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THE MEASUREMENT OF THE MOBILITY OF STRUCTURES AT ACOUSTIC FREQUENCIES

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SUMMARY

This paper describes the development of a technique to measure the response of structures to vibrational input forces and moments. So far, simple structures with known properties have been used but it is intended to apply the technique in the design of an interface which isolates vibrationally the gearbox of a helicopter from the cabin at acoustic frequencies.

In a first experiment, no moments were applied. The main object was to demonstrate that measurements could be made on two components of a structure with sufficient accuracy to correctly predict measurements on the assembled structure. The two components were a beam and a dynamic absorber.

In a second experiment a special block was attached to a beam to enable forces and moments to be applied and translations and rotations to be measured. The particular mobilities measured were chosen so that they could be easily compared with theory and also with measurements without a block.

1 INTRODUCTION

In the current generation of helicopters, the internal noise spectrum is dominated by spectral components at gearbox teeth meshing frequencies which typically lie at a few hundred hertz. Unfortunately, measures such as the addition of absorptive material inside the cabin are not very effective in reducing the internal noise at these frequencies without introducing a considerable weight penalty. The end objective of the work described in this paper is to achieve a reduction of the internal noise by designing an interface to isolate vibrationally the gearbox of a helicopter from the cabin. Such an interface may only need to be effective at selected frequencies.

The proposed approach is to find the vibrational properties of the gearbox and of the cabin at their respective points of attachment. These properties are described by the translational and rotational responses to force and moment inputs. In this paper the ratio of a response to the input which gives rise to it at a given frequency will be referred to as a mobility. The term direct mobility is used to imply that the input and response are measured at the same point. Furthermore, they are measured in the same direction if the input is a force or about the same direction if the input is a moment. Other mobilities are referred to as cross mobilities. The responses used are acceleration and angular acceleration. Displacement, velocity and their angular counterparts are, of course, simply related to these. The basis for this approach is that when mobility data for the separate components of a structure are known those for the assembled structure can be predicted.

So far, effort has been concentrated on developing the measuring technique. The need for this phase of the work to be carried out carefully is highlighted by the results of a survey carried out in the UK (Ref 1). A set of structures was circulated for test amongst several laboratories where mobility measurements were a part of the usual work. A comparison of results from the experiments for the normal acceleration response to a unit normal input force at selected points on the test structures showed considerable scatter. The problems of correct measurement are even more severe when the input of moments and the measurement of angular acceleration are also required. In developing the technique it has therefore been considered important to use test structures for which theory is available.

For a general structure, the application of forces and moments at a point and the measurement of translations and rotations in or about each of three orthogonal directions requires attachment of a special block to the structure. By using various excitation points on the block combinations of forces and moments can be applied and by combining readings from accelerometers on the block translations and rotations can be measured. Refs 2-4 describe applications of such a block. Normally the block would be chosen to be rigid since this facilitates the removal of the dynamic effects of the block from the results. In principle, though, the block could be flexible provided the appropriate properties were known.

In a first experiment, described in section 2, the structure used consisted of two components where the only important reaction between the two would be the normal force of reaction. One of these components was a free-free beam and the other was a dynamic absorber. The single-direction translational mobility measurements made on the two separately were then used to predict the mobility of the connected structure.

In a second experiment, described in section 3, a block was attached to a thin free-free beam to enable the measurement of translational and rotational mobilities. These measurements were restricted to those corresponding to an input force normal to the beam and to an input moment about a line in the plane of the beam and normal to the centre line. Not only is the theory available for the bending waves excited by such inputs, but the particular mobilities obtained could also be measured without the use of a block, enabling the effects of the block to be assessed.

2 TRANSLATIONAL MOBILITY MEASUREMENTS ON A BEAM AND DYNAMIC ABSORBER

2.1 Experimental details

Firstly, a thin free-free beam was forced in a direction normal to its plane at each of two points on its centre line. For each position of forcing the response was measured at both points using accelerometers aligned with the direction of forcing. Measurements were then made on a dynamic absorber and subsequently on the beam and absorber combined.

Excitation was provided by an electromagnetic shaker driven by band limited white noise. Accelerometer and force gauge readings were fed through charge amplifiers to a Fourier analyser which used digital fast Fourier transforms. The frequency range used for analysis was 0-400 Hz with an analysis bandwidth of 0.78 Hz. Sufficient samples were taken to ensure adequately converged results.

The beam used was a uniform aluminium beam 0.99 m long, 37 mm wide and 64 mm deep. Damping material was added to provide damping values more typical of a built-up structure. With very low values of damping the input force goes through an extremely small and sharp minimum at resonance and very narrow bandwidths must be used for analysis in the neighbourhood of the resonance. The beam was suspended by threads which included a section of rubber and it behaved effectively as a free-free beam over the frequency range of interest. The two positions at which forcing was to be applied were on the beam centre line at positions 0.30 m and 0.64 m from one end. Thirty gram accelerometers were attached at the corresponding points on the opposite face of the beam.

Force measurements were made using an impedance head because no suitable force gauge was available. This was fixed to the beam at the desired position of excitation and the exciter was connected to the head by means of a rod aligned with the direction of excitation. This rod was about 80 mm long with a 15 mm section of plane wire which was included because its lateral flexibility reduces the transmission of moments and lateral forces. Previously a longer connecting rod had been used but transverse vibrations in the rod caused apparent extra resonances in the transfer function. Response measurements were made for forcing at each of the two excitation points in turn and the two direct and the two cross mobilities were derived.

The dynamic absorber used is illustrated in Fig 1 and consisted of a steel strip 16 mm wide and 0.91 mm thick with two similar masses of 35 g at its ends. A means of fixing to the beam at the midpoint of the strip was also included. The absorber was attached to the impedance head, which in turn was attached directly to the exciter, and the translational mobility of the absorber at the point of excitation was measured.

Finally, the absorber was fixed to the beam at the accelerometer position 0.30 m from one end and orientated so that its 'arms' were transverse to the beam as illustrated in Fig 2. The intention was that the only important reaction between the absorber and the beam would be the normal force. The same measurements were carried out as for the beam on its own so that two direct and two cross mobilities were obtained.

2.2 Results

The excitation and response points at 0.30 m and 0.64 m from the end of the beam will be referred to as points 1 and 2 respectively. The absorber was attached to the beam at point 1. Let Y_{11} and Y_{22} denote the direct mobilities for the beam alone at points 1 and 2 respectively and Y_{12} denote the cross mobility for forcing at point 2 and acceleration measurement at point 1. Because of reciprocity the cross mobility function Y_{21} with forcing at point 1 and measurement at point 2 should equal Y_{12} and this was verified experimentally. Let Y_a denote the mobility of the absorber and denote by Y'_{11}, Y'_{22}, Y'_{12}, Y'_{21} the mobilities corresponding to Y_{11}, Y_{22}, Y_{12} and Y_{21} after the absorber has been fixed to the beam. Detailed measurements of Y_{11}, Y_{12}, Y_{21} and Y_{22} are given in Ref 5 and are shown to be in very good agreement with theory. The same reference gives the method for predicting the responses of a total assembly when measurements on the individual components of the assembly are known. The prediction for the current set-up gives the following relationship for the mobilities of the beam with and without the absorber present:

$$\begin{pmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{pmatrix}^{-1} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}^{-1} + \begin{pmatrix} Y_a^{-1} & 0 \\ 0 & 0 \end{pmatrix} .$$

This assumes that the only reaction between the beam and the absorber is the normal force of reaction.

Fig 3 shows the mobility Y_a of the absorber on its own and Fig 4 shows the direct mobilities, Y_{11} and Y'_{11}, at point 1 of the beam with and without the absorber attached.† In Fig 5 the prediction for Y'_{11} using the mobilities of the beam and absorber separately is compared with the measurement. Agreement is seen to be good. The solid line in Fig 6 shows the direct mobility at point 2, that is at a point where the absorber is not attached. Unlike the direct mobility at point 1, there is no antiresonance at the frequency to which the absorber is tuned although the mobility has still been substantially reduced from that for the beam alone. The dashed line shows the prediction from the separate measurements on the beam and absorber. The agreement in the neighbourhood of the absorber frequency is not as good as elsewhere. The likely reason for this is apparent when considering the equation for deriving Y'_{22} :

$$Y'_{22} = Y_{22} - Y_{12}^2 \left(Y_a + Y_{11}\right)^{-1} .$$

Since the absorber frequency was a resonant frequency of the beam, Y_{11}, Y_{12} and Y_{22} are very large whereas Y_a is small and large quantities are being differenced to provide a result which is not large. Under these circumstances inaccuracies in the measurements around the peaks of Y_{11}, Y_{22} and Y_{12}, which is precisely where the inaccuracies are greatest, may well lead to discrepancies like those shown around the absorber frequency in Fig 6. The predicted and measured values of Y'_{12} and Y'_{21} agreed well.

3 TRANSLATIONAL AND ROTATIONAL MOBILITY MEASUREMENTS ON A BEAM

3.1 Experimental details

In the work described here two blocks were designed so that the application of forces to them would exert forces and moments on the beam to which they were attached in such a sense as to excite bending waves. Fig 7 shows the blocks, denoted by block A and block B. The latter block in particular is similar to the one used by Ewins and Gleeson[3]. Each block was the same width as the beam and the transducers were mounted so that their point of attachment lay in the plane normal to the beam through its centre line. Point P on the centre line of the beam is the position at which mobility measurements were required. Glue and two bolts through the beam into the block at positions across the width of the beam

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† The vertical scales are expressed in dB . A mobility is converted to dB by the expression 20 $\log_{10}$ (mobility/reference value of the mobility).

were used to attach the block to the beam. Since it was assumed that the block moved rigidly with respect to the point P the method of connection was important. The particular difference between block A and block B was chosen in order to study whether it was better to spread the contact forces and moments over a larger area or to concentrate them over a more confined area.

Whereas the blocks were made of aluminium a steel beam was chosen for this experiment to keep the ratio of the mass of the block to that of the beam to a minimum. Clearly if the block was massive compared with the structure to which it was attached most of the input force would be used in shaking the block leading to poor quality results. The accelerometers were considered to be part of the block, the dimensions of which are shown in Fig 7. The beam was 50 mm wide, 25 mm deep and 1.2 m long and the position chosen for P was at 0.22 m from one end.

For a given block the force gauge was fixed to the block at position 1 (Fig 7) and random excitation was applied in the same way as described in section 2. The force and the sum of the accelerometer readings were passed to the Fourier analyser and the ratio obtained. This was repeated with the difference between the accelerometer readings replacing the sum. The whole exercise was then repeated with the force gauge at position 2. The total frequency range covered was 0-2400 Hz with an analysis bandwidth of 3.125 Hz and sufficient samples were taken to achieve converged results. Results from the bottom 20 Hz or so were disregarded because of low frequency support modes of the beam. The required mobilities were derived from the measurements using the relationships derived in the next subsection.

After the measurements with the two blocks were completed a further test was performed without a block. Two accelerometers were placed 25 mm apart on the centre line of the beam with the point P midway between them. Excitation was then applied through a force gauge placed in turn at each of the two corresponding positions on the opposite side of the beam. For each force position the sum of the accelerometer readings was passed to the Fourier analyser and the ratio with the force obtained. This was repeated with the difference of the accelerometer readings. From these measurements it was possible to obtain the same mobilities as those found using the block by the method described in the following subsection.

3.2 Measurement analysis

Fig 8 is an exploded diagram of the block and the beam, and indicates the resultant forces and moment of reaction between them which are present when motion is confined to the (x, z) plane. The resultants at P of the forces acting on the block at the interface are the force R in the z direction and the force Q in the $-x$ direction. There is also a couple C about the y axis. The resultant forces and couple at P on the beam are equal and opposite. The point P is assumed to have the acceleration $(0, 0, a)$ so that only the component normal to the beam is considered non-negligible. The angular acceleration is denoted by $(0, -X, 0)$. The accelerations measured by the accelerometers will be denoted by α_{11} and α_{21} if the force is being input at position 1 and by α_{12} and α_{22} for forcing at position 2. For the analysis the spacing 2ℓ between the force gauges and between the accelerometers is required and also the height g of the centre of gravity above P. The mass and moment of inertia about the centre of gravity of the block will be denoted by m and I_G respectively.

The acceleration of the centre of gravity of the block is $(-\chi g, 0, a)$. If the force F_1 is the only external force acting then the equations of motion of the centre of gravity are:-

$$ma_1 = F_1 - R_1 \ , \tag{1}$$

$$m\chi_1 g = Q_1 \ , \tag{2}$$

and

$$I_G\chi_1 = F_1\ell - C_1 - Q_1 g \ , \tag{3}$$

where the suffix 1 on the quantities is to denote that forcing is at position 1. There is a further relationship connecting a_1, χ_1, R_1 and C_1 which comes from the equation of motion for the beam, namely:

$$\begin{pmatrix} a_1 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} R_1 \\ C_1 \end{pmatrix} \ , \tag{4}$$

where the matrix with entries Y_{11}, Y_{12}, Y_{21}, Y_{22} is the mobility matrix which is to be determined. Similar equations can be derived for forcing at position 2 with suffix 1 replaced by suffix 2 and ℓ replaced by $-\ell$. Elimination of the forces and moments of reaction from the equations yields

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} a_1/F_1 & a_2/F_2 \\ \chi_1/F_1 & \chi_2/F_2 \end{pmatrix} \left\{ \begin{pmatrix} 1 & 1 \\ \ell & -\ell \end{pmatrix} - \begin{pmatrix} m & 0 \\ 0 & I_p \end{pmatrix} \begin{pmatrix} a_1/F_1 & a_2/F_2 \\ \chi_1/F_1 & \chi_2/F_2 \end{pmatrix} \right\}^{-1} \ , \tag{5}$$

where $I_p = I_G + mg^2$ is the moment of inertia of the block about P. This equation can be used in conjunction with the relationships:

$$a_i = \frac{\alpha_{1i} + \alpha_{2i}}{2} \quad \text{and} \quad \chi_i = \frac{\alpha_{1i} - \alpha_{2i}}{2\ell} \qquad \text{for } i = 1, 2 \ ,$$

to derive the mobility matrix from the measurements.

It is worth noting that if the block is shaken without the beam attached then

$$a_i/F_i = 1/m \quad \text{and} \quad \chi_i/F_i = \ell/I_G \qquad \text{for } i = 1, 2 \ ,$$

from which it is possible to obtain values for m and I_G.

The analysis of the measurements made without a block will now be described. Suppose that the point P is at $x = x_0$, that the first excitation/accelerometer position is at $x = x_0 - \Delta$ and that the second is at $x = x_0 + \Delta$. Let α_{1i} and α_{2i} be the acceleration responses at positions 1 and 2 for a force input F_i at position i where $i = 1$ or 2. Then, if a_i and χ_i are the corresponding acceleration and angular acceleration at P respectively,

$$a_i = \frac{\alpha_{1i} + \alpha_{2i}}{2} + 0(\Delta) \quad \text{and} \quad \chi_i = \frac{\alpha_{1i} - \alpha_{2i}}{2\Delta} + 0(\Delta) .$$

The force F_1 at position 1 is equivalent to a force F_1 applied at P plus a couple $F_1\Delta$ applied about P to a first order in Δ . The force F_2 at position 2 can be similarly cast as a force and a couple.

Hence
$$\begin{pmatrix} a_1 & a_2 \\ \chi_1 & \chi_1 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} F_1 & F_2 \\ F_1\Delta & -F_2\Delta \end{pmatrix} ,$$

so that
$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a_1/F_1 & a_2/F_2 \\ \chi_1/F_1 & \chi_2/F_2 \end{pmatrix} \begin{pmatrix} 1 & 1/\Delta \\ 1 & -1/\Delta \end{pmatrix} ,$$

to the first order in Δ . The accuracy here will be dependent on the ratio of Δ to a wavelength. To check this a calculation for the derived mobilities was performed using generated accelerometer readings from the theory for forcing at the two positions and estimating the mobilities according to the equations just given. These estimates using the values of Δ and x corresponding to the experiment were found to be virtually indistinguishable from exact calculations for the mobilities at P to well above the frequency range of interest.

3.3 Results

The derived measurements for the mobilities of the beam at P are presented in Figs 9 to 12. In Figs 9 to 11, the results obtained using no block, block A and block B are presented separately and the theoretical prediction for zero damping is repeated in each picture. The format of Fig 12 is similar but the measurement of the acceleration response for unit moment input is compared with the measurement of angular acceleration for unit force input instead of with theory. This is to check the reciprocity that should exist in the measurements.

The level of agreement between the mobility measurements with no block and theory gives confidence in both. The agreement with the measurements made with the blocks is also reasonable below about 1500 Hz except for the angular acceleration response to unit moment input. The discrepancy for this particular mobility is not well understood although the measured results are expected to suffer from greater inaccuracies than for the other mobilities. This is because the derivation essentially involves the differencing of accelerometer readings which may be of a similar level and then subtracting the difference for one force position from that for the other. This again may involve subtraction of quantities of similar levels. However, the errors seem to be of a systematic rather than of a random nature.

The reasons for the discrepancy between theory and measurement above 1500 Hz when the blocks are used became apparent by studying Fig 13. This figure shows the mobilities of block A and the beam combined, which are obtained by setting m and I_p equal to zero in equation (5). By comparison with the top pictures in Figs 9 to 11 it can be seen that the block has introduced an extra mode at about 2 kHz and so the block is no longer behaving as a totally rigid attachment. It was originally thought that the reason for this was the level of the finite stiffness of the connection, but block A and block B would be expected to have very different characteristics in this respect. Indeed, the results for block A and block B are similar throughout with block A results tending to show marginally better agreement with theory.

By comparing the measurements in Fig 13 with the middle pictures in Figs 9 to 11 it can be seen that extra spiky peaks have been introduced by the block removal calculations. These must be spurious and may have been generated either by inaccuracies in the numerical work because of the large dynamic range involved or because resonant peaks were not measured in sufficient detail to give the required precision.

4 CONCLUSIONS

In this paper aspects of the technique of mobility measurement have been studied by experimenting, in the main, with free-free beams which have known properties. In the first experiment described, the quality of the measurements on a beam and a dynamic absorber separately was sufficiently good to successfully predict the measurements on the structure formed by connecting them together. In the second experiment, a block was attached to a free-free beam and certain translational and rotational mobilities were measured. Whilst the particular mobilities found were also obtained without a block this would not have been possible for other mobilities and the object was to assess the limitations of using such a block. For the configuration used, results were found to be good up to about 1500 Hz for all but the angular acceleration response to unit moment input. Results above this frequency were incorrect because the presence of the block introduced an extra resonance at about 2 kHz.

Sufficient confidence in the measurement technique has been obtained to progress to the measurement of the full mobility matrix at a point on a beam and subsequently to a helicopter.

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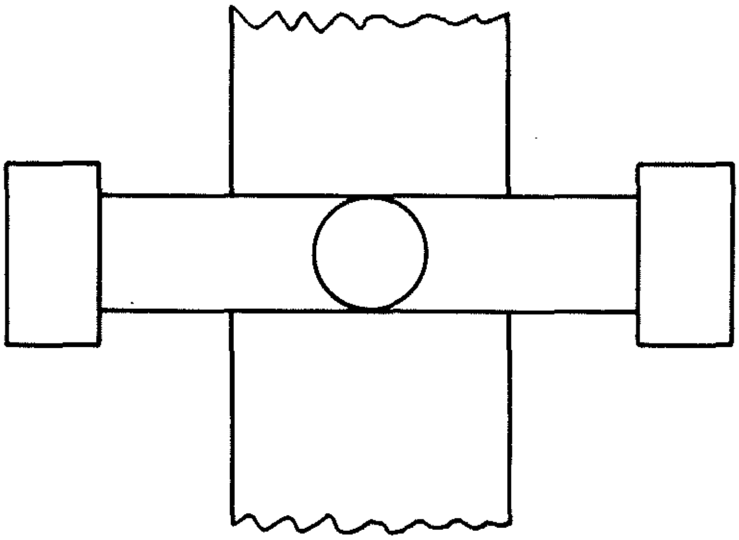
Side view

Fig 1 Absorber



Plan view

Fig 2 Absorber attached to beam

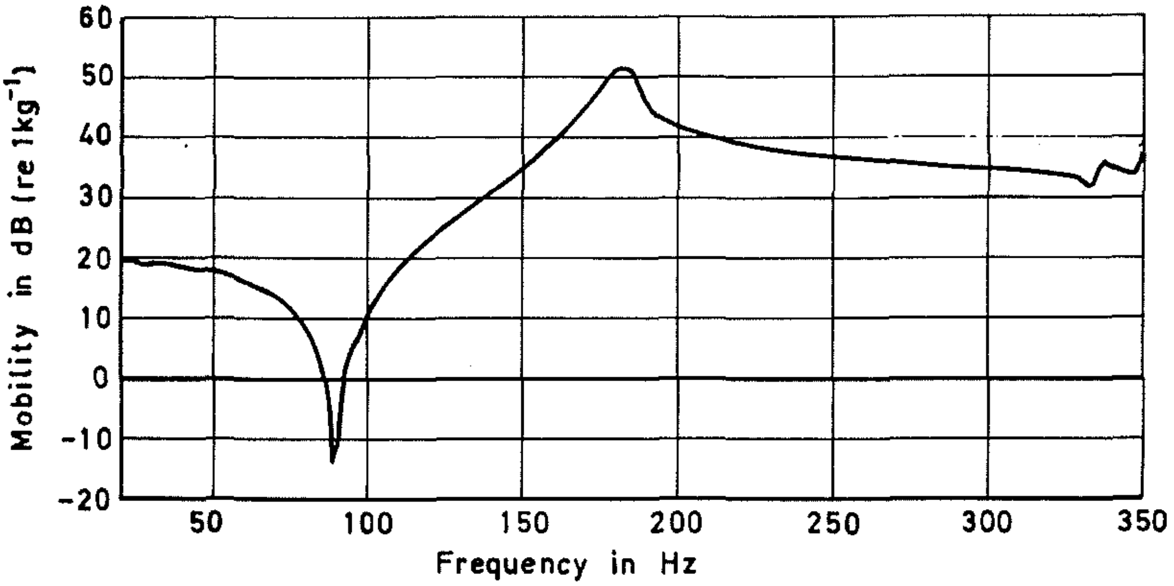


Fig3 Mobility of the absorber

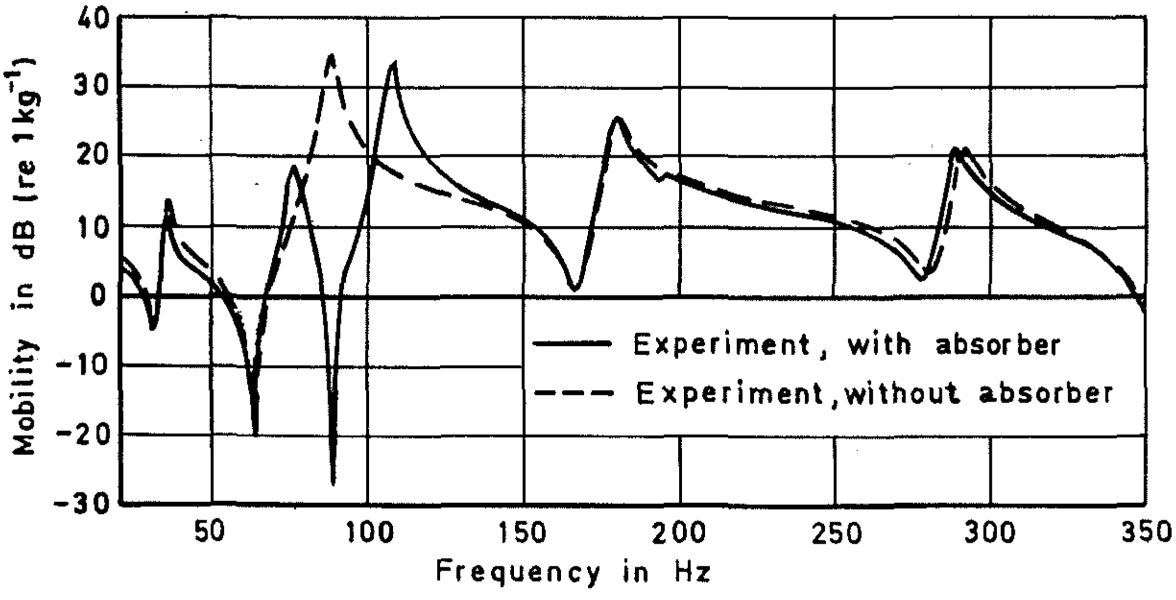


Fig 4 Direct mobility at point 1 of beam with and without absorber

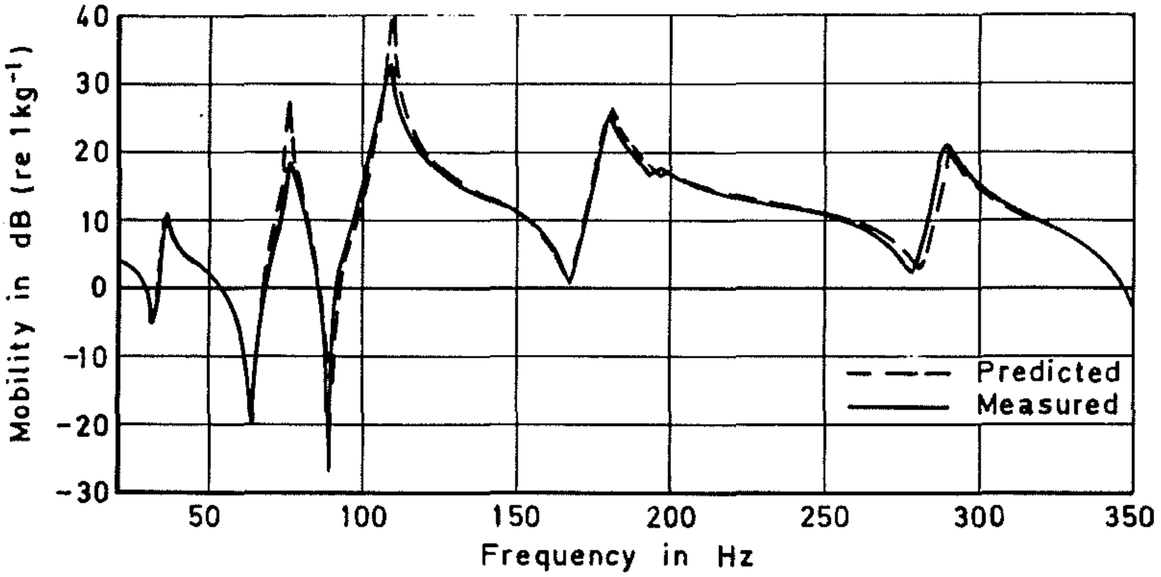


Fig 5 Predicted and measured direct mobility at point 1 with absorber

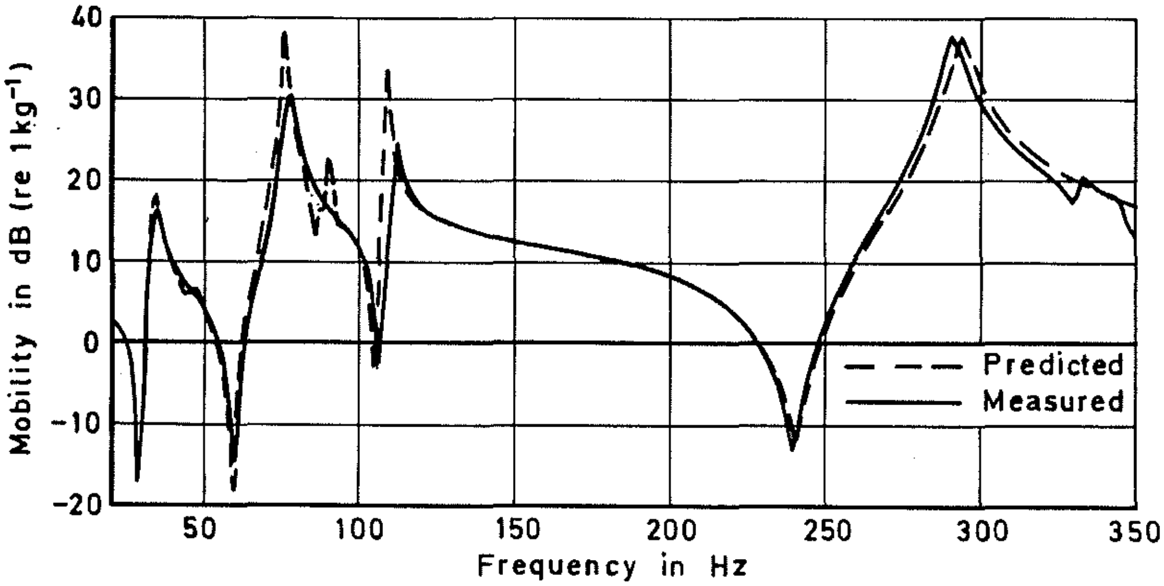
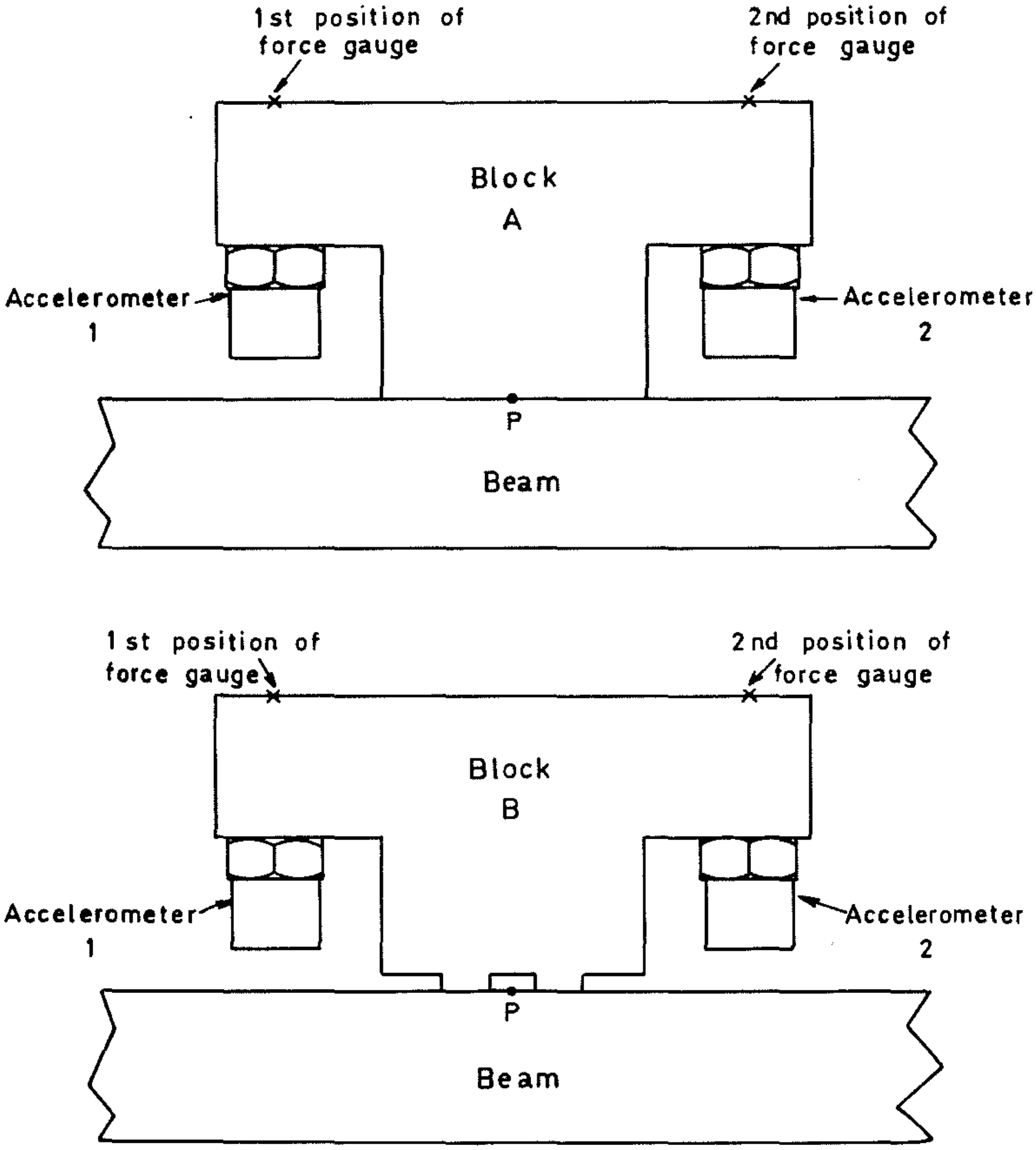


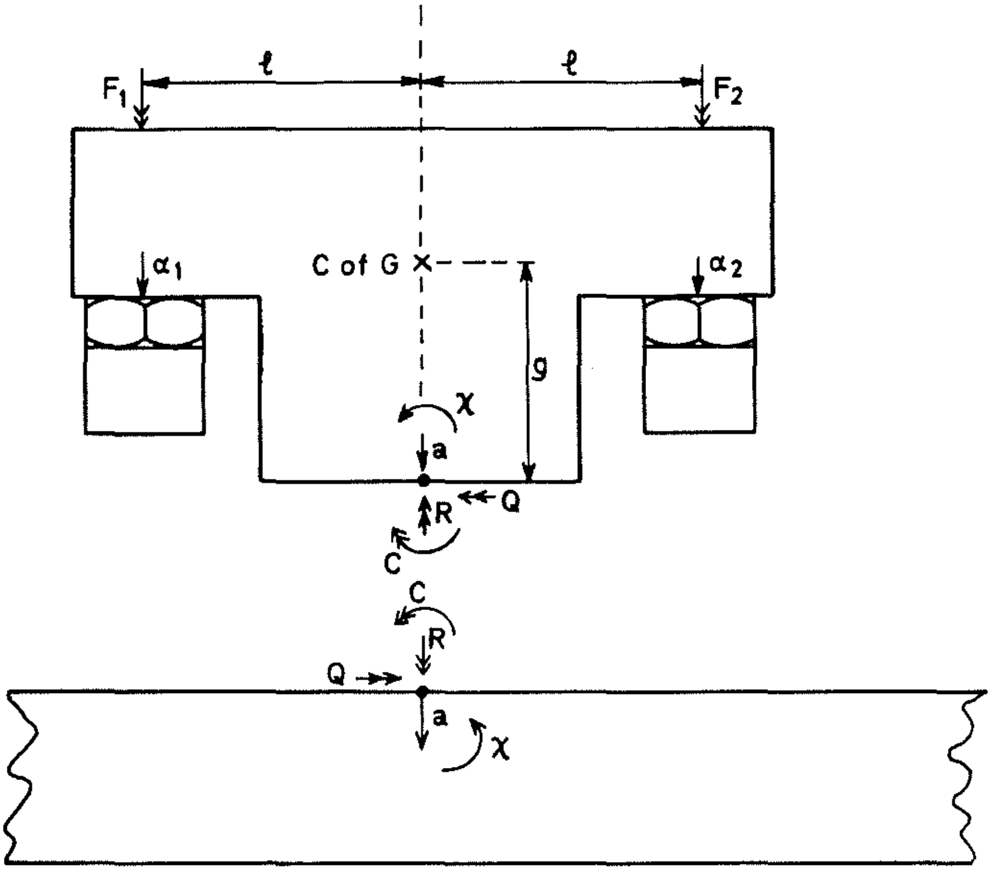
Fig 6 Predicted and measured direct mobility at point 2 with absorber

Fig 7



Scale :– 1:1

Fig 7 Sketches of the blocks used



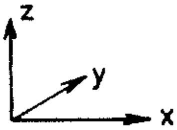


Fig 8 Dynamics of block and beam

Fig 9

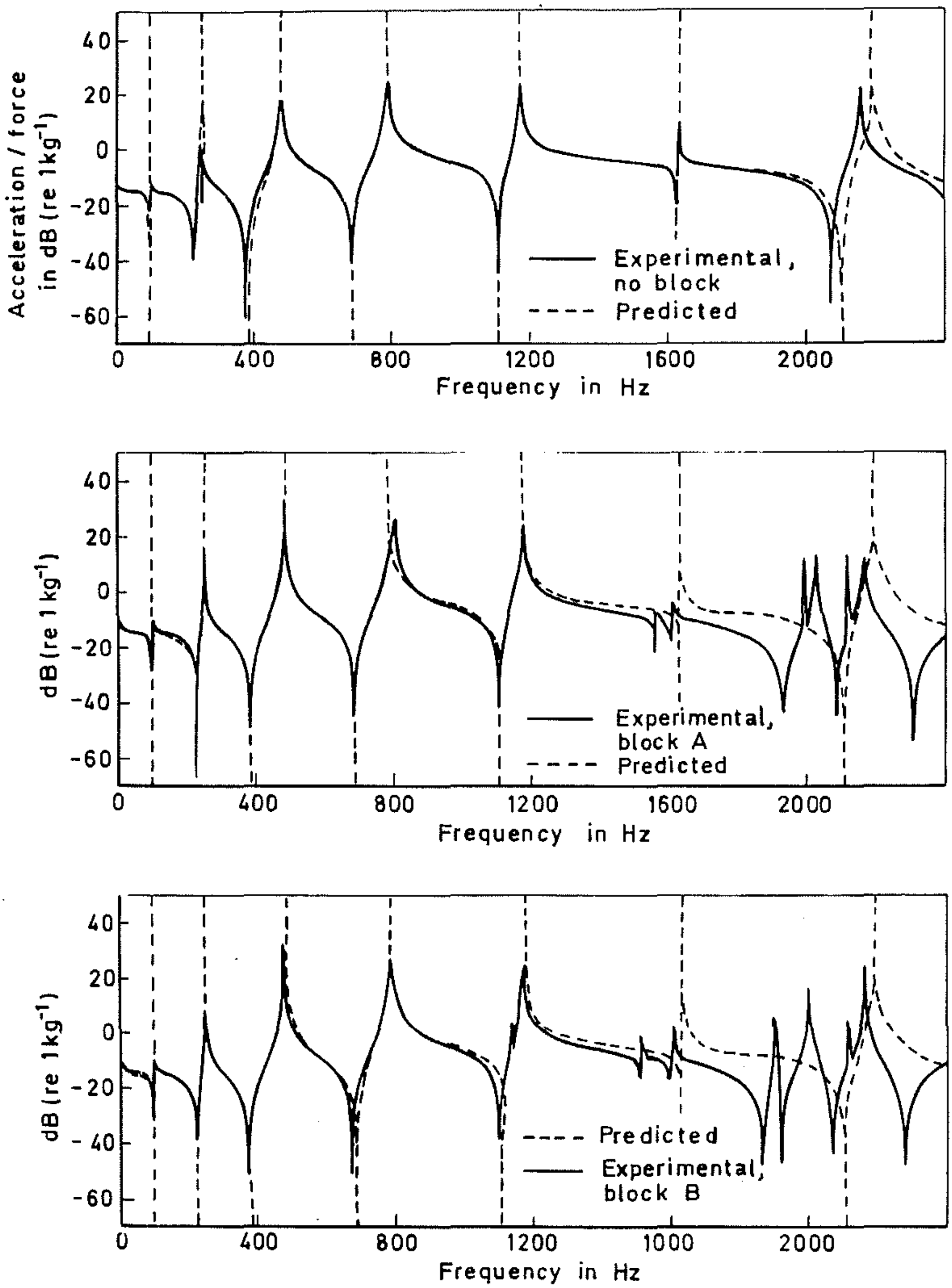


Fig 9 The ratio of acceleration to force

Fig 10

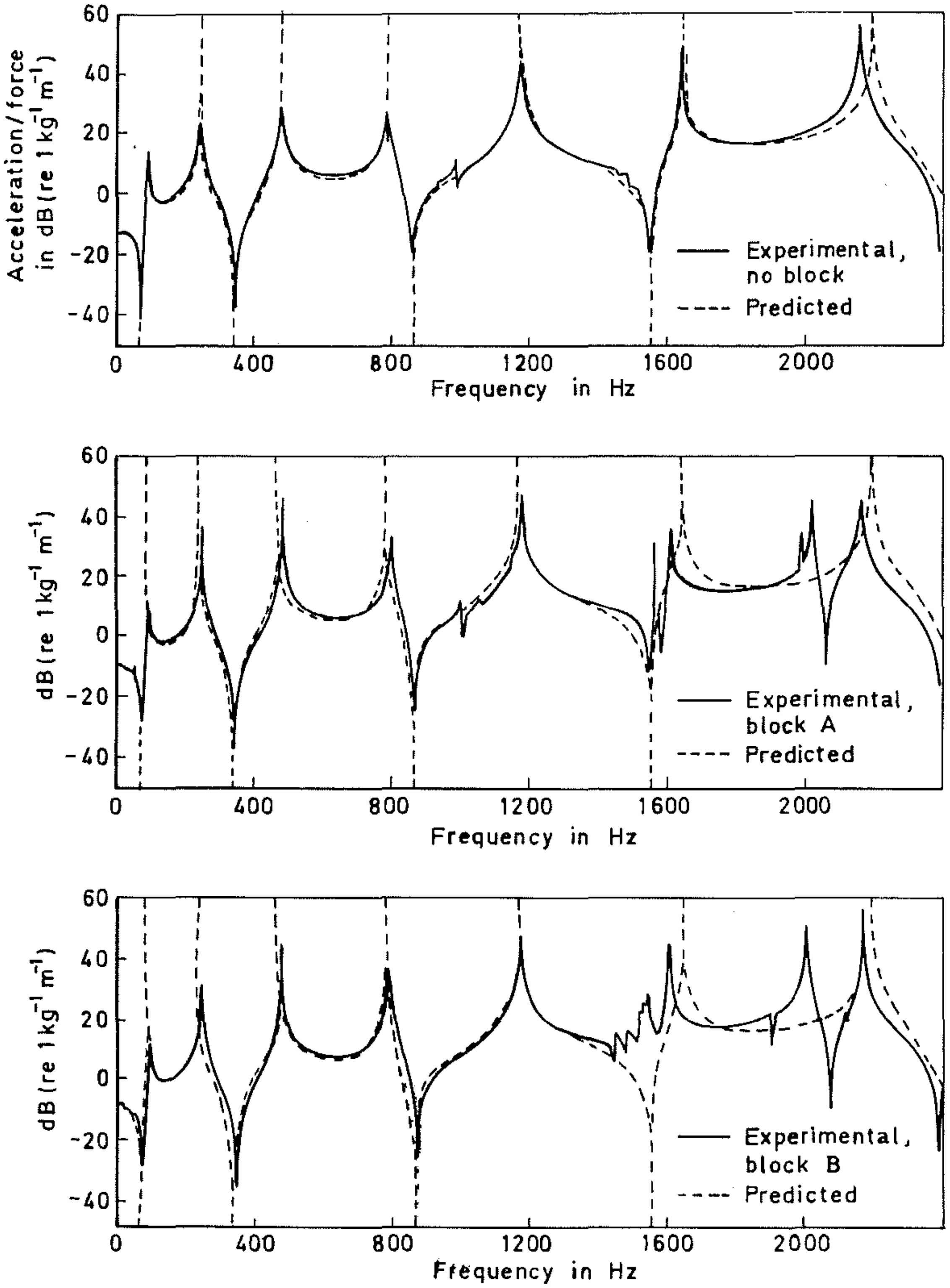


Fig 10 The ratio of angular acceleration to force input

Fig 11

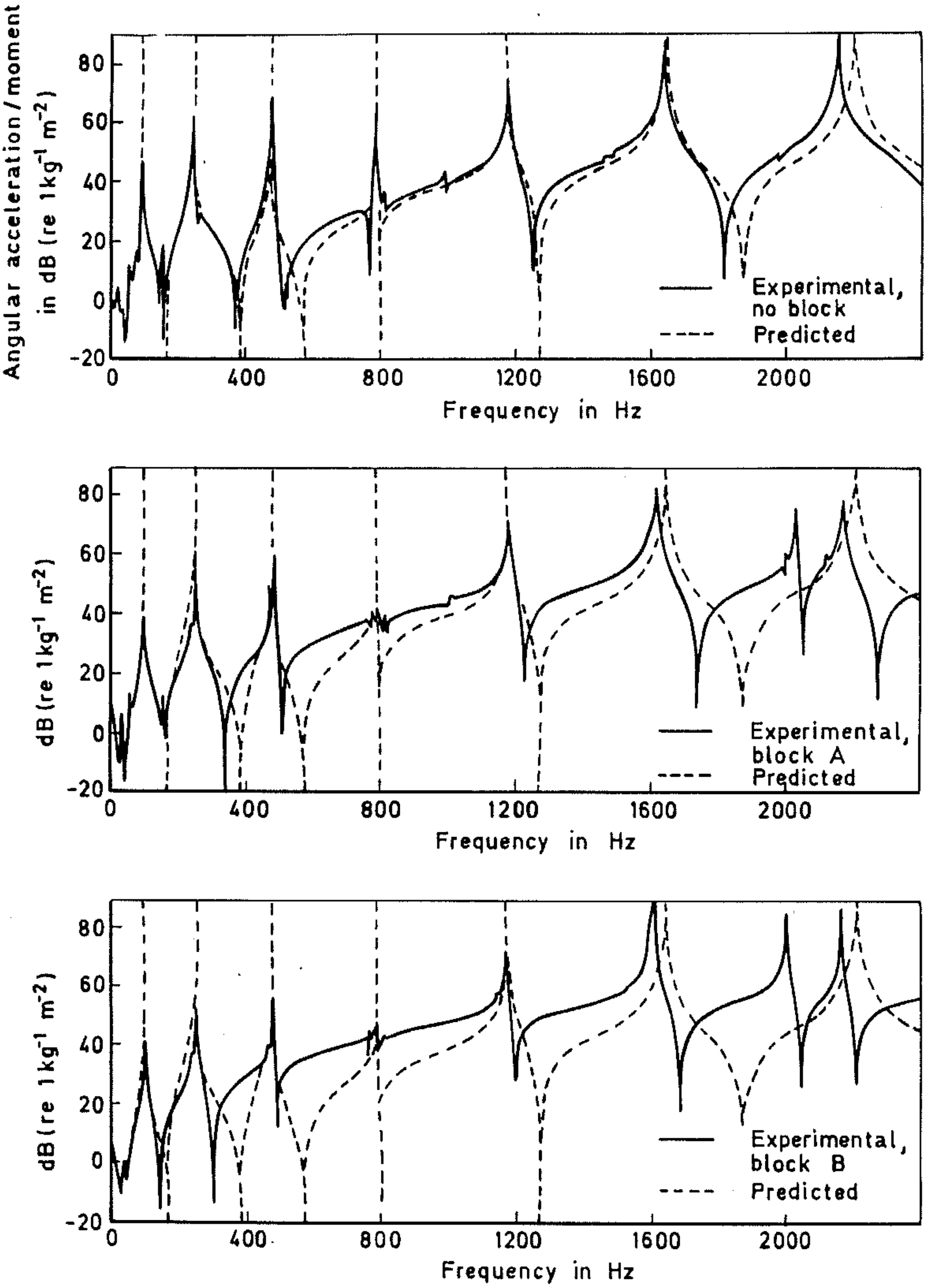


Fig 11 The ratio of angular acceleration to moment

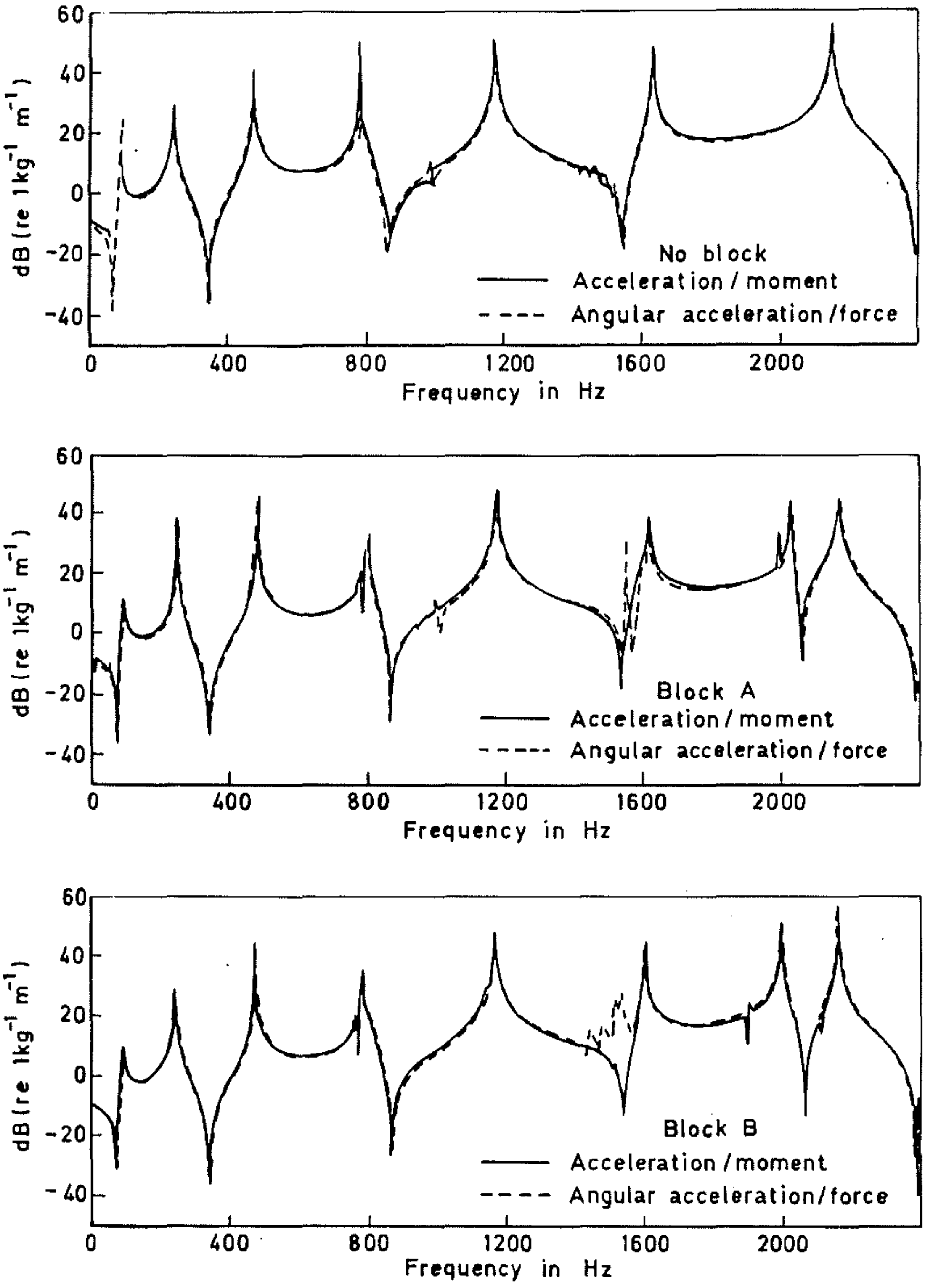


Fig 12 Reciprocity of the measurements

Fig 13

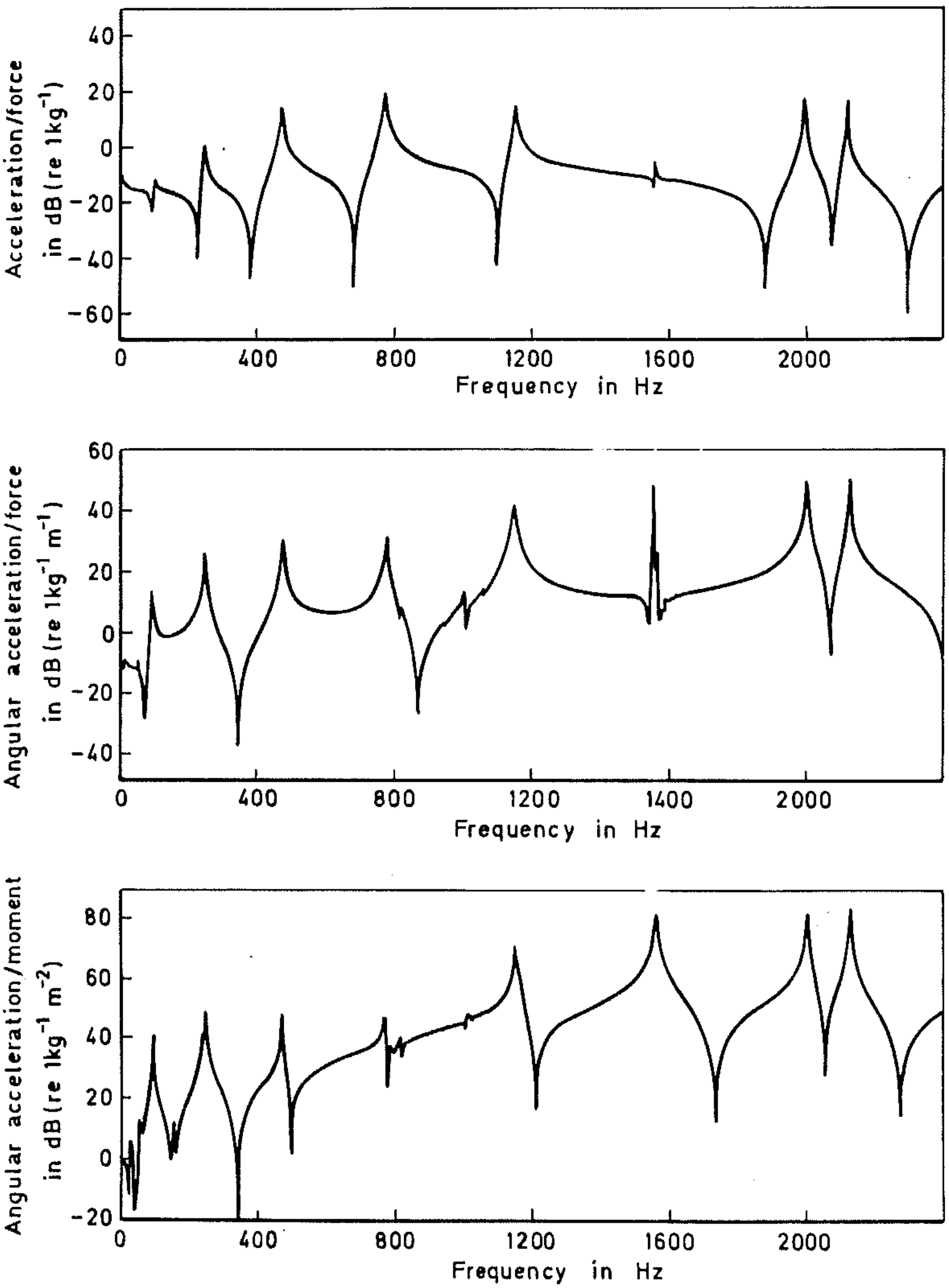


Fig 13 Mobilities of beam and block A together