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AN ALGEBRIC MODELIZATION OF HELICOPTER ROTOR

by

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SUMMARY

The paper presents a Romanian made Average Blade Theory (ABT) helicopter model based on a theoretically calculated formula of helicopter rotor average forces and moments. The context leading to the previously mentioned model is pointed out. On the other hand it is outlined how an intended for simulation model could be used as a development tool for rotor/helicopter design as well as for educational purposes.

1. The context leading to modelization approach

The modelization to be presented was initially intended for a Romanian made full flight helicopter simulator. In order to meet simulation fidelity requirements the model was conceived and developed to describe the most general flight situation : longitudinal + lateral + vertical flight. Transition from hovering flight was equally modelized.

But, the concrete form of the model to be implemented was severely restricted due to following limitations:

(a) low computing power of the target computer which was initially intended to host simulation model; consequently, a high integration step was requested, so as high frequency phenomena has to be neglected;

(b) missing of data bases concerning flight tests and aerodynamic local nonlinearities; only massic, geometric, gravimetric, functional, overhaul, maintenance data were available and are used by the implemented model;

(c) poor documentation sources existing in the country concerning modelization of a helicopter general flight situation;

2. Description of ABT approach

All these limitations eliminated from the very beginning the possibility to use an up-to-date theory as BET (blade element theory). As well known, in accordance with BET, the rotor aerodynamic forces are on-line evaluated in several points along the blade, then these local forces are appropriately summed (let's say integrated) so as an instantaneous force is on-line calculated for each blade several times during one rotation of the rotor. Of course, an important amount of mathematical computations are expected in BET modelization.

In our case, the computing power was the main restriction. Consequently, a new approach directed towards minimization of the on-line calculations was necessary. So, the along the blade integration of local aerodynamic forces, which is on-line performed in BET model, is replaced by algebric formula given by the hand made integration of the same aerodynamic forces. Of course, the hand calculated formula are given under certain assumptions concerning, for instance, lift coefficient linearity, average values of certain parameters (chord) or induced velocity distribution low.

Despite these simplificatory hypothesis the obtained formula are rather complicated, because all linear and angular velocities components, as well as longitudinal and lateral cyclic control are taken into account. The powerful coupling between lateral and longitudinal motions, specific for helicopter dynamics is pointed out by these formula (see fig.1). On the other hand formula will lead to the average value of rotor forces and moments. Indeed, the along the blade integration and calculation of the mean value during a blade rotation leads obviously to an average value of rotor forces and moments, periodic components of rotor forces being eliminated. So, the Average Blade Theory (ABT) modelization was born. An algebraic non-linear model has resulted, including dependence until the fourth degree of flapping coefficients and adimensional speeds.

3. General presentation of the model

The model is mainly a theoretically derived one. It is based on fundamental dynamics and aerodynamics laws which are applied in the particular case of the blade, fuselage and helicopter itself. The flight general situation is treated in a coherent and unitary theory, the model having appropriate options so as it can be easily amended in accordance with human pilot opinions or recommendations during the simulation set up. The model correcting capabilities compensate the effect of calculus hypothesis which are quite restrictive so as to lead to an analytic modelization (hand calculated closed formula).

a.Systems of axes

The axis systems are quite usual in helicopter theory. The ground system locally horizontal (G), helicopter body axis system (B) rotated with traditional attitude angles as against (G), the shaft system (S) rotated in pitch as against (B), disc system (D) rotating with rotor angular velocity as against (S) and blade system (b) rotated with flapping angle $\beta=a_0+a_1*\cos\Psi+b_1*\sin\Psi$ as against (D). Positive senses both for helicopter attitude angles, flapping angles, as well as for rotor controls (A_1,B_1) are those inducing a positive angular rate, so as traditionally accepted positive senses are sometimes ignored. This approach is due to necessity to develop a coherent theory respecting all natural transformations, without supplementary amendments and conventions.

b.Speeds field on blade

More precisely, the model is based on a complete evaluation of air velocity field on the blade, taking into account a general flight situation (climbing+forward+sideslip). Implementation aspects were taken into consideration at this level: so, forward and side-velocity were not composed in a single horizontal velocity because of undetermination existing at low speeds when it is difficult to evaluate numerically the proper direction of horizontal velocity vector (due to numerical errors in calculating the angle between horizontal velocity vector and helicopter longitudinal axis). Due to this approach concerning the blade velocities field, the model includes two horizontal adimensinal speeds denoted by μ_{01} (corresponding to traditional notation of forward speed μ) and μ_{02} (corresponding to the adimensional side velocity).

Induced velocity was considered also for a general situation (climbing + forward flight). Induced velocity mean values is calculated as a result of a quadratic bidimensional interpolation of experimental characteristics supplied by [2] - see fig.2 - adimensional transversal velocity through rotor versus rotor longitudinal velocity at constant values of rotor mean induced velocity. In case of experimental characteristics interpolation with ellipses, the interpolation results were stored in a 32760 locations bidimensional array, which is used for on- line calculation of mean induced velocity as a function of transversal and longitudinal rotor velocities. One must emphasized that, due to its adimensional form, the interpolation result is available for any helicopter rotor intended to be simulated in case of constant induced velocity hypothesis. Additionally, options are provided so as these induced velocity calculations can be upgraded by using Mayer-Drees induced velocity distribution (see fig.3). Moreover, on its turn Mayer-Drees modelization is amended to be available for low-forward speeds when it is expected vanishing of induced velocity dependence on azimuth and remaining only radial dependence or even a mean value available for whole rotor.

c.Rotor forces calculation

By using air and induced velocity field on blade, the aerodynamic local forces are calculated in a lift coefficient linearity hypothesis. Then, hand made integration of analytic forces expressions has leaded to the value of total lift and drag on blade. One must emphasize the important effort to analytically integrate each force along the blade, calculating their harmonics until second order both for blade lift and drag. Calculations are made in a periodic first order flapping hypothesis and, at the same time, contribution of cyclic longitudinal (B_1) and cyclic lateral (A_1) controls are evaluated (a feathering angle law $\theta=\theta_0+A_1*\cos\Psi+B_1*\sin\Psi$ is considered). Finally, mean forces are calculated in helicopter body axis using transformations of all forces harmonics from blade axis (b) to helicopter body axis (B) eliminating periodic components. It must be pointed out that this extensive approach leads naturally to those formula of rotor forces and moments which are obtained usually by particular approaches (see [1]). Indeed, for instance, moments due to blade excentricity result as a particular case of a general procedure in moments calculations. In the same framework, rotor forces and moments dependence both on control angles (A_1, B_1) and general speed distribution (μ_{01},μ_{02}) are implicitly derived (see fig.1).

4. Rotor modelization formula validation

The first question arising when the formula of fig.1 are seen is as follows: how can we verify that this formula are correct. The answer is given if we see what it happens when the rotor axes are turned with 90 degrees. In this case the formula must remain the same if longitudinal velocity is replaced by the lateral one, the lateral velocity is replaced by the longitudinal one with minus sign, the angular roll velocity is replaced by the pitch angular velocity, a.s.o. Practically, when the displacement is done along y axis instead of x axis of the rotor, due to rotor simetry related to x and y displacements, the force formula must remain the same if x axis parameters/states are replaced with the y axis parameters, and the y axis parameters are replaced with the x axis parameters with minus sign.

More precisely, using formula of X and Y rotor forces the following relationship must take place:

$$X(u,v,w,p,q,r, ,A_1 , B_1) = Y(v,-u,w,q,-p,r, , B_1 , -A_1)$$

$$Y(u,v,w,p,q,r, ,A_1 , B_1) = -X(v,-u,w,q,-p,r, , B_1 ,-A_1)$$

the forces expressions from fig.1 being in total agreement with these consistency requirements.

Also, particular cases (e.g. $,\mu_{02}=0,p=0,q=0$ or $A_1=0$ or $B_1=0$, etc.) of these forces formula are in very good agreement with formula given by bibliography [1] for these particular situations (forward flight, helicopter stability analysis). Moreover, control (A_1,B_1) and flapping coefficients (a_0,a_1,b_1) characteristics versus longitudinal adimensional velocity in stabilized flight are in very good agreement with experimental similar characteristics invoked in bibliography [1]. In fact, one of the main results of Mayer-Drees distribution is the good agreement between experimental and theoretically determined relationship between lateral flapping coefficient versus flight speed (see fig.4).

Besides this let's say, static validation, a dynamic validation was equally performed by using experienced pilots' opinions. The right modelization of the coupling between lateral and longitudinal channels was outlined in this manner. For instance, it was observed that roll oscillations of the PUMA helicopter (flying without auto-pilot) are very well compensated by quick forward-rearward motions, as it happens in the real flight.

Another validation criteria refers to helicopter (rotor + fuselage) behavior as specified in helicopter standards (as MIL-H-8501). In this framework, the pitch angular rate and normal acceleration responses to a stick rearward motion were verified to be in good agreement with MIL-H-8501 requirements.

5. Modelization capabilities and limitations

The main feature of the model is its low cost, resulting from the usage of a simplified theoretic support instead of expensive flight test data or aerodynamic tunnel measurements. As a consequence of ABT conceptual approach, the model has to require minimal input data concerning those parameters not easily available or needing expensive flight tests for determination (stability derivatives, blade elasticity effects). So, costs of the in flight measurements on real helicopter are considerably reduced. Moreover, due to the generality of the developed model, the costs of the model development activity are considerably reduced (a new helicopter model is now set up by a single person in six months, while initially the basic model was developed by a single person in three years).

On the other hand, from a simulator point of view, the low cost is induced by the usage of a low computing power machine. Besides this, as a specific aspect for Romania the low cost results as a consequence of the low cost of the national labor force.

So, the ABT concept avoids the important number of calculus as well as expensive experimental/test data which are required by the up-to-date BET. Moreover, because ABT manipulates mean values and not instantaneous ones as BET method, ABT concept requires a less integration frequency, reducing once more the computational power needed by BET modelization. Of course, a less fidelity in high-frequency phenomena simulation is expected in case of ABT method. It is clear that due to algebric approach, high frequency phenomena are neglected, as well as elastic behavior of the blade. Taking into account this last aspect, capabilities are provided for Romanian made simulators in order to simulate vibrations and high-frequency oscillations. So, main computer is driving the amplitude and frequency of analogic oscillators coupled to motion system.

Also, ABT does not include local nonlinear aspects (e.g. limitations in lift coefficient), these aspects being taken into account for the on simulator implemented model in a global manner (e.g. reducing of rotor global lift when linear calculus leads to high values of rotor lift or discontinuity in roll moments when critic Mach values are identified on blade tip, a.s.o.).

Educational features are also provided by the ABT modelization in case of integrated model (rotor + fuselage+tail rotor). The most important are as follows:

- possibility to calculate controls for a stationary flight at any horizontal flight speed into the flight envelope; algebric form simplifies considerably calculations because the system responds instantaneously to variation of searched values of controls. Stationary flight controls are calculated by solving the non-linear implicit algebric system resulting from dynamic equations in case of stationary flight (accelerations and angular rates are zero). A direct searching method is used to identify the hypercube containing equilibrium flight controls and then a Newton-Raphson method is used (numerical derivation is used to get the partial derivatives values), so as a 0.01% relative error is provided in control calculations.

- one of the most interesting educational feature of the model is to identify the linear model corresponding to a particular stabilized flight situation. Damping, time constants and delays can be identified using this induced linear model. Educational effects are enhanced (reaching even scientific staff interest) by the possibility of using three different identification manners: (a) appropriately (e.g. impulse) exciting of non-linear model and identifying the linear model using, for instance, a least-squares method, (b) by direct analytic derivation of those analytic non-linear formula calculated for implemented model and then numerical calculation of linear model matrix (it is outlined the important human effort which is involved for this version of linear model calculation), and (c) by numerical derivation of forces and moments in stationary flight conditions.

Unfortunately, validation of the calculated linear model was never performed because reference data (experimentally determined) were not available as previously mentioned (see modelization context). Only comparison between methods was available as a validation instrument for the calculated linear models.

- for the full flight simulator purposes the model includes also capabilities to simulate global effects of neglected phenomena (e.g. rotor traction delay effect due to blade elasticity is simulated by traction low-pas filtering, helicopter low frequency oscillations in starting procedure is simulated by introducing a low frequency rotating force proportional to A_1 and B_1 controls, a.s.o.).

- in terms of model testing capabilities, response to impulse and step commands are available, so as it can be verified accordance with helicopter dynamic behavior standards (see MIL-H- 8501) as well as accordance with concrete features of helicopter intended to be simulated. Additionally, flight controls dependence on forward velocity can be determined. Test capabilities are also completed by parameter on-line recording and off-line graphic (selective) presentation of evolutions (or parts of evolution) involved by the test procedures.

All these educational features of the proposed model are very well appreciated by the main end user of Romanian simulators, the national flying school. Of course other more refined features are available for the personnel the most interested to better know the helicopter (e.g. activation and inhibition of certain models as engines or fuselage aerodynamics, or modifying locations of model state variables in order to see their effects on helicopter static forces and moments - "static" means frozen exercise).

Another remarcable capability of the model is its flexibility, capability to implement the input data modifications. All basic input parameters (massic, gravimetric, inertial, etc) are read for an input file as well as the options concerning induced velocity law to be selected (Mayer Drees or constant velocity), the horizontal tail modelization or the taking into account of different state variables influence in aerodynamic moments of the fuselage (pitch angular rate, vertical rate, etc.).

On the other hand the algebric form leads to an instantaneous rotor response (flapping coefficients as well as rotor forces/moments) to the control stick commands which can provide a certain difficulty for a pilot flying a such helicopter model. That's why, for the full flight simulator, angular rate feedbacks are provided in order to mask this undesirable instability.

Another deficiency is related to the model capability to simulate some non-linear features. For instance, evolutions at high incidence are not well simulated. Thrust limitation at high blade incidence, as well as rotor torque strongly increasing in autorotative flight, require a better modelization. More clear, in autorotative flight if the collective pitch is increased the modeled rotor does not stop. This was quite suprising, because autorotative flight is well simulated when collective pitch is acted between reasonable limits. The effect is that in autorotation landing, when collective pitch is increased at highest value (in order to stop the descent near the ground), the modeled helicopter begin to climb, which does not really happen. Local modifications are necessary, but this is not satisfactory from the scientist point of view.

6. Conclusions

In fact, it is obvious that ABT is less performant than BET, having some conceptual limits. But, as previously mentioned, the Romanian simulators main end user is the national flying school, so as requirements in accurate reproducing of unusual (acrobatic) evolutions are more relaxed. Consequently, non-linear aspects not covered by ABT model (and "excited" by acrobatic evolutions), are not required by a school exercise. Also, restrictions in a very accurate quantitative simulation of the helicopter features (e.g. a certain vertical speed in autorotation at a certain collective pitch angle and forward speed) are also more relaxed because of non-specific character of the training process in a flying school. So, as usually, the problem is what do we want to simulate and if financial and conceptual effort is justified by the intended simulation objectives.

The proposed modelization is in fact a conceptual approach having more than simulation goals: it is probably the most efficient way to achieve the desired cost-effective simulation, then to advice the potential end users about simulation educational efficiency (being an end user oriented modelization), and finally, to promote the advanced educational/schooling techniques in Romania.

Preliminary design of helicopter autopilot seems also possible based on determination of the linear model. Also, estimation of the helicopter performances can be done based only on the input structural data (masses, geometry, etc.). However, educational features are effective at least for a schooling process in the flying or polytechnical schools.

Anyway, the proposed model must be understood in the context of the general effort to override the technologic and documentation lacking existing in the Eastern Europe countries, and especially in Romania. On the other hand, the paper presents the level of helicopter flight simulation existing in Romania so as interested reader can have a reference starting point for a scientific productive discussion. In this context Romanian scientists are open to any good advice or scientific co-operation directed toward this modelization improvement.

Finally, I want to emphasize that main idea of this paper is the ABT model can be hardly improved. In fact SIMULTEC Institute is looking for co-operation in attempting this objective.

$$\underline{X}_{RO} = \{a_1\theta_0/3 + 3a_1\lambda_R/4 - a_0b_1/6 - a_0\underline{q}/6 + \underline{p}\lambda_R/2 + \underline{p}\theta_0/6 + \mu_{01}[\delta_R/2a_R - \lambda_R\theta_0/2 + a_1^2/4 + a_0^2/4 - b_1\underline{q}/16 + a_1\underline{p}/16] + \mu_{02}[a_0/2(3\lambda_R - 3\theta_0/2) + a_1b_1/4 + 7b_1\underline{p}/16 - 5a_1\underline{q}/16] + \mu_{01}\mu_{02}[a_0a_1 - \theta_0b_1/2] + \mu_{02}^2\ [a_0b_1 + a_1\theta_0/2]\} + T(A_1, B_1)$$

$$\underline{Y}_{RO} = \{b_1\theta_0/3 + 3b_1\lambda_R/4 + a_0a_1/6 + a_0\underline{p}/6 + \underline{q}\lambda_R/2 + \underline{q}\theta_0/6 + \mu_{02}\ [\delta_R/2a_R + a_0^2/4 + b_1^2/4 - \lambda_R\theta_0/2 + b_1\underline{q}/16 - a_1\underline{p}/16] + \mu_{01}[-a_0/2\ (3\theta_0/2 - 3\lambda_R) + a_1b_1/4 + 7a_1\underline{p}/16 - 5b_1\underline{p}/16] + \mu_{01}\mu_{02}[(a_0b_1 + a_1\theta_0)/2] - \mu_{01}^2\ [a_0a_1 - b_1\theta_0/2]\} + T(B_1, -A_1)$$

$$Z_{RO} = \theta_0(1+3\mu_R^2/2)/3 + \lambda_R'/2 - (\mu_{01}\underline{p} + \mu_{02}\underline{q})/4$$

$$L_{RO} = y_R Z_{RO} - z_R Y_{RO} - M_b e_R b_R \Omega_R^2 x_{cg} b_1/2 - e_R Z_{P2}/2$$

$$M_{RO} = z_R X_{RO} - x_R Z_{RO} + M_b e_R b_R \Omega_R^2 x_{cg} a_1/2 + e_R Z_{P1}/2$$

$$N_{RO} = x_R Y_{RO} - y_R X_{RO} + \delta_R/4a_R - \theta_0\lambda_R/3 - \lambda_R^2 - [(b_1+\underline{q})^2 + (a_1+\underline{p})^2]/8 + \mu_{01}[\underline{p}\theta_0/6 - \lambda_R a_1/2 + a_0(b_1+\underline{q})/3] + \mu_{02}[\underline{q}\theta_0/6 - \lambda_R b_1/2 - a_0(a_1+\underline{p})/3] - (\mu_{01}a_1 + \mu_{02}b_1)^2/8 - \mu_R^2[(a_0^2 + (a_1^2 + b_1^2)/4 - \delta_R/4a_R]/4 - \{A_1[(b_1+\underline{q})/4 - a_0\mu_{01}/3 - \lambda_R\ \mu_{02}/2 - a_1\mu/4 + b_1(\mu_{01}^2 + \mu_{02}^2)/8] + B_1[-(a_1+\underline{p})/4 - a_0\mu_{02}/3 + \lambda_R\ \mu_{01}/2 + b_1\mu/4 + a_1(\mu_{01}^2 - \mu_{02}^2)]\}$$

$$Z_{P1} = -\mu_{02}\theta_0 + (b_1+q)/3 - a_0\mu_{01}/2 - \mu_{02}\lambda_R - \mu a_1/2 + (\mu_{01}^2 - \mu_{02}^2)b_1/4$$

$$Z_{P2} = \mu_{01}\theta_0 - (a_1+\underline{p})/3 + a_0\mu_{02}/2 + \mu_{01}\lambda_R + \mu b_1/2 + (\mu_{01}^2 - \mu_{02}^2)a_1/4$$

$$a_0 = \gamma\{[\theta_0(1/4 + \mu_R^2/2) + (\lambda_R - \mu_{02}A_1 + \mu_{01}B_1)/3 + (\underline{p}\mu_{01} + \underline{q}\mu_{02})/6] - M_b x_{cg}/(\Omega^2 B)\}/[2(1 + M_b e x_g R^2/B)]$$

$$a_1 = B_1 + [(3/4\lambda_R' + \theta_0)8\mu_{01}/3 - a_0\mu_{02}4/3\ ]/[1-\Delta\mu^2/2] - [\underline{p} + 16\underline{q}/\gamma + 8(d\underline{p}/dt + A_1\varepsilon)/\gamma]/[1-\Delta\mu^2/2]$$

$$b_1 = -A_1 + [-(3/4\lambda_R' + \theta_0)8\mu_{02}/3 + a_0\mu_{01}4/3\ ]/[1+\Delta\mu^2/2 - [\underline{q} + 16\underline{p}/\gamma + 8(d\underline{q}/dt + B_1\varepsilon)/\gamma]/[1+\Delta\mu^2/2]$$

where : $\mu_{01} = u/\Omega R$; $\mu_{02} = v/\Omega R$; $\Delta\mu^2 = \mu_{01}^2 - \mu_{02}^2$

$\lambda_R = (w - V_i)/\Omega R$; $\underline{p} = p/\Omega$; $\underline{q} = q/\Omega$

$\varepsilon = M_b e x_g R^2/B$; $\gamma = -\rho a_R C_R R^4$; $\lambda_R' = \lambda_R - \mu_{02}A_1 + \mu_{01}B_1$

Fig.1 Rotor forces and movements (dependence on ciclic controls A_1, B_1 not included for presentation simplicity).

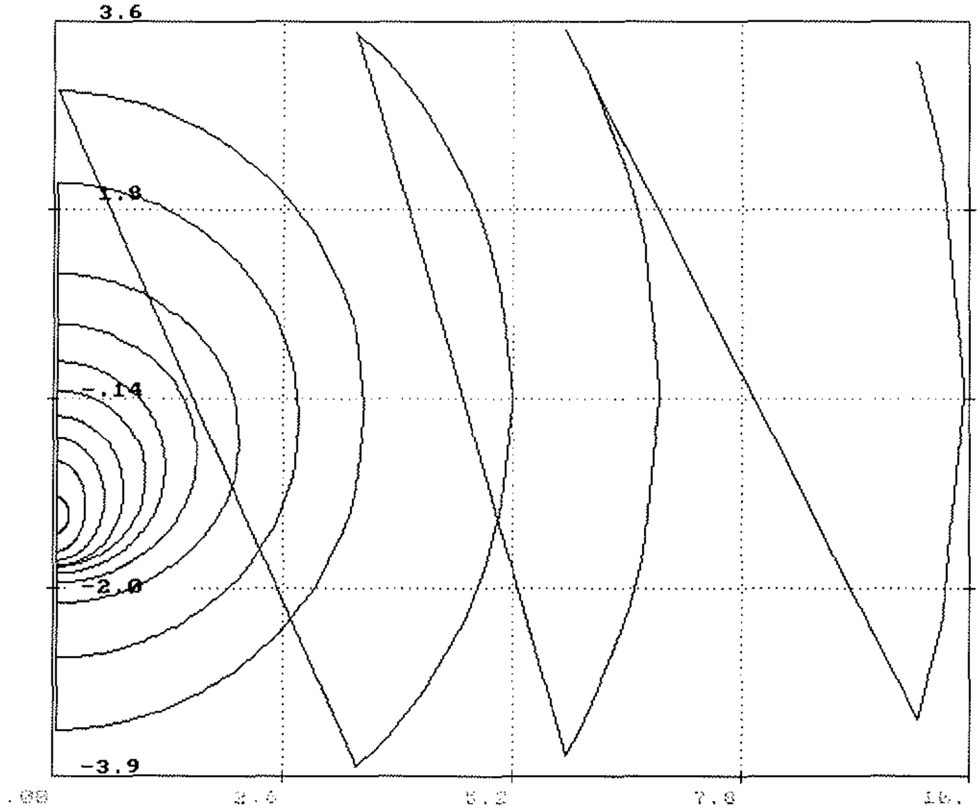

Fig.2 Adimensional transversal velocity through rotor versus adimensional longitudinal velocity for constant induced velocity characteristics

$$V_i(\underline{x},\psi) = V_{i0} + V_{ic}\underline{x}\cos\psi - V_{is}\underline{x}\sin\psi$$

where :

ψ = azimuth angle for rotor

$V_{ic} = [4/3(1-1.8\mu_{01}^2) - 2\mu_{02}]\ V_{i0}/(1-3/2\mu_R^2)$

$V_{is} = [2\mu_{01} + 4/3(1-1.8\mu_{02}^2)]\ V_{i0}/(1-3/2\mu_R^2)$

$\mu_R^2 = \mu_{01}^2 + \mu_{02}^2$

Fig.3 Mayer-Drees induced velocity law

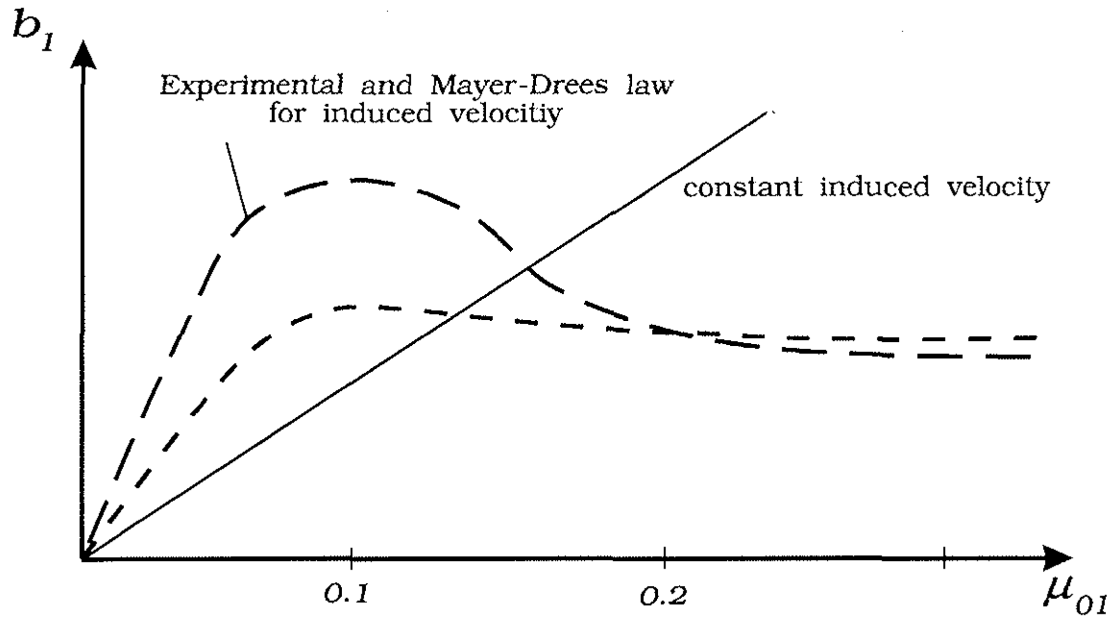

Fig.4 Lateral flapping b_1 coefficient versus adimensional forward speed.

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