z = C_{z_e} - C_{z_s}$

$\theta_s = 12.5 - 4.5\,\mathcal{M}$

Stall delay: $\Delta \tau_s = 10$

$d = 0.275 - 0.25\,\mathcal{M}$

$b = 0.047 + 0.135\,\mathcal{M} - (0.03 + 0.2\,\mathcal{M})\,\Delta C_z$

$s = 10\,\pi / 360$

$\sigma = b - d s$

$\sqrt{r} = 0.10 + 0.20\,\Delta C_z^2$

$a = 0.10 + 0.15\,\Delta C_z^2$

$E = 0.05\,\Delta C_z - 0.10\,\Delta C_z^2$

VI - 3 VR7 lift : uncompressible flow

$$\begin{cases} \dot C_{z_1} + \lambda C_{z_1} = \lambda C_{z_\ell} + (\lambda s + \sigma)\dot\theta + s\ddot\theta \\ \ddot C_{z_2} + a\dot C_{z_2} + r C_{z_2} = -(r\Delta C_z + E\dot\theta) \\ C_z = C_{z_1} + C_{z_2} \end{cases}$$

$C_{z_\ell} = -22/3000\ \theta\sqrt{|\theta|} + 0.102\ \theta + 0.16$

$C_{z_a} = C_{z_\ell}$ $\quad \theta \le 9°$

$C_{z_a} = 0.88 + 0.069\ (\theta - 9) - 0.0135\ (\theta - 9)^{1.6}$ $\quad 9 \le \theta \le 16°$

$C_{z_a} = 1.0593$ $\quad 16° \le \theta$

$C_{z_b} = 0.77 - 0.00237\ (\theta - 16)^2$ $\quad \theta \le 16°$

$C_{z_b} = 0.77 + 0.0006\ (\theta - 16)^2$ $\quad 16° \le \theta$

$C_{z_s} = C_{z_\ell}$ $\quad \theta \le 9°$

$C_{z_s} = C_{z_a}$ $\quad 9° \le \theta \le 16°$

$C_{z_s} = C_{z_b}$ $\quad 16° \le \theta$

$\Delta C_z = C_{z_\ell} - C_{z_s}$

$\lambda = 0.25$

$b = 0.06 + 0.003\theta - 0.3\Delta C_z$

$s = 0.103$

$\sigma = b - \lambda s$

$\sqrt{r} = 0.35 + 0.65\Delta C_z^2$

$a = 0.6 + 1.8\Delta C_z^2$

$E = -0.63\ \Delta C_z^2$

Stall delay : $\Delta\tau_s = 10$

Reattachment delay : $\Delta\tau_a = 10$

VI - 4 General model for lift , based on OA 9 tests , $M \in [0, 0.9]$

$$\begin{cases} \dot C_{z_1} + \lambda C_{z_1} = \lambda C_{z_\ell} + (\lambda s + \sigma)\dot\theta + s\ddot\theta \\ \ddot C_{z_2} + a\dot C_{z_2} + r C_{z_2} = -(r\Delta C_z + E\dot\theta) \\ C_z = C_{z_1} + C_{z_2} \end{cases}$$

$C_{z_\ell} = C_{z_0} + p_0\ \theta$

$C_{z_s} = C_{z_\ell}$ $\quad \theta \le \theta_s$

$C_{z_s} = C_{z_0} + p_0\ \theta_s + c\left[e^{h(\theta - \theta_s)} - 1\right] + p_1\ (\theta - \theta_s)$ $\quad \theta_s \le \theta$

$\Delta C_z = C_{z_\ell} - C_{z_s}$

$\theta_s = 15\ (1 - M^2)$

$C_{z_0} = 0$

$p_0 = 0.100\ (1 - M^8)/\sqrt{1 - M^2}$

$p_1 = 0.1\ M^4$

$c = 0.70\ (1 - M)$

$h = -M^3 + 1.5M^2 - 0.5$

Stall Delay $\Delta\tau_s = 10$

$\lambda = 0.25 - 0.15M^2$

$b = 0.105 - 0.08M + 0.1\ \Delta C_z$

$s = 0.08\ (1 + M^2)$

$\sigma = b - \lambda s$

$\sqrt{r} = 0.1 + 0.65\Delta C_z - 1 + 1/(0.65\Delta C_z + 1)$

$a = 0.15 + 0.45\ \Delta C_z^2$

$E = -0.08\ \Delta C_z^2$

VI 5 - General model for moment, based on OA9 tests , $\mathcal{M} \in [0, 0.9]$

$$\begin{cases} C_{m_1} = C_{m\ell} + (s+\sigma)\dot{\theta} + s\ddot{\theta} \\ \ddot{C}_{m_2} + a\dot{C}_{m_2} + r C_{m_2} = -\left[r\,\Delta C_m + E\dot{\theta} \right] \\ C_m = C_{m_1} + C_{m_2} \end{cases}$$

$$C_{m\ell} = C_{m_0} + p\theta \qquad \theta \leq \theta_s - 6$$

$$C_{m\ell} = C_{m_0} + p\theta + M_{ax}\,(\theta - \theta_s + 6)^2 \qquad \theta_s - 6 \leq \theta \leq \theta_s$$

$$C_{m\ell} = C_{m_0} + p\theta_s + 36\,M_{ax} + (p + 12\,M_{ax})(\theta - \theta_s) \qquad \theta_s \leq \theta$$

$$C_{m_s} = C_{m\ell} \qquad \theta \leq \theta_s$$

$$C_{m_s} = C_{m_0} + p\theta_s + 36\,M_{ax} + (p + 12\,M_{ax})(\theta - \theta_s) + c\left[1 - e^{-h(\theta - \theta_s)}\right] \qquad \theta_s \leq \theta$$

$$\Delta C_m = C_{m\ell} - C_{m_s}$$

$$\theta_s = 15\,(1 - \mathcal{M}^2)$$

$$C_{m_0} = -0.005 - 0.05\, e^{50(\mathcal{M}-1)^2}$$

$$p = 0$$

$$M_{ax} = 0.00025\left[1 + 2/\pi\ \mathrm{Arctg}[8(0.6 - \mathcal{M})]\right]$$

$$c = 0.09 + 0.08\, e^{-30(\mathcal{M}-0.6)^2} + 0.035\left[1 + 2/\pi\ \mathrm{Arctg}[18(0.55 - \mathcal{M})]\right]$$

$$h = -0.40 - 0.21\ \mathrm{Arctg}[22(0.45 - \mathcal{M})]$$

$$b = -\pi^2/360\,[1 + 1.4\,\mathcal{M}^2]$$

$$s = -\pi^2/360\,\left[-0.47 - 1.8/\pi\ \mathrm{Arctg}[15(\mathcal{M} - 0.7)]\right]$$

$$\sigma = b - s$$

$$\sqrt{r} = 0.1 + 0.65\,\Delta G - 1 + 1/(0.15\,\Delta G + 1)$$

$$a = 0.15 + 0.45\,\Delta G^2$$

$$E = 0.02\,\Delta G^2$$

Stall delay $= \Delta\tau_s = 10$