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NOVEL CONCEPT FOR REALIZING INDIVIDUAL BLADE CONTROL (IBC) FOR HELICOPTERS

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ABSTRACT

In order to increase the acceptance of helicopters, the German Aerospace Center (DLR) has for many years been researching on technologies for active rotor control which contribute to the solution of different problems of helicopters (e.g. high noise emissions, high cabin vibrations and power consumption, low maximum cruising speed, high blade loads). These inherent problems are primarily resulting from asymmetric airflow on the rotor during forward flight as well as – depending on flight attitude and forward speed – from interactions of the blade tip vortices with following rotor blades (BVI) or other components of the helicopter. Active rotor control superimposes additional control signals on the pilot signal, changing the pitch angles of the rotor blades to alleviate the aforementioned problems. Many different technical approaches to active rotor control are well-known. A concept significantly diverging from the approaches existing up to date is the multiple-swashplate control system which is introduced in this paper as an alternative to the established active rotor control concepts. As an introduction to the subject, this paper first outlines a short motivation for active rotor control and provides a brief overview of the state of the art and the different approaches. Advantages and disadvantages of the different technologies are mentioned. The paper provides an overview of the function, underlying principles and design of the essential components of the multiple-swashplate control system for IBC applications and the integration into the rotor test rig of the DLR. The main focus is on the principles of primary rotor control as well as Higher Harmonic Control (HHC) and Individual Blade Control (IBC) using electro-hydraulic actuators in the nonrotating frame. The last part of this paper outlines the current state of the project and discusses experiments planned once the necessary hardware is available.

1. INTRODUCTION

The ability to take-off and land vertically, to hover and the excellent low speed flight performances and handling qualities in comparison to other VTOL aircraft are the main reasons why helicopter conquered their market and cannot be replaced by any other aircraft. Today, helicopters cover a wide range of applications in military as well as in civil operations; for example, emergency medical services (EMS) missions.

On the other hand, helicopters still suffer from many problems that hinder a further increase in their market share and limit the acceptance of helicopters in the public as well as for operators. The most important problems are:

- high level of vibrations,
- high noise generated by the rotor,
- high power required in high speed forward flight,
- low range and limited speed of flight.

Since the middle of the last century, a dramatic reduction in vibration level has been achieved by passive as well as actively controlled vibration systems [1]. A further significant reduction through passive means does not seem to be feasible. The level of 0.02 *g* recommended by NASA seems to be out of reach. Vibrations are not only a concern of pilot and passenger comfort, they also give rise to an increase in maintenance effort and cost. A study of the US Army and Sikorsky, for example, showed that helicopters equipped with rotor in-plane vibration absorbers (bifilars) have 10% lower life cycle costs than helicopters without [2].

In contrast to vibrations, noise radiated by the helicopter is more relevant to the public and hence is a strong certification issue. In 2001, the allowable noise limits have been lowered by several decibels depending on the flight conditions and take-off or landing weight [3]. The main rotor contributes significantly to the overall noise of helicopters. Whereas blade vortex interaction (BVI) noise is important in descent flight, dominant effects in forward flight are blade wake interaction noise, trailing edge noise, and high speed impulsive noise. Thickness and loading noise are a general problem during all flight segments. A proper design of blades can help to reduce rotor noise signature significantly. But active rotor control can further reduce the noise level even for rotors having an inherent silent passive design.

Besides other objectives, recent wind tunnel tests in 2009, for example, also demonstrated the potential of IBC to improve rotor performance [4]. During these tests, rotor power reductions of about 5% were found using 2/*rev* IBC inputs at high forward speed ($\mu = 0.4$).

So, active rotor control technology is capable to alleviate all of the aforementioned problems. A survey of the different well-known active control systems and some selected results are given in [5] and [6].

Active rotor control systems can generally be divided into two categories, depending on the location of the actuators which can either be located below the swashplate in the nonrotating frame (fuselage) called Higher Harmonic Control (HHC) or the rotating rotor frame which are referred to as Individual Blade Control (IBC) (see Figure 1). IBC-Systems can be further distinguished by how and where the actuators are integrated.

Due to the significant differences in their functional principles, HHC and IBC systems also differ regarding their respective advantages and disadvantages. As HHC systems do not need electrical or hydraulic slip rings to transfer energy and signals between the nonrotating and the rotating frame, they are mechanically less complex. Furthermore, the actuators are not subjected to any centrifugal forces from the rotation of the rotor, special consideration of rotor hub and blade design is not necessary.

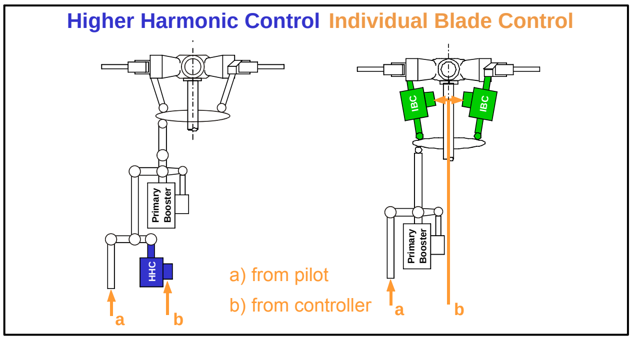

Figure 1 Comparison of HHC and IBC concepts

However, one serious disadvantage of HHC, due to mechanical constraints of the swashplate, is that only a limited range of control frequencies can be transmitted into the rotating frame. These are limited to harmonic signals with frequencies of n times the number of blades, N, times the rotational frequency of the rotor, Ω, and $(nN\pm1)\Omega = m\Omega$, with n being a positive integer. Those frequencies are abbreviated as “m per rev” (*m*/*rev*).

Hence, on a helicopter with four blades, only the control frequencies 3/*rev*, 4/*rev*, and 5/*rev* can be realized in the rotating frame using HHC. It has been shown in various investigations that the 2/*rev* frequency is very useful in terms of noise and power reduction. This frequency cannot be controlled by HHC for such rotors, which can be clarified by the following example. An exemplary time history of the corresponding 2/*rev* blade pitch signals is shown in Figure 2. Additionally for the time $t = 0$ (blade 1 at 0° azimuth) the current blade pitch angles of all four blades are plotted. Since the pitch angles are negative for the blades 1 and 3 but positive for the blades 2 and 4, the swashplate would have to incorporate an additional degree of freedom (DOF) (i.e. bending) for this so called reactionless control mode to be feasible, see illustration on the right.

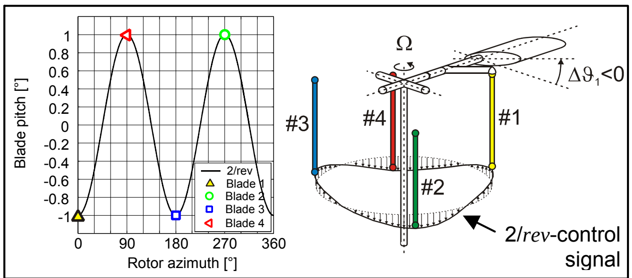

Figure 2 2/*rev* control signal on a 4-bladed rotor

This disadvantage of HHC only applies for rotors with more than three rotor blades like those featured on most medium and large helicopters and this limitation can only be overcome by IBC, which can realize arbitrary control frequencies and changes of blade pitch angles.

On a rotor with up to three rotor blades, the three DOF of the swashplate are opposed to a maximum of three blade pitch angles – hence arbitrary IBC signals can even be realized by a HHC system. However, this aspect is merely theoretical since helicopters with only three rotor blades or less are, in most cases, too small to make the installation of a HHC-system economically viable[1].

The first investigations regarding active rotor control are based on HHC and go back to the years between 1952 and 1961 [7], [8], [9]. Those theoretical studies were addressed to the simulation of the effects of stall or speed enhancements. First flight tests with the aim to reduce fuselage vibration levels using 2/*rev* HHC signals were conducted in 1961 [10]. Further flight tests were conducted in 1982 [11]. Besides successful reduction of vibration levels and without significant increases in control loads these studies for the first time indicated the potential of HHC for rotor performance enhancement. In 1994 wind tunnel tests using HHC on a scaled BO105 rotor were conducted with the aim of vibration and noise reduction [12]. The results showed a noise reduction of 6 *dB* and considerable vibration reductions, but a simultaneous reduction was not reached, because – contrary to the vibrations – the noise level strongly depends on the angle of attack of the rotor disk against the incoming flow. For this reason, in most cases the reduction of one of the targeted parameters led to an increase of the other, and therefore, a trade-off between noise and vibration would have to be made.

Early theoretical and experimental investigations of IBC were conducted in the early 1980s [13], [14], [15] and showed great potential for gust and stall alleviation and enhancement of lead-lag damping characteristics. Successful wind tunnel tests with IBC systems were conducted using a BO 105 and an UH-60A full-scale rotor [16], [17], [4]. These tests showed the enormous significance of the 2/*rev* control frequency[2] for the reduction of vibrations and noise as well as for rotor performance enhancement. For the first time simultaneous reductions of noise (-6 *dB*) and vibrations (-75%) were demonstrated. The tests also showed that not only BVI noise, which propagates underneath the rotor disk, but also in-plane noise can successfully be reduced. Further results included rotor performance enhancements up to 5% and the use of IBC for in-flight tracking. At the same time a BO 105 [18] and later a CH-53G [19], [20] were equipped with blade-root IBC systems and tested in flight.

The potential of IBC was impressively demonstrated in the aforementioned experiments. Moreover, IBC is applicable for many different purposes and promises to alleviate some of the main problems of helicopters. Nevertheless, IBC can be realized by means of several technologies whose pros, cons, and maturity level differ significantly. For instance, blade root actuation IBC with hydraulic actuators in the rotating frame needs electric and hydraulic slip rings. In case of HHC, the power requirements for actuation are relatively high since the whole rotor blade is actuated and effects on the whole mechanical control chain must be taken into account during the design phase. An advantage of blade root IBC is that the blades themselves don’t have to be redesigned, but this is far outweighed by the high complexity and weight penalty of such a system.

Today, a trend towards blade-integrated actuators has emerged. This includes discrete flaps as well as actuators distributed over the blade. Active trailing edge flaps (see Figure 3) have been successfully tested on a BK117 by Eurocopter [21] and on a MD900 rotor in the Ames wind tunnel by Boeing [22]. The results were quite similar to those achieved by blade root IBC.

[1] Except for CH-47: two 3-bladed rotors

[2] A HHC-system can realize this control frequency on a rotor with up to three blades, but not on a 4-bladed rotor.

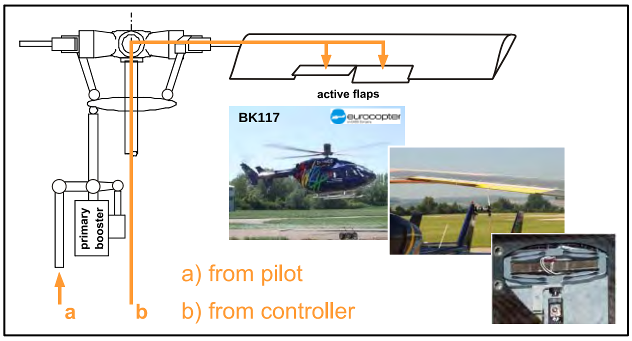

Figure 3 Concept of active trailing edge flaps

The more sophisticated but less mature concepts with distributed piezo-microfiber actuators, integrated in the blade skin (see Figure 4) or on the blade spar, lead to a continuous blade twist. A comparison of two concepts can be found in [23]. Sufficient wind tunnel test results are not yet available. An advantage of this concept compared to active flaps is the complete lack of mechanical parts and the homogeneity of the blade in terms of mass distribution, etc.. However, the issue of blade maintainability is still unsolved.

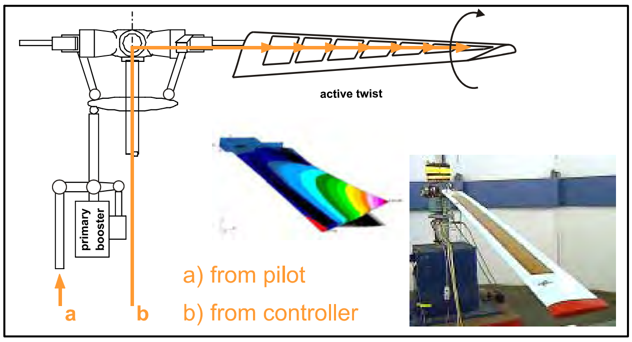

Figure 4 Concept of active twist

Both concepts, active flaps as well as active twist, considerably increase the complexity of rotor blade design. This is due to the already demanding task of accurately tuning the eigenfrequencies of the blade, high centrifugal loads, elastic blade deformations, influences of climatic conditions and many more reasons. Notice that rotor blades are generally rated for endurance strength and moreover downtimes and costs for actuator maintenance would be of very limited acceptance to operators. Consequently it must still be proven that these systems (based on piezo-electric actuators) will be maintenance free.

2. Concept Of Individual Blade Control Via Multiple Swashplates

The special disadvantages of the concepts of active rotor control introduced in section 1 are both evident and serious. Even though HHC uses a simple design and could easily be developed to production standard, it still lacks essential benefits – especially if simultaneous reductions of several target parameters are concerned. IBC generally has higher benefits because arbitrary control signals, optimized for noise and vibration reduction as well as for rotor performance enhancement, can be realized, but it comes with a significantly higher technical risk due to actuators in the rotating frame. Smart piezo-actuators as used with active flaps or active twist blades also pose many concerns which are still unsolved (i.e. costs, maintenance issues, durability etc.).

In 2008 the DLR was issued a patent for the multiple-swashplate rotor system called META (Mehrfachtaumelscheibensteuerung) [24]. Based on the principle of HHC, META achieves full IBC capability for helicopters with more than three rotor blades without using actuators in the rotating frame. As full IBC capability with a conventional swashplate equipped with the same actuation system than for the concept of HHC is only achievable if the number of rotor blades is less or equal to the DOF of the swashplate, the number of blades is limited to a maximum of three per swashplate. If the number of rotor blades is increased, further DOF are necessary. In the META control system, these DOF are introduced via additional swashplates which are arranged in the same reference plane. Hence, for helicopters with four to six blades, which would cover most helicopters on the market today, a second swashplate is sufficient, seven to nine blades would necessitate a third one, and so on. The principle of rotor control using the META control system is illustrated in Figure 5.

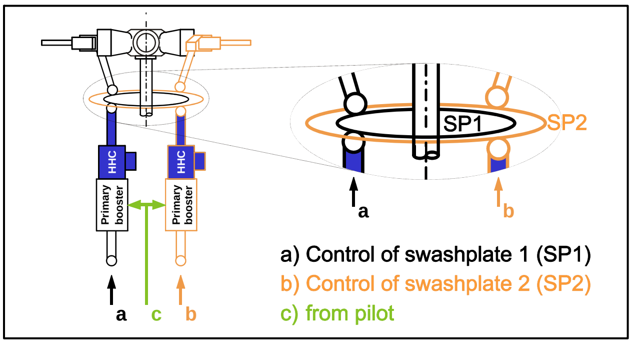

Figure 5 Principle of the META control concept

Just like HHC, the signals for the HHC actuators are superimposed on the pilot signals (primary control) in the nonrotating frame and directly transferred to one of the swashplates (SP). Within the limits of the control authority of the HHC actuators any arbitrary position of the swashplates can be achieved, thus making full IBC possible for every swashplate and therefore for the whole rotor system. The META concept thereby employs advantages known from earlier HHC projects. The rotor can still be designed conventionally and the costs for maintenance of rotor components do not increase compared to conventional rotors without IBC capability. The swashplate is a well proven and technically mature system.

The principle of the META control system for a 6-bladed rotor is exemplified in Figure 6. For the sake of simplification, each of the swashplates seen in the picture is actuated by three separate boosters. Both green and the white rotor blades are connected to the inner swashplate, the red blades are connected to the outer one. For clarification the diagrams below show the time histories of the pitch angles of the six rotor blades as well as of the booster strokes during "true IBC" for one rotor revolution. In this example blade 1 exhibits a constant pitch angle, while blade 2 performs a 1/*rev* variation, and so on. The sequences of the booster strokes depicted below are steady and unproblematic.

Since the mid-1970 the DLR's Institute of Flight Systems (FT) operates its own rotor test rig (RVS) for scaled rotors and wind tunnel helicopter models which has been successfully employed in numerous wind tunnel tests. Within the current national aeronautical research program "Vollaktive Rotorsteuerung" (fully active rotor control, VAR) the RVS is enhanced by a fully functional META system. The basis of this system is a 40% Mach-scaled BO 105 model rotor which is already available.

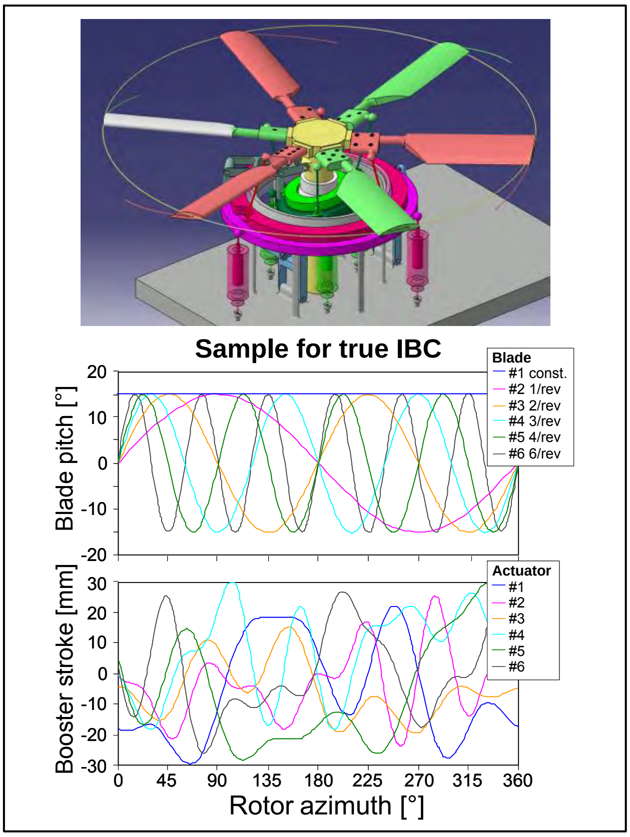

Figure 6 6-bladed rotor with META; example for “true IBC“

The actuation of the multiple swashplates is thereby carried out by combined electro-hydraulic actuators in the nonrotating frame. In the further course of this paper the current state of the project will be described in detail. The main focus is on the principles of primary rotor control (electrical, from pilot) and HHC/IBC control (hydraulic) using actuators in the nonrotating frame. These principles differ fundamentally from those of other IBC-systems and therefore require new control laws.

3. Control Laws

3.1. Control laws in the rotating frame

The control inputs which have to be superimposed on the primary rotor control in the rotating frame to reduce vibrations and noise, alleviate stall-effects or enhance rotor performance, have – as mentioned before – already been investigated and validated in numerous HHC/IBC tests. It was demonstrated that, with specific harmonic control signal frequencies of n times the rotor’s rotational frequency Ω the aforementioned phenomena can be alleviated. It is also possible to use other periodic signals individual to each blade (which, for example, only influence the blade pitch angle ϑ_m at a specific azimuthal position ψ_m) for that purpose [17].

If the time history of the pitch angle ϑ_m of a rotor blade in the rotating frame is described as a Fourier series with a limited number of harmonics, the first harmonic describes the control equation of a conventional swashplate. The remaining harmonics are represented by the individual blade pitch term $\Delta\vartheta_{m,IBC}(t)$:

(1) $$\vartheta_m = \Theta_0 + \Theta_C \cdot \cos\psi_m + \Theta_S \cdot \sin\psi_m + \Delta\vartheta_{m,IBC}(t)$$

Here $\psi_m = \Omega t + 2\pi(m-1)/N$ is the individual azimuthal position of each blade $m = 1 \ldots N$, and N is the number of blades. The Fourier coefficients Θ_0, Θ_C, and Θ_S correspond to the collective and cyclic primary control of a conventional rotor control system using a swashplate. The individual portion of the blade pitch angle itself can be any arbitrary control signal in the time domain, different for each blade. When applying IBC, this signal is superimposed on the primary controls directly at the blade itself (blade root actuation, active flap and active twist). With the META system, the control signals – individual to every blade – are generated in the nonrotating frame and therefore have to be transmitted to the rotating frame via the swashplates. The determination of the required swashplate positions is not trivial and thus the determination of the control signals for the electro-hydraulic actuators poses the actual challenge.

3.2. Control laws in the nonrotating frame

The generalized control law for a rotor control via a swashplate is [25], [26]:

(2) $$\theta = \theta_0 + \theta_C \cdot \cos\psi + \theta_S \cdot \sin\psi + \theta_2$$

If only harmonic control signals are to be used in the rotating frame, the Fourier coefficients of the control law in the nonrotating frame can generally be written as:

(3) $$\theta_0 = \frac{1}{N}\sum_{m=1}^{N}\vartheta_m(t) = 0 \qquad \text{except } p = nN,$$

(4) $$\theta_C = \frac{2}{N}\sum_{m=1}^{N}\vartheta_m(t)\cdot\cos\psi_m(t) = 0 \qquad \text{except } p = nN\pm1,$$

(5) $$\theta_S = \frac{2}{N}\sum_{m=1}^{N}\vartheta_m(t)\cdot\sin\psi_m(t) = 0 \qquad \text{except } p = nN\pm1,$$

(6) $$\theta_2 = 0 \qquad \text{except } p = nN\pm N/2$$

Here n is an arbitrary positive integer and p is the rotor harmonic number. θ_2 is the so called reactionless control mode, which only occurs on rotors with more than three blades and an even number of rotor blades. It is nonzero for $p = nN\pm N/2$ and not assessable with a conventional swashplate, as shown in section 1.

If equations (3) to (6) are applied to a 4-bladed rotor and a pure 2/*rev* control signal is introduced into the rotating frame, only the differential control mode θ_2 exhibits a value unequal to zero. Since the reactionless control mode does not apply to rotors with three rotor blades, all rotor harmonics can be transmitted to the rotating frame via the META control system. Moreover, this configuration has full IBC capability, since the number of blades equals the three DOF of the swashplate connected to them [27]. In this case the control coefficients (equations (3) to (5)) also become terms dependent on time or rotor azimuth.

3.3. Development of control laws for META

The control law described in subsection 3.2 based on the coefficients θ_0, θ_C, and θ_S, is not suited to achieve IBC by means of the META system, as the required command inputs are the corresponding actuator lengths in the nonrotating frame. In order to determine those necessary parameters, the whole kinematic path between blade pitch angles and actuator lengths was formulated for one of the swashplates. Since the two swashplates only differ in terms of actuator positions in the nonrotating frame and azimuth of the corresponding rotor blades, the equations for the second swashplate are almost identical to the first, and the algorithms are adaptable by adding a phase angle. A schematic illustration of the principal kinematic correlations for one of the swashplates is shown in Figure 7.

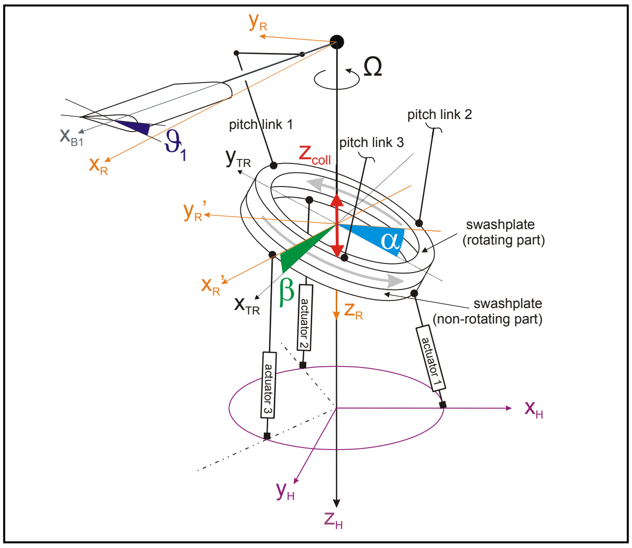

Figure 7 Illustration of the swashplate kinematics

Every change of the blade pitch angle ϑ_m requires a corresponding displacement of the respective pitch link connected to the blade's pitch horn. Therefore, the swashplate has to be shifted vertically along the rotor axis (z_{coll}) and – via cyclic control – tilted about its longitudinal and lateral axis, expressed by the tilting angles α and β when needed. The position of the swashplate must thereby always satisfy the pitch angles of the three attached rotor blades. Then, the corresponding actuator lengths underneath the swashplate result from the distances between their pivot points on the swashplate and the baseplate. With three actuators and three rotor blades per swashplate there always is one unique position of the swashplate which satisfies all kinematic constraints. Since the META system will be operated on the test bed with a 4-bladed rotor (two blades per swashplate), in order to obtain a unique solution, either a third virtual rotor blade has to be added (additional constraint), or the DOF of the swashplate have to be reduced[3].

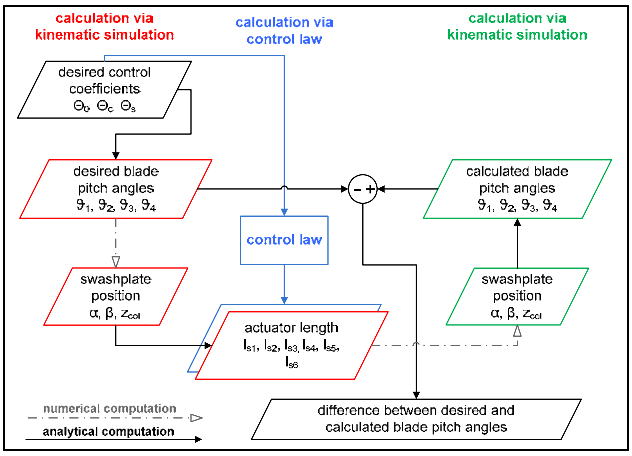

Figure 8 Flow diagram of the kinematic simulation

Figure 8 shows a flow diagram of the single steps to develop the control laws suitable for the kinematic simulation of the META system for any arbitrary individual control signal for each blade. The red path on the left side represents the "top down" calculation, i.e., the calculation of actuator strokes from the time-dependent control coefficients Θ_0, Θ_C, and Θ_S.

These coefficients can analytically be computed into the actual blade pitch angles ϑ_m for every single blade. Then, the actual blade pitch angles are transformed into the positions of both swashplates defined by z_{coll}, α, and β. This determination of the time-dependent swashplate position corresponding to the individual blade pitch angles is obtained by solving the equation system formulated for its kinematic paths. Since these formulations contain transcendental terms, an analytical way to solve them is almost futile, so a rather time-consuming numerical calculation is used. With the knowledge of the swashplate position, the lengths of all actuators can then be determined analytically again.

For reasons of feasibility and real-time capability an approximated control law – defined in section 4, and represented by the blue path in the middle of Figure 8 – is used to replace the mathematically exact method of the whole kinematic path in practice. The green path on the right side represents the "backwards" or "bottom up" calculation of the individual blade pitch angles from the actuator lengths obtained before. Again, one part of this calculation can easily be determined analytically, the other one is better solved numerically. By comparison of the desired and the calculated blade pitch angles the congruence of the two calculation "directions" as well as of the kinematic simulation and the approximated control law can be investigated and verified. The following section deals with the derivation of such a simplified control law, suitable to sufficiently approximate the exact kinematic path (blue path) of the META control system for the RVS.

4. DEVELOPMENT OF REAL-TIME CAPABLE CONTROL FOR META

4.1. Primary conventional control

The kinematic simulation for the META system (see red path in Figure 8) is – within numerical accuracy limits – mathematically exact, but, as shown in subsection 3.3, it uses iterative and time-consuming algorithms which renders it unsuitable for real-time applications. For this reason it is necessary to sufficiently simplify the calculation of the vector of actuator lengths $\vec{l}_S$ from the control coefficients while keeping the resulting approximation errors as low as possible. In most cases a linear approximation leading to a constant control matrix $\underline{\underline{M}}_{St}$ (for each swashplate) is used for this purpose:

(7) $$\vec{l}_S = \underline{\underline{M}}_{St} \cdot \vec{\theta} + \vec{l}_{S,0}$$

Multiplication of this 3x3 control matrix and the 3x1 control vector consisting of the three Fourier coefficients θ_0, θ_C, and θ_S leads to the actuator lengths. By adding the reference lengths (for example in the neutral position) of the actuators (vector $\vec{l}_{S,0}$) the absolute lengths of the actuators l_{S1}, l_{S2}, and l_{S3} are obtained.

To calculate this control matrix: first, several different trim states of the rotor – and therefore Fourier coefficients θ_0, θ_C, and θ_S, and corresponding actuator lengths l_{S1}, l_{S2}, and l_{S3} – are measured (or simulated via the kinematic simulation); second, the system is linearized and third, the elements of matrix $\underline{\underline{M}}_{St}$ are derived by transposing equation (7). Using such a relatively simple control matrix inevitably leads to linearization errors which increase with the

[3] For example the tilt angle about the axis connecting the pivot points of the two pitch links (of the two rotor blades per swashplate) can be set to zero.

deviation of the swashplate's position from its reference position i.e. the linearization point of the system. The reason for this are the kinematic nonlinearities in the mechanical system.

Typically these errors are corrected recursively – for example, during operation of the RVS the current blade pitch angles are measured at the rotor and then transformed via FFT (Fast Fourier Transform) into corresponding control coefficients, which are displayed in the control room. If necessary, corrections are made manually. Another way to minimize linearization errors is the constant recalculation or adaptation of the control matrix e.g. new linearization of the system with every different trim condition [28]. As this process is very time-consuming, it is not viable for the operation of a rotor during the experiments.

4.2. Polynomial approach of higher order

The linearization error occurring with the use of a linearized control matrix described in subsection 4.1 exceeded – in some simulation cases – 1° of blade pitch amplitude. Since blade pitch amplitudes for HHC are mostly smaller (with some rare exceptions), 1st order approximations like in equation (7) were consequently deemed unsuitable for HHC operation. Only when the HHC amplitudes are very small and the system is re-linearized at the current trim condition such control matrices should be used. Hence, an approach with a control matrix containing nonlinear elements is suggested. Comprehensive studies have shown that a control matrix which approximates the actuator lengths as polynomials of the 5th order – and additionally incorporates linear combinations of all control coefficients – achieves the best correlation with the mathematically exact solution for a wide range of rotor trim states. The corresponding control-equation is as follows:

(8) $$\vec{l}_S = \underline{\underline{M}}_{HO} \cdot \left(\theta_0 \; \theta_C \; \theta_S \; \cdots \; \theta_0^5 \cdot \theta_C^5 \cdot \theta_S^5\right)^T + \vec{l}_{S,0}$$

Again, the actuator lengths are obtained by multiplying the control matrix $\underline{\underline{M}}_{HO}$ with a coefficient vector and adding the reference lengths of the actuators. The coefficient vector consists of all possible linear combinations of the control coefficients θ_0, θ_C and θ_S. The constant control matrix for this equation consequently has three rows and 215 columns. For its calculation: first, the exact kinematic simulation (see Figure 8, red path) is used to generate data sets containing the actuator lengths l_{S1}, l_{S2}, and l_{S3} corresponding to every possible combination of control coefficients θ_0, θ_C, and θ_S imposed on the rotor (within kinematic constraints). These data sets are then processed by a multiple regression algorithm which calculates the 645 elements of the control matrix as well as the reference lengths of the actuators. It should be noted that the computational effort increases exponentially with the number of measured/simulated data as well as with the number of columns in the matrix.

In various simulation runs at varying rotor trim conditions it could be shown that the nonlinear control law according to equation (8) represents the exact nonlinear kinematics of the system far better than a simple control law with a control matrix of 1st order. To compare the different approaches the standard deviation σ averaged for all N blades between the desired and the resulting blade pitch angles $\Delta\vartheta_m^{Basis}(\psi_k)$ of one rotor revolution (with a resolution of K = 256 azimuthal positions) was calculated:

(9) $$\sigma_\vartheta(\theta_0, \theta_C, \theta_S) = \sqrt{\sum_{m=1}^{N}\sum_{k=1}^{K} \frac{\left(\Delta\vartheta_m^{Basis}(\psi_k)\right)^2}{N\,K}}$$

Figure 9 shows a comparison of the determined standard deviations and maximum pitch angle errors (primary rotor control) vs. simulated airspeed for control matrices with different orders of approximation polynomials used in the calculations. It is clearly visible that control matrices with an approximation order of three or higher reproduce the swashplate's kinematics relatively well throughout the entire simulated flight regime (with errors below the measurement accuracy of the RVS of 0.04°).

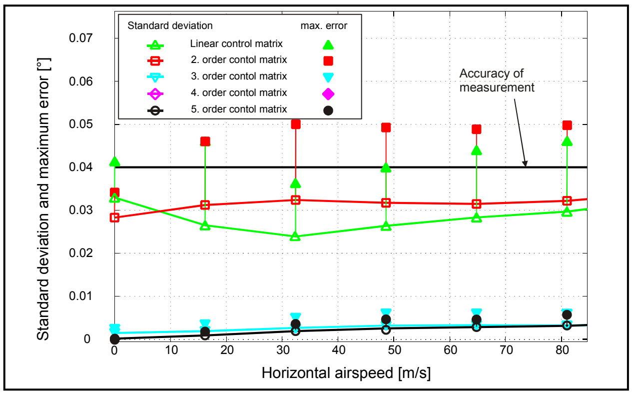

Figure 9 Averaged standard deviation and max. error of blade pitch angles vs. simulated airspeed

However, with simultaneous primary and HHC control inputs, the maximum errors increased dramatically when approximations of the 3rd or 4th order were used. For this reason a control matrix corresponding to a 5th order approximation was chosen. Then manual corrections or recalculations of the control matrix are no longer needed and the HHC control is sufficiently exact for all rotor trim states. A sensitivity analysis to determine those of the 215 elements of the coefficient vector which are insignificant enough to be omitted is currently underway to further make simplifications of the control algorithms possible.

4.3. HHC control

To realize HHC control via the control matrix, the amplitudes and phases of the control signals with frequencies of *n/rev* are converted into time-dependent HHC coefficients using equations (3) to (5). These coefficients can then be superimposed to those of the primary control:

(10) $$\vec{\theta} = \vec{\theta}^{Basis} + \vec{\theta}^{HHC}$$

Before employing the resulting control coefficients – representing the sum of primary and HHC control – in equation (8) the coefficient vector with 215 elements has to be calculated. Since the actuators for the swashplates on the RVS have both an electric (for primary control) and a hydraulic part (for HHC/IBC, see subsection 6.2) the resulting control signals also have to be split into static and dynamic part to operate the META system.

As the hardware for the META system is still in production (see section 6) and hence not yet available for tests, simulations were used to determine the control accuracy of the system. They showed the significant advantages of a nonlinear 5th order approximation, compared to a conventional control matrix of lower order. In all simulated control cases the standard deviation was about five times smaller than that encountered using a linear control matrix while controlling HHC/IBC signals.

4.4. Tip path plane splitting

In the course of the VAR program, one goal is to investigate the feasibility of splitting the tip path plane (TPP) of the rotor using the IBC functionality. This capability of IBC is later to be applied in a possible follow-up test in the German-Dutch Wind Tunnel (DNW) to generate vertically spaced blade tip vortices (instead of vortices in the same plane). A significant reduction of BVI noise during landing approaches is expected.

To achieve this effect on a 4-bladed rotor, the two pairs of blades opposed to each other are subjected to different 1/*rev* or 2/*rev* (if applicable) control signals, thus effectively creating two 2-bladed rotors with different tip path planes. Here, a great advantage of the META system is the fact that alternating rotor blades are hinged on different swashplates which makes TPP splitting relatively easy to achieve.

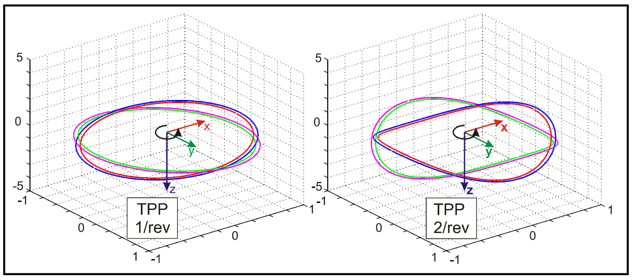

Figure 10 Different forms of TPP splitting

For pure primary control, a 1/*rev* splitting of the tip path planes can be realized by differential tilting of the two swashplates. The resulting tip path planes are illustrated in Figure 10 (left side). To obtain the same effect with 2/*rev* HHC, a 180° phase shift between the control signals for the two swashplates has to be introduced, so that the blade pitch variations of two consecutive blades are inversely phased, see Figure 10 (right side). Due to the fact that TPP splitting is a special application of HHC control, this functionality is easily implemented in the META control system.

4.5. Blade tracking

To be able to avoid the time consuming process of conventional blade tracking for the scaled rotor on the RVS, a functionality was implemented and incorporated into the META control system which makes corrections of the static blade pitch angles during the operation of the rotor – hence "in-flight tracking" – possible. Compared to the aforementioned functions of the META control system, where all blades have the same cyclic amplitudes (HHC), partly with different phases (TPP splitting), this is a real IBC application. Since the developed control software (see section 5) does not yet provide full IBC capabilities (development is underway) the correction of single individual blade pitch angles is obtained using a real-time capable approximation procedure.

The change of the transfer function between the actuator stroke and the blade pitch is usually nonlinear throughout the operational envelope of the rotor, and depends on all three control coefficients θ_0, θ_C und θ_S, i.e., the position of the swashplate (see subsection 4.2). Since the dependency on θ_0 is by far the strongest, this coefficient was included into the approximation procedure while the cyclic coefficients were neglected. The transfer function between actuator stroke Δl_S and change in blade pitch $\Delta\vartheta$ is as follows (pitch link positioned exactly above the pivot point of the corresponding actuator):

(11) $$m_{Tr} = \Delta l_S / \Delta\vartheta_{Tr}$$

Figure 11 shows this transfer function m_{Tr} as a function of collective pitch θ_0 – a function, which can be approximated by a polynomial of 3rd order.

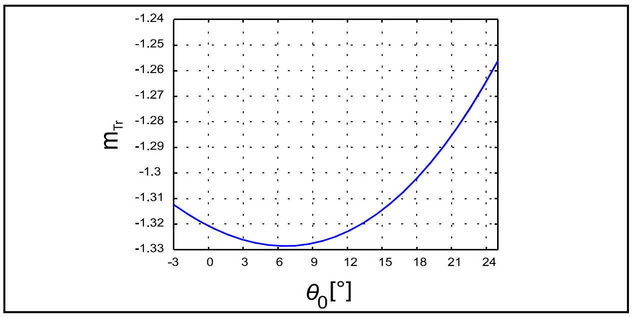

Figure 11 Transfer function between actuator stroke and blade pitch angle vs. collective pitch

The actual tracking of the 4-bladed rotor used is achieved by collective shifting and simultaneous tilting of the swashplate. Hereby it is possible, for example, to change the pitch angle of blade 1 without (approximately) influencing the pitch angle of blade 3 on the opposite side. A schematic illustration of this procedure is sketched in Figure 12. While the pivot point of the left pitch link changes its position along the rotor axis by a collective (Δz_{coll}) and a cyclic (Δz_{tilt}) part, the pivot point of the pitch link on the right side remains in its prior position.

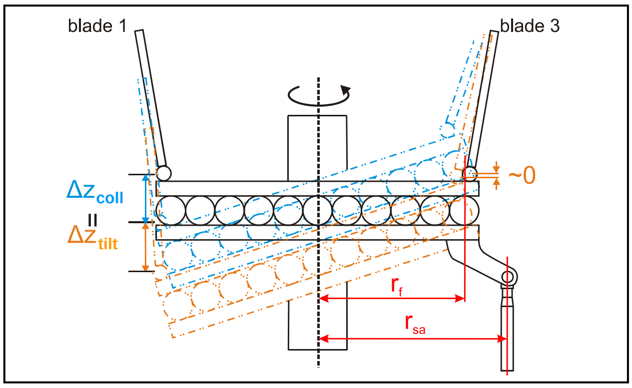

Figure 12 Approximation procedure for in-flight tracking

To implement this approximated tracking procedure for the respective swashplates a so called "tracking term" l_{Sk}^{Tr} is added to every actuator length l_{Sk} calculated from equation (8):

(12) $$l_{Sk}^{Tr} = a_{Tr} \cdot \left\{ r_f / r_{sa} + \cos\left[\psi - \psi_{1k} - (k-1)\cdot\varphi_S\right] \right\} \qquad k = 1,2,3$$

In equation (12) ψ_{1k} represents the angle the rotor has to be turned for the pitch link of blade 1 to be positioned exactly over actuator *k*. For the first swashplate, with actuators on the rotor azimuths 0°, 120° and 240° these angles are illustrated in Figure 13. φ_S in equation (12) is the angular distance between the actuators in the nonrotating reference frame (here: 120°). The first term (r_f / r_{sa}, see Figure 12) within the large brackets represents the collective shift, the second one the tilting of the swashplate. The term a_{Tr} is the "tracking amplitude" of the respective actuator and is obtained as follows:

(13) $$a_{Tr} = m_{Tr} \cdot \Delta\vartheta_{Tr} / 2$$

If tracking is conducted using the procedure described above, the swashplate performs an additional 1/*rev* movement oscillating with the rotor superimposed on the swashplate's static position resulting from primary control. If tracking of more than one blade and different changes of static pitch are essential, the "tracking terms" for the blades can be superimposed.

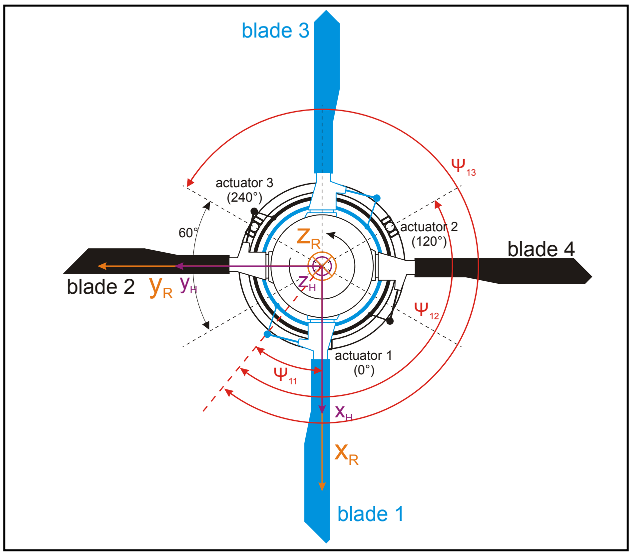

Figure 13 Illustration of ψ_{1k}

5. RVS Integration

5.1. Real-time control system

To achieve real-time control capabilty for the scaled rotor on the RVS the matrix-control algorithms described in the subsections 4.1 to 4.5 were modeled in software and compiled into real-time code. A real-time processor (RTP) system is used as a control computer and is accessed via a conventional PC, see Figure 14.

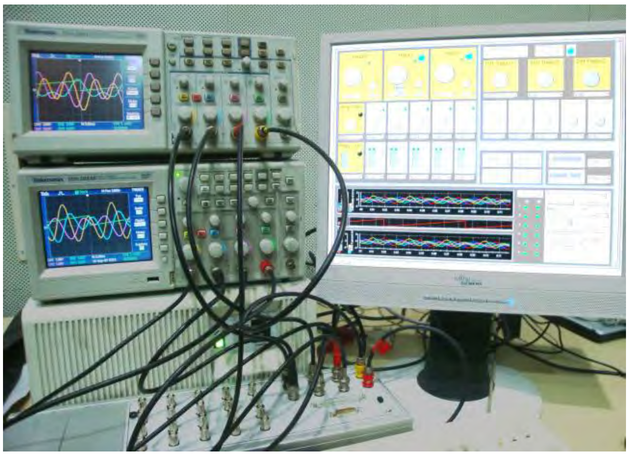

Figure 14 Experimental setup with RTP system

On the left side, the control signals for the six electro-hydraulic actuators are monitored. Directly beneath the oscilloscopes the RTP system is located, including D/A-A/D converters and a dedicated I/O board. All parameters of the turning rotor (in the preliminary setup, the rotation is simulated by an artificial trigger signal) are sampled with 256 samples/*rev*. Hence, all actuator strokes are calculated at the same rate. The nominal rotational frequency of the scaled rotor used on the RVS is 17.5 *Hz*. Thus, the real-time control system has to be able to recalculate the whole software model at a rate of 256x17.5 *Hz* = 4.48 *kHz* and to transmit the results to the hardware.

A shaft encoder, integrated into the RVS and connected to the rotor, generates a 1/*rev* signal to synchronize the azimuthal position of the scaled rotor with the control software, and a 256/*rev* signal to trigger the calculations described in section 4 and to determine the current rotor azimuth which is needed for these calculations.

In order to verify the software model used for real-time control, the actuator strokes obtained were matched against those calculated with the exact kinematic simulation. Even though differences between both calculations shouldn't exist, slightly deviations with a maximum of 0.05 *mm* occurred. A possible source of this error is assumed to be the compilation of the software into real-time code and is currently unter further investigation. But, since those deviations are smaller than the maximum positioning accuracy of the actuators, the resulting control errors can be considered negligible.

5.2. Graphical user interface for META control

For manual operation of the real-time control software and thus the META system a GUI was created to enable the user to manipulate and monitor all generated control signals online (see Figure 15). On the upper left there are control elements for the primary rotor control, below the user can enter the desired amplitudes and phases for HHC operation. On the right side there are several more input dials for easy generation of typical IBC signals. Blade tracking (see section 4.5) and TPP splitting (see sections 4.4) are already implemented; possibly new functions could be included in the future. The lower part of the GUI contains plots for displaying the electrical and hydraulic actuator strokes as well as warning lights, e.g., for reading the mechanical actuator limits.

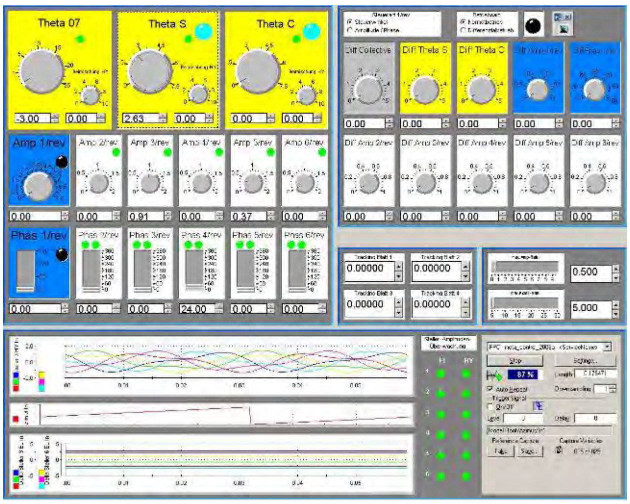

Figure 15 META control GUI

6. Hardware Development And Assembly

The basis for the assembly of the META system is the rotor test rig (ROTEST) of the DLR's Institute of Flight Systems. It consists of a fully instrumented six components rotor balance for measurement of forces, moments and torque of the overall assembly, a hydraulic motor for driving the rotor, a transmission unit, a slip-ring for transmitting signals between the rotating and nonrotating frame and the dynamic system, which will be mounted on top of the rotor balance (see Figure 16). The dynamic system itself is consists of the rotor shaft, swashplates, actuators, pitch links, rotor hub, and rotor blades. ROTEST is mounted onto a support sting. The rotor system including the BO 105 rotor hub center has a height of approximately 525 *mm* from the top of the rotor-balance.

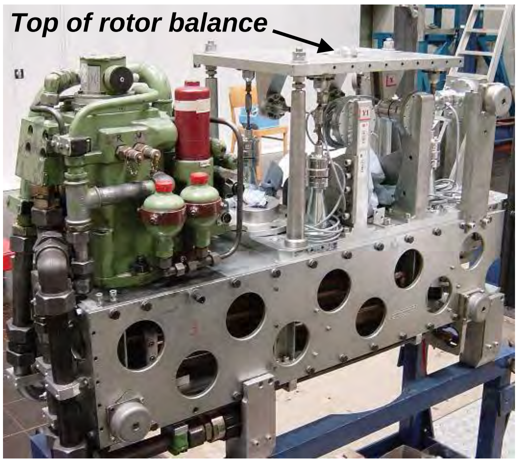

Figure 16 ROTEST (rotor test rig)

One goal in the modification process of the RVS was to reuse as many parts as possible – such as the rotor hub and the blades. Even though there are no geometric restrictions regarding the total height of the installed system, an increased height of the rotor shaft will lower the eigenfrequencies of the test rig, which in turn may lead to serious resonance problems at the RPM of operation.

For the strength analysis of the dynamic components different load cases from earlier test campaigns with and without HHC were analysed. The forces on the critical cross-sections of the respective components were calculated using these load cases and the components were adequately dimensioned using a safety factor. The most critical part in the test rig system with respect to bending is the rotor shaft.

The assembled META system (including swashplates, rotor shaft system, electro-hydraulic actuators and bearing brackets) on top of the rotor balance has a considerably higher mass than the present conventional swashplate system, which influences the eigenfrequencies of the overall assembly. Due to the complexity of the system these eigenfrequencies can only be obtained by experiment using the fully assembled system – an approach, which of course holds certain risks.

6.1. The META system

The META system devised for IBC on a 4-bladed rotor mounted on the RVS mainly consists of two swashplates concentrically and gimbal mounted in the same reference plane, whose kinematic are completely independent from each other (see Figure 17).

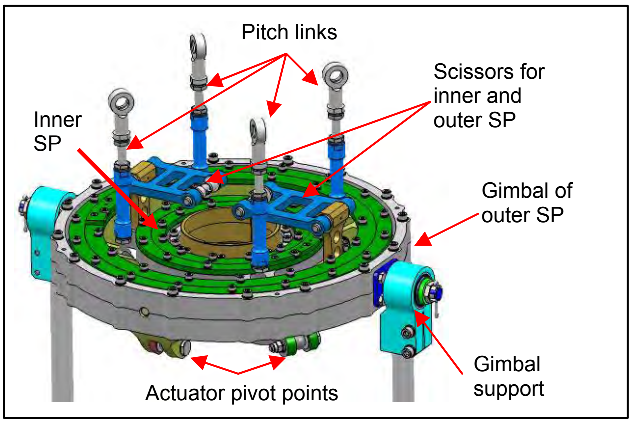

Figure 17 Gimbal mounted META system

The inner swashplate is gimbal mounted via a sliding sleeve around the rotor shaft, whereas the outer swashplate is mounted via an outer gimbal connected to the top of the balance. The pivot points of the pitch links are located on the rotating outer side of the inner swashplate – and vice versa for the outer swashplate.

The pivot points of all pitch links as well as the pivot points of the actuators underneath the swashplates are located on respective common radiuses which results in equal control strokes for both swashplates. Although this is not absolutely necessary for the control concept of META, it facilitates the design and construction of the testbed system as only one type of actuator is needed.

Since the pivot points of the actuators are not in the same reference plane as those of the pitch links and both swashplates are gimballed around the same reference point, a change of actuator length leads to both horizontal and vertical movement of the respective pivot points with respect to the rotor shaft.

In order to keep the construction small, the META system was designed such that a collision of both swashplates can only occur if the tilting angle between both swashplates exceeds 6°. This particular position cannot occur during normal operation, even when the swashplates oscillate in phase opposition with maximum amplitudes of 4° and a common "primary control" baseline position. Only if the electric part of the actuators introduces an additional tilt angle difference of 2° collisions are possible. For this reason, several proximity sensors are installed on specific points of both swashplates. When collision danger is imminent they trigger an emergency shutdown of the system and an immediate synchronization of the swashplate positions.

Altogether, the system was designed for a maximum collective pitch of 19° with simultaneous maximum cyclic pitch of 12° (1/*rev* amplitude) and an additional HHC pitch angle manipulation with a maximum amplitude of 4° - as long as these values are not exceeded, no collisions or mechanical lock-ups can occur in the dynamic system.

6.2. Electro-hydraulic actuators

As mentioned before, electro-hydraulic actuators are used to position the swashplates. The design of those actuators is based on actuators build by MBB (now the German part of Eurocopter, ECD) in the 1980s which have been used in various HHC-campaigns [12].

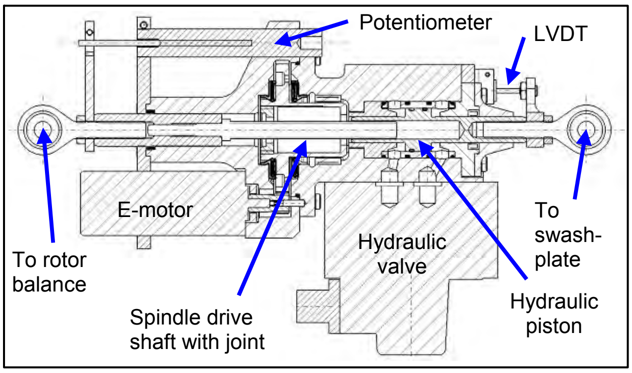

Figure 18 Electro-hydraulic META-actuator cutaway

To fulfill the kinematic requirements of the META system, the electric part of the actuators is capable of a ±20 *mm* (quasi-steady) stroke, while the hydraulic component can carry out strokes of up to ±4 *mm* with frequencies of 1-6/*rev* (approx. 17.5-105 *Hz*). The actuator force required to hold a position or to actuate the swashplate is estimated at

2000 *N*, about half the maximum force of the actuators' hydraulic component.

The general design of the combined electro-hydraulic actuators is illustrated in Figure 18 (tilted 90°). The electric part is visible on the left side whereas the hydraulic components are located on the right. An electric stepper motor with position control drives a spindle (flank lead P = 1 mm/*rev*) via a pinion/sprocket gear with a transmission ratio of 1:7.5. The spindle travel is measured by a linear potentiometer.

The hydraulic part of the actuator is controlled by a hydraulic valve with integrated control electronics supplied with a maximum charge pressure of 210 *bar* (3045 *psi*). The maximum possible volume flow rate of the valve is 26 *l/min*, the maximum for the actuator is at 7 *l/min*. The piston stroke is measured by a separate Linear Variable Differential Transformer (LVDT) sensor and controlled by a hardware controller specifically designed and built for this actuator system.

6.3. Overall system

The compact overall assembly of the META system is illustrated in Figure 19. Underneath the two swashplates the six electro-hydraulic actuators are mounted upright and are equally spaced circumferentially around the rotor shaft. The actuators are alternately connected to the inner and outer swashplate and are each mounted on a bearing bracket, which are instrumented with resistive strain gauges (RSG) for measuring the control loads in the system. The rotor hub electronics are located between the rotor hub and the swashplates, and is used for signal conditioning (e.g. preamplification) and transmission of measured parameters from the rotating to the nonrotating frame (e.g. blade pitch torque, pitch link loads etc.).

The cable ring keeps the electric cables (e.g. blade cables) in place and protects them from centrifugal loads. The hydraulic piping (not depicted) consists of one feed line and one return line per actuator and has to be mounted onto the upper plate of the rotor balance, without introducing any additional loads (e.g. forces and moments) into the system so as not to adversely affect the measurements.

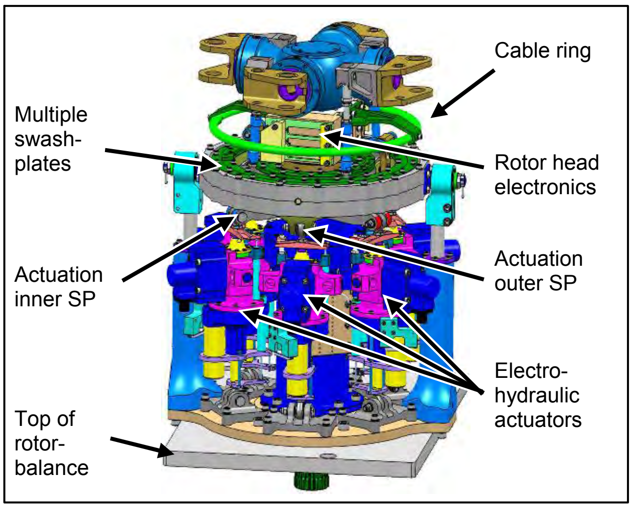

Figure 19 The complete META system for operation on the RVS

6.4. System Tests

The newly developed control laws (see sections 3 and 4) will be tested in 2010 – initially without, then with rotor blades – using the real-time control system introduced in subsection 5.1. During those verification tests the safety precautions (emergency shutdown in case of imminent swashplate collision or exceedance of safe actuator strokes) will be thoroughly tested as well. When these tests will be finished preliminary HHC/IBC test runs will be conducted to demonstrate the full control capability of the system. This phase includes primary control signals, single harmonics (2-6/*rev*), and mixed mode HHC as well as demonstrating full IBC capability with various control signals like blade tracking or TPP splitting (see subsections 4.4 and 4.5), etc. for each rotor blade.

7. CONCLUSIONS AND OUTLOOK

A novel concept for realizing individual blade control for helicopters with four or more blades and without actuators in the rotating frame – the so called multiple swashplate control system META – invented and patented by DLR was presented as an alternative to the established active rotor control concepts. This technology, based on the HHC control concept, requires the derivation of completely different IBC control laws for the actuation of the multiple swashplates from the nonrotating system, compared to any other IBC concept with actuators all located in the rotating frame. The research activities on META are made within the frame of the national aeronautical research program "Vollaktive Rotorsteuerung" (fully active rotor control, VAR) funded by the German ministry of economics (BMWi). The single components of the META system are currently in production and are planed to be integrated into DLR's rotor test rig (RVS) in 2010.

After finishing the construction and integration process into the RVS, function tests of the system (rotating and nonrotating) are planned. Since all these tests will be conducted in the rotor hall of the DLR's Institute of Flight Systems instead of a wind tunnel, the effect of TPP splitting with regards to reductions of BVI noise cannot be investigated during the course of these tests. Altogether, these tests are intended to demonstrate the capabilities and the potential of this rotor control system and determine its dynamic limitations. Since the control functions introduced in subsection 4.5 make in-flight tracking of single rotor blades possible, this capability will be tested with the aim of developing a controller for automatic in-flight tracking in hover mode, which can then also be verified on the RVS.

The aim of this project is to prove the META concept by demonstrating the feasibility of such an active rotor control system. Therefore, the design and the mechanical complexity of the overall system for the RVS are acceptable. However, in case of a potential integration of a META system on a real helicopter control system other technical aspects get more important. Besides its technical advantages – compared to the established IBC systems (no necessary redesign of the blades or the rotor system above the swashplates, no signal or energy transfer into the rotating system, no active components in the centrifugal field) – a META system could only be successfully established or offered as an alternative, when there are considerable advantages in terms of complexity, weight, reliability, and safety and it is simultaneously providing a significant benefit, e.g., in the form of reduced costs (acquisition, maintenance, operation) for the operator. Though we are convinced of the possibility to fully demonstrate the technically and economically viability of this new IBC concept, DLR will not pursue this goal alone. Reason for that is that an important part of this work is of industrials' responsibility and that the economical advantages that this technology promises are high enough to incite private companies industrializing this technology.

For this reason, in a follow-up campaign DLR plans first, to perform wind tunnel test with ROTEST – fully equipped with a META system – and second, to accomplish an integration study together with a helicopter manufacturer or a qualified supplier to verify the aforementioned questions with special attention to a reduced complexity. Before that, the overall system and the developed control laws have to be tested successfully in the rotor test hall of DLR, planned in 2011.

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ARC = Aeronautical Research Council

ERF = European Rotorcraft Forum

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