FOURTEENTH EUROPEAN ROTORCRAFT FORUM

Paper No. 67

A TECHNIQUE FOR THE TUNING OF HELICOPTER FLIGHT CONTROL SYSTEMS

M.A. Manness
D.J. Murray-Smith

Department of Electronics and Electrical Engineering
University of Glasgow
GLASGOW G12 8QQ, SCOTLAND

20-23 September, 1988
MILANO, ITALY

ASSOCIAZIONE INDUSTRIE AEROSPAZIALI
ASSOCIAZIONE ITALIANA DI AERONAUTICA ED ASTRONAUTICA

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M.A. Manness
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D.J. Murray-Smith

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University of Glasgow

ABSTRACT

Helicopter flight control systems can be successfully tuned using sensitivity functions. Systematic adjustments to control system parameters are made on the basis of responses measured from the system to be tuned. The importance of the method lies in its ability to optimize controllers in the presence of unmodelled dynamics such as those of the rotor. The adjustment procedure will yield a controlled system whose dynamics are as close to those of a reference model as the plant dynamics will allow.

1) INTRODUCTION

Much effort is currently being expended in attempts to design helicopter flight control systems using active control technology (ACT). The motivation behind this work is largely a desire to produce battlefield helicopters with better handling qualities and more extensive flight envelopes. Indeed, the lethality of the modern battlefield demands improved helicopter performance and ease of operations. Some of the "nap of the earth" manoeuvres required for a successful mission will only be possible through the use of ACT in control system design. The helicopter community is relying on its ability to successfully design control configured vehicles (CCV's) in order to meet these requirements. However, at the heart of every design will be a mathematical model of the raw plant dynamics and the outcome of the design process will largely be determined by the accuracy with which the plant dynamics are known.

There are numerous examples of fixed wing aircraft which have had to have their multivariable control systems redesigned because the of unsatisfactory performance of the original design [1]. Problems with the implementation of ACT will be more severe on rotorcraft than on fixed wing aircraft because the dynamics of the helicopter plant, particularly those of the rotor, are not as well understood nor modelled. As the bandwidths of flight control systems are extended to higher frequencies, unmodelled high order dynamics can degrade the performance of the controlled system [2]. Unmodelled rotor and actuator dynamics will be stumbling blocks for many types of control theory such as modal control because these dynamics can have adverse effects on stability. These unmodelled dynamics create a need for tuning flight control systems which will become increasingly important as ACT matures.

The need to adjust the flight systems of helicopters as a result of preliminary flight tests is not a new problem. However, the trial and error techniques which are presently being used will undoubtedly be inadequate for full-authority fly-by-wire flight controllers. The high level of system integration which is implied by the use of ACT will obscure the relation of each adjustable parameter of the controller to the overall dynamics of the system. Making a series of test flights in order to find the best set of controller parameters using trial and error techniques is not only costly, but also inefficient. The fundamental difficulty with the trial and error approach is that it suffers from a lack of quantitative information concerning the parameters which should be adjusted and the amount by which they should be changed.

Sensitivity functions provide information which is useful concerning two aspects of the tuning problem. First, since sensitivity functions provide a measure of the change in the system response which will result from changes in control system parameters, their amplitudes indicating which parameters are significant in terms of the system response. By tuning parameters which significantly affect the system's dynamics, adjustments can be kept to a minimum. The second aspect of the tuning problem is that of knowing how parameters will affect system response and this information is also provided by the sensitivity functions. As will be shown, time domain optimization can be performed by using state variable sensitivities to determine the required shifts in controller parameters.

2) THE PARAMETER ADJUSTMENT ALGORITHM

The adjustment algorithm is based on the idea that from the sensitivity functions it is possible to predict the changes which will occur to the system response as a result of changing the parameter values, α_i, in the flight control system. The prediction of what the modified state response will be is generated using Newton–Raphson techniques. The predicted system response after parameter shifts, to second order accuracy, is given by,

$$\underline{x}_m(t) = \underline{x}_a(t) + \left[\frac{\partial \underline{x}_a(t)}{\partial \alpha} \right] \underline{\Delta A}_1 + \left[\frac{\partial^2 \underline{x}_a(t)}{\partial \alpha \partial \beta} \right] \underline{\Delta A}_2 \tag{1}$$

Where $\underline{x}_m(t)$ is the predicted or modified system response vector

$\underline{x}_a(t)$ is the measured or actual system response vector

$[\partial \underline{x}_a(t)/\partial \alpha]$ is the matrix of first order sensitivity funtions of the actual system response

$[\partial^2 \underline{x}_a(t)/\partial \alpha \partial \beta]$ is the matrix of second order sensitivity functions of the actual system response

$\underline{\Delta A}_1 = [\ \Delta\alpha_1 \ \ \Delta\alpha_2 \ \ \ldots \ \ \Delta\alpha_n\ ]^T$

$\underline{\Delta A}_2 = [\ \Delta\alpha_1\Delta\alpha_1 \ \ \Delta\alpha_1\Delta\alpha_2 \ \ \ldots \ \ \Delta\alpha_n\Delta\alpha_n\ ]^T$

$\Delta\alpha_i$ are the changes in the values of the control system parameters α_i

In order to systematically tune a flight control system, there must be a criterion by which improvements in system response can be measured. Sensitivity functions allow the prediction of what the modified system response will look like but this information is virtually useless if it is not possible to quantitatively say that one response is better than another. For each set of possible control system parameter values, a figure of merit must be assigned to the system response. By comparing the figures of merit associated with various sets of parameter perturbations, it is possible to identify the changes which will lead to the greatest improvement in response.

The figures of merit used to compare the possible sets of parameter values can take many forms. This paper describes the use of a figure of merit which is calculated using the Least Integral Error Square Performance Index which attempts to tune the system response towards an *'ideal'* system response. It is a model reference technique.

2.1) Least Integral Error Square Performance Index

The objective of the Least Integral Error Square Performance Index is to make the system responses as similar as possible to desired or *'ideal'* system responses. These *'ideal'* responses can be generated by any means, but are in general just the responses which one generates from the simulation model used for the design of the control system [3]. When the simulation model and the actual system are excited by the same inputs, there will be differences in the responses which one observes from the two systems. With the Least Integral Error Square Performance Index, the difference between the two signals is squared and summed to yield a measure of the difference between the two response signals. The difference between these two signals is squared in order to avoid positive and negative excursions of the difference signal cancelling each other.

In the past, single–input single–output systems have been successfully tuned using a Least Integral Error Square Performance Index. For single–input, single–output systems, the time domain adjustment algorithm is given by Winning et.al. [4]. The primary constraint on the expansion of the single–input single–output tuning theory towards multivariable systems is in the generation of sensitivity functions. For single–input single–output systems, sensitivity functions can be generated in real time by either a sensitivity cosystem [5] or the signal convolution method [4],[6],[7],[8]. The possibility of using a sensitivity cosystem with a Least Integral Error Square Performance Index to tune an Advanced Boiling Water Reactor with multivariable controller has been successfully explored by Winkelman [3]. Unfortunately, dynamic modelling of helicopters is not sufficiently accurate for a sensitivity cosystem to be employed to tune flight control systems. Indeed, lack of information concerning the helicopter plant produced the need for tuning in the first place and one of the fundamental constraints on the project is that the plant must be treated as a *'black box'*. The decision to use signal convolution techniques is based on this fact and forces one to accept that tuning will not be performed in real time. For multivariable systems, only a sensitivity cosystem can yield the state variable sensitivity functions in real time.

Assuming that the first and second order sensitivity functions, along with the system response, are known, then by careful manipulation of the control system parameters, α_i, it is possible to decrease the residual error, $\underline{R}_e(t)$, between the actual system response, $\underline{x}_a(t)$, and a desired response, $\underline{x}_d(t)$,

$$\underline{x}_d(t) = \underline{x}_a(t) + \left[\frac{\partial \underline{x}_a(t)}{\partial \alpha} \right] \underline{\Delta A}_1 + \left[\frac{\partial^2 \underline{x}_a(t)}{\partial \alpha \partial \beta} \right] \underline{\Delta A}_2 + \underline{R}_e(t) \qquad (2)$$

Adopting the notation of Equation 1 for the modified system response, $\underline{x}_m(t)$, it is possible to express Equation 2 in terms of the projected residual error vector, $\underline{R}_e(t)$.

$$\underline{R}_e(t) = \underline{x}_d(t) - \underline{x}_m(t) \qquad (3)$$

The performance index to be minimized becomes the time integral of the inner product of the residual error vector.

$$J = \int_0^T \left[\underline{R}_e(t), \underline{R}_e(t) \right] dt \qquad (4)$$

For helicopter applications, it is necessary to tune with regards to dynamics which are excited by all four pilot inceptors. Therefore, the performance index must be capable of optimizing with regards to more than one input signal. For example, if the actual aircraft system has undesirable phugoid and pitching characteristics, then it is beneficial to be able to tune the controller with respect to these two modes simultaneously to avoid improving one mode at the possible expense of another. If the phugoid dynamics are excited by an input confined to the longitudinal inceptor and the pitching dynamics are excited by an input on the vertical inceptor, then it makes sense to stimulate the system with separate inputs on each of these inceptors. The response to each of these separate inputs is then to be used as data for the tuning process. It is beneficial to take the data in two distinct test sequences rather than using the two inputs in close succession in a single test. The problem with performing a single data run in which the longitudinal inceptor is excited followed by the vertical inceptor is that the fast pitch mode excitation would occur at a time when the phugoid was already excited. The phugoid mode would then add dynamics to the fast pitch section of the measured time history and this would create a bias towards phugoid tuning at the expense of fast pitch tuning. Therefore, the Least Integral Error Square Performance Index, J, which has been used in the study is a sum over h distinct time histories.

$$J = \sum_{h} \left\{ \int_0^T \left[\underline{R}_e(t),\ \underline{R}_e(t) \right] dt \right\} \tag{5}$$

By minimizing Equation 5, it is possible to find the set of controller parameters α_i which is optimal in terms of minimizing the difference between the desired response and the actual system response. In practice, the process requires a small number of iterations to obtain the closest fit to the desired response.

When one remembers that the residual error vector is a function of the control system parameters, α_i, Equations 5 helps to make it clear that the α_i's must be optimized in a three dimensional sense. Not only must changes in parameters, $\Delta\alpha_i$, be chosen to minimize the relative error on a state with time, they must also be chosen to minimize error across the various output states and with respect to several test manoeuvres as well. This is particularly true in a highly coupled system such as a helicopter because each control parameter may influence each state in a fully integrated, multivariable flight control system.

2.2) Minimization Routine Theory

The search for the set of control system parameter values which minimizes the performance index of Equation 5 is performed on the computer using the routine E04JAF of the Numerical Algorithm Group (NAG) Libraries [9]. This algorithm determines values of the performance index with various parameter sets subject to bounds on the parameters [10]. The algorithm makes use of quasi–Newton methods during its search of sets of parameter values.

It was decided at an early stage in the development of the adjustment algorithm that perturbations to the designed controller parameter values should be kept to a minimum. This led to the decision to bound the range over which parameter values could migrate. The motivation behind this decision was that during the tuning process attempts were to be made to maintain the underlying control strategy as much as possible. There seemed little point in letting the tuning process move the dynamics of the controlled system significantly away from those which it was designed to have. The easiest method of accomplishing this objective was to restrict control system parameter movement. The optimization methods used [9], [10], allow bounds to be placed on the range over which controller parameters may be adjusted.

3) HELICOPTER MODEL AND FLIGHT CONTROL SYSTEM DESIGN

Testing of the tuning procedure has been conducted using a computer simulation model of a single rotor helicopter. The control system developed for these tests helps to show that the tuning process can be applied to controllers with various and even unconventional structures. Representations of the helicopter plant have been supplied by the Royal Aerospace Establishment (Bedford). The HELISTAB software package [11],[12] generates a system matrix, [A], and input distribution matrix, [B], given an initial flight condition. The helicopter plant is represented by the linear state space canonical form,

$$s\, \underline{X}(s) = [A]\, \underline{X}(s) + [B]\, \underline{U}(s) \tag{6}$$

Modal control theory has been used to design an acceleration demand flight path controller. Following the work of Parry and Murray–Smith [13], a feedback matrix is designed such that the closed loop eigenstructure satisfies two criteria. First, the eigenvalues are chosen such that the system will have stability characteristics consistent with good handling qualities. Second, the eigenvectors are chosen to minimize the amount of coupling in the system. By minimizing the principal angles between desired (perfectly decoupled) eigenvectors and those which are achievable with the given plant dynamics, coupling in the system is minimized in a least squares sense [11]. The motivation behind attempting to decouple the longitudinal and lateral dynamics is that the pilot's workload will be eased.

The design of the controller was carried out for a flight condition of 80.0 knots forward speed. The acceleration demand controller is designed such that the pilot will pulse his inceptors in order to change between one trimmed state and another. Although it is appreciated that this input strategy has disadvantages, the controller is illustrative of the extent to which ACT can be used to configure a pilot's inputs. This example also helps to show that the tuning procedure is widely applicable to closed loop systems. The structure of the controller was designed to regulate the three linear fuselage accelerations (vertical acceleration, $\dot{w}$; forward acceleration, $\dot{u}$; lateral acceleration, $\dot{v}$) and the roll rate, p, in earth axes. The following diagram shows the structure of the flight path controller as implemented using the state space plant of Equation 6.

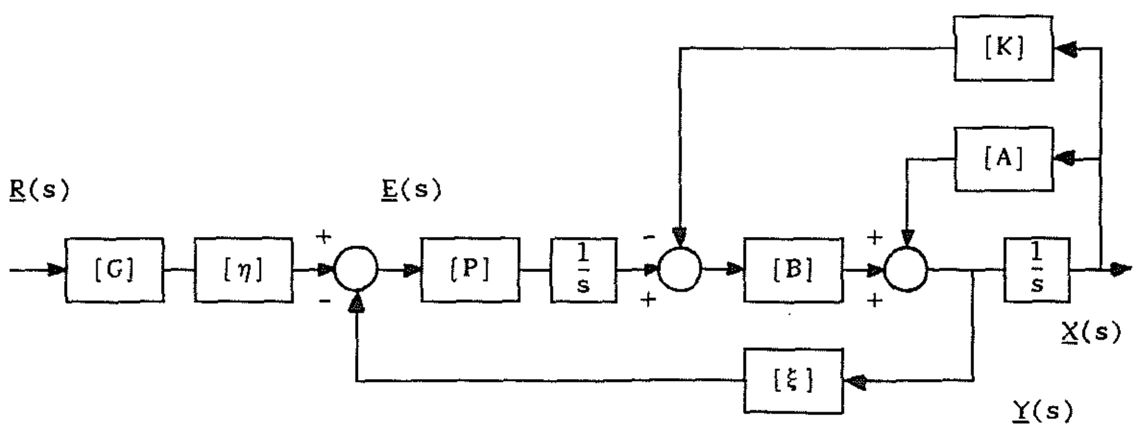

Figure 1: The Structure of the Flight Path Controller.

Pilot inputs are converted from inceptor displacements to earth axis acceleration demands by the diagonal pilot input gain matrix, [G], and then into body axis demands by the Euler angle relationships of the conversion matrix, $[\eta]$. In the above, the acceleration feedback matrix, $[\xi]$, is used to select the particular quantities from the acceleration vector, $\underline{Y}(s)$, which are to be compared with the pilot inputs in body axes. The gains of the feedback matrix, [K], and the compensator matrix, [P], are chosen to yield the desired eigenstructure.

The equation governing the dynamics of the controlled system is,

$$\underline{X}(s) = \left\{ s[I] - [A] + [B][K] + [B][P][\xi] \right\}^{-1} [B]\frac{1}{s}[P][\eta][G] \; \underline{R}(s) \qquad (7)$$

The closed loop transfer function matrix is given by,

$$[W(s)] = \left\{ s[I] - [A] + [B][K] + [B][P][\xi] \right\}^{-1} [B]\frac{1}{s}[P][\eta][G] \qquad (8)$$

The eigenstructure assignment in the design method of Parry and Murray–Smith [13] is based on a single feedback matrix, $[K_p]$. The flight path controller shown above is given the same eigenstructure by equating the characteristic polynomials of the two system's to yield Equation 9.

$$[K_p] = [K] + [P][\xi] \qquad (9)$$

It is possible in this manner to preserve the decoupling of modes as designed and at the same time allow the structure of the controller to be modified to accommodate different control strategies.

4) CALCULATION OF SENSITIVITY FUNCTIONS

Systematic tuning of helicopter flight control systems relies on the information provided by sensitivity functions. The problem at hand dictates the use of signal convolution techniques [4],[6],[7],[8] for the calculation of sensitivity functions. Not only does the signal convolution method work without knowledge of the plant, but it also helps to minimize the amount of flight testing which is necessary. Although the theory is developed with respect to a flight path controller designed using modal control theory, the signal convolution method can be applied to other controller structures designed by other methods. The signal convolution method of generating sensitivity functions is in no way linked to the control strategy; however, the structure of the flight controller will affect details concerning the implementation of the theory. The following theory shows the equations governing the signal convolution method as applied to the controller of Section 4.

The equations which describe the sensitivity functions are found by successive implicit differentiations of the system equation (Equation 7) with respect to the control system parameters. Assuming that the parameters to be tuned are located in either the feedback matrix, [K], or the compensator matrix, [P], then the first order state sensitivities can be shown to be given by,

$$\frac{\partial \underline{X}(s)}{\partial \alpha_i} = [W(s)][G]^{-1}[\eta]^{-1}[P]^{-1} \times$$

$$\left\{ - \frac{\partial [K]}{\partial \alpha_i} \underline{Y}(s) - \frac{\partial [P]}{\partial \alpha_i} [\xi] \underline{Y}(s) + \frac{\partial [P]}{\partial \alpha_i} [\eta] [G] R(s) \right\} \tag{10}$$

At this point it is convenient to consider the sensitivity function equation as a product of the closed loop transfer function matrix, [W(s)], and a sensitivity signal vector, $\underline{Z}^1{}_{\alpha i}(s)$.

$$\frac{\partial \underline{X}(s)}{\partial \alpha_i} = [W(s)] \underline{Z}^1_{\alpha_i}(s) \tag{11}$$

where,

$$\underline{Z}^1_{\alpha_i}(s) = [G]^{-1}[\eta]^{-1}[P]^{-1} \times$$

$$\left\{ - \frac{\partial [K]}{\partial \alpha_i} \underline{Y}(s) - \frac{\partial [P]}{\partial \alpha_i} [\xi] \underline{Y}(s) + \frac{\partial [P]}{\partial \alpha_i} [\eta][G] R(s) \right\} \tag{12}$$

The preceeding equations show that the sensitivity signals are generated by applying signals taken directly from the system, namely $\underline{Y}(s)$ and $\underline{R}(s)$, to a sensitivity filter which has a form which depends only upon the controller and its structure. Therefore, if one is able to estimate the closed loop tranfer function matrix, [W(s)], then the sensitivity functions can be generated without precise knowledge of the system plant.

For the purposes of time domain optimization, Equation 11 is transformed into a convolution integral,

$$\frac{\partial \underline{x}(t)}{\partial \alpha_i} = \int_0^t [w(\tau)] \underline{z}^1_{\alpha_i}(t-\tau) \, d\tau \tag{13}$$

In the simulation trials which have been performed, observable system signals have been recorded and subsequently fed through software representations of the sensitivity filters, thereby generating the sensitivity signals, $\underline{z}_{\alpha i}{}^1(t)$. The linear convolution of data sequences representing the time histories of the impulse response function matrix, [w(t)], and $\underline{z}_{\alpha i}{}^1(t)$ can be carried out efficiently using Fast Fourier Transform (FFT) techniques [15].

The second order sensitivity functions are calculated using the same techniques as were used to calculate the first order sensitivity functions. Since all of the control matrices are first order in the control system parameters, all terms containing second order partial derivatives of the control system matrices will be zero. Thus,

$$\frac{\partial^2 [K]}{\partial \alpha_j \partial \alpha_i} = \frac{\partial^2 [P]}{\partial \alpha_j \partial \alpha_i} = 0 \tag{14}$$

After simplication, the second order sensitivity functions of the state variables of the flight path controller are,

$$\frac{\partial^2 \underline{X}(s)}{\partial\alpha_j \partial\alpha_i} = - [W(s)][\bar{G}]^{-1}[\eta]^{-1}[P]^{-1} s \times$$

$$\left\{ \left[\frac{\partial [K]}{\partial\alpha_i} + \frac{\partial [P]}{\partial\alpha_i}[\xi] \right] \frac{\partial \dot{X}(s)}{\partial\alpha_j} + \left[\frac{\partial [K]}{\partial\alpha_j} + \frac{\partial [P]}{\partial\alpha_j}[\xi] \right] \frac{\partial X(s)}{\partial\alpha_i} \right\} \quad (15)$$

Once again, the time domain calculations involve the convolution of the impulse response function matrix with a sensitivity signal. The use of second order sensitivities gives a much tighter control over how the parameters are adjusted than is possible with just first order sensitivity information.

One of the benefits of using signal convolution techniques on flight test data is that the convolution integral will help to filter system noise. This is of obvious importance for helicopter applications.

4.1) Identification of the Closed Loop Transfer Function Matrix

The signal convolution technique requires the identification of the impulse response function matrix. It has been shown that because of the possibility of each input coupling into each output state in an unknown multivariable plant, [w(t)] cannot be identified simultaneously with the sensitivity signals [8]. Since the state response to each inceptor must be determined individually, the identifitication of [w(t)] precludes real time tuning using signal convolution techniques. In addition, it must be stated that [w(t)] strictly exists only for linear systems. However, through judicious choice of inputs, nonlinear plants such as that of a helicopter can be made to operate in a small region about an operating point, thereby reducing nonlinear effects. Both pulses and step inputs have been used to identify [w(t)].

5) PARAMETER ADJUSTMENT RESULTS

The tuning procedure has been used to successfully tune the responses of a system with rotor flapping dynamics included in the plant description towards a system which ignores rotor dynamics. Both systems were simulated by fully nonlinear models and only a small subset of the controller parameters was allowed to change. In order to avoid driving the responses into nonlinearities, a doublet input of 10% amplitude was used on each inceptor and the length of time history was restricted to 1.0 seconds (sampled at 64 Hz.). The improvement in performance index with one pass of the adjustment algorithm was 17%, reducing the performance index from 1.17×10^{-3} to 1.00×10^{-3}. The value of the performance index is indicative that the original error between the responses was small.

In order to highlight the improvements which can be made, Figure 2 shows the state responses to a pulse input on the collective. The desired response (solid line) is generated by a linear simulation of an eighth order plant with the designed control matrices. The actual response (asterisks) is generated by the same simulation model but with 3 compensator parameters perturbed. The problem for the adjustment algorithm is to retrieve the original, designed parameter values. After one pass of the tuning process, the adjusted response is shown as the dashed line which corresponds to a reduction of the performance index from 2.97×10^{-1} to 6.59×10^{-4}, an improvement of over 99%. Most of

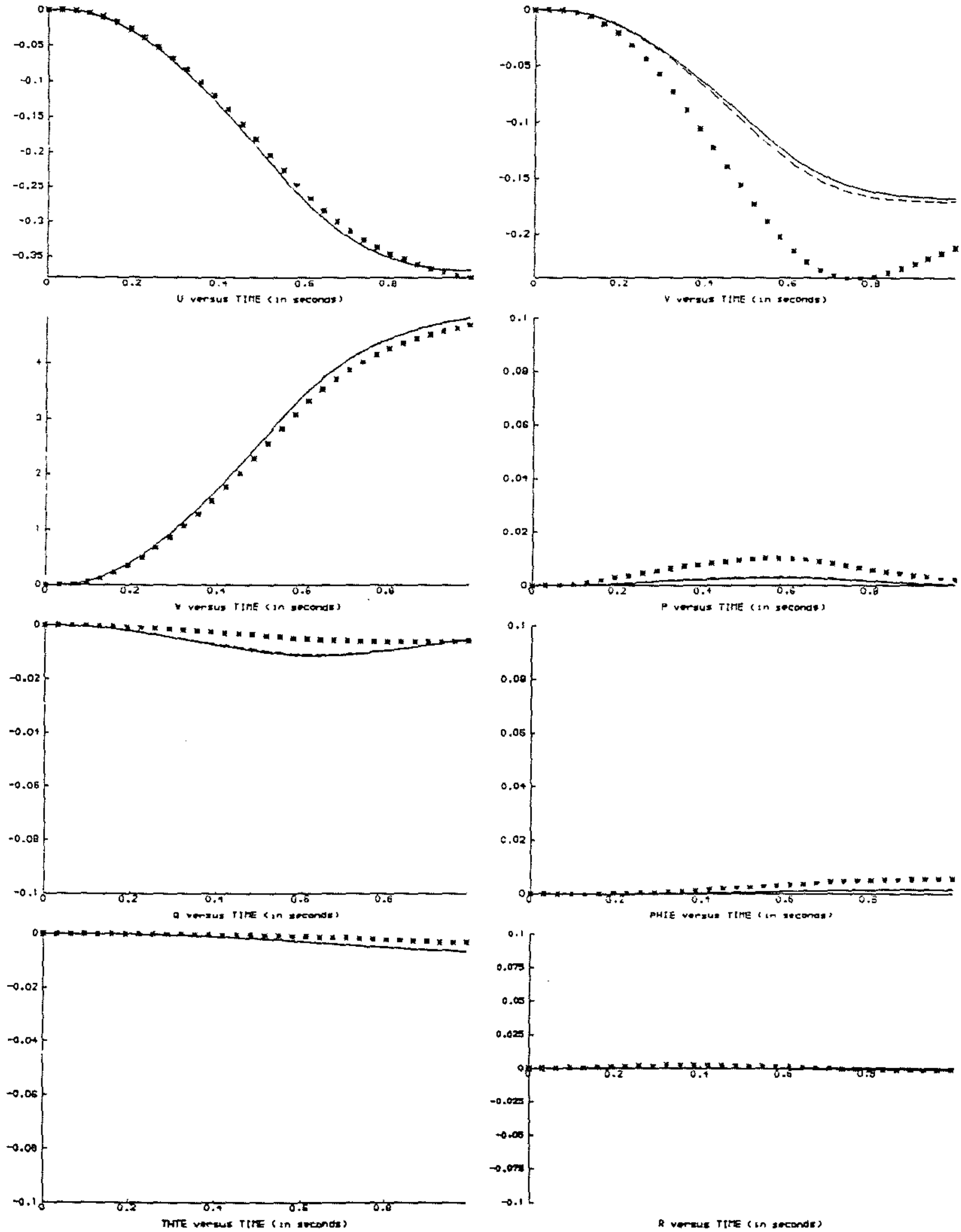

Figure 2: State responses to a vertical inceptor pulse (—— desired response, *** untuned response, – – tuned response).

the adjusted responses are coincident with the desired responses. Of the three perturbed parameters, the perturbations have been reduced from 10% to 0.7%, 0.9%, and 0.4% for p_{11}, p_{22}, and p_{33}. respectively.

6) CONCLUSIONS

Although the Least Integral Error Square Performance Index, can be used to successfully tune controller parameters, there are problems associated with model reference tuning for flight control systems. One of the fundamental concerns regarding the use of this performance index is how does one ensure that the *'ideal'* response is in fact ideal. By using the design model to generate the *'ideal'* signal, one is probably coming as close to ideal as possible provided that care has been taken to ensure that the response of the design model with controller satisfies the performance specifications for the design. The second concern over the use of a Least Integral Error Square Performance Index is that the system response may only be tuned for the input sequences used during the tuning process. In helicopter applications it will be impossible to tune the controller with the infinite set of pilot inputs which may be used throughout the flight envelope. It is therefore important that tuning on helicopters relies on input signals which are representative of as many manouevres as possible.

Research into a performance index which directly measures improvements in handling quality characteristics is being undertaken. By tuning handling qualities directly, both of the disadvantages mentioned above can be overcome. Results of this work will be reported in the near future.

7) ACKNOWLEDGEMENTS

The authors gratefully acknowledge the assistance provided by the Royal Aeospace Establishment (Bedford) in this research, particularly for the provision of the HELISTAB flight mechanics modelling program. M. Manness is being supported by the Government of the Province of British Columbia, Canada, by the University of Glasgow, and by the Committee of Vice-Chancellors and Principals of the Universities of the United Kingdom through the Overseas Research Scheme.

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